A Better Conditioned Domain Wall Operator

Punch Line:

Precondition $D_{DW} P$ from the right:

-► $D_{DW} P A$

- A is a diagonal preconditioning Matrix
- P contains chiral projectors

Contents

Define...

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{pmatrix}$$

$$D_+ = b\,D_w + 1 \qquad D_- = c\,D_w - 1$$

$$P_+ = (1+\gamma_5)/2 \qquad P_- = (1-\gamma_5)/2$$

$$D_\alpha = D\,P\,A\,P^{-1} =$$

$$\begin{pmatrix} D_+\left(P_- + \alpha P_+\right) & \alpha D_- P_- & 0 & -m D_- P_+ \\ \alpha D_- P_+ & \alpha D_+ & \alpha D_- P_- & 0 \\ 0 & \alpha D_- P_+ & \alpha D_+ & \alpha D_- P_- \\ -m D_- P_- & 0 & \alpha D_- P_+ & D_+\left(P_+ + \alpha P_-\right) \end{pmatrix}$$

Why $D_{DW}PA$ ...

... rather than

$D_{DW}A$

?

From 5D to 4D

Get 4D Propagator in 5D:

$$D^5_{OV} = \begin{pmatrix} D^4_{OV} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From 5D to 4D

$$L\,D_{DW}(m)\,P\,R(m) \;=\; L\,D_{DW}(1)\,P\,R(1)\;D_{OV}^{5}(m) \qquad (1)$$

$$D_{OV}^{5}(m) \;=\; R(1)^{-1}\,P^{-1}\,D_{DW}^{-1}(1)\,D_{DW}(m)\,P\,R(m) \qquad (2)$$

$$R(1)\,D_{OV}^{5}(m)\,R(m) \;=\; P^{-1}\,D_{DW}^{-1}(1)\,D_{DW}(m)\,P \qquad (3)$$

$$R(1)\,D_{OV}^{5}(m)\,R^{-1}(m) = \begin{pmatrix} D_{OV}^{4}(m) & 0 & 0 & 0 \\ B_1(m) & 1 & 0 & 0 \\ B_2(m) & 0 & 1 & 0 \\ B_3(m) & 0 & 0 & 1 \end{pmatrix} \qquad (4)$$

From 5D to 4D with α

$$R(1)\, D_{OV}^{5}(m)\, R^{-1}(m) = \begin{pmatrix} D_{OV}^{4}(m) & 0 & 0 & 0 \\ \alpha^{-1} B_1(m) & 1 & 0 & 0 \\ \alpha^{-1} B_2(m) & 0 & 1 & 0 \\ \alpha^{-1} B_3(m) & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$$P^{-1} D_{DW}^{-1}(1)\, D_{DW}(m)\, P \;\; x_i = s_i \quad (2)$$

$$D_{DW}(m)\, P \;\; x_i = D_{DW}(1)\, P \;\; s_i, \qquad i = 1, \ldots, L_s \quad (3)$$

Propagator x_1 *independent of* α

$D_\alpha P = D P A =$

$$\begin{pmatrix} Q_- c_- & \alpha Q_+ & 0 & 0 \\ 0 & \alpha Q_- & \alpha Q_+ & 0 \\ 0 & 0 & \alpha Q_- & \alpha Q_+ \\ Q_+ c_+ & 0 & 0 & \alpha Q_- \end{pmatrix}$$

$$c_+ = P_+ - m P_- \qquad c_- = P_- - m P_+$$

$$Q_+ = \gamma_5 D_w (b P_+ + c P_-) + 1 \qquad Q_- = \gamma_5 D_w (b P_- + c P_+) - 1$$

