

Nucleon structure from 2+1-flavor dynamical domain-wall lattice QCD at physical mass

Shigemi Ohta^{*†‡} for LHP+RBC+UKQCD

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Domain-wall fermions (DWF) lattice Quantum Chromodynamics (QCD):

- preserves both **chiral and flavor symmetries**,
- started by RIKEN-BNL-Columbia Collaboration 22 years ago, using purpose-built parallel supercomputers.

Joint RBC+UKQCD Collaborations have been generating **2+1-flavor dynamical DWF** ensembles:

- for more than a decade, and at **physical mass** for several years,
- with a range of momentum cuts off, 1-3 GeV, and volumes $m_\pi L \sim 4$.

We have been calculating **pion, kaon, $(g - 2)_\mu$, and nucleon electroweak matrix elements**.

An update.

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RBC/UKQCD $N_f = 2 + 1$ -flavor dynamical DWF ensembles¹ with $a^{-1} = 1.730(4)$ and $2.359(7)$ GeV :

- $L \sim 5.5$ fm, with pion mass of $139.2(4)$ and $139.2(5)$ MeV respectively,
- $m_\pi L \sim 4$, small volume corrections.

Chiral and continuum limit with good flavor and chiral symmetries (and so renormalizations wherever needed):

- meson decay constants: $f_\pi = 130.2(9)$ MeV, $f_K = 155.5(8)$ MeV, $f_K/f_\pi = 1.195(5)$;
- quark mass: $m_s^{\overline{\text{MS}}(3\text{GeV})} = 81.6(1.2)$ MeV, $m_{ud}^{\overline{\text{MS}}(3\text{GeV})} = 3.00(5)$ MeV, $m_s/m_{ud} = 27.34(21)$;
- chiral condensate $\Sigma^{1/3}(\overline{\text{MS}}, 3\text{GeV}) = 0.285(2)_{\text{stat.}}(1)_{\text{pert.}}$ GeV;
- kaon mixing parameter: $B_K^{\overline{\text{RGI}}} = 0.750(15)$, $B_K^{\overline{\text{MS}}(3\text{GeV})} = 0.530(11)$,
- $K_{l3}^2 f_+(0) = 0.9685(34)_{\text{stat.}}(14)_{\text{FV}}$, $|V_{us}| = 0.2233(5)_{\text{exp.}}(9)_{\text{lat}}$;
- $SU(2)$ low-energy constants³ $B^{\overline{\text{MS}}}$, f , $\Sigma^{1/3, \overline{\text{MS}}}$, $f_\pi/f = 1.064(2)(5)$, $l_{1,2,3,4,7}$;
- $SU(3)$ -breaking ratios for D- and B-mesons⁴, $|V_{cd}/V_{cs}|$, $|V_{td}/V_{ts}|$;
- BSM kaon mixing⁵ are also being calculated, testing the SM, or constraining the BSM.

Contribute to **determining SM parameters** from meson calculations and **constraining the BSM**⁶.

¹T. Blum et al., RBC and UKQCD Collaborations, Phys.Rev. D93 (2016) 074505, arXiv:1411.7017 [hep-lat].

²D. Murphy et al., RBC and UKQCD Collaborations, PoS LATTICE2014 (2015) 369

³P.A. Boyle et al., RBC and UKQCD Collaborations, Phys.Rev. D93 (2016) 054502, arXiv:1511.01950 [hep-lat].

⁴P.A. Boyle et al., RBC and UKQCD Collaborations, arXiv:1812.08791 [hep-lat].

⁵P.A. Boyle et al., arXiv:1812.04981 [hep-lat].

⁶S. Aoki et al., Eur.Phys.J. C77 (2017) 112 DOI: 10.1140/epjc/s10052-016-4509-7 e-Print: arXiv:1607.00299 [hep-lat].

In contrast, systematics in the baryon sector is not well understood yet:

- Proton mean squared charge radius, though Lamb shift discrepancy might have been resolved⁷,
- Nucleon axial charge, g_A ,
- Nucleon electroweak form factors, $F_V(q^2)$, $F_T(q^2)$, $F_A(q^2)$, $F_P(q^2)$,
- Nucleon structure functions and parton distribution functions,
- Proton spin puzzle,

despite potentials for new physics:

- dark matter via g_T and g_S ,
- neutron electric dipole moment,
- proton decay,
- $n\bar{n}$ mixing, ...

⁷N. Beznin, T. Valdez, M. Horbatsch, A. Marsman, A.C. Vutha, E.A. Hessels, Science 06 Sep 2019: Vol. 365, Issue 6457, pp. 1007-1012 DOI: 10.1126/science.aau7807.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N} F_T(q^2) \right] u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_5 \gamma_\mu F_A(q^2) + \gamma_5 q_\mu F_P(q^2) \right] u_n e^{iq\cdot x},$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2} F_2, G_M = F_1 + F_2.$$

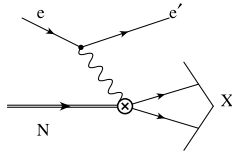
Related to

- mean-squared charge radii, $F_V = F_V(0) - \frac{1}{6}\langle r_E^2 \rangle Q^2 + \dots$
- anomalous magnetic moment, $F_2(0)$,
- $g_A = F_A(0) = 1.2732(23)g_V$ ($g_V = F_V(0) = G_{\text{Fermi}} \cos \theta_{\text{Cabibbo}}$).

