Studies on meson-baryon interactions in the HAL QCD method with all-to-all propagators

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APLAT (Online), August 4, 2020

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- Motivation
- HAL QCD method
- 1st step: NK interactions
- 2nd step: Nπ interactions with Δ sources

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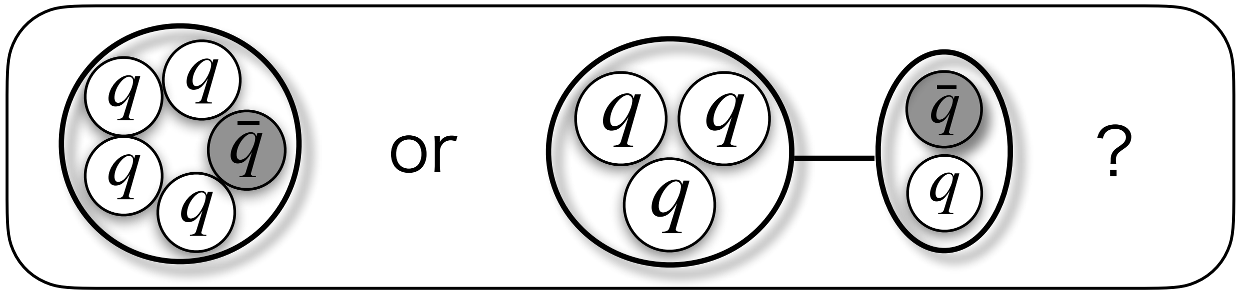
Motivation

Backgrounds

- Various **exotic hadrons** have been found from experiments (X, Y, Z, P_c, etc.).
- Although there have been lots of theoretical and experimental approaches to explain such hadrons, they are still not understood well.



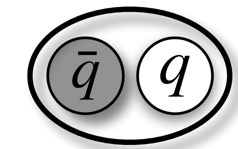
- On the other hand, QCD may describe all hadrons.

Our ultimate goal: reveal the properties of all hadrons including **exotic hadrons** from lattice QCD

Motivation

Recent studies on resonances in lattice QCD

meson-meson scatterings → mesonic resonances



Finite volume method

well investigated

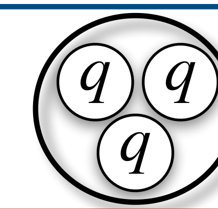
- ρ [M. Werner et al., 2019]
- σ, f_0, f_2 [R. Briceno et al., 2018]
- κ, K^* [G. Rendon et al., 2020]

HAL QCD method

I=1 P-wave $\pi\pi$ → ρ

(cf. Y. Akahoshi's talk)

meson-baryon scatterings → baryonic resonances



Finite volume method

I=3/2 P-wave $N\pi$ → Δ

[S. Paul et al., 2018]
[C. W. Andersen et al., 2017]

HAL QCD method

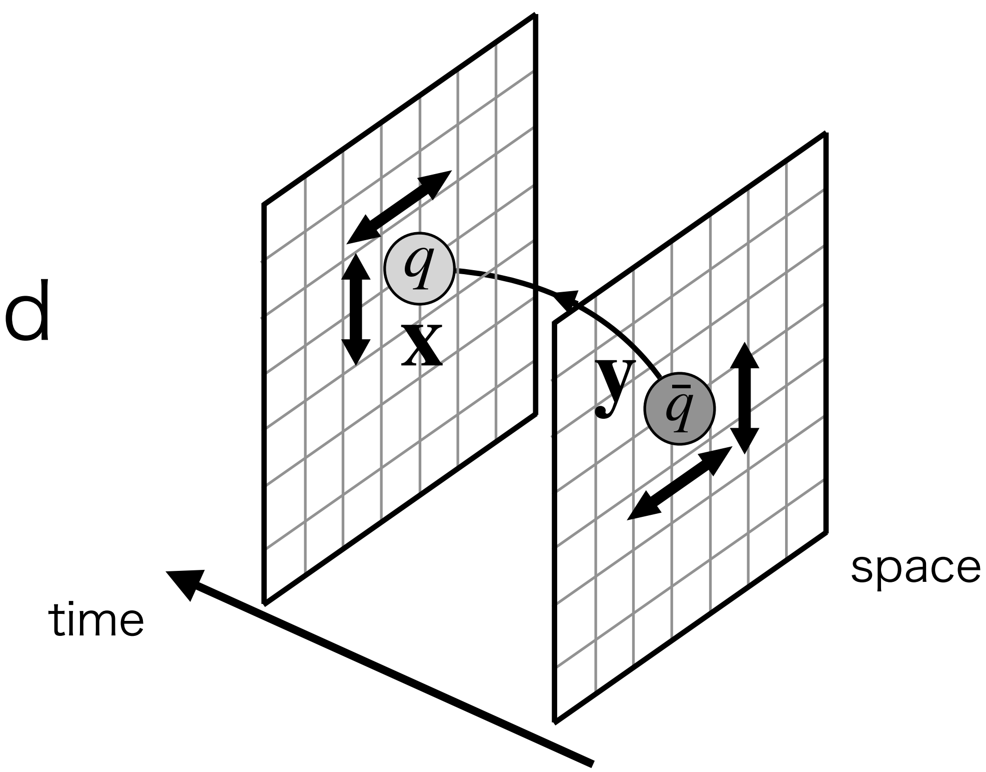
none

But this method may be efficient for meson-baryon systems!

Motivation

All-to-all propagators in the HAL QCD method

- To investigate meson-baryon scatterings that have resonances, we need **all-to-all propagators**.
- **One-end trick** [M. Foster, C. Michael, 1999]
  : very efficient for the HAL QCD method with all-to-all propagators.
  (cf. Y. Akahoshi's talk)



As a first step ···

- **S-wave NK scatterings** ← check the effectiveness of the one-end trick for meson-baryon systems
- **I=3/2 P-wave Nπ scatterings** ← extract Δ resonance

Motivation

Our plan

: today's talk

S-wave NK (LO)

↓ Examinations of the effectiveness of the **one-end trick** for meson-baryon systems

I=3/2 P-wave Nπ with Δ source (LO) ← We are here

↓ Simulation at a heavy pion mass to see Δ as a **bound state**

I=3/2 P-wave Nπ with Δ and Nπ sources (NLO)

↓ Simulation near the physical point using the **one-end trick** to see Δ as a **resonance**

Other resonances or pentaquarks

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HAL QCD method

Ideas of HAL QCD method

[N. Ishii, S. Aoki, T. Hatsuda, 2007]

NBS wave function

$$\Psi^W(\mathbf{r}) = \langle 0 | O_1(\mathbf{x}+\mathbf{r},0) O_2(\mathbf{x},0) | 2H, W \rangle$$

($|2H, W\rangle$ ⋯two-hadron states with energy W)

($O_1(\mathbf{x}+\mathbf{r},0)$, $O_2(\mathbf{x},0)$: hadron op.)

($W = \sqrt{k^2 + m_1^2} + \sqrt{k^2 + m_2^2}$)

partial wave decomposition

→ $$\Psi^{W,l}(r) \underset{r\to\infty}{\propto} \frac{\sin(kr - \frac{l}{2}\pi + \delta^l(k))}{kr} e^{i\delta^l(k)}$$

phase shift

→ $$\left(\frac{k^2}{2\mu} - H_0\right)\Psi^W(\mathbf{r}) = \int d^3r' \; \underline{U(\mathbf{r},\mathbf{r}')}\Psi^W(\mathbf{r}')$$

non-local potential

We can obtain a **potential for two-hadron states** from NBS wave functions

HAL QCD method

HAL QCD method on lattice

4-pt correlation function

(source op.)

$$F(t, \mathbf{r}) = \langle 0 | O_1(\mathbf{x}+\mathbf{r}, t+t_0) O_2(\mathbf{x}, t+t_0) \, \underline{\bar{J}(t_0)} \, | 0 \rangle$$

$$\mathbf{1} = \sum_n |2H, W_n\rangle\langle 2H, W_n| + \cdots$$

$$= \sum_n \underline{\Psi^{W_n}(\mathbf{r})} \, \langle 2H, W_n | \bar{J}(t_0) | 0 \rangle \, e^{-W_n t} + \cdots$$

$$(W_n = \sqrt{k_n^2 + m_1^2} + \sqrt{k_n^2 + m_2^2})$$

NBS wave function

It is hard to extract a ground state if the system contains **baryons**.

HAL QCD method

Time-dependent HAL QCD method

[N. Ishii et al., 2011]

R-correlator

$(\Delta W_n = W_n - m_1 - m_2)$

$$R(t,\mathbf{r}) = \frac{F(t,\mathbf{r})}{e^{-m_1 t}e^{-m_2 t}} = \sum_n \underline{A_n\ \Psi^{W_n}(\mathbf{r})\ e^{-\Delta W_n t}} + (inelastic)$$

elastic term satisfies

$$\sum_n \left(\underline{\frac{k_n^2}{2\mu}} - H_0\right) \underline{A_n \Psi^{W_n}(\mathbf{r})\ e^{-\Delta W_n t}} = \sum_n \int d^3r'\ U(\mathbf{r},\mathbf{r}')\ \underline{A_n \Psi^{W_n}(\mathbf{r})\ e^{-\Delta W_n t}}$$

$$= \frac{1+3\delta^2}{8\mu}\Delta W_n^2 + \Delta W_n + O(\Delta W_n^3)$$

$(\delta = \frac{m_1 - m_2}{m_1 + m_2})$

$$\left(\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right)R(\mathbf{r},t) \underset{t\to\infty}{\simeq} \int d^3r'\ U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) + O(\Delta W_n^3)$$

Once the inelastic states are supressed, we can derive the potential even when the excited states remain in $R(t,\mathbf{r})$.

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1st step: NK interactions

About NK systems

- $K^+p\ (I=1)$, $K^+n - K^0p, (I=0)$
- S-wave NK ➡ $J^P = 1/2^-$
- **no quark annihilation diagrams** in NK system

➡ all-to-all propagators play a role in **increasing statistics**

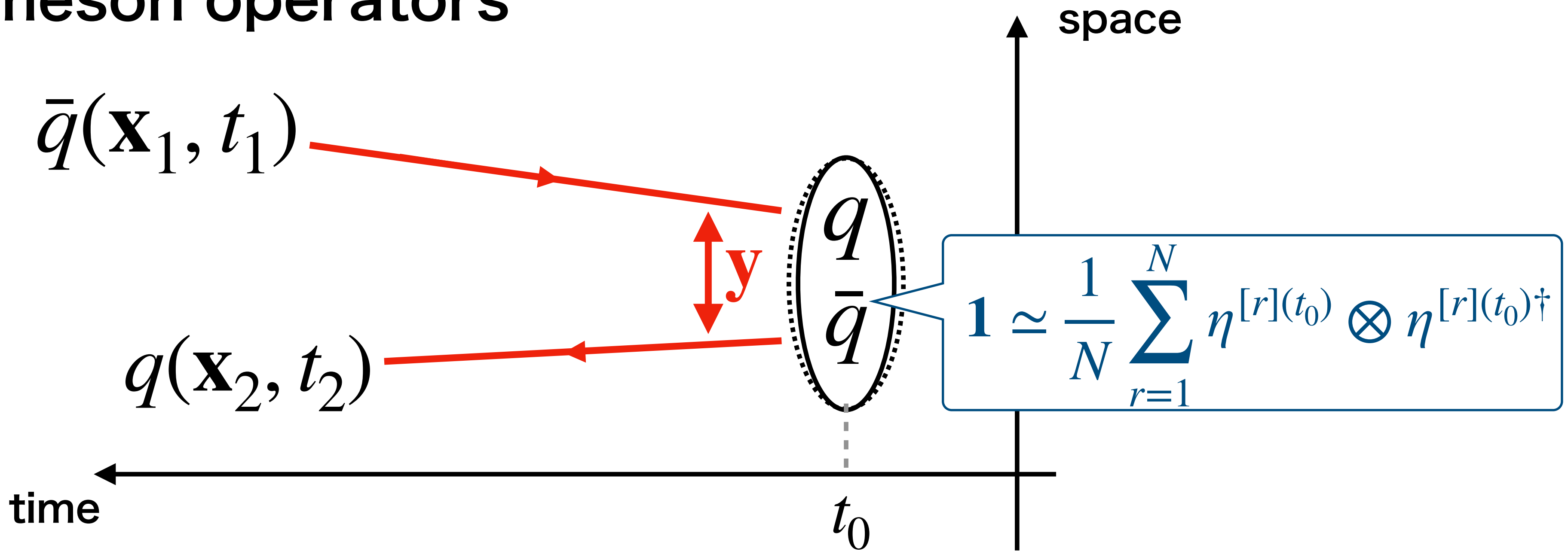
- NK for $I(J^P) = 0(1/2^-), 1(1/2^-)$: candidates for the channels of $\Theta^+(1540)$ pentaquark [LEPS Collab., 2003]

1st step: NK interactions

Brief review of one-end trick

[M. Foster, C. Michael, 1999]

: stochastic method that can be applied to only **meson operators**



$$\simeq \frac{1}{N}\sum_{r=1}^{N} \underline{(D^{-1}\eta^{[r](t_0)})(\mathbf{x}_1, t_1)} \otimes \left(\underline{(D^{-1}\gamma_5\Gamma^\dagger\eta^{[r](t_0)})^\dagger(\mathbf{x}_2, t_2)}\gamma_5\right)$$

