Bottomonium resonances with $I = 0$ from lattice QCD static potentials

[2]**Lasse Mueller**, [1]Pedro Bicudo, [1]Nuno Cardoso, [2]Marc Wagner

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[1]Universidade de Lisboa

[2]Goethe-Universität Frankfurt am Main

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Study heavy-heavy-light-light tetraquarks with lattice QCD using the Born Oppenheimer approximation

- heavy quarks are regarded as static color charges
- potential in presence of two light quarks is computed using Lattice QCD and utilized as an effective potential

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→ Extension of work already published recently

[P. Bicudo, M. Cardoso, N. Cardoso, M. Wagner, Phys. Rev. D 101, 034503 (2020), arXiv: 1910.04827 [hep-lat]]

Quantum numbers

Consider two channels:

- Quarkonium channel $\bar{Q}Q$
- Heavy-light meson-meson channel, $\bar{M}M$ with $M = \bar{Q}q$

Quantum numbers

- J^{PC} : total angular momentum, parity and charge conjugation of the respective system.
- $S^{PC}_{Q/q}$: spin of $\bar{Q}Q/\bar{q}q$ and corresponding parity and charge conjugation.
- $\tilde{J}^{PC}$: total angular momentum excluding the heavy $\bar{Q}Q$ - spins and corresponding parity and charge conjugation. (for Quarkonium $\tilde{J}^{PC} = L^{PC}$).

Assumptions and symmetries

- Heavy quark spins are conserved quantities
  $\rightarrow$ represented by a scalar wave function $\psi_{\bar{Q}Q}(\mathbf{r})$
- Only considering the lightest decay channel which corresponds to two parity negative mesons
- $\bar{Q}Q$ state with angular momentum $L_{\bar{Q}Q}$ can only decay into a $\bar{M}M$ state with $S^{PC}_q = 1^{--}$ and $L_{\bar{M}M} = L_{\bar{Q}Q} \pm 1$
  $\rightarrow$ represented by a 3-component wavefunction $\vec{\psi}_{\bar{M}M}(\mathbf{r})$

Coupled channel Schroedinger equation

$\Rightarrow$ The wave function of the SE has 4-components $\psi(\mathbf{r}) = \left(\psi_{\bar{Q}Q}(\mathbf{r}), \vec{\psi}_{\bar{M}M}(\mathbf{r})\right)$

Resulting Schroedinger equation

$$\left(-\frac{1}{2}\mu^{-1}\left(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\mathbf{L}^2}{r^2}\right) + V(\mathbf{r}) + 2m_M - E\right)\psi(\mathbf{r}) = 0 \qquad (1)$$

where $\mu^{-1} = \mathrm{diag}(1/\mu_Q, 1/\mu_M, 1/\mu_M, 1/\mu_M)$ and

$$V(\mathbf{r}) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\mathrm{mix}}(r)\,(1 \otimes \mathbf{e}_r) \\ V_{\mathrm{mix}}(r)\,(\mathbf{e}_r \otimes 1) & V_{\bar{M}M,\parallel}(r)\,(\mathbf{e}_r \otimes \mathbf{e}_r) + V_{\bar{M}M,\perp}(r)\,(1 - \mathbf{e}_r \otimes \mathbf{e}_r) \end{pmatrix} \qquad (2)$$

$V_{\bar{Q}Q}(r)$, V_{mix}, $V_{\bar{M}M,\parallel}$ and $V_{\bar{M}M,\perp}$ can be related to lattice results for static potentials from QCD.

Static potentials from lattice QCD

Treat heavy quarks as static quarks with frozen positions at $\mathbf{0}$ and $\mathbf{r}$.
Lattice computation of string breaking with optimized operators:
[G. S. Bali, H. Neff, T. Duessel, T. Lippert, and K. Schilling (SESAM), Phys. Rev. D 71, 114513 (2005), arXiv:hep-lat/0505012 [hep-lat]], [J. Bulava, B. Hörz, F. Knechtli, V. Koch, G. Moir, C. Morningstar, and M. Peardon, Phys. Lett. B 793,493 (2019), arXiv:1902.04006 [hep-lat]]