$\langle r_E^2 \rangle$ and g_A , in particular, are being revised:

- $\sqrt{\langle r_E^2 \rangle} = 0.875(6)$ fm from electron scattering, 0.8409(4) and 0.833(10) from μ and e Lamb shift;
- $g_A/g_V = 1.264(2)$ pre 2002 (“cold neutron,”) 1.2755(11) post, (“ultra cold neutron.”)

The latter, with Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, determines much of nuclear physics, such as primordial and neutron-star nucleosyntheses.

Deep inelastic scatterings  : $\left| \frac{\mathcal{A}}{4\pi} \right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}, W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

unpolarized: $W^{\{\mu\nu\}}(x, Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu \right) \left(P^\nu - \frac{\nu}{q^2} q^\nu \right) \frac{F_2(x, Q^2)}{\nu},$

polarized: $W^{[\mu\nu]}(x, Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\sigma}{\nu^2} g_2(x, Q^2) \right),$

with $\nu = q \cdot P, S^2 = -M^2, x = Q^2/2\nu.$

Traditionally, moments of the structure functions, $F_i(x, Q^2)$, are accessible on the lattice:

$$2 \int_0^1 dx x^{n-1} F_1(x, Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),$$

$$2 \int_0^1 dx x^n g_2(x, Q^2) = \frac{1}{2n+1} \sum_{q=u,d} [e_{2,n}^q(\mu^2/Q^2, g(\mu)) d_n^q(\mu) - 2e_{1,n}^q(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2),$$

- c_1, c_2, e_1 , and e_2 are the **Wilson coefficients** (perturbative),
- $\langle x^n \rangle_q(\mu), \langle x^n \rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is transversity, $\langle 1 \rangle_{\delta q}(\mu) = \langle P, S | \bar{q} i \gamma_5 \sigma_{\mu\nu} q | P, S \rangle,$
- and scalar density g_S .

Now we have alternative lattice path to **PDFs**^{8 9 10}.

⁸T. Ishikawa, Y. Q. Ma, J. W. Qiu and S. Yoshida, Phys. Rev. D **96**, 094019 (2017).

⁹X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. Lett. **120**, 112001 (2018).

¹⁰T. Izubuchi, X. Ji, L. Jin, I. W. Stewart and Y. Zhao, arXiv:1801.03917 [hep-ph].

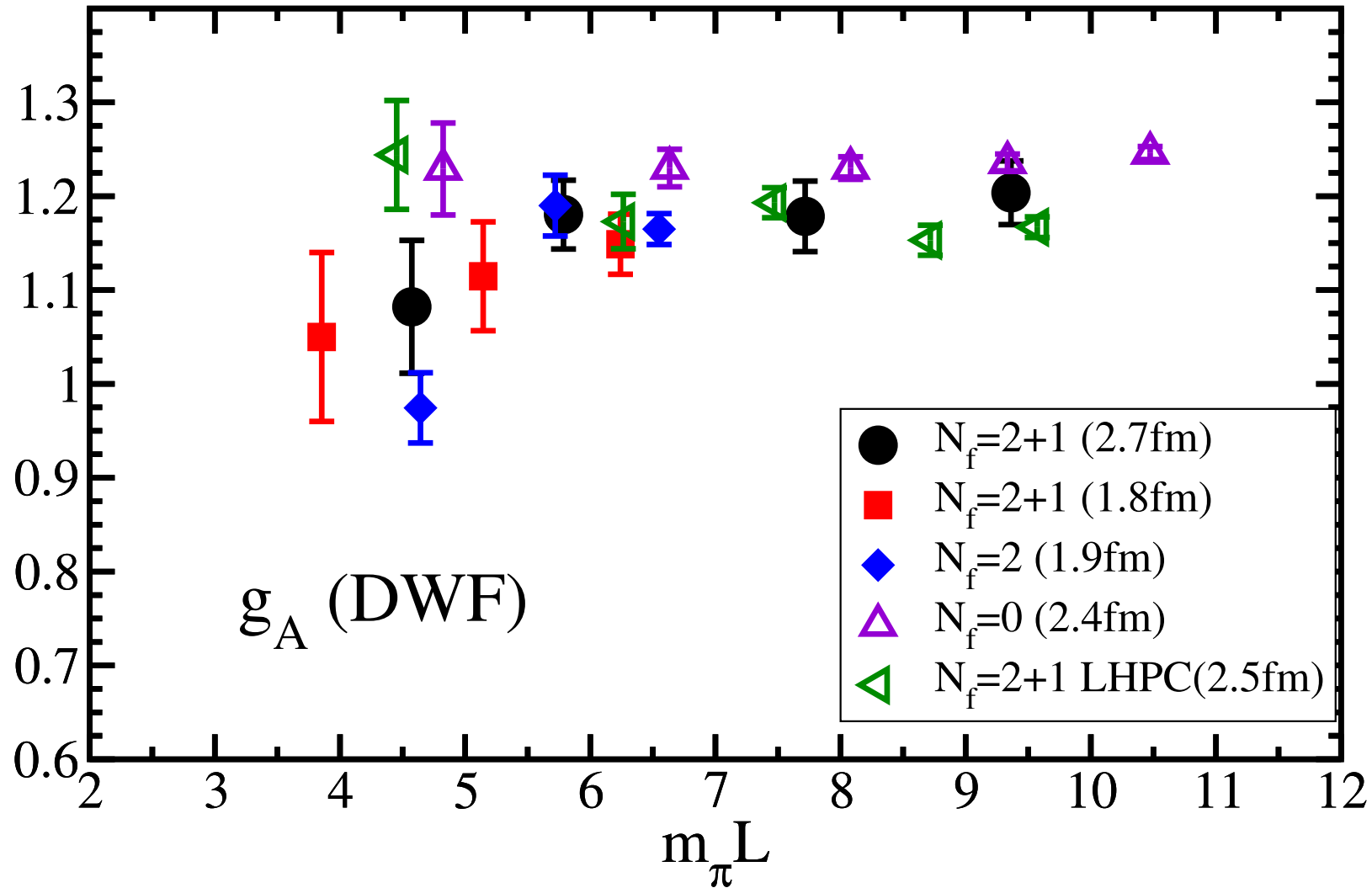
On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c$, ratio of two- and three-point correlators such as $\frac{C_{3\text{pt}}^{\Gamma,O}(t_{\text{sink}}, t)}{C_{2\text{pt}}(t_{\text{sink}})}$ with