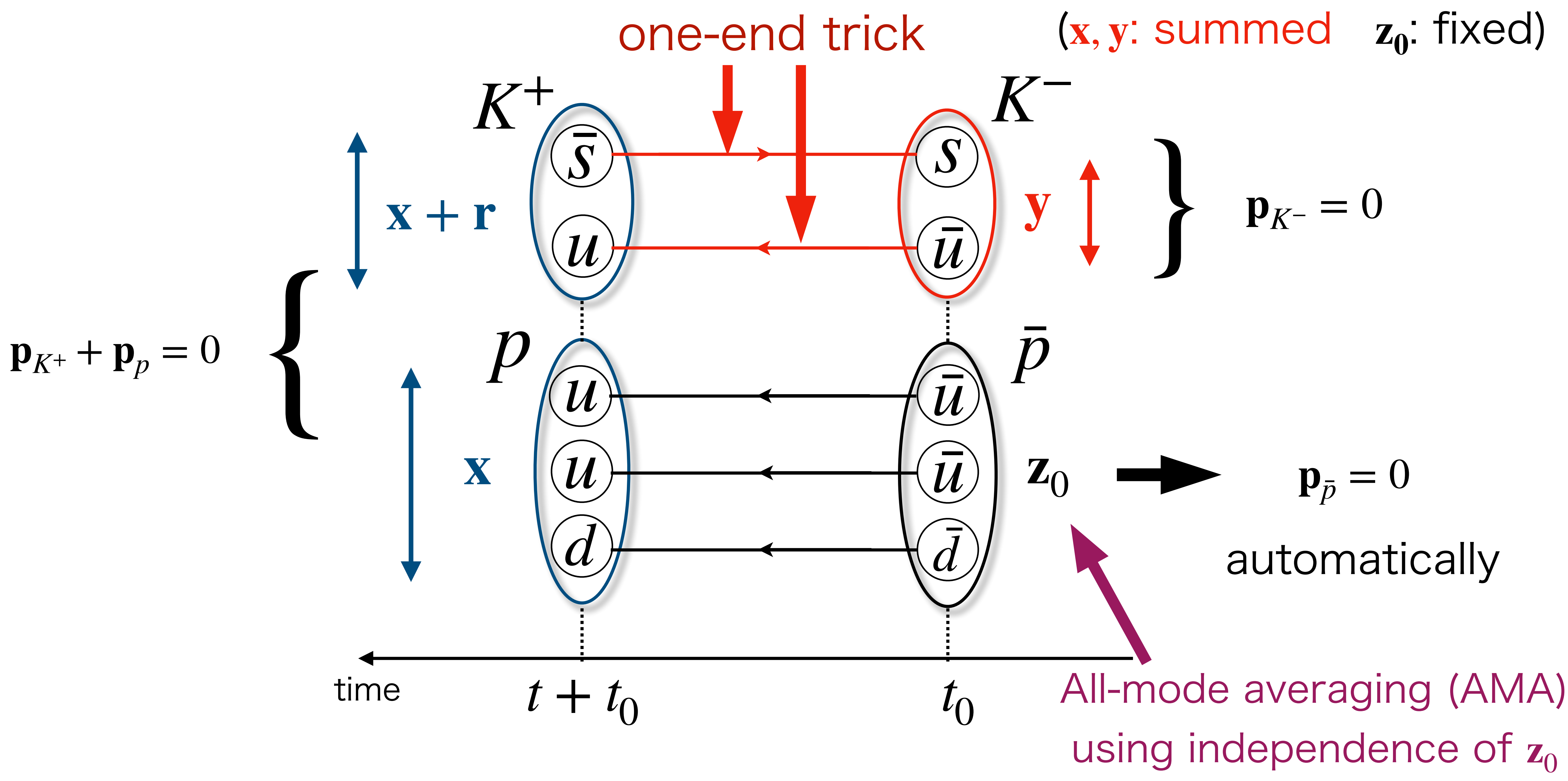
➡ Solving $D\psi^{[r](t_0)} = \eta^{[r](t_0)}$ and $D\xi^{[r](t_0)} = \gamma_5\Gamma^\dagger\eta^{[r](t_0)}$,

we can calculate 2 all-to-all propagators using 1 noise vector!

1st step: NK interactions

Strategy for calculating all-to-all propagators in NK

- One example of quark contractions for I=1



[E. Shintani et al., 2015]

1st step: NK interactions

Setup

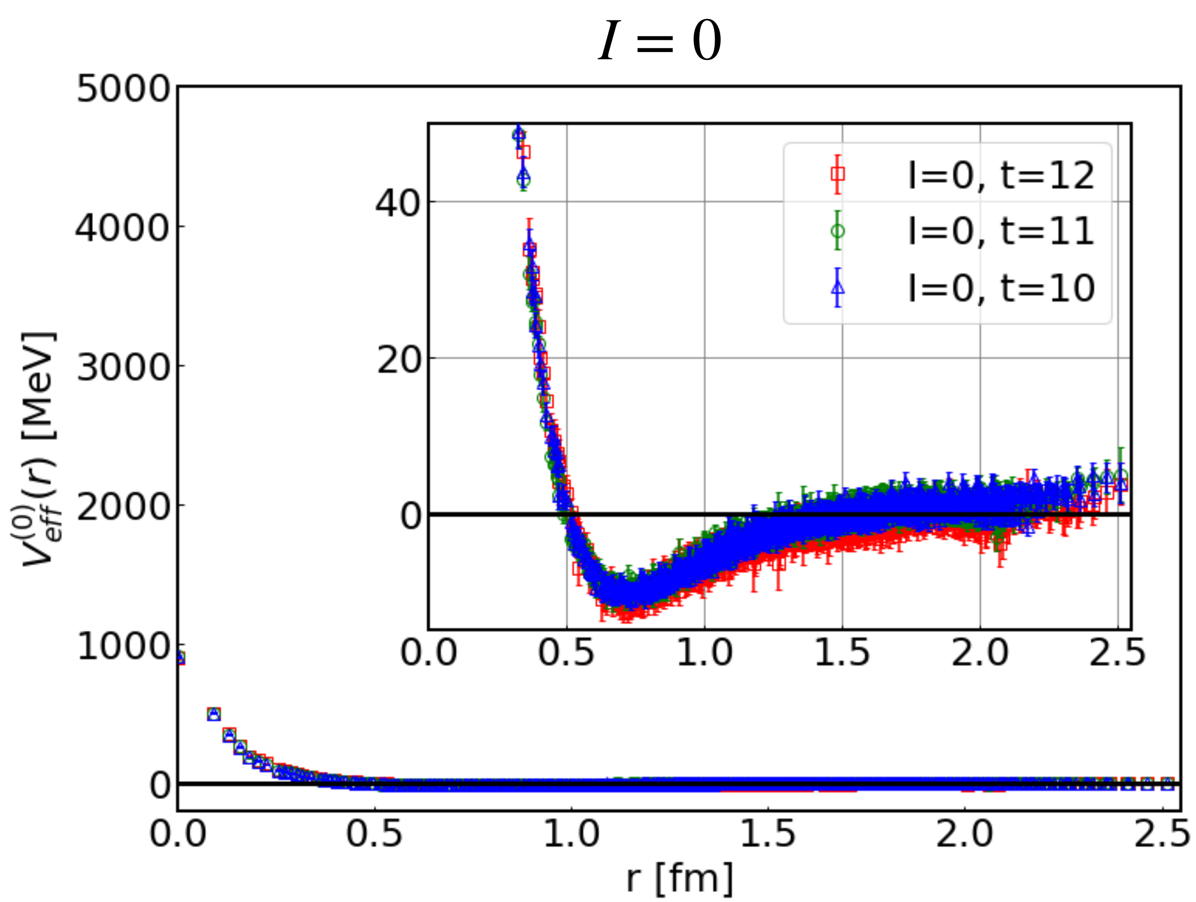
- PACS-CS, (2+1)-flavor configurations (gauge fixed, **400 conf.**)
- $a = 0.0907$ fm on $32^3 \times 64$ **lattices at** $m_\pi \approx 570$ **MeV,** $m_K \approx 710$ MeV and $m_N \approx 1400$ MeV
- smearing quarks at the source
- **leading order analysis** in the derivative expansion of the non-local potential

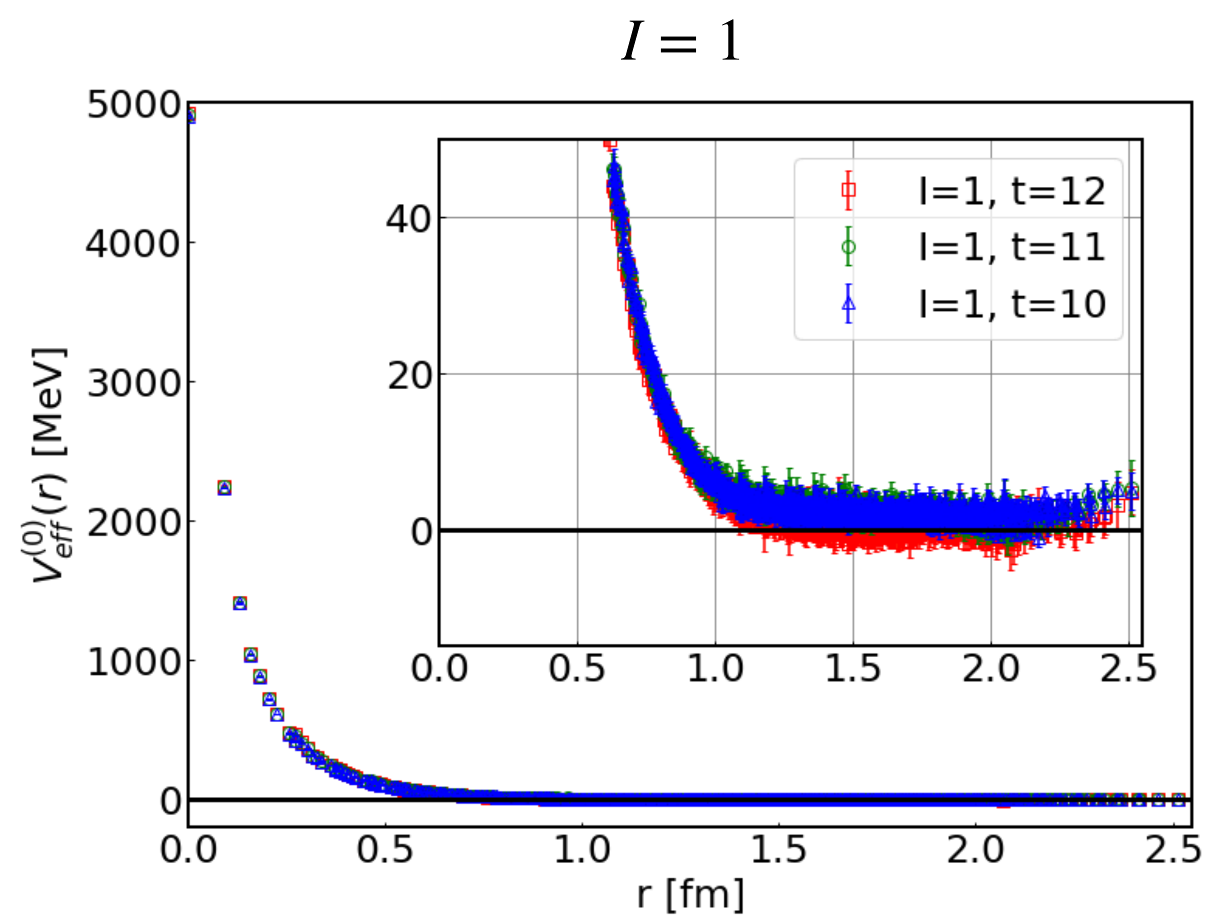
$$V_0^{LO}(r) = \frac{1}{R(\mathbf{r},t)}\Big(\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\Big)R(\mathbf{r},t) + O(\Delta W_n^3)$$

1st step: NK interactions

Results: potentials

(t=10 (blue), 11(green), 12(red))





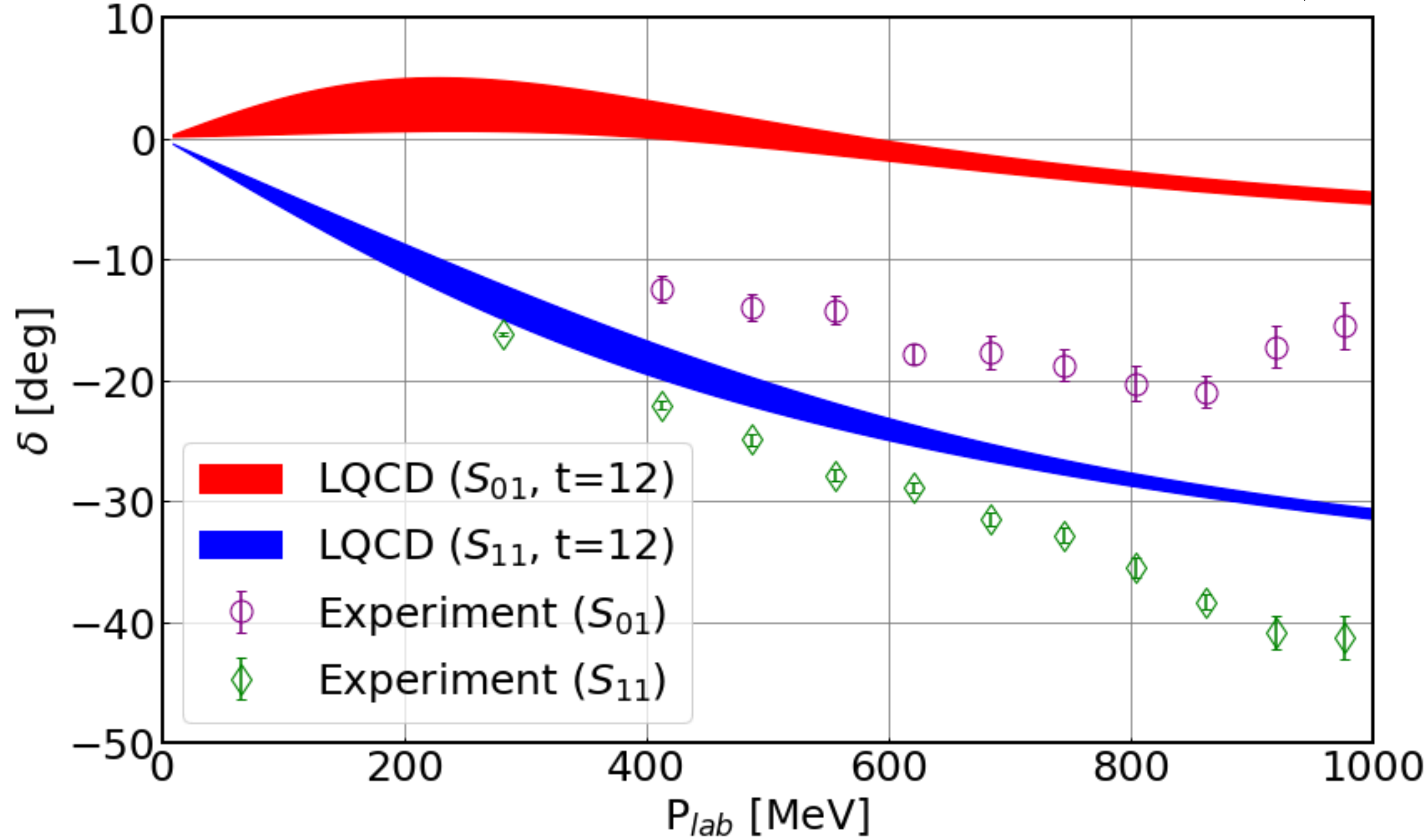
- independent of t

  ➡ inelastic states are suppressed, LO analysis is good
- repulsive core for both $I = 0$ and $I = 1$
- shallow attractive pocket at middle distances for $I = 0$
- both potentials go to zero at long distances

1st step: NK interactions

Results: phase shifts

(red: $I = 0$, blue: $I = 1$)



Experiment: INS GW Data Analysis Center [SAID] (http://gwdac.phys.gwu.edu/)

- consistent qualitatively with the experimental ones
  → one-end trick is good for meson-baryon systems
- no bound or resonant states
  → We could not find $\Theta^+(1540)$ at $m_\pi \simeq 570$ MeV.