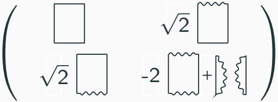
$$C(t) = \begin{pmatrix} \langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{Q\bar{Q}} \rangle & \langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{M\bar{M}} \rangle \\ \langle \mathcal{O}_{M\bar{M}} | \mathcal{O}_{Q\bar{Q}} \rangle & \langle \mathcal{O}_{M\bar{M}} | \mathcal{O}_{M\bar{M}} \rangle \end{pmatrix} \tag{3}$$

$$\mathcal{O}_{Q\bar{Q}} = (\Gamma_Q)_{AB} \qquad \left(\bar{Q}_A(\mathbf{0})\; U(\mathbf{0};\mathbf{r})\; Q_B(\mathbf{r})\right) \tag{4}$$

$$\mathcal{O}_{M\bar{M}} = (\Gamma_Q)_{AB}(\Gamma_q)_{CD} \qquad \left(\bar{Q}_A(\mathbf{0})\; u_D(\mathbf{0})\; \bar{u}_C(\mathbf{r})\; Q_B(\mathbf{r}) + (u \to d)\right) \tag{5}$$

$$\langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{Q\bar{Q}} \rangle_U \propto \left\langle \mathrm{tr}\left(V_t^\dagger(\mathbf{r},\mathbf{0}) U_{\mathbf{r}}(t,0) V_0(\mathbf{r},\mathbf{0}) U_{\mathbf{0}}^\dagger(t,0) \right) \right\rangle_U \tag{6}$$

$$\langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{M\bar{M}} \rangle_U \propto \left\langle \mathrm{tr}\left(\Gamma_Q M^{-1}_{(\mathbf{0},t);(\mathbf{r},t)} U_{\mathbf{r}}(t,0) V_0(\mathbf{r},\mathbf{0}) U_{\mathbf{0}}^\dagger(t,0) \right) \right\rangle_U \tag{7}$$

$C(t) =$ 

— gauge transporter
〰 light u and d quark propagators

Talk by Marco Catillo on Thu. 16:20-16:40 "From QCD string breaking to quarkonium spectrum"

Relating V(r) to static potentials from lattice QCD

From C(t) the potentials can be extracted in the limit of large Euclidean time separations:

$$[C(t)]_{ij} \propto \sum_k a_k(r) \mathrm{e}^{-V_k(r)t} \qquad \text{for} \quad t \to \infty \tag{8}$$

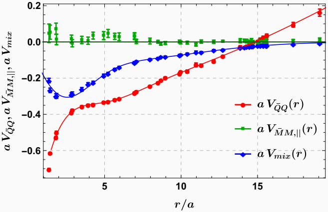
One can derive a relation between these $V_k(r)$ and $V_{\bar{Q}Q}(r)$, $V_{mix}(r)$ and $V_{\bar{M}M}(r)$.

$$V_{\bar{Q}Q}(r) = \cos^2(\theta(r)) V_0^{\Sigma_g^+}(r) + \sin^2(\theta(r)) V_1^{\Sigma_g^+}(r)$$

$$V_{\bar{M}M,\parallel}(r) = \sin^2(\theta(r)) V_0^{\Sigma_g^+}(r) + \cos^2(\theta(r)) V_1^{\Sigma_g^+}(r)$$

$$V_{\mathrm{mix}}(r) = \cos(\theta(r)) \sin(\theta(r)) \left(V_0^{\Sigma_g^+}(r) + V_1^{\Sigma_g^+}(r) \right)$$

$$V_{\bar{M}M,\perp}(r) = V^{\Pi_g^+}(r) = 0$$



where $V_0^{\Sigma_g^+}(r)$ denotes the ground state potential and $V_1^{\Sigma_g^+}(r)$ its first excitation. We use existing results from

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Coupled channel Schroedinger equation for resonances

We expand $\psi_{\bar{Q}Q}(\mathbf{r})$ in terms of $\tilde{J}$ eigenfunctions and project the SE to definite angular momentum. For $\tilde{J}=0$ we receive two coupled equations

$$\left(\mathcal{H}_{\tilde{J}} + (2m_M - E)\,\mathbb{1}_{3\times 3}\right)\begin{pmatrix} u_{0,0}(r) \\ \chi_{1\to 0,0}(r) \end{pmatrix} = \; -\begin{pmatrix} V_{\mathrm{mix}}(r) \\ V_{\bar{M}M,\parallel}(r) \end{pmatrix} kr\, j_1(kr) \tag{9}$$

and for $\tilde{J}>0$ we receive two sets of three coupled equations

$$\left(\mathcal{H}_{\tilde{J}} + (2m_M - E)\,\mathbb{1}_{3\times 3}\right)\begin{pmatrix} u_{\tilde{J},\tilde{J}_z}(r) \\ \chi_{\tilde{J}-1\to\tilde{J},\tilde{J}_z}(r) \\ \chi_{\tilde{J}+1\to\tilde{J},\tilde{J}_z}(r) \end{pmatrix} = -\,\mathbf{V}_{\tilde{J}-1\to\tilde{J}}(r)\, kr\, j_{\tilde{J}-1}(kr) \tag{10}$$

$$" = -\,\mathbf{V}_{\tilde{J}+1\to\tilde{J}}(r)\, kr\, j_{\tilde{J}+1}(kr). \tag{11}$$

to be solved numerically with boundary conditions

$$u_{\tilde{J},\tilde{J}_z}(r) = 0 \quad \text{and} \quad \chi_{L\to\tilde{J},\tilde{J}_z}(r) = i t_{L\to\tilde{J},\tilde{J}_z}\, kr\, h_L^{(1)}(kr) \quad \text{for} \quad r\to\infty. \tag{12}$$