$$C_{2\text{pt}}(t_{\text{sink}}) = \sum_{\alpha,\beta} \left(\frac{1 + \gamma_t}{2} \right)_{\alpha\beta} \langle N_\beta(t_{\text{sink}}) \bar{N}_\alpha(0) \rangle,$$

$$C_{3\text{pt}}^{\Gamma,O}(t_{\text{sink}}, t) = \sum_{\alpha,\beta} \Gamma_{\alpha\beta} \langle N_\beta(t_{\text{sink}}) O(t) \bar{N}_\alpha(0) \rangle,$$

give a plateau in t for a lattice bare value $\langle O \rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1 + \gamma_t)/2$ or $(1 + \gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

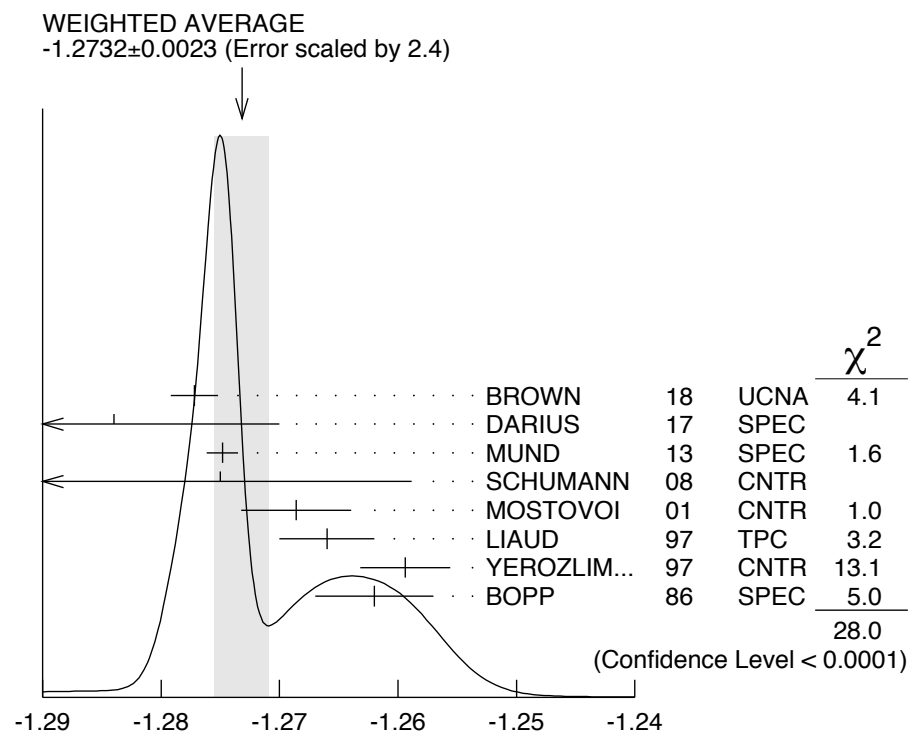
A bit of back ground: In 2007 Takeshi Yamazaki reported **unexpectedly large deficit** in lattice calculation ¹¹:



¹¹T. Yamazaki *et al.* [RBC+UKQCD Collaboration], Phys. Rev. Lett. **100**, 171602 (2008).

Why?

Difficult history¹²



The lifetime has been almost monotonically increasing since the first measurement > 21 minutes^{13 14}: the more recent peak from the ultra-cold neutrons, $1.2764(6)$ ¹⁵ and $1.2772(20)$ ¹⁶, appears more reliable¹⁷.

Lattice calculations appear to follow a parallel path.

¹²M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.

¹³A.H. Snell and L.C. Miller in APS Washington Meeting, Spring 1948.

¹⁴Dirk Dubbers, arXiv:1807.07026 [hep-ph].

¹⁵B. Märkisch et al., arXiv:1812.04666 [nucl-ex].

¹⁶M. A.-P. Brown et al. (UCNA Collaboration) Phys. Rev. C 97, 035505.

¹⁷A. Czarnecki, W.J. Marciano, and A. Sirlin, Phys.Rev.Lett. 120 (2018) 202002.