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2nd step: Nπ interactions with Δ sources

About Nπ systems

- P-wave Nπ ➡ $J^P = 3/2^+$
- I=3/2, $J^P = 3/2^+$ Nπ ➡ Δ(1232)

3-pt correlation function

$$F_{\alpha j_z}(\mathbf{r}, t) = \langle \pi^+(\mathbf{r}+\mathbf{x}, t) N_\alpha(\mathbf{x}, t) \bar{\Delta}^{++}_{j_z}(t_0) \rangle$$

where

$$\bar{\Delta}^{++}_{+3/2}(t_0) = -\sum_{\mathbf{y}} \epsilon_{abc}(\bar{u}_b(\mathbf{y}, t_0)\Gamma_+ \bar{u}_c^T(\mathbf{y}, t_0))\bar{u}_{a,0}(\mathbf{y}, t_0)$$

$$\bar{\Delta}^{++}_{+1/2}(t_0) = -\frac{1}{\sqrt{3}}\sum_{\mathbf{y}} \epsilon_{abc}[\sqrt{2}(\bar{u}_b(\mathbf{y}, t_0)\Gamma_z \bar{u}_c^T(\mathbf{y}, t_0))\bar{u}_{a,0}(\mathbf{y}, t_0) + (\bar{u}_b(\mathbf{y}, t_0)\Gamma_+ \bar{u}_c^T(\mathbf{y}, t_0))\bar{u}_{a,1}(\mathbf{y}, t_0)]$$

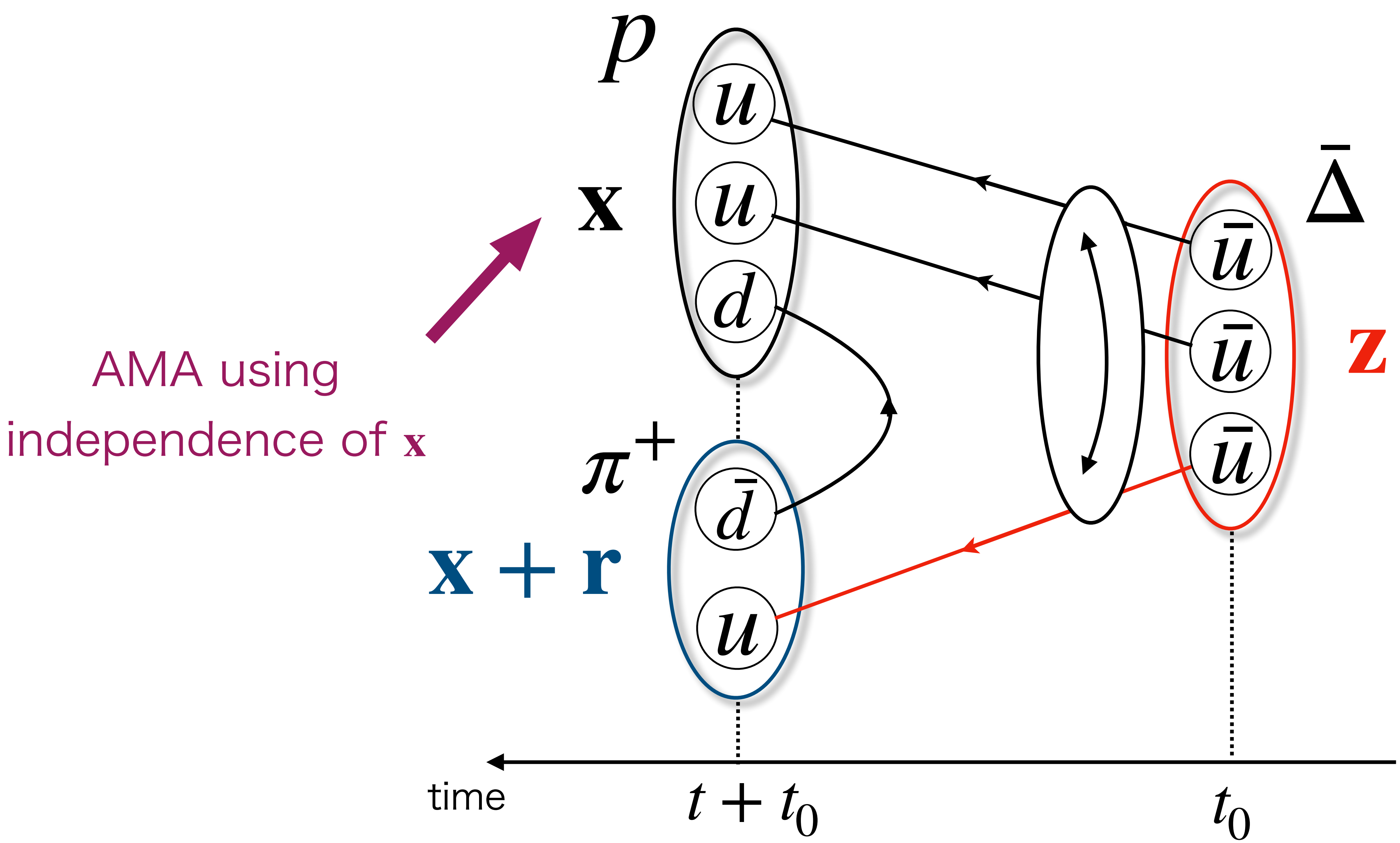
$$\bar{\Delta}^{++}_{-1/2}(t_0) = \frac{1}{\sqrt{3}}\sum_{\mathbf{y}} \epsilon_{abc}[\sqrt{2}(\bar{u}_b(\mathbf{y}, t_0)\Gamma_z \bar{u}_c^T(\mathbf{y}, t_0))\bar{u}_{a,1}(\mathbf{y}, t_0) + (\bar{u}_b(\mathbf{y}, t_0)\Gamma_- \bar{u}_c^T(\mathbf{y}, t_0))\bar{u}_{a,0}(\mathbf{y}, t_0)]$$

$$\bar{\Delta}^{++}_{-3/2}(t_0) = \sum_{\mathbf{y}} \epsilon_{abc}(\bar{u}_b(\mathbf{y}, t_0)\Gamma_- \bar{u}_c^T(\mathbf{y}, t_0))\bar{u}_{a,1}(\mathbf{y}, t_0)$$

$$\left(\Gamma_\pm = \frac{1}{2}C(\gamma_2 \pm i\gamma_1),\ \Gamma_z = \frac{-i}{\sqrt{2}}C\gamma_3\right)$$

2nd step: Nπ interactions with Δ sources

Quark contraction diagrams



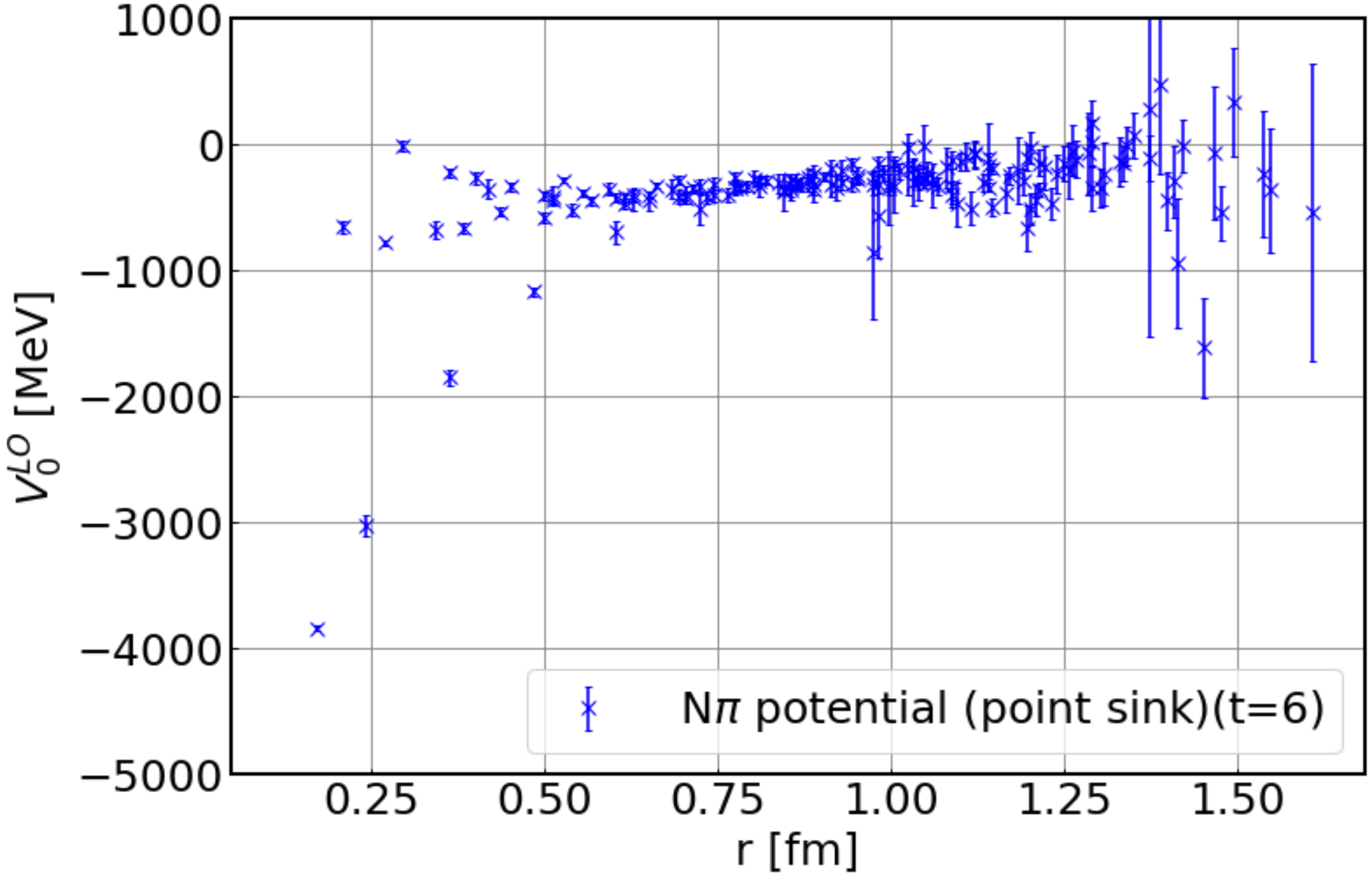
⟵ : (conventional) stochastic estimation

$\mathbf{z}$: summed, $\mathbf{x}$: fixed, $\mathbf{r}$: spatial coord. of NBS w.f.

2nd step: Nπ interactions with Δ sources

Quark-antiquark pair and sink smearing

- We usually use point quark sinks, but ⋯



It is impossible to fit this potential!

What does this behavior come from?

2nd step: Nπ interactions with Δ sources

Quark-antiquark pair and sink smearing

- quark-antiquark pair has a singular behavior in short distances according to OPE

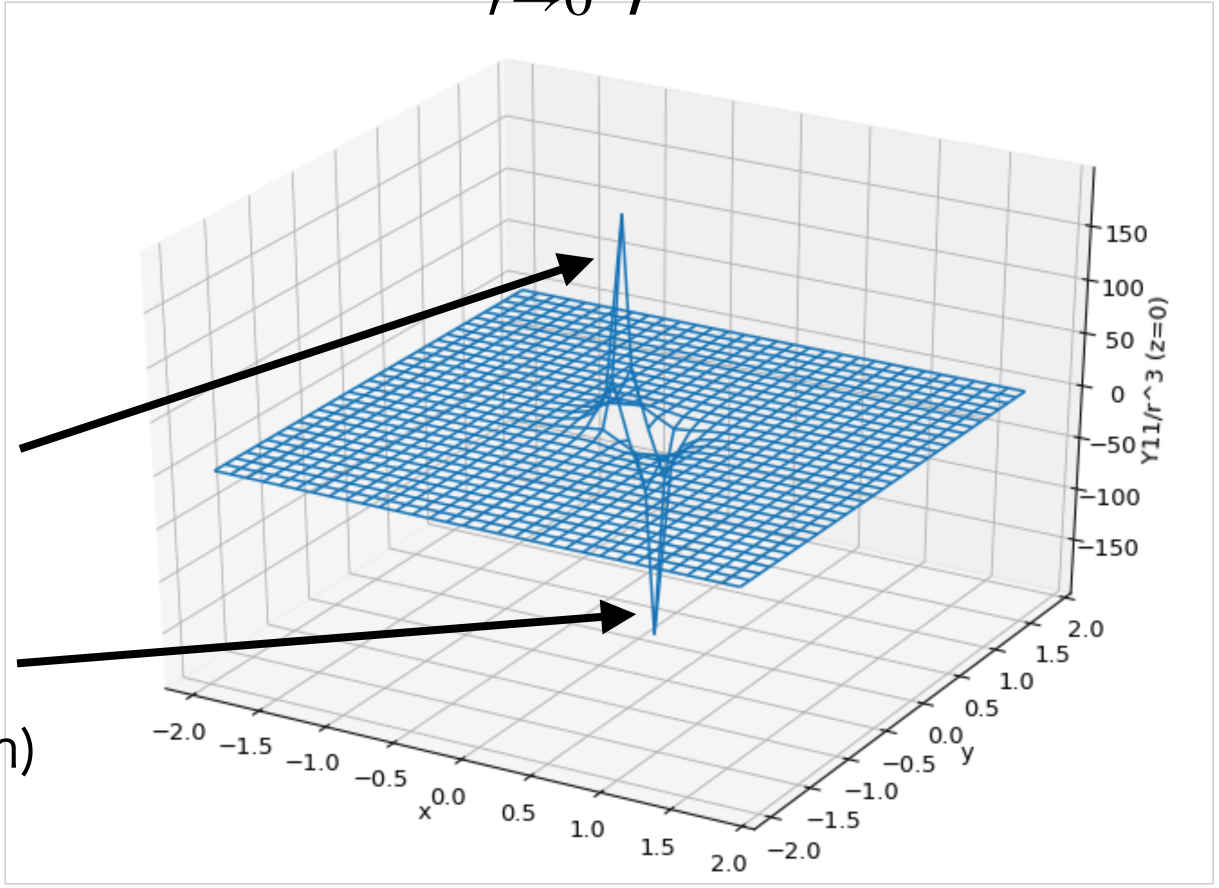
$$F(\mathbf{r},t) \sim \langle q(\mathbf{r})\bar{q}(\mathbf{0})\rangle \underset{r\to 0}{\propto} \frac{1}{r^3} \quad \Rightarrow \quad V(r) \underset{r\to 0}{\propto} \frac{1}{r^2}$$

- we consider P-wave

$$F(\mathbf{r},t) \propto \frac{1}{r^3} Y_{1,m}(\Omega)$$

sharp structure produces the spreading behavior!