This will yield the t-matrix and s-matrix for $\tilde{J}>0$

$$t_{\tilde{J},\tilde{J}_z} = \begin{pmatrix} t^{\tilde{J}-1}_{\tilde{J}-1\to\tilde{J},\tilde{J}_z} & t^{\tilde{J}+1}_{\tilde{J}-1\to\tilde{J},\tilde{J}_z} \\ t^{\tilde{J}-1}_{\tilde{J}+1\to\tilde{J},\tilde{J}_z} & t^{\tilde{J}+1}_{\tilde{J}+1\to\tilde{J},\tilde{J}_z} \end{pmatrix}, \quad s_{\tilde{J},\tilde{J}_z} = 1 + 2it_{\tilde{J},\tilde{J}_z} \tag{13}$$

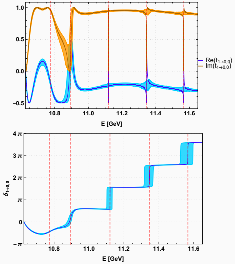
Scattering amplitude and phase shifts

Solved SE for $\tilde{J} \leq 3$ using two independent methods:

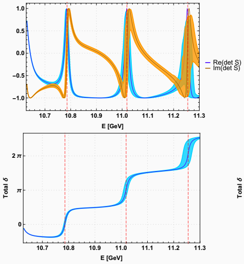
- Discretization of spacetime rewriting the SE as a system of linear equations $M(E)\mathbf{x} = \mathbf{b}$, solved by Matrix inversion
- 4th order Runge-Kutta algorithm

Propagating the errors of the lattice data by resampling and computing the 16th and 84th percentile.

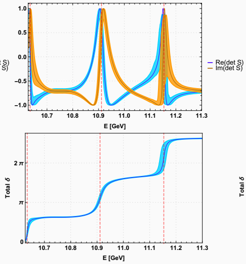
scattering phase: $\quad \mathrm{e}^{2i\delta_{L\to\tilde{J},\tilde{J}_z}} = 1 + 2it_{L\to\tilde{J},\tilde{J}_z} \qquad \mathrm{e}^{2i\delta_{\tilde{J},\tilde{J}_z;\mathrm{total}}} = det(s_{\tilde{J},\tilde{J}_z})$



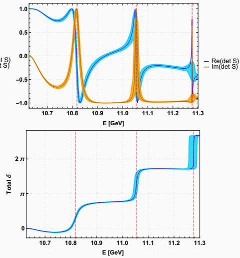
$\tilde{J}^{PC} = 0^{++}$



$\tilde{J}^{PC} = 1^{--}$



$\tilde{J}^{PC} = 2^{++}$



$\tilde{J}^{PC} = 3^{--}$

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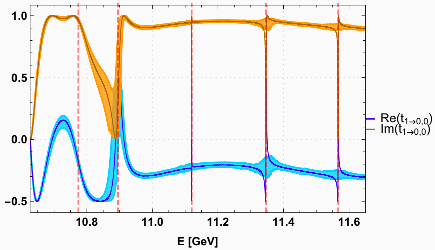

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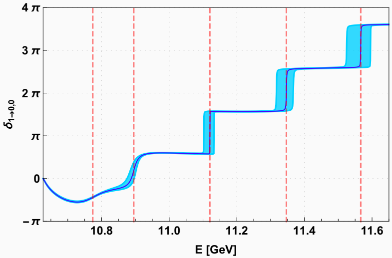

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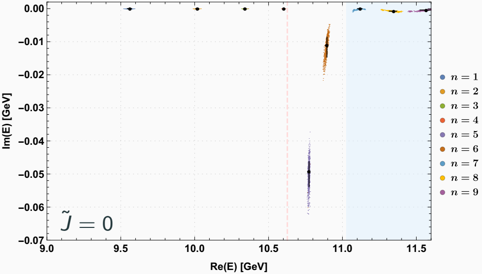
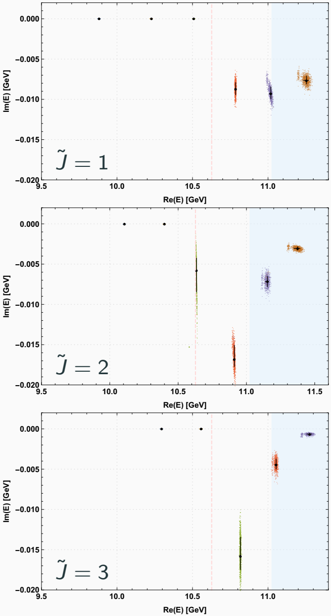
Polepositions in the complex plane