Why?

Difficult history:

Non-relativistic quark model: $5/3$. Very bad, but some “large- N_c ” conform?

And with absurd “relativistic” correction: $5/4$, really?

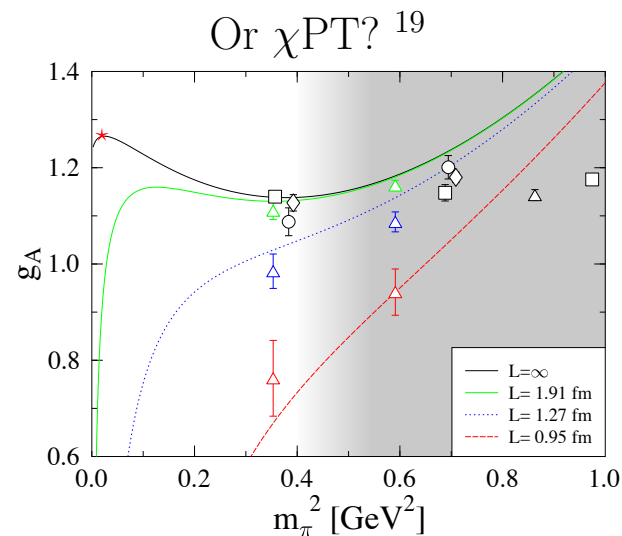
Without pion,

MIT bag model: 1.09, as good(!) as lattice but when experiment was 1.22.¹⁸

With only pion,

Skyrmion: 0.61(!) with a peculiar geometry but when experiment was 1.23.

Accurate reproduction of the ‘pion cloud’ geometry seems essential.



¹⁸Assuming a growth rate of 0.001 per year.

¹⁹A. A. Khan *et al.*, PoS LAT **2005**, 349 (2006).

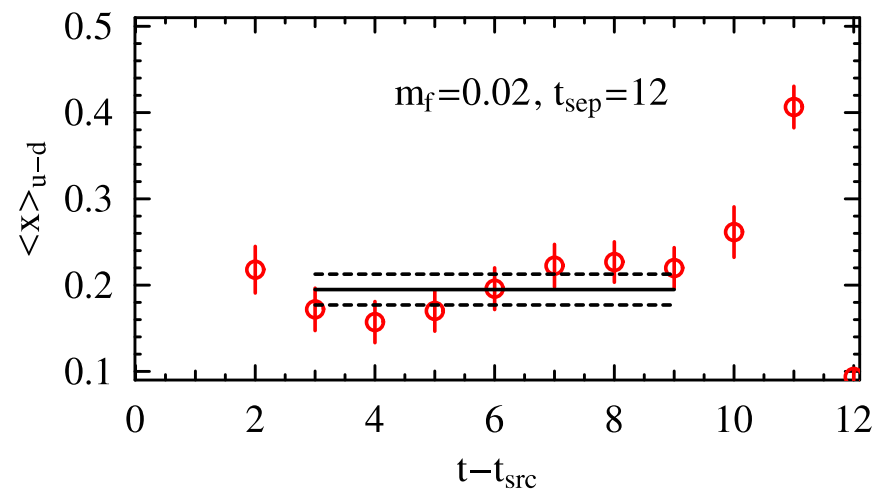
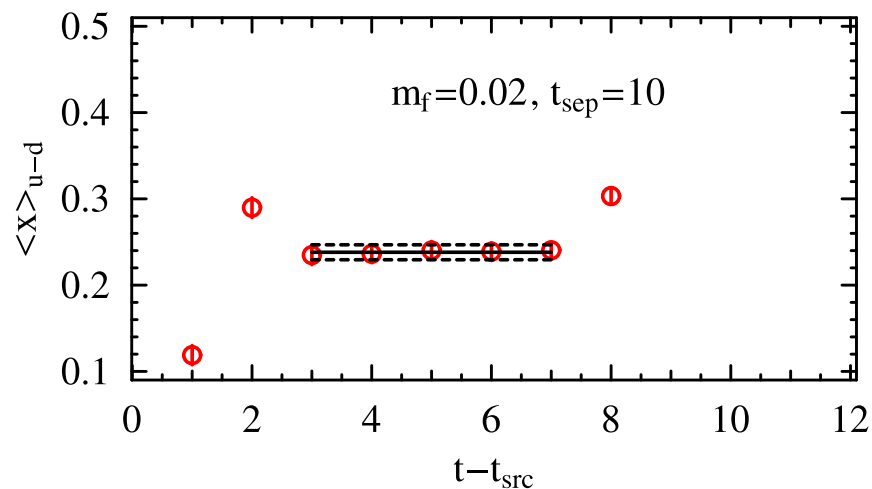
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.