(The same thing happens in I=1 P-wave $\pi\pi$ system)



One of the solutions to this problem: **sink smearing**

2nd step: Nπ interactions with Δ sources

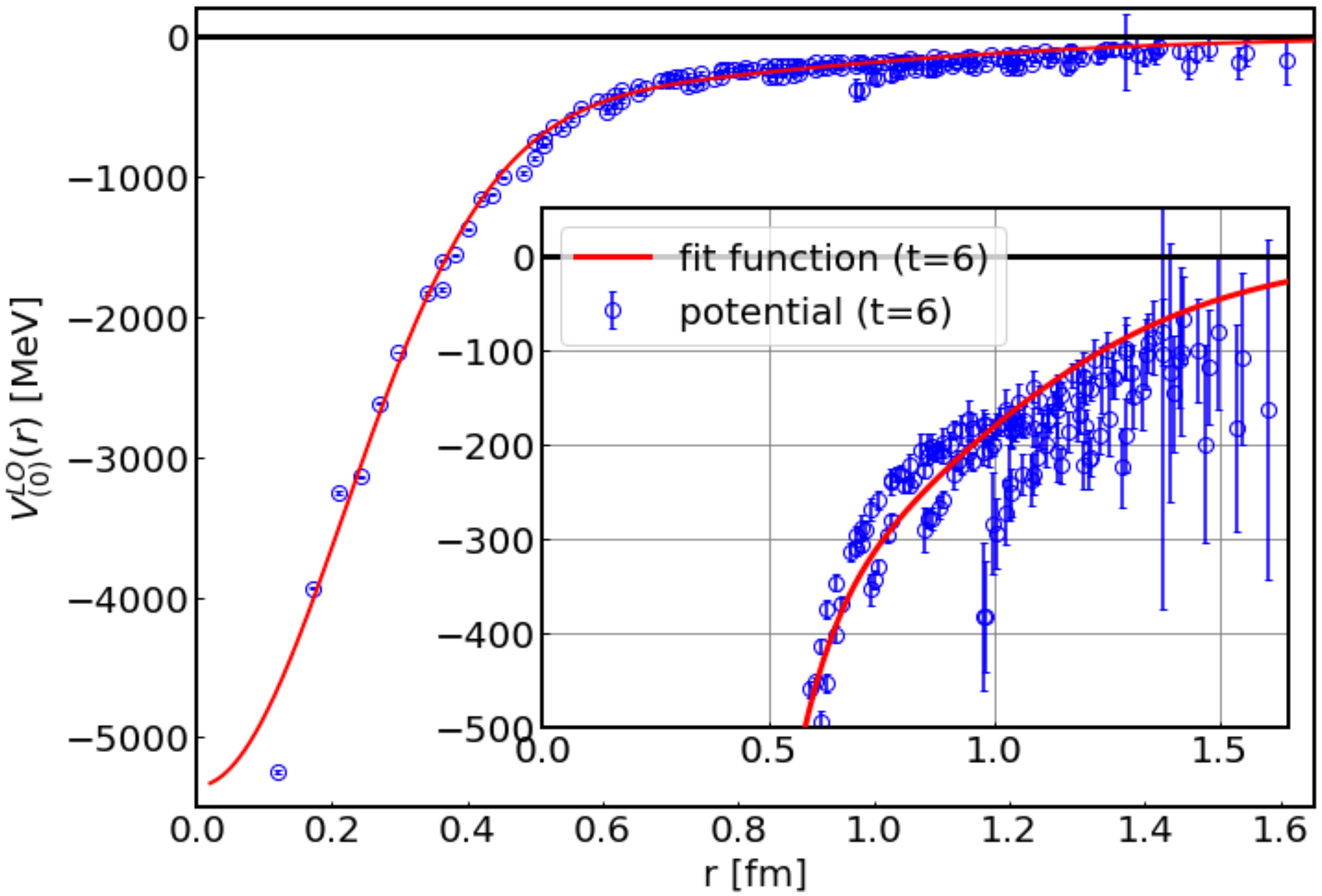
Setup

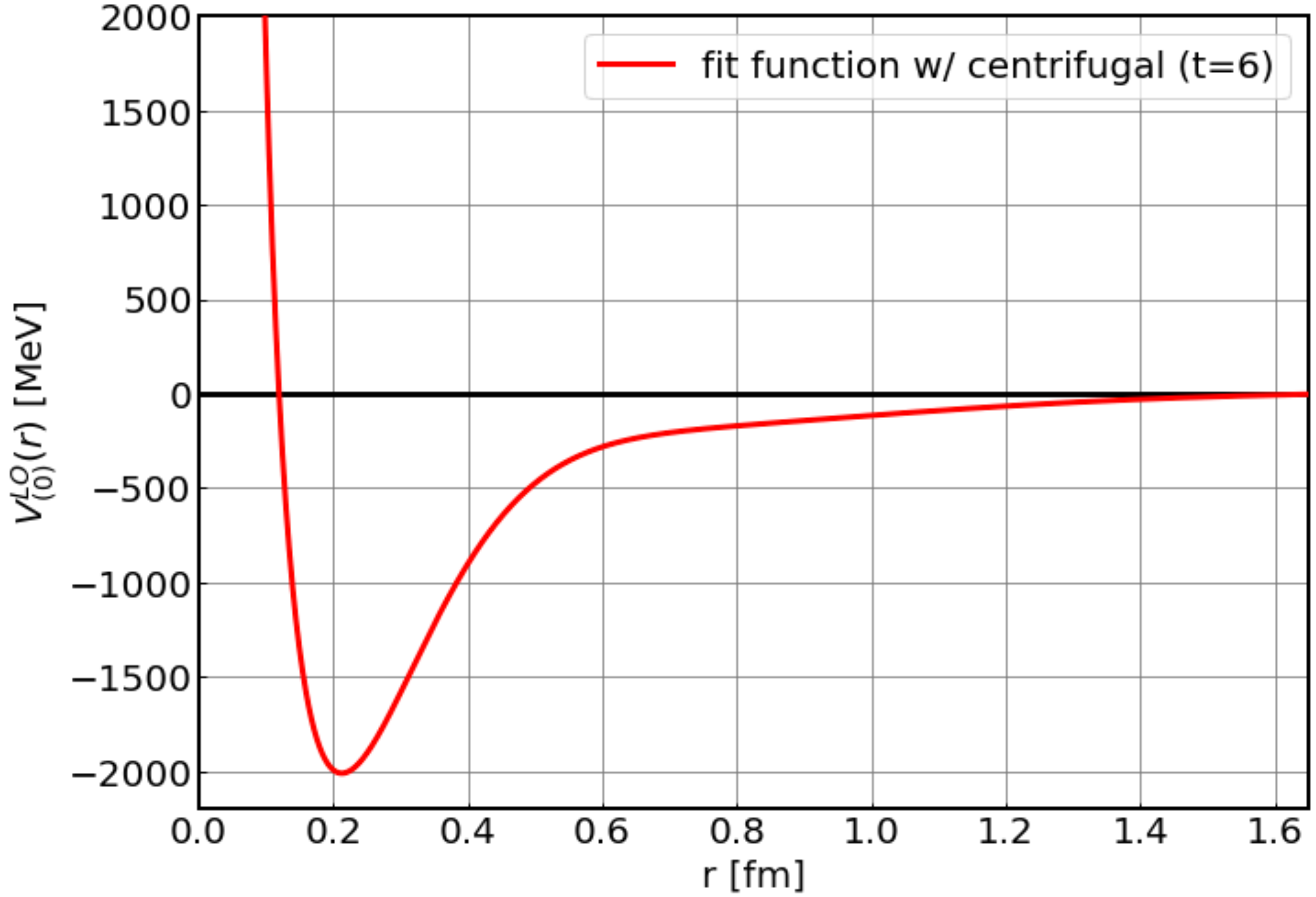
- CP-PACS+JLQCD, (2+1)-flavor configurations (gauge fixed, **50 conf.**)
- $a = 0.12$ fm on $16^3 \times 32$ **lattices at** $m_\pi \simeq 870$ **MeV**
- smearing quarks at the source and **the sink**
- leading order analysis in the derivative expansion
- $m_N \simeq 1820$ MeV, $m_\Delta \simeq 2030$ MeV from 2pt functions

➡ $E_b \simeq 660$ MeV (?)

2nd step: $N\pi$ interactions with Δ sources

Results: potentials





the spreading behavior disappears!

→ binding energy: $E_b = 729 \pm 11$ MeV

← larger than 660 MeV due to the smallness of the volume

Conclusions

- We investigate meson-baryon interactions in the HAL QCD method with all-to-all propagators.
- We study S-wave NK interactions and see the effectiveness of the one-end trick for meson-baryon systems.
- We are analyzing I=3/2 P-wave Nπ interactions with Δ sources at a heavy pion mass to see Δ as a bound state.

Future works

- Nπ with Δ sources on a larger volume at a lighter pion mass
  ➡ Δ as a bound state
- 3rd step: Nπ with Δ and $N\pi$ sources (NLO) near the physical point using the one-end trick
  ➡ Δ as a resonance
- 4th step: Other systems
  ➡ other resonances, pentaquarks

Back up

Two main methods to analyze hadron scatterings

Finite volume method

[M. Lüscher, 1991]

: extract phase shifts using boundary condition in the finite volume

- good at meson-meson systems
- difficult for systems including baryons

hard to extract the energy in such systems

HAL QCD method

[N. Ishii, S. Aoki, T. Hatsuda, 2007]

: derive interaction potentials from the NBS wave functions $\Psi^W(\mathbf{r})$

- very efficient for systems including baryons

no need to extract ground states

- very large computational cost

need for the dependences of the relative position between 2 hadrons

Detailed numerical setups

S-wave NK

- take summation of 4 timeslices t_0 to increase statistics
- each component of 4-pt corr. is projected onto A_1^+ irreps
- smear quark sources using smearing function $f_{A,B}(\mathbf{x})$ at $(A, B) = (1.2,0.19)$ for u quarks and $(A, B) = (1.2,0.25)$ for s quarks

$$f_{A,B}(\mathbf{x}) = \begin{cases} Ae^{-B|\mathbf{x}|} & (|\mathbf{x}| < \frac{L-1}{2}) \\ 1 & (|\mathbf{x}| = 0) \\ 0 & (|\mathbf{x}| \geq \frac{L-1}{2}) \end{cases}$$

P-wave Nπ

- take summation of 32 timeslices t_0 to increase statistics
- 4-pt corr. is projected onto H_g irreps
- use smearing function $f_{A,B}(\mathbf{x})$ at $(A, B) = (1.0,0.38)$ for source quarks and $(A, B) = (1.0,1/0.7)$ for sink quarks

Detailed numerical setups

Others

$$\eta^{(s_{dil})}(\mathbf{x}) = \begin{cases} \eta(\mathbf{x}) & (x+y+z \equiv s_{dil} \pmod 2) \\ 0 & (x+y+z \equiv s_{dil}+1 \pmod 2) \end{cases}, \ s_{dil} = 0,1,$$

- diluted indices in one-end trick: time, color, spinor, s2 (spatial)
- AMA: 8 spatial points (0,0,0),(0,0,L/2) $\cdots$ (L/2,L/2,L/2), $\epsilon = 10^{-4}$

About configurations

- CP-PACS/JLQCD, (2+1)-flavor confs.: [CP-PACS/JLQCD Collab., 2006]

  renormalization-group improved Iwasaki gauge action

  + nonperturbatively O(a) improved Wilson-clover quak action

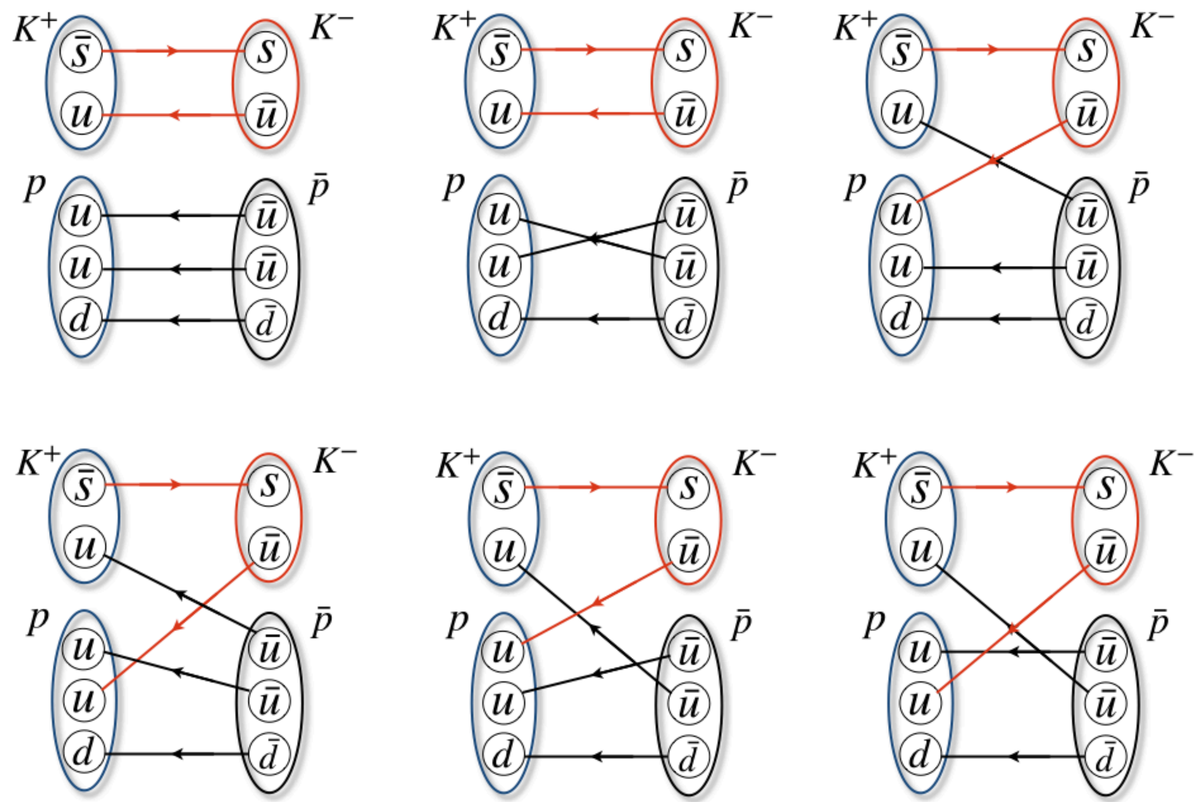
- PACS-CS, (2+1)-flavor confs.: [PACS-CS Collab., 2009]