Red Black Preconditioning

$$3\,Options:$$

1. $I_{rr} - D_{rb} I_{bb}^{-1} D_{br}$

2. $1_{rr} - I_{rr}^{-1} D_{rb} I_{bb}^{-1} D_{br}$

3. $1_{rr} - D_{rb} I_{bb}^{-1} D_{br} I_{rr}^{-1}$

Red Black Preconditioning with α

$$With\ B = P^{-1} A P :$$

1. $I_{rr} B_{rr} - D_{rb} I_{bb}^{-1} D_{br} B_{rr}$ *improves with α*

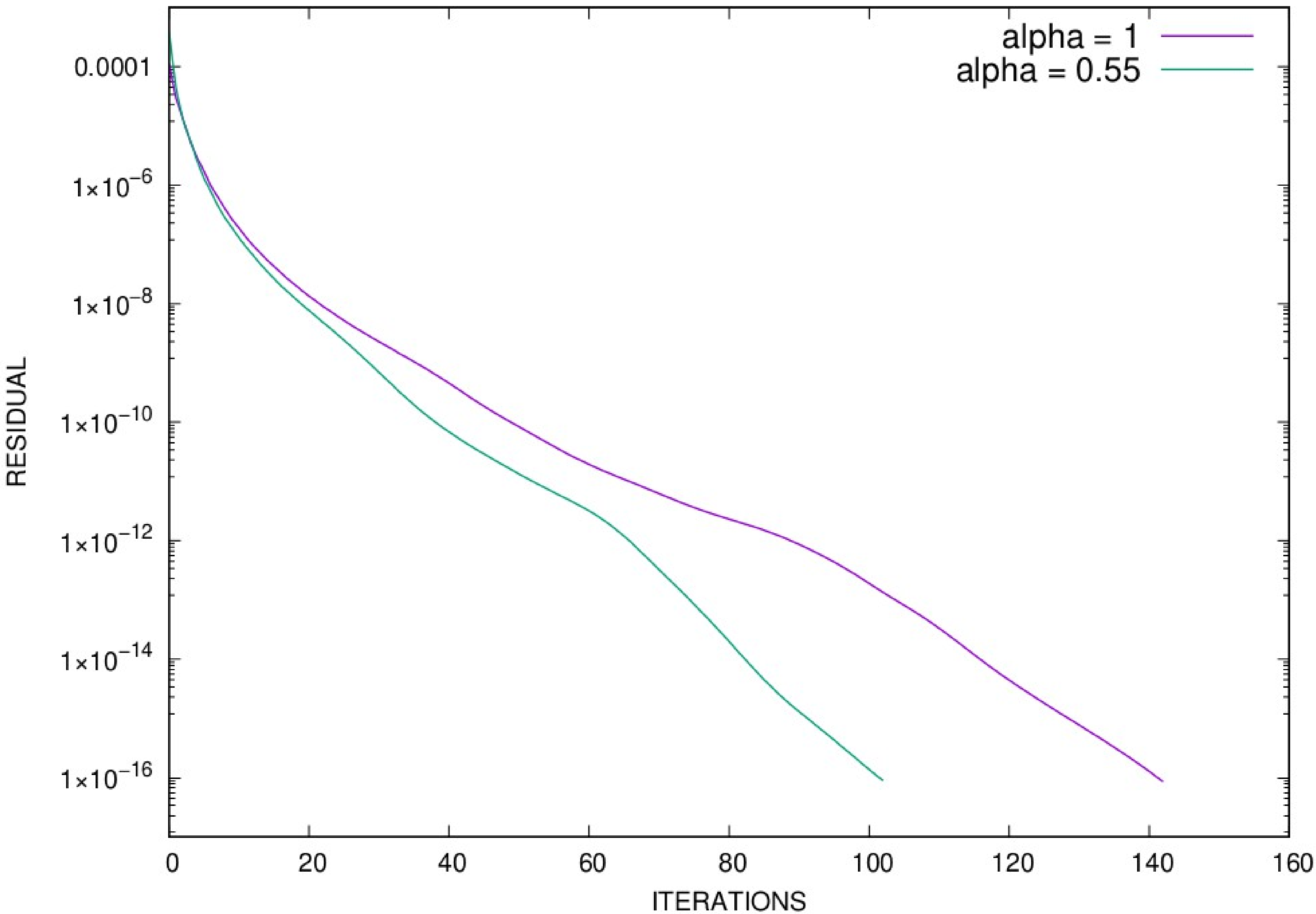
2. $1_{rr} - B_{rr}^{-1} I_{rr}^{-1} D_{rb} I_{bb}^{-1} D_{br} B_{rr}$ *improves with α*