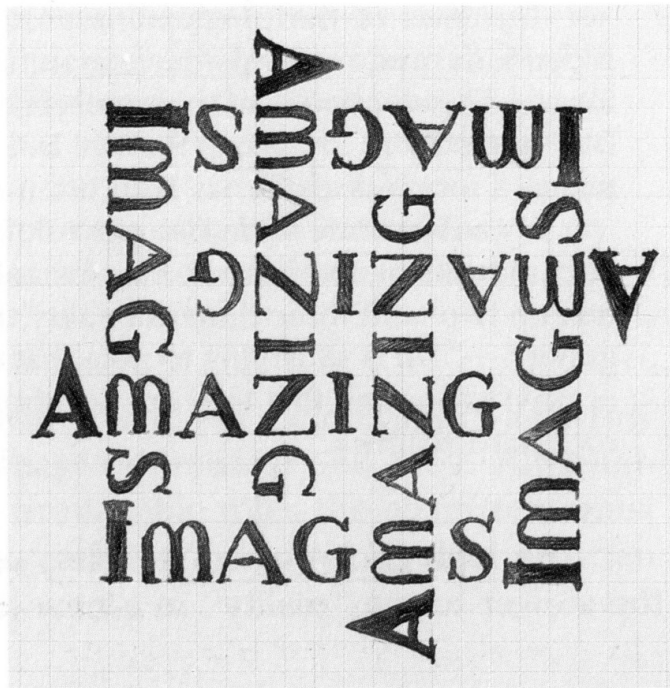


- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

The “AMA” trick²⁰ helped a lot. It provides $\times 10\text{--}20$ acceleration by allowing

- cruder,
- but cheaper,

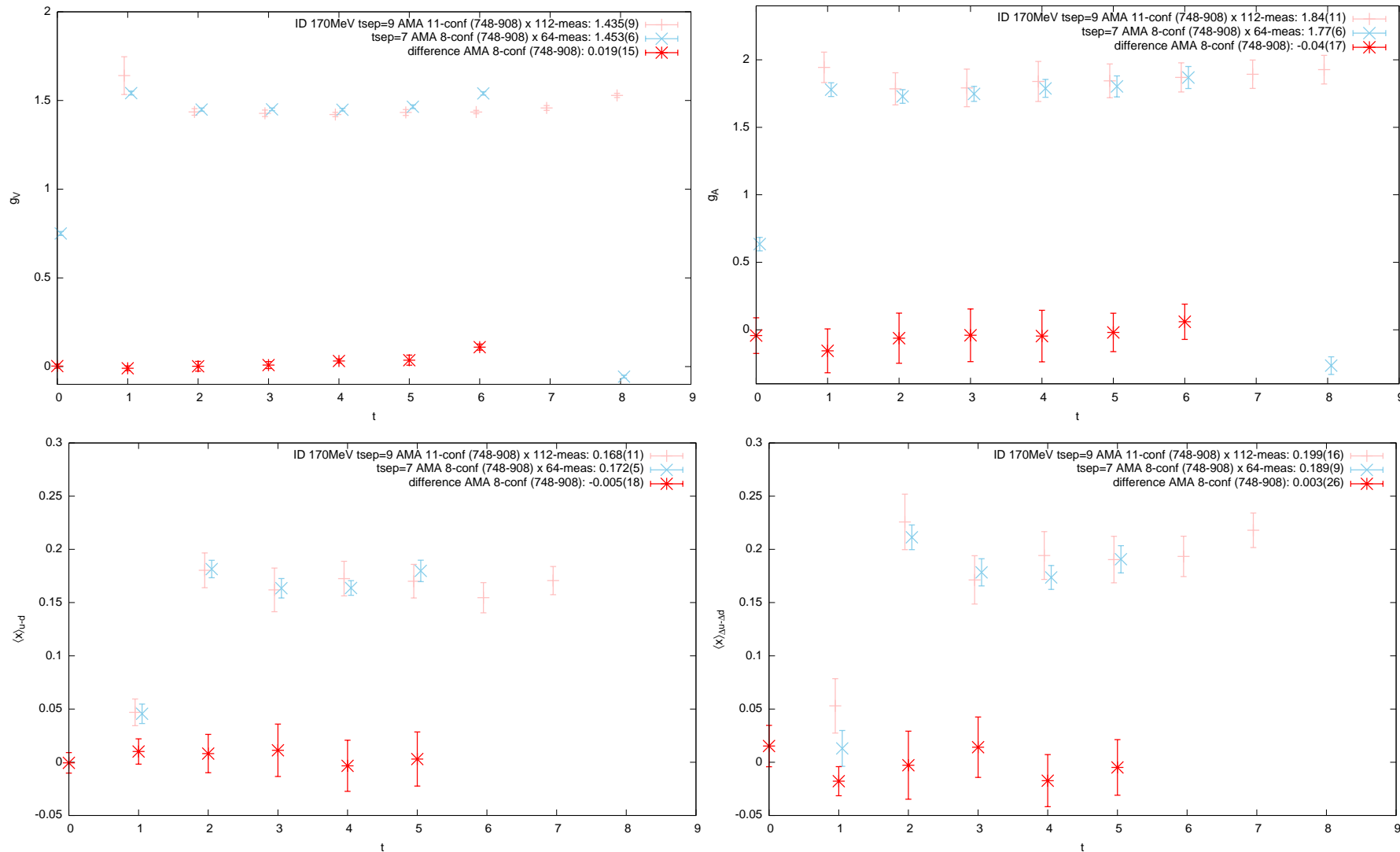
independent statistical sampling at much higher frequency, by taking advantage of point-group symmetries of the lattice to organize many such cruder but independent and equivalent measurements:



$$\langle O \rangle_{\text{AMA}} = \frac{1}{N_{\text{sloppy}}} \sum_s^{N_{\text{sloppy}}} \langle O \rangle_{\text{sloppy}}^s + \frac{1}{N_{\text{accurate}}} \sum_a^{N_{\text{accurate}}} \left(\langle O \rangle_{\text{accurate}}^a - \langle O \rangle_{\text{sloppy}}^a \right)$$

²⁰E. Shintani, R. Arthur, T. Blum, T. Izubuchi, C. Jung and C. Lehner, Phys. Rev. D **91**, 114511 (2015).