  renormalization-group improved Iwasaki gauge action

  + nonperturbatively O(a) improved Wilson-clover quak action

All quark contraction diagrams in NK

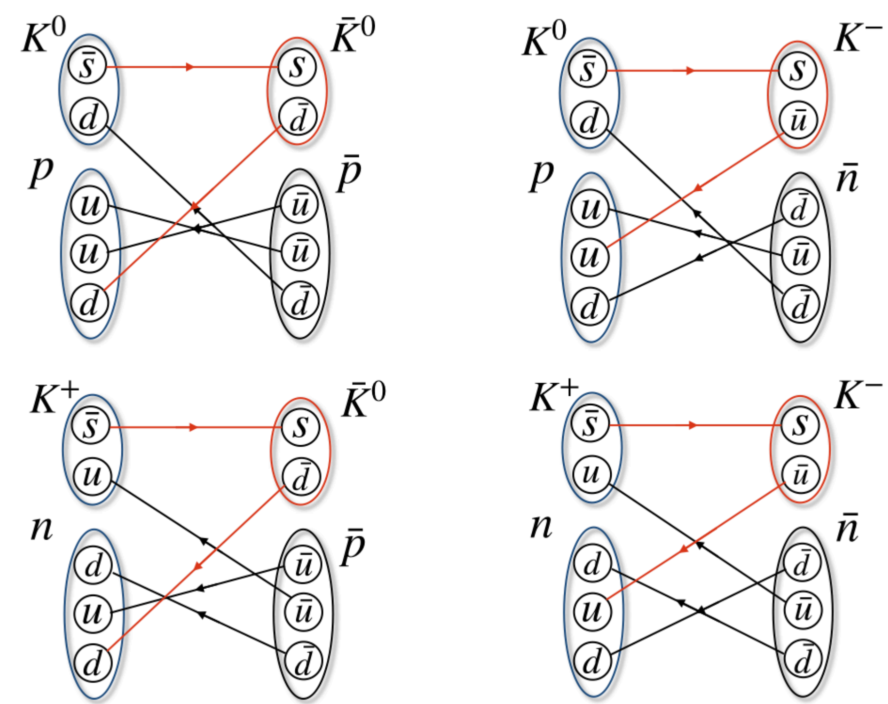
I=1



I=0

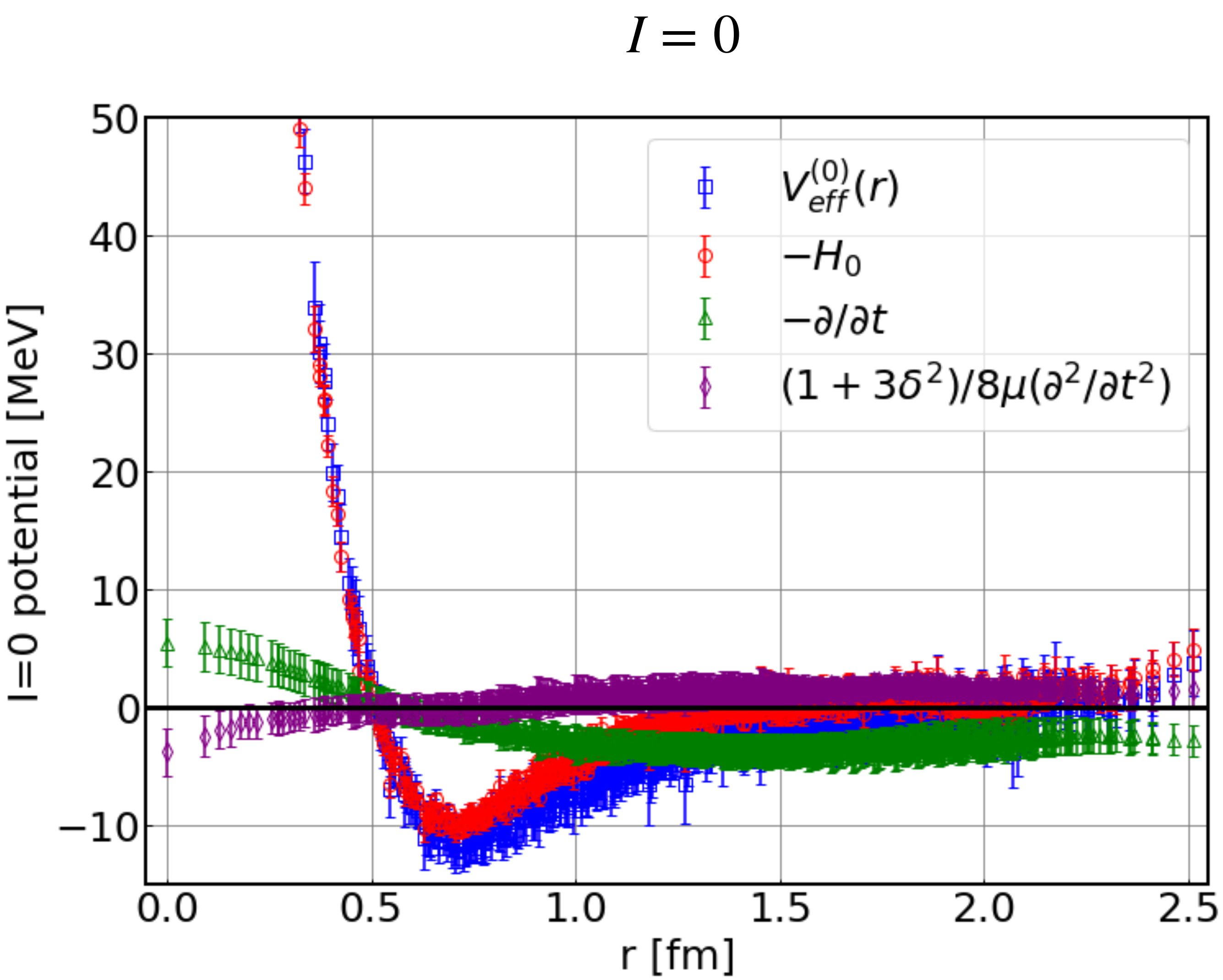
I=1 diagrams with different coefficients