- Analytic continuation of our scattering problem to the complex plane
- Poles found using a Newton-Raphson shooting algorithm.
- Pole positions are related to masses and decay width via

$$m = \mathrm{Re}(E) \qquad \text{and} \qquad \Gamma = -2\,\mathrm{Im}(E)$$

Comparison to the experiment

from poles of $t_{\tilde{J},\tilde{J}_z}$				from experiment			
$\tilde{J}^{PC}$	n	Re(E)[GeV]	Im(E)[MeV]	name	m[GeV]	Γ[MeV]	$I^G(J^{PC})$
0^{++}	1	9.563^{+11}_{-17}	0	$\eta_b(1S)$	9.399(2)	10(5)	$0^+(0^{+-})$
				$\Upsilon_b(1S)$	9.460(0)	≈ 0	$0^-(1^{--})$
	2	10.018^{+8}_{-10}	0	$\eta_b(2S)_{\text{BELLE}}$	9.999(6)	-	$0^+(0^{+-})$
				$\Upsilon(2S)$	10.023(0)	≈ 0	$0^-(1^{--})$
	3	10.340^{+7}_{-9}	0	$\Upsilon(3S)$	10.355(1)	≈ 0	$0^-(1^{--})$
	4	10.603^{+5}_{-6}	0	$\Upsilon(4S)$	10.579(1)	21(3)	$0^-(1^{--})$
	5	10.774^{+4}_{-4}	$-49.3^{+3.0}_{-4.6}$	$\Upsilon(10750)_{\text{BELLE II}}$	10.753(7)	36(22)	$0^-(1^{--})$
	6	10.895^{+7}_{-10}	$-11.1^{+2.4}_{-3.6}$	$\Upsilon(10860)$	10.890(3)	51(7)	$0^-(1^{--})$
	7	11.120^{+13}_{-18}	$-0.0^{+0.0}_{-0.2}$	$\Upsilon(11020)$	10.993(1)	49(15)	$0^-(1^{--})$
1^{--}	1	9.882^{+3}_{-4}	0	$\chi_{b0}(1P)$	9.859(1)	-	$0^+(0^{++})$
				$h_b(1P)$	9.890(1)	-	$?^?(1^{+-})$
				$\chi_{b1}(1P)$	9.893(1)	-	$0^+(1^{++})$
				$\chi_{b2}(1P)$	9.912(1)	-	$0^+(2^{++})$
	2	10.228^{+3}_{-3}	0	$\chi_{b0}(2P)$	10.233(1)	-	$0^+(0^{++})$
				$\chi_{b1}(2P)$	10.255(1)	-	$0^+(1^{++})$
				$h_b(2P)_{\text{BELLE}}$	10.260(2)	-	$?^?(1^{+-})$
				$\chi_{b2}(2P)$	10.267(1)	-	$0^+(2^{++})$
	3	10.508^{+3}_{-3}	0	$\chi_{b1}(3P)$	10.512(2)	-	$0^+(0^{++})$
	4	10.786^{+2}_{-3}	$-8.7^{+0.8}_{-0.7}$				
	5	11.019^{+6}_{-9}	$-9.3^{+0.8}_{-0.6}$				
	6	11.255^{+13}_{-20}	$-7.6^{+0.5}_{-0.4}$				
2^{++}	1	10.107^{+3}_{-3}	0	$\Upsilon(1D)$	10.164(2)	-	$0^-(2^{--})$
	2	10.400^{+3}_{-3}	0				
	3	10.635^{+2}_{-1}	$-5.8^{+1.5}_{-2.6}$				
	4	10.911^{+4}_{-6}	$-16.8^{+1.7}_{-0.9}$				
	5	11.153^{+9}_{-15}	$-7.2^{+0.6}_{-0.6}$				
3^{--}	1	10.296^{+2}_{-3}	0				
	2	10.557^{+4}_{-3}	0				
	3	10.818^{+2}_{-3}	$-15.8^{+2.4}_{-1.6}$				
	4	11.054^{+6}_{-9}	$-4.4^{+0.7}_{-0.5}$				

bound states

resonances

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bound states

resonances

Conclusion and Outlook

We

- obtain resonances that match the experimentally found states $\Upsilon(10750)_{\mathrm{BELLE\ II}}$ and $\Upsilon(10860)$.
- find indications that $\Upsilon(11020)$ might be an D-wave state
- were able to make predictions for resonances with $\tilde{J} > 0$ which may be found in the future by the experiment

Outlook:

- **Aim: Reduce systematic errors as much as possible**
  $\rightarrow$ **next step: include heavy spin effects to reduce the systematic error**
- include decay channels with to a negative parity and a positive parity heavy-light meson
  $\rightarrow$ more realistic predictions up to around 11.5 GeV
- perform a dedicated lattice QCD computation of the static potentials