With the AMA we established no excited-state contamination is present in any of our 170-MeV calculations:



When compared with the same configurations, the difference is always consistent with 0.

$A_1 \langle 1|O|0 \rangle \sim 0$ for any observable we look at: A_1 is negligible for these small $\langle 1|O|0 \rangle$.

In agreement with many other groups' experiences in controlling this systematics.

More recently from RBC: results from a heavier and coarser "I24" ensembles

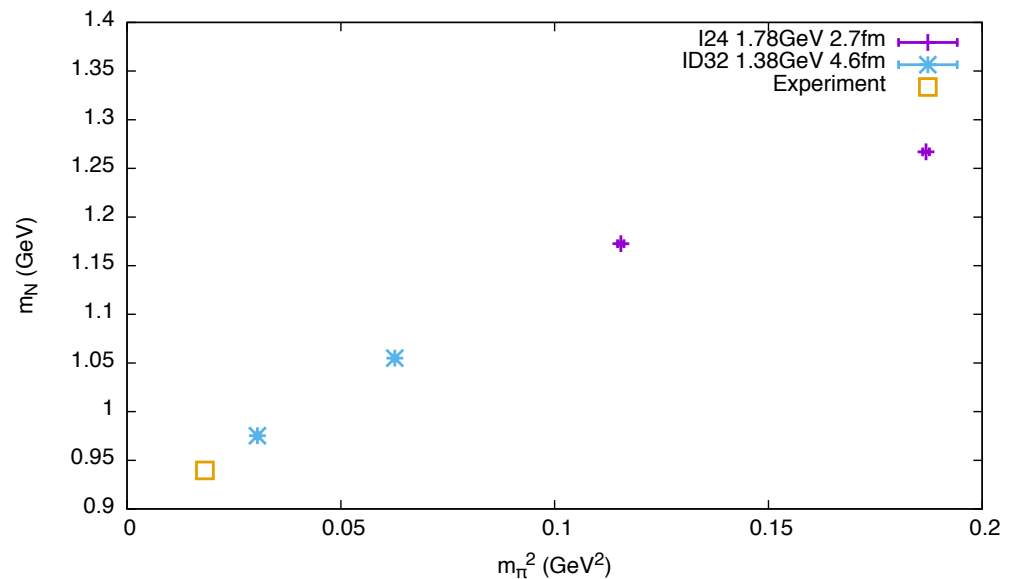
- with Iwasaki \times dislocation-suppressing-determinant-ratio (DSDR) gauge action at $\beta = 1.75$, $a^{-1} = 1.378(7)$ GeV, and pion mass of about 249 and 172 MeV.

We also improved AMA statistics for "I24" ensembles

- with Iwasaki gauge action at $\beta = 2.13$, corresponding the inverse lattice spacing of $a^{-1} = 1.7848(5)$ GeV, and pion mass values of about 432 and 340 MeV.

From these we estimate the nucleon mass:

a^{-1} [GeV]	$m_q a$	$m_N a$	m_N [GeV]
1.378(7)	0.001	0.7077(08)	0.9752(11)
	0.0042	0.76557(16)	1.0550(20)
1.7848(5)	0.005	0.6570(9)	1.1726(16)
	0.01	0.7099(5)	1.2670(09)



Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t} |0\rangle + A_1 e^{-E_1 t} |1\rangle + \dots,$$

resulting in dependence on source-sink separation, $t_{\text{sep}} = t_{\text{sink}} - t_{\text{source}}$,

$$\langle 0 | O | 0 \rangle + A_1 e^{-(E_1 - E_0) t_{\text{sep}}} \langle 1 | O | 0 \rangle + \dots$$

Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1 | Q | 0 \rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

We can optimize the source so that A_1 is small, and we take sufficiently large t_{sep} : Indeed with AMA we established there is no excited-state contamination present in any of our 170-MeV calculations.

In summary, nucleon in RBC+UKQCD ID32 ensembles:

Nucleon mass extrapolates linearly in pion mass squared, m_π^2 , $m_N = 0.950(5)$ GeV at the physical point.

- This compares well with the average of proton and neutron mass experimental values, 0.938918747(6) GeV.
- The result also constrains non-linear dependence of nucleon mass on pion mass squared.

Isovector vector charge, g_V , renormalizes to unity in the chiral limit:

- This narrowly constrains excited-state contamination in the Gaussian smearing.

The ratio of the isovector axial-vector to vector charge, g_A/g_V , shows a deficit of about ten percent.

- This is in agreement with some other major lattice numerical calculations using different actions but with similar lattice spacings and quark masses.
- The origin of this deficit is still to be understood.

Good signals for isovector tensor and scalar couplings that agree well with our earlier results and by other groups with different actions.

Long story short, by 2017: deficit in nucleon g_A/g_V calculated in lattice QCD [with small volumes and heavy mass](#).