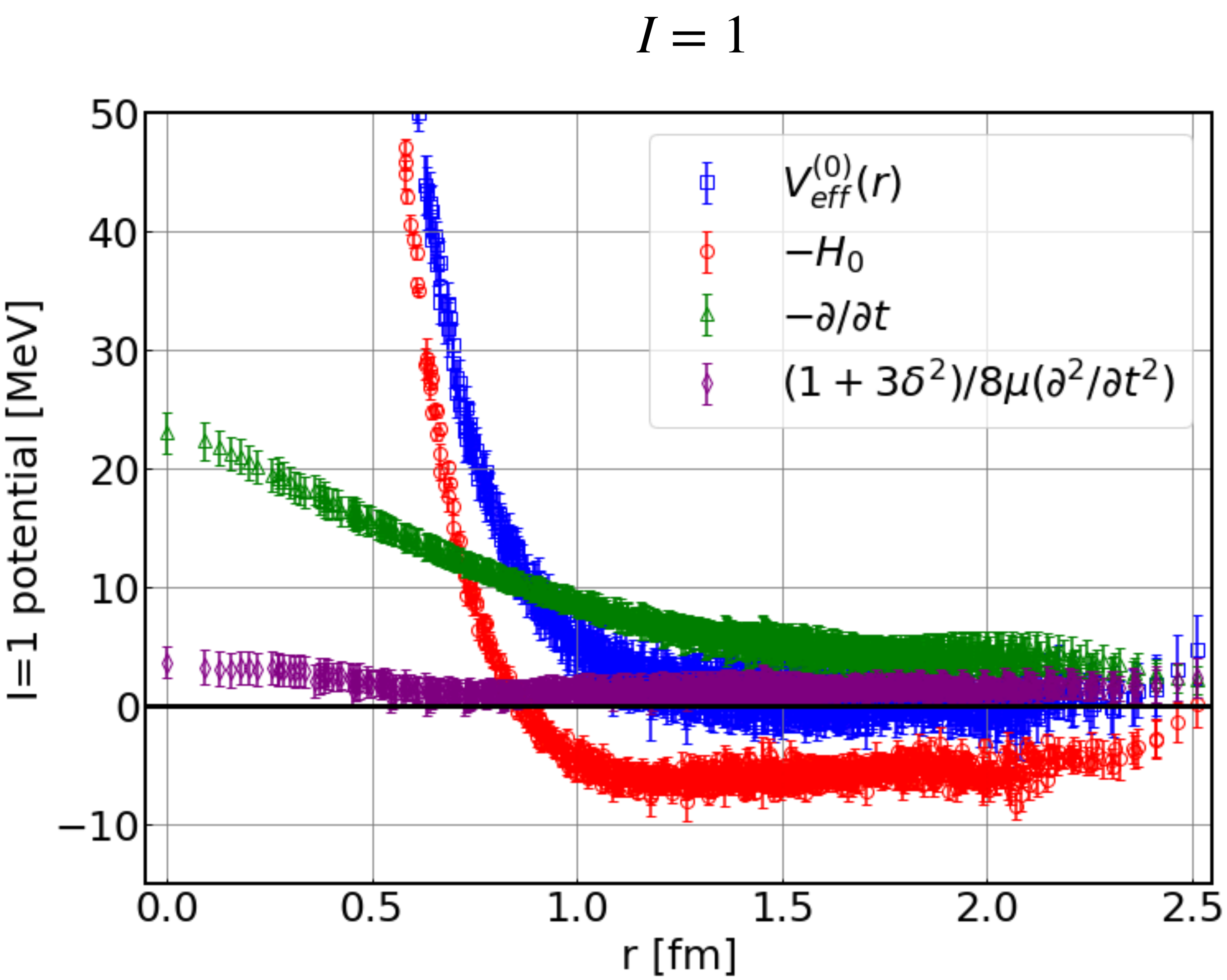
+



NK potentials and their breakups

- t=12

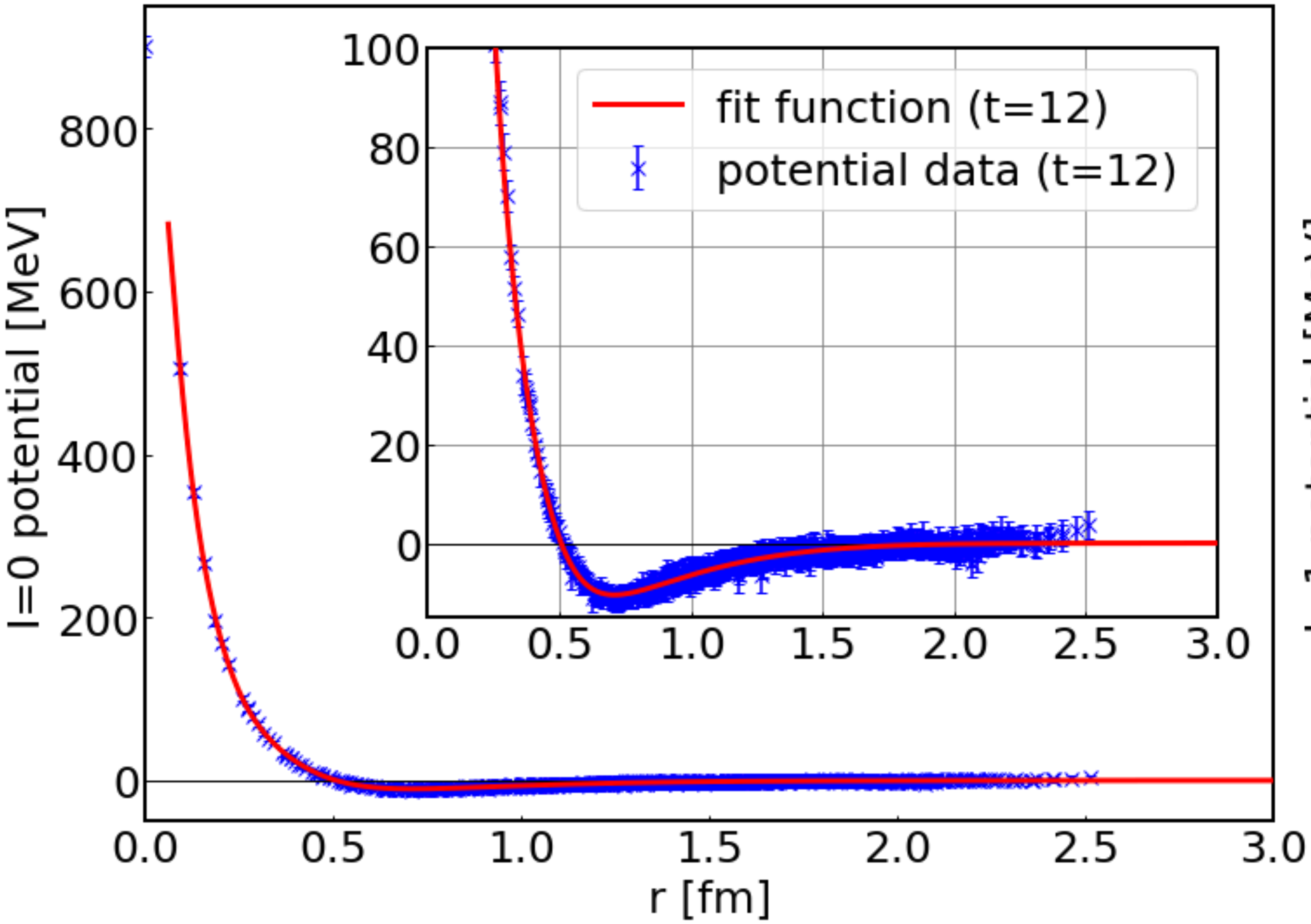




Fitting results for NK interactions

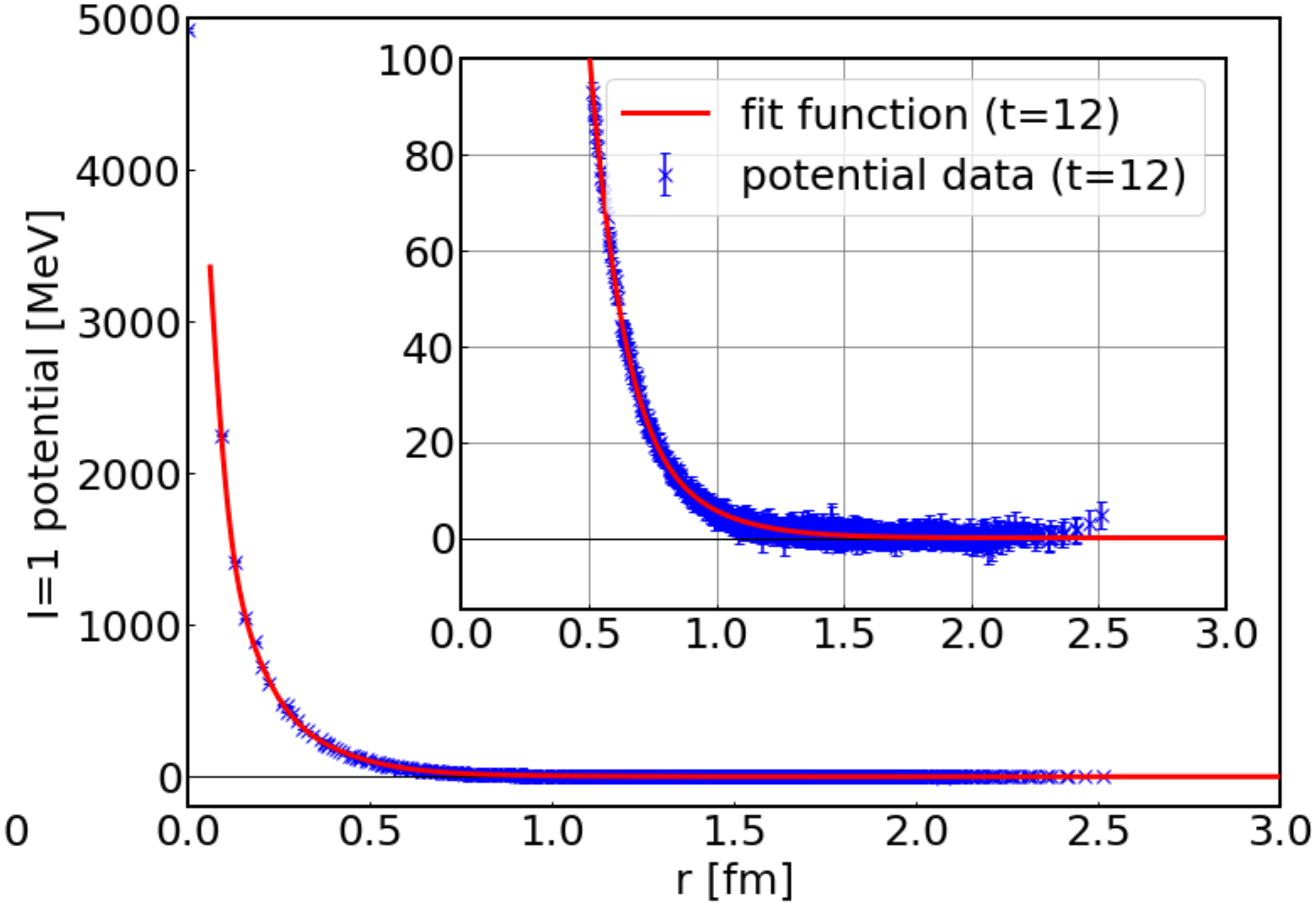
- fitting function: 4 Gaussians

$I = 0$



$\chi^2/dof = 0.27(0.09)$

$I = 1$



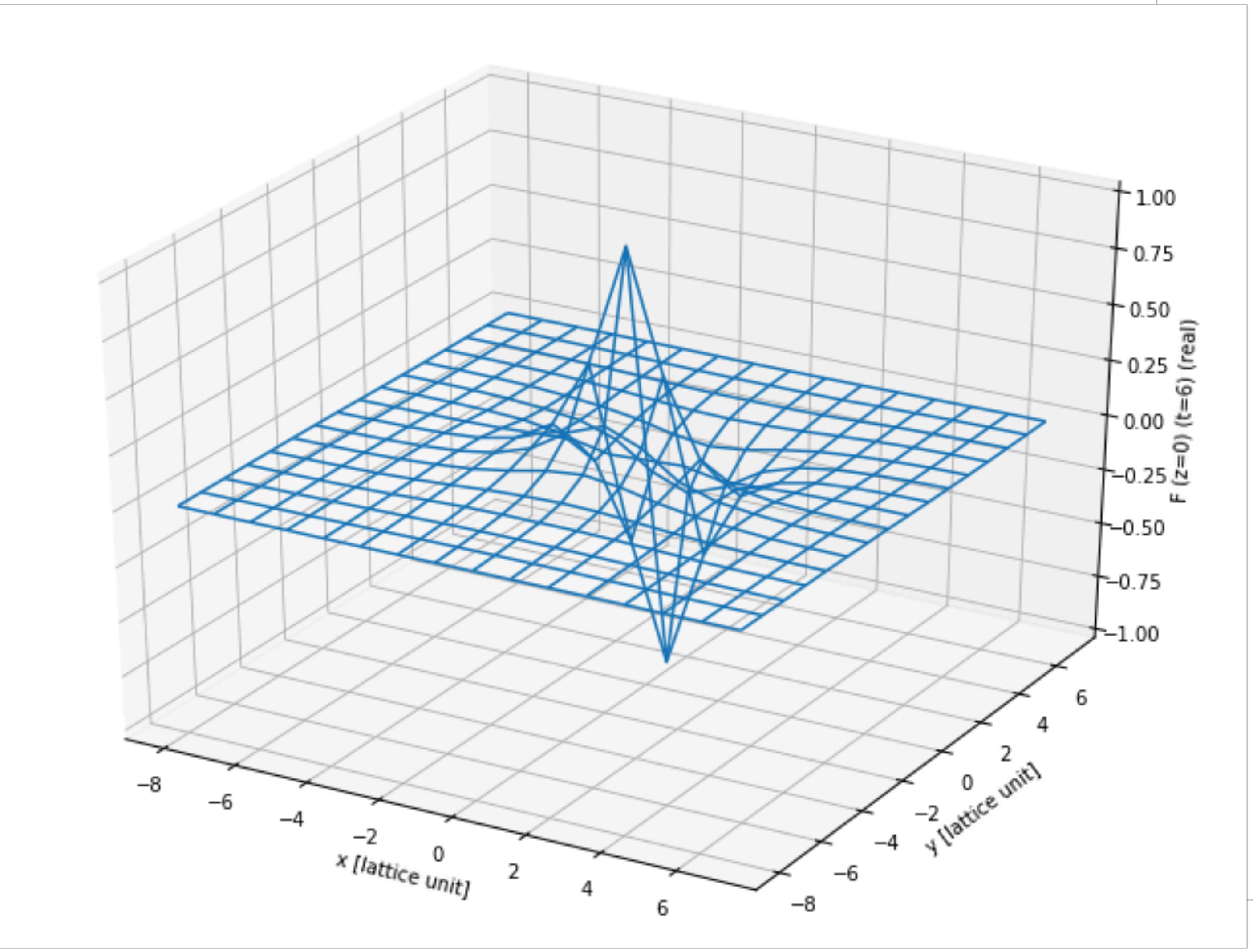
$\chi^2/dof = 0.65(0.19)$

Sink smearing and the singular behavior

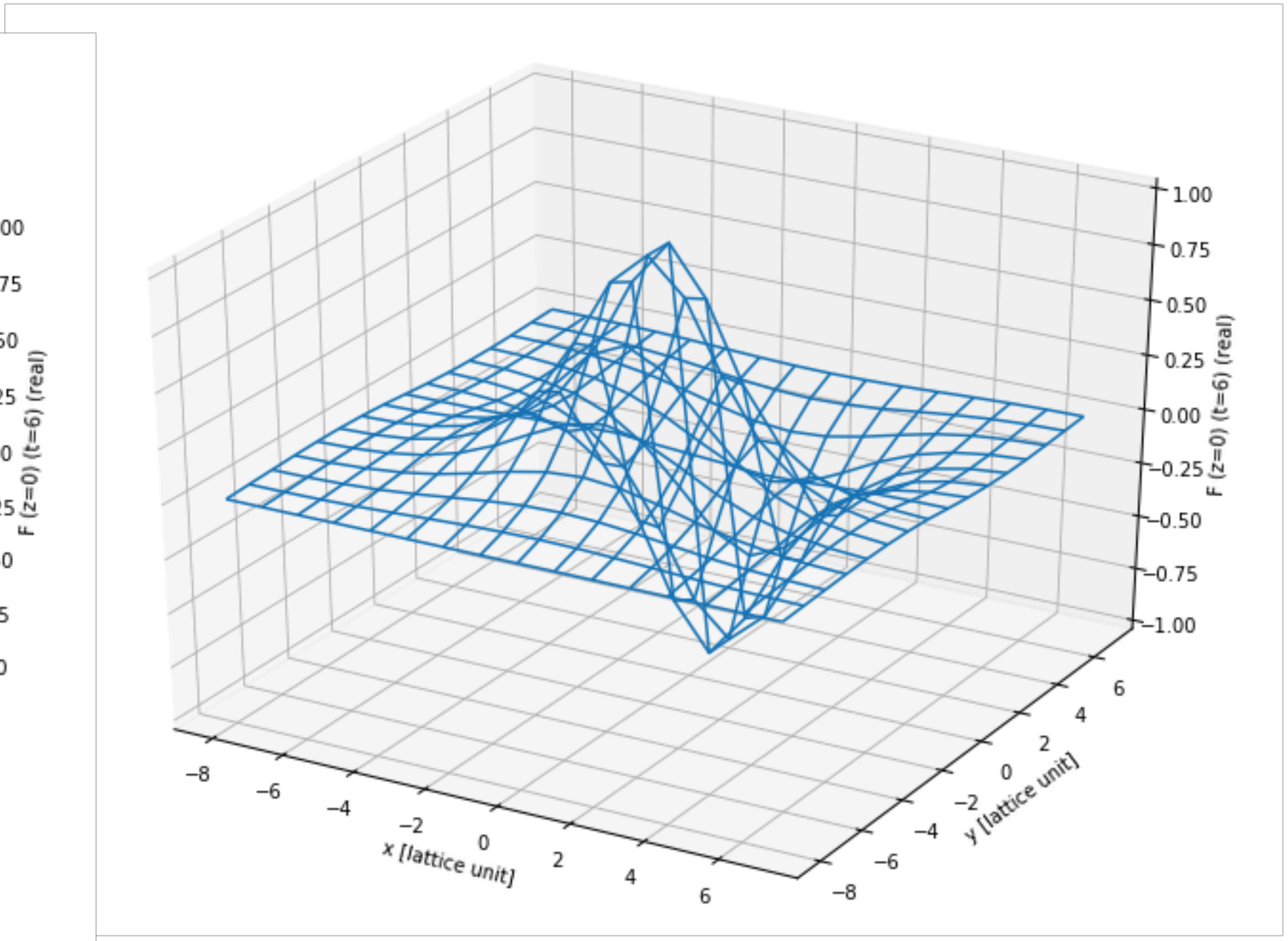
3pt function at z=0, t=6 and $\alpha = 0$ (real part)

(normalized s.t. maximum value=1.0)

w/ point quark sinks

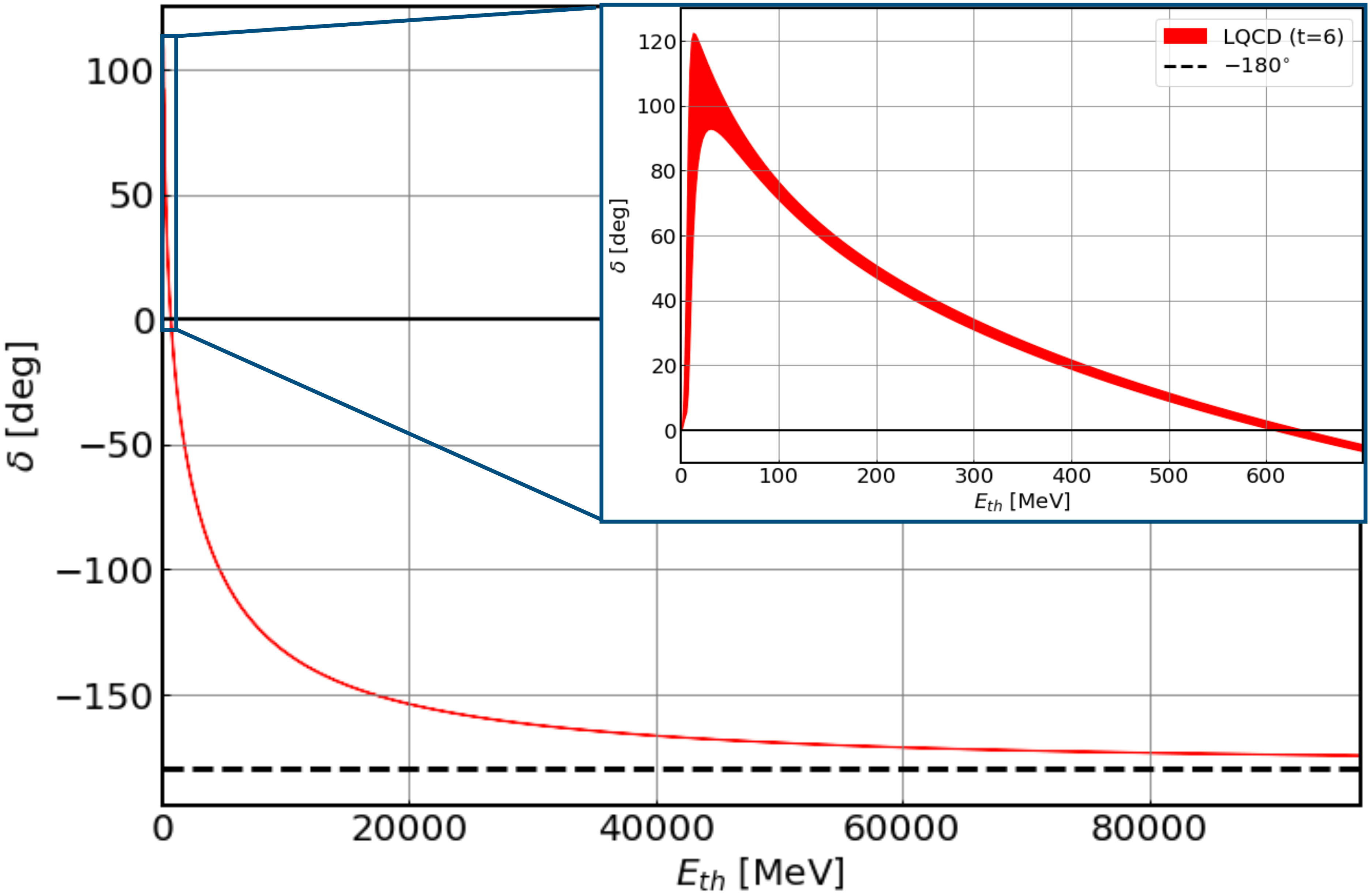


w/ smeard quark sinks



- the sharp structure is smeared by the smeard quark sinks

Phase shift results for Nπ interactions with Δ sources



- sharp rise followed by gradual fall
- $\delta(E_{th} \to \infty) \to -180^\circ$ ➡ **1 bound state**