3. $1_{rr} - D_{rb} I_{bb}^{-1} D_{br} I_{rr}^{-1}$ *cancels α*

Red Black Preconditioning

$$\begin{pmatrix} 1 & 0 \\ D_{br} I_{rr}^{-1} & 1 \end{pmatrix} \begin{pmatrix} I_{rr} & 0 \\ 0 & I_{bb} - D_{br} I_{rr}^{-1} D_{rb} \end{pmatrix} \begin{pmatrix} 1 & I_{rr}^{-1} D_{rb} \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ D_{br} I_{rr}^{-1} & 1 \end{pmatrix} \begin{pmatrix} I_{rr} & 0 \\ 0 & I_{bb} \end{pmatrix} \begin{pmatrix} I_{rr}^{-1} & 0 \\ 0 & I_{bb}^{-1} \end{pmatrix} \begin{pmatrix} I_{rr} & 0 \\ 0 & I_{bb} - D_{br} I_{rr}^{-1} D_{rb} \end{pmatrix} \begin{pmatrix} 1 & I_{rr}^{-1} D_{rb} \\ 0 & 1 \end{pmatrix}$$



alpha = 1
alpha = 0.55
0.0001
1×10−6
1×10−8
1×10−10
1×10−12
1×10−14
1×10−16
RESIDUAL
0
20
40
60
80
100
120
140
160
ITERATIONS

Results

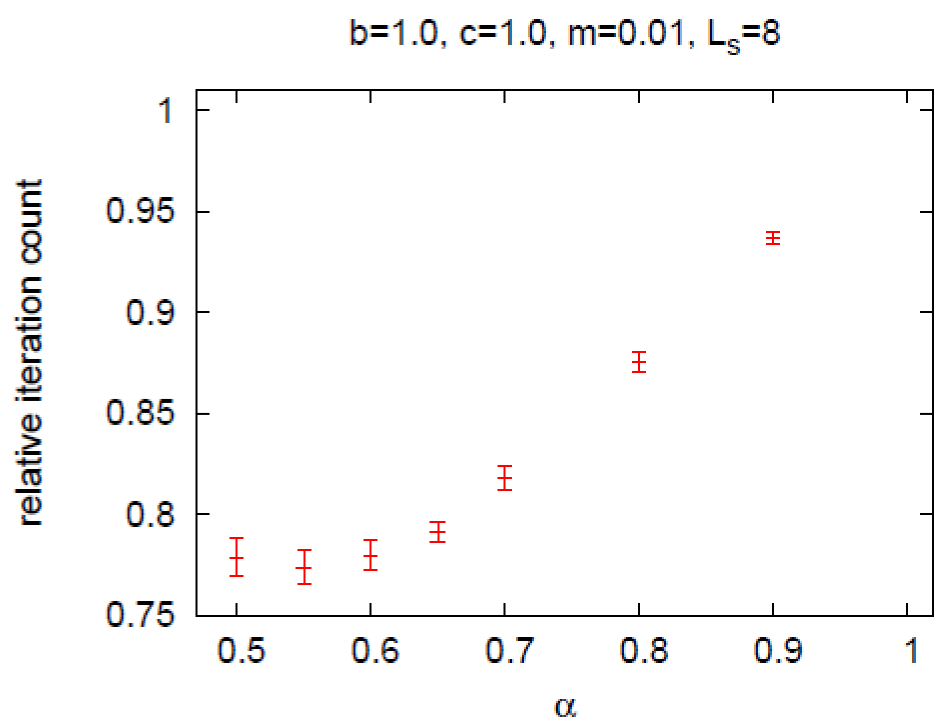


Figure 6: Relative iteration count $n_i(\alpha)/n_i(\alpha = 1)$ for 3 gauge fields of size $16^3 \times 32$.