Yet a validation of lattice QCD: As of Lattice 2017, with similar quark mass and lattice cuts off,

- Calculations with overlap-fermion valence quarks on RBC+UKQCD DWF ensembles: ~ 1.2 ²¹,
- Wilson-fermion unitary calculations now agree too once $O(a)$ systematics is removed:
 - PACS, $1.16(8)$ ²²,
 - QCDSF ~ 1.1 ²³,
- and even a Wilson valence on HISQ, PNDME²⁴, ~ 1.2 ,
- except the then latest DWF valence²⁵ on HISQ staggered ensembles after an extrapolation.

g_A from different actions “blindedly” agree with deficits once $O(a)$ systematics is removed,

²¹J. Liang, Y. B. Yang, K. F. Liu, A. Alexandru, T. Draper and R. S. Sufian, arXiv:1612.04388 [hep-lat].

²²A parallel talk by Tsukamoto at Lattice 2017, Granada; K. I. Ishikawa *et al.* [PACS Collaboration], Phys. Rev. D **98**, no. 7, 074510 (2018) doi:10.1103/PhysRevD.98.074510 [arXiv:1807.03974 [hep-lat]].

²³J. Dragos *et al.*, Phys. Rev. D **94**, no. 7, 074505 (2016) doi:10.1103/PhysRevD.94.074505 [arXiv:1606.03195 [hep-lat]].

²⁴T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H. W. Lin and B. Yoon, Phys. Rev. D **94**, 054508 (2016) [arXiv:1606.07049].

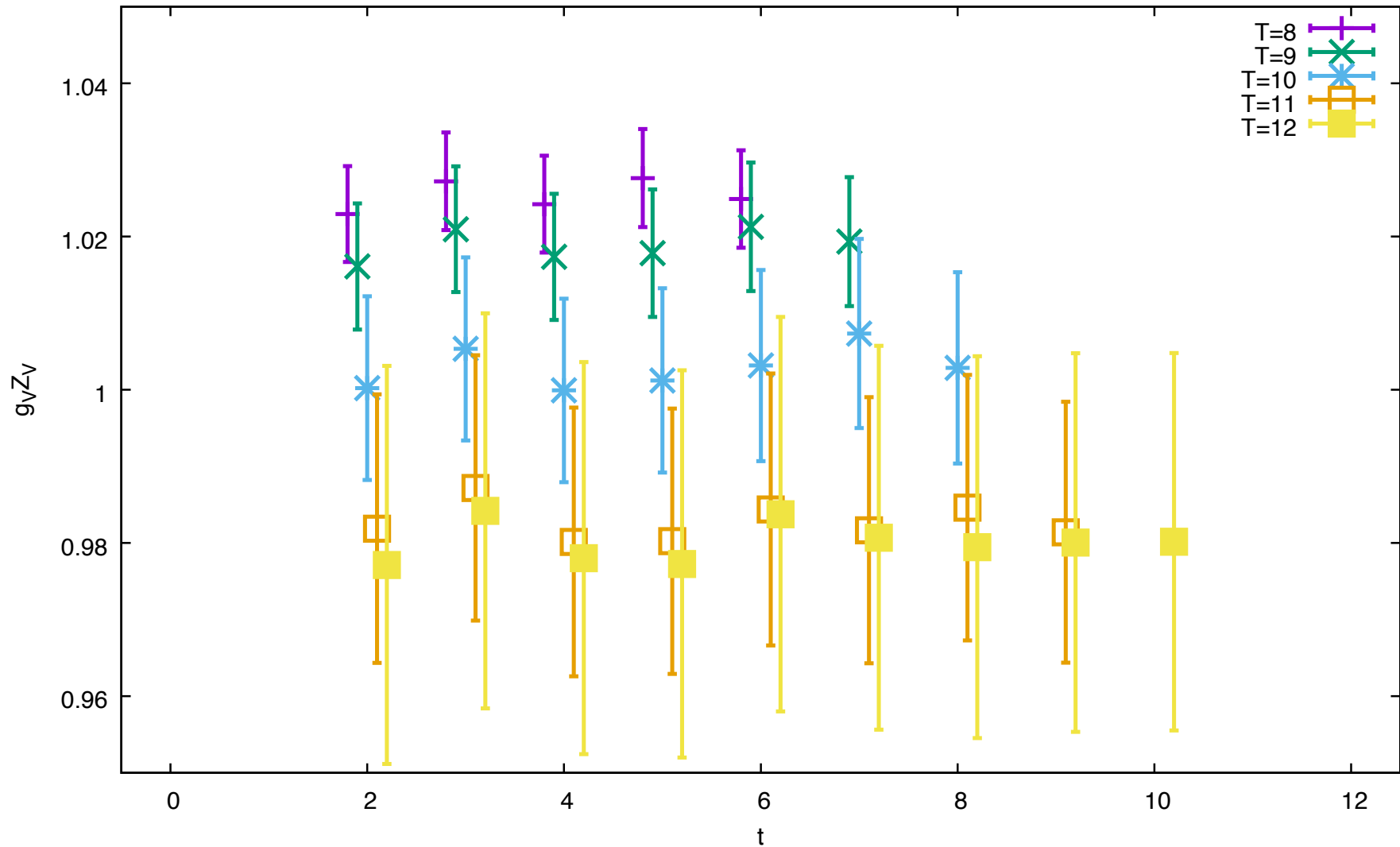
²⁵E. Berkowitz *et al.*, arXiv:1704.01114 [hep-lat]; C. C. Chang *et al.*, Nature **558**, no. 7708, 91 (2018) [arXiv:1805.12130 [hep-lat]].