stochastic estimation

$\eta(x)_{\underset{a}{\alpha}}$ ··· noise vector that satisfies

$$\begin{cases} \langle\langle \eta(x)_{\underset{a}{\alpha}} \eta^*(y)_{\underset{b}{\beta}} \rangle\rangle = \delta_{xy}\delta_{ab}\delta_{\alpha\beta} \\ \eta(x)_{\underset{a}{\alpha}} \eta^*(x)_{\underset{a}{\alpha}} = 1 \ (\textbf{for all } x, a, \alpha) \end{cases}$$

Propagator D^{-1} can be written as

$$q(x)_{\underset{a}{\alpha}} \longleftarrow \bar{q}(y)_{\underset{b}{\beta}} = D^{-1}(x,y)^{ab}_{\alpha\beta} = \sum_{c,\gamma,z} D^{-1}(x,z)^{ac}_{\alpha\gamma} \underline{\delta_{zy}\delta_{cb}\delta_{\gamma\beta}}$$

$$= \sum_{c,\gamma,z} D^{-1}(x,z)^{ac}_{\alpha\gamma} \underline{\langle\langle \eta(z)_{\underset{c}{\gamma}} \eta^*(y)_{\underset{b}{\beta}} \rangle\rangle}$$

$$= \langle\langle (\underbrace{D^{-1}\eta}_{\equiv\psi})(x)_{\underset{a}{\alpha}} \eta^*(y)_{\underset{b}{\beta}} \rangle\rangle = \langle\langle (\psi(x)_{\underset{a}{\alpha}} \eta^*(y)_{\underset{b}{\beta}} \rangle\rangle$$

stochastic estimation

$$\Leftrightarrow D^{-1}(x,y)^{ab}_{\alpha\beta} = \lim_{N_r\to\infty} \frac{1}{N_r}\sum_{r=1}^{N_r} \psi_{[r]}(x)_{\substack{\alpha\\a}} \eta^*_{[r]}(y)_{\substack{\beta\\b}}$$

$$\left(\psi\cdots\textbf{solution}\ \ \sum_{b,\beta,y} D(x,y)^{ab}_{\alpha\beta}\psi(y)_{\substack{\beta\\b}} = \eta(x)_{\substack{\alpha\\a}}\right)$$

Therefore, D^{-1} can be estimated by

$$D^{-1}(x,y)^{ab}_{\alpha\beta} \approx \frac{1}{N_r}\sum_{r=1}^{N_r} \psi_{[r]}(x)_{\substack{\alpha\\a}} \eta^*_{[r]}(y)_{\substack{\beta\\b}}$$

noisy estimation: very noisy ← $\eta(x)_{\substack{\alpha\\a}}$ itself has $O(1)$ error

→ this noise can be reduced by using "dilution"

stochastic estimation (+ dilution)

ex) time dilution

decompose the noise vector

$$\eta(x)^{\alpha}_{a} = \sum_{j=0}^{N_t-1} \eta^{(j)}(x)^{\alpha}_{a} \quad \text{where} \quad \eta^{(j)}(x)^{\alpha}_{a} = \begin{cases} \eta(x)^{\alpha}_{a} & (\text{for } j = t) \\ 0 & (\text{for } j \neq t) \end{cases}$$

$$\begin{bmatrix} \eta(t=0) \\ \eta(t=1) \\ \eta(t=2) \\ \vdots \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \eta(t=0) \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{=\eta^{(0)}(t)} + \underbrace{\begin{bmatrix} 0 \\ \eta(t=1) \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{=\eta^{(1)}(t)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \eta(t=2) \\ 0 \\ \vdots \end{bmatrix}}_{=\eta^{(2)}(t)} + \cdots$$

stochastic estimation (+ dilution)

ex) time dilution

$$D^{-1}(x,y)^{ab}_{\alpha\beta} = \sum_{c,\gamma,z} D^{-1}(x,z)^{ac}_{\alpha\gamma} \langle\langle \underset{c}{\eta(z)_\gamma} \underset{b}{\eta^*(y)_\beta} \rangle\rangle$$

$$= \sum_{c,\gamma,z} D^{-1}(x,z)^{ac}_{\alpha\gamma} \sum_{j,k=0}^{N_t-1} \underline{\langle\langle \underset{c}{\eta^{(j)}(z)_\gamma} \underset{b}{\eta^{(k)*}(y)_\beta} \rangle\rangle}$$

$j \neq k$ terms are noisy parts, not signals

$$\rightarrow \sum_{c,\gamma,z} D^{-1}(x,z)^{ac}_{\alpha\gamma} \sum_{j=0}^{N_t-1} \underline{\langle\langle \underset{c}{\eta^{(j)}(z)_\gamma} \underset{b}{\eta^{(j)*}(y)_\beta} \rangle\rangle}$$

stochastic estimation (+ dilution)

ex) time dilution

$$D^{-1}(x,y)^{ab}_{\alpha\beta} = \sum_{j=0}^{N_t-1} \langle\langle (\psi^{(j)}(x)_{\underset{a}{\alpha}}\eta^{(j)*}(y)_{\underset{b}{\beta}}\rangle\rangle$$

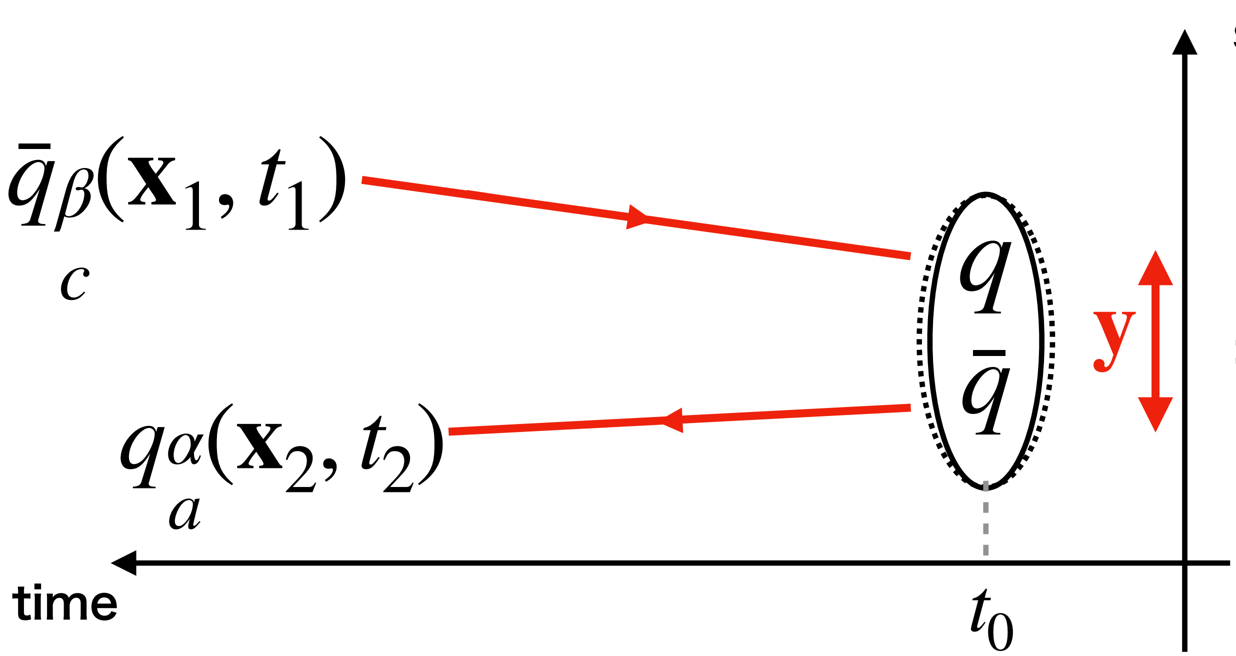
$$\left(\sum_{b,\beta,y} D(x,y)^{ab}_{\alpha\beta}\psi^{(i)}(y)_{\underset{b}{\beta}} = \eta^{(i)}(x)_{\underset{a}{\alpha}} \right)$$

Therefore,

$$D^{-1}(x,y)^{ab}_{\alpha\beta} \approx \frac{1}{N_r}\sum_{r=1}^{N_r}\sum_{j}\psi^{(j)}_{[r]}(x)_{\underset{a}{\alpha}}\eta^{(j)*}_{[r]}(y)_{\underset{b}{\beta}}$$

one-end trick

[M. Foster, C. Michael, 1999]



$$= \sum_{\mathbf{y}} D^{-1}(\mathbf{x}_1, t_1; \mathbf{y}, t_0)\Gamma D^{-1}(\mathbf{y}, t_0; \mathbf{x}_2, t_2)$$

$$= \sum_{\mathbf{y}} \sum_{\mathbf{z}, t_z, t_y} D^{-1}(\mathbf{x}_1, t_1; \mathbf{z}, t_z)\underline{(\delta_{t_z, t_0}\delta_{t_y, t_0}\delta_{\mathbf{z},\mathbf{y}})}\Gamma D^{-1}(\mathbf{y}, t_y; \mathbf{x}_2, t_2)$$

$$= \langle\langle \eta^{(t_0)} \otimes \eta^{(t_0)\dagger} \rangle\rangle \simeq \frac{1}{N}\sum_{r=1}^{N} \eta^{[r](t_0)} \otimes \eta^{[r](t_0)\dagger}$$

$$(\eta^{[r](t_0)}_{a,\alpha}(\mathbf{x}, t_x) = \delta_{t_x, t_0}\Xi^{[r]}_{a,\alpha}(\mathbf{x}))$$

$$(\langle \Xi^{[r]}_{a,\alpha}(\mathbf{x})\Xi^{[r]\dagger}_{b,\beta}(\mathbf{y})\rangle = \delta_{a,b}\delta_{\alpha,\beta}\delta_{\mathbf{x},\mathbf{y}})$$

$$\simeq \sum_{\mathbf{y}} \sum_{\mathbf{z}, t_z, t_y} D^{-1}(\mathbf{x}_1, t_1; \mathbf{z}, t_z)\left(\frac{1}{N}\sum_{r=1}^{N} \eta^{[r](t_0)}(\mathbf{z}, t_z) \otimes \eta^{[r](t_0)\dagger}(\mathbf{y}, t_y)\right)\Gamma D^{-1}(\mathbf{y}, t_y; \mathbf{x}_2, t_2)$$

one-end trick

[M. Foster, C. Michael, 1999]

Using γ_5 hermiticity,

$$\frac{1}{N}\sum_{r=1}^{N}\left(\sum_{\mathbf{z},t_z} D^{-1}(\mathbf{x}_1,t_1;\mathbf{z},t_z)\eta^{[r](t_0)}(\mathbf{z},t_z)\right)\otimes\left(\sum_{\mathbf{y},t_y}\eta^{[r](t_0)\dagger}(\mathbf{y},t_y)\Gamma\gamma_5 D^{-1\dagger}(\mathbf{y},t_y;\mathbf{x}_2,t_2)\gamma_5\right)$$

$$=\frac{1}{N}\sum_{r=1}^{N}\underline{(D^{-1}\eta^{[r](t_0)})(\mathbf{x}_1,t_1)}\otimes\left(\underline{(D^{-1}\gamma_5\Gamma^\dagger\eta^{[r](t_0)})^\dagger(\mathbf{x}_2,t_2)}\gamma_5\right)$$

Therefore, solving 2N linear equations

$$\begin{cases} \underline{D\psi^{[r](t_0)}=\eta^{[r](t_0)}} \\ \underline{D\xi^{[r](t_0)}=\gamma_5\Gamma^\dagger\eta^{[r](t_0)}} \end{cases}$$

$$\sum_{\mathbf{y}} G(\mathbf{x}_1,t_1;\mathbf{y},t_0)\Gamma G(\mathbf{y},t_0;\mathbf{x}_2,t_2)\simeq\frac{1}{N}\sum_{r=1}^{N}\underline{\psi^{[r](t_0)}(\mathbf{x}_1,t_1)}\otimes(\underline{\xi^{[r](t_0)\dagger}(\mathbf{x}_2,t_2)}\gamma_5)$$

All-mode averaging (AMA)

general idea: Covariant approximation averaging (CAA)

$O[U] \cdots$ observable that is covariant under symmetry G

$$\Leftrightarrow O[U^g] = O^g[U] \text{ for all } g \in G$$

(ex) $G \cdots$ translation $x \to x + a$)

We define

$$O_G[U] = \frac{1}{N_G}\sum_{g\in G} O[U^g] = \frac{1}{N_G}\sum_{g\in G} O^g[U]$$

($N_G \cdots$ number of the element of G)

This variable satisfies

$$\langle O[U]\rangle = \langle O_G[U]\rangle \quad (\because \langle O[U^g]\rangle = \langle O[U]\rangle)$$

All-mode averaging (AMA)

general idea: Covariant approximation averaging (CAA)

$O^{(appx)}[U] \cdots$ approximation of G which reduces computational cost

and we introduce

$$O_G^{(appx)}[U] = \frac{1}{N_G} \sum_{g \in G} O^{(appx)}[U^g] = \frac{1}{N_G} \sum_{g \in G} O^{(appx)g}[U]$$

All-mode averaging (AMA)

general idea: Covariant approximation averaging (CAA)

Improved estimator is defined by

$$O^{(imp)}[U] = O[U] - O^{(appx)}[U] + O_G^{(appx)}[U]$$

and this satisfies

$$\langle O^{(imp)}[U]\rangle = \langle O[U]\rangle - \langle O^{(appx)}[U]\rangle + \underline{\langle O_G^{(appx)}[U]\rangle}$$

$$= \langle O^{(appx)}[U]\rangle$$

$$= \langle O[U]\rangle$$

All-mode averaging (AMA)

All-mode averaging

$$O^{(AMA)} = O[S^{(all)}[U]]$$

$$O_G^{(AMA)} = \frac{1}{N_G} \sum_{g \in G} O[S^{(all)g}[U]]$$

where

$$(S^{(all)}b)_i = \sum_{k=1}^{N_\lambda} \frac{1}{\lambda_k} (\psi_k^\dagger b)(\psi_k)_i + (f_\epsilon(H)b)_i$$

spectral decompposituon for low mode

$$f_\epsilon(H)b = \sum_{i=1}^{N_{CG}} (H^i) c_i$$

relaxed stopping criterion in the CG method

All-mode averaging (AMA)

strategy for all-mode averaging w/o low modes

$C(U; \mathbf{z}_0)$: correlation function at gauge conf. U with the hadron source operator at $\mathbf{z}_0$

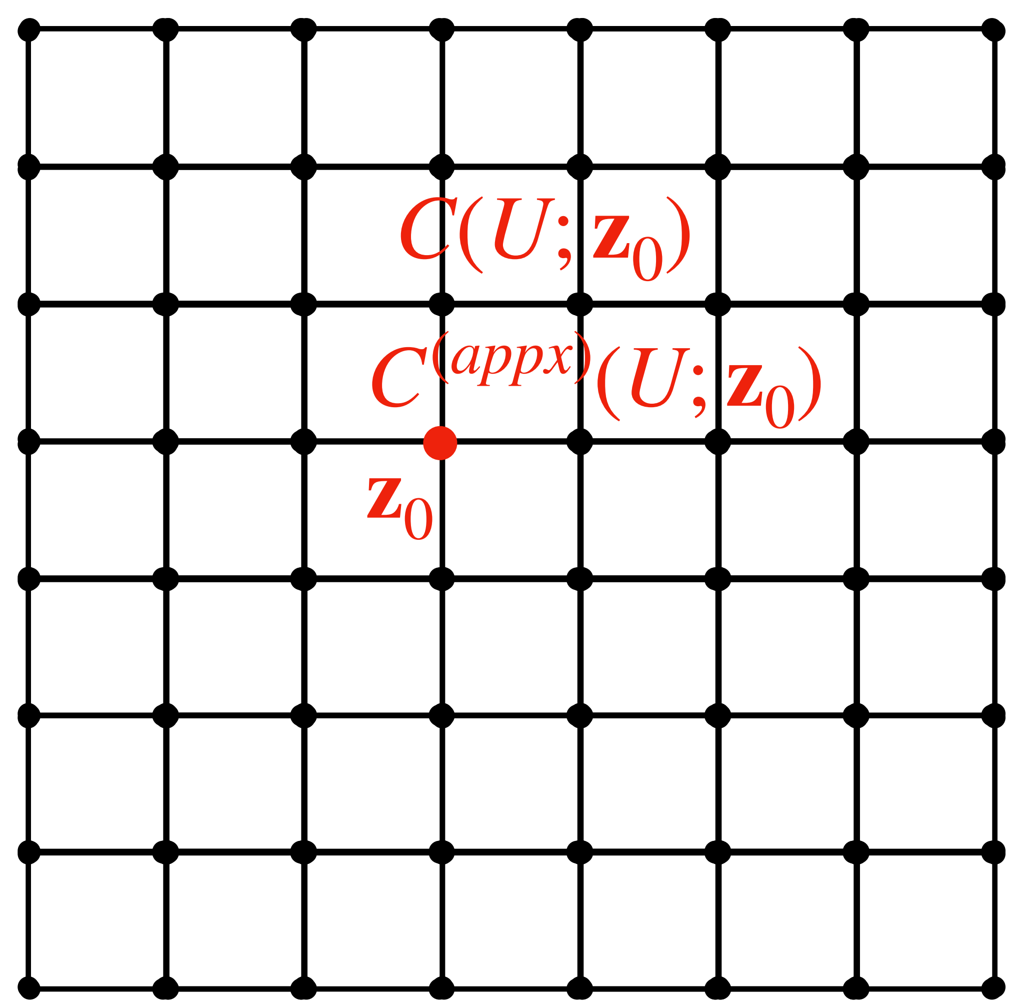
$C^{(appx)}(U; \mathbf{z}_i)$: approximated correlation function at gauge conf. U with the hadron source operator at $\mathbf{z}_i$

by relaxing stopping condition $||D\psi - s||/||s|| < \epsilon$ in BiCG solver

All-mode averaging (AMA)

strategy for all-mode averaging w/o low modes

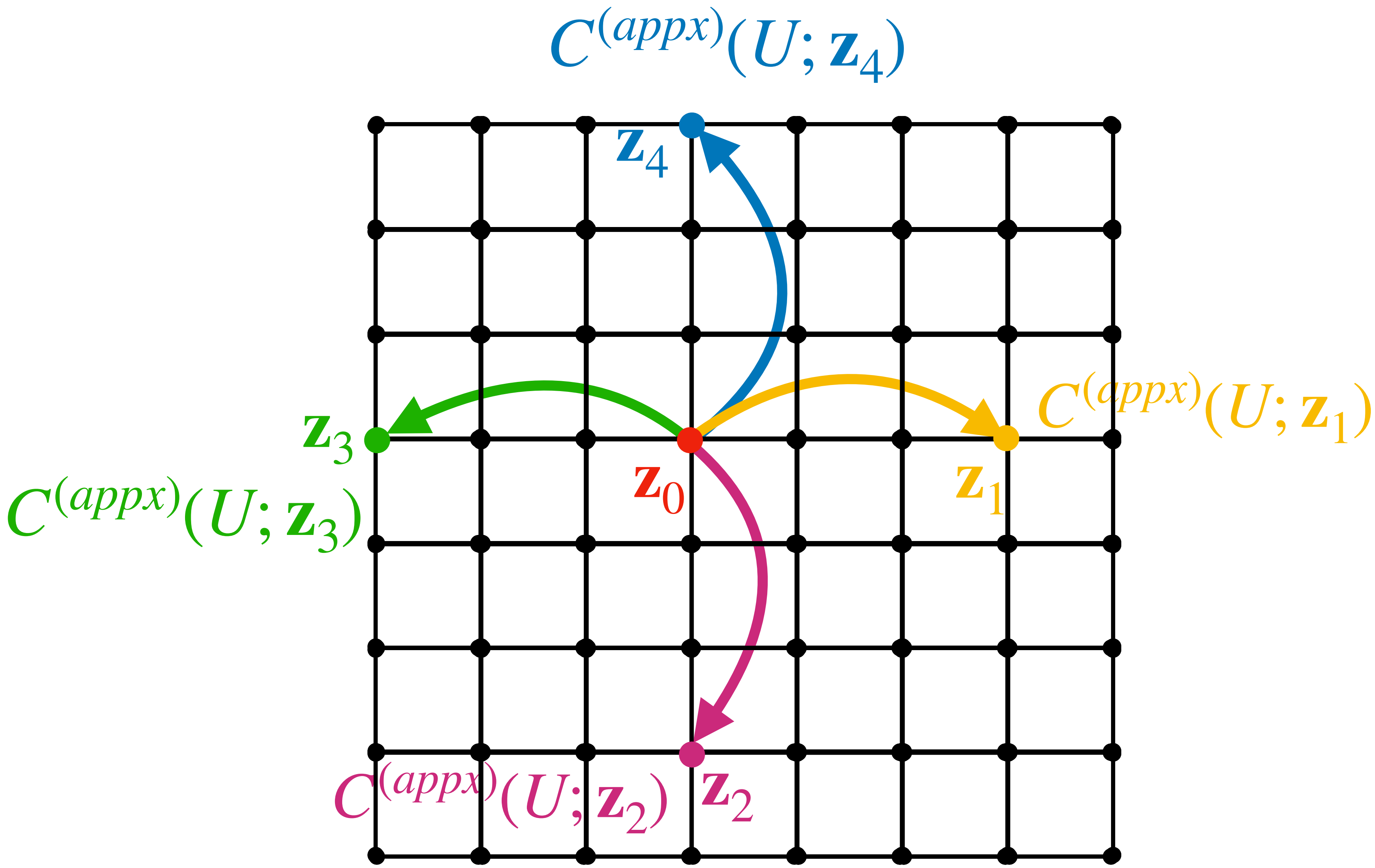
1. For each gauge conf., we calculate $C(U;\mathbf{z}_0)$ and $C^{(appx)}(U;\mathbf{z}_0)$ for some $\mathbf{z}_0$.



All-mode averaging (AMA)

strategy for all-mode averaging w/o low modes

2. Tranlate $\mathbf{z}_0$ and calculate $C^{(appx)}(U;\mathbf{z}_i)$ at each source point.



All-mode averaging (AMA)

strategy for all-mode averaging w/o low modes

3. The improve estimator is constructed from $C(U;\mathbf{z}_0)$

and $\{C^{(appx)}(U;\mathbf{z}_i)\}_{i=0,1\cdots N_s}$

$$C^{(imp)}(U) = C(U;\mathbf{z}_0) - C^{(appx)}(U;\mathbf{z}_0) + \frac{1}{N_s}\sum_{i=1}^{N_s} C^{(appx)}(U;\mathbf{z}_i)$$

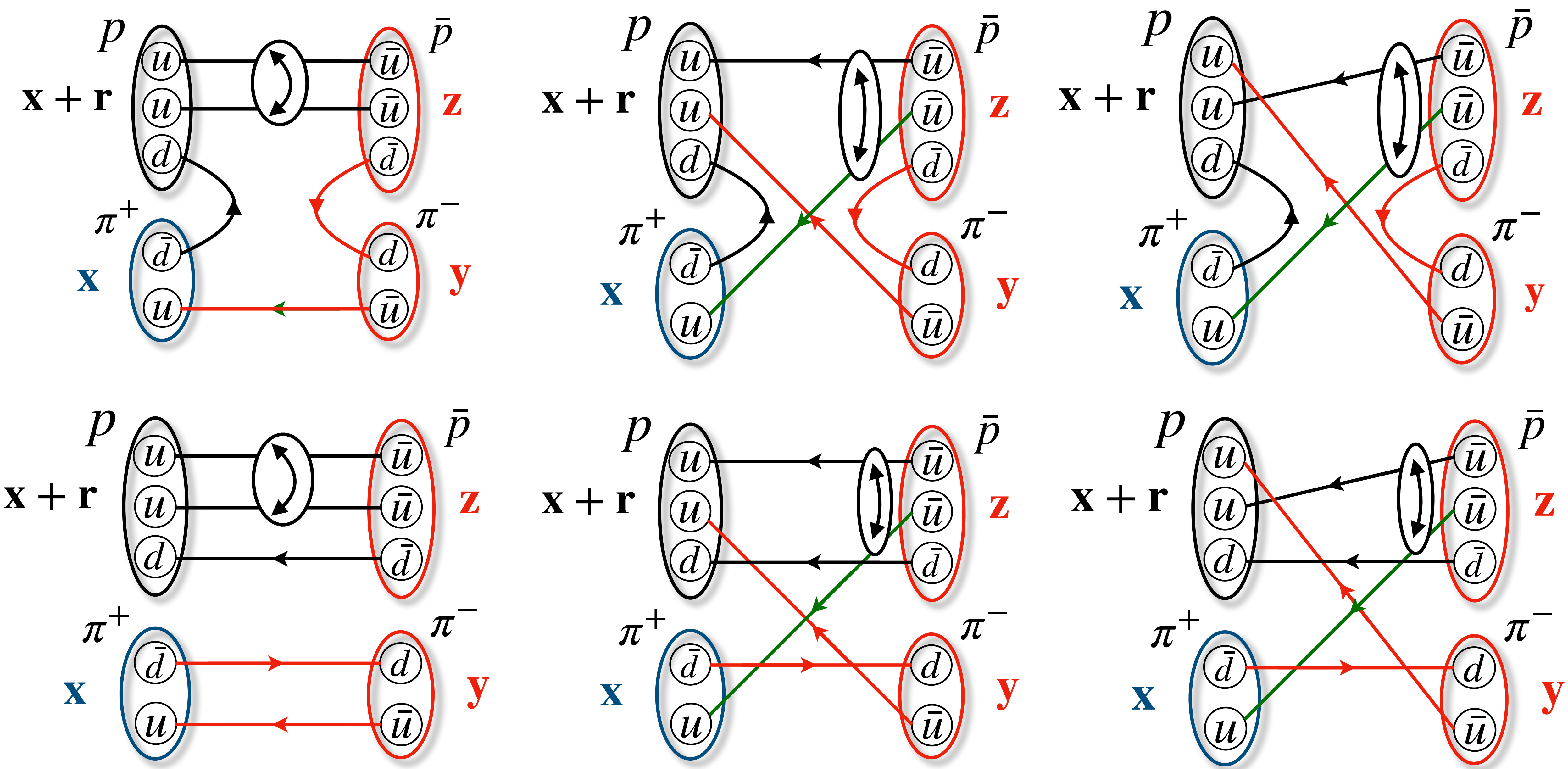
this satisfies

$$\langle C^{(imp)}(U)\rangle = \langle C(U;\mathbf{z}_0)\rangle - \langle C^{(appx)}(U;\mathbf{z}_0)\rangle + \underline{\frac{1}{N_s}\sum_{i=1}^{N_s} \langle C^{(appx)}(U;\mathbf{z}_i)\rangle}$$

$$= \langle C^{(appx)}(U;\mathbf{z}_0)\rangle$$

$$= \langle C(U;\mathbf{z}_0)\rangle$$

Quark contractions in I=3/2 Nπ with Nπ sources



point-to-all + stochastic+ one-end trick