2018: I reported results for isovector quark bilinears: vector charge g_V , $O = \bar{q}\gamma_t q$, axial charge g_A , $O = \bar{q}\gamma_5\gamma_z q$, transversity, g_T , $O = \bar{q}\gamma_5\gamma_z\gamma_t q$, and scalar coupling, g_S , $O = \bar{q}q$, from RBC+UKQCD “48I” ensemble:

- with Iwasaki gauge action at $\beta = 2.13$, $a^{-1} = 1.730(4)$ GeV, and pion mass of about 139.2(4) MeV,
- 130 configurations at trajectory (620-980)/20 and (990-2160)/10
 - except 1050, 1070, 1150, 1170, 1250, 1270, and 1470,
- each deflated with 2000 low-lying eigenvalues,
- each with $4^4 = 256$ AMA sloppy calculations unbiased by 4 precision ones.

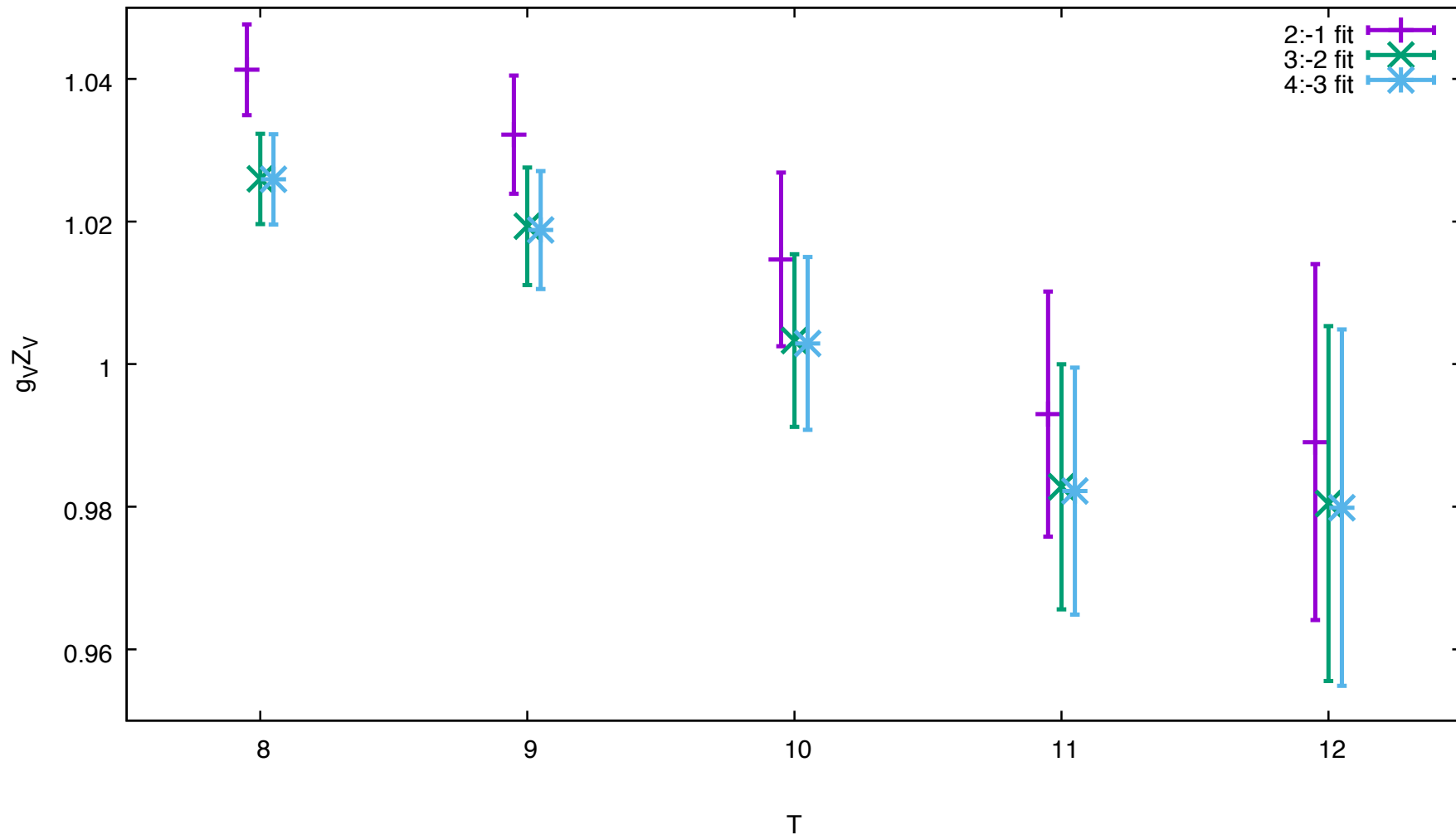
with similar Gaussian smearing as in earlier RBC studies.

Isvector vector charge, g_V , renormalized with meson-sector $Z_V^{\text{meson}} = 0.71076(25)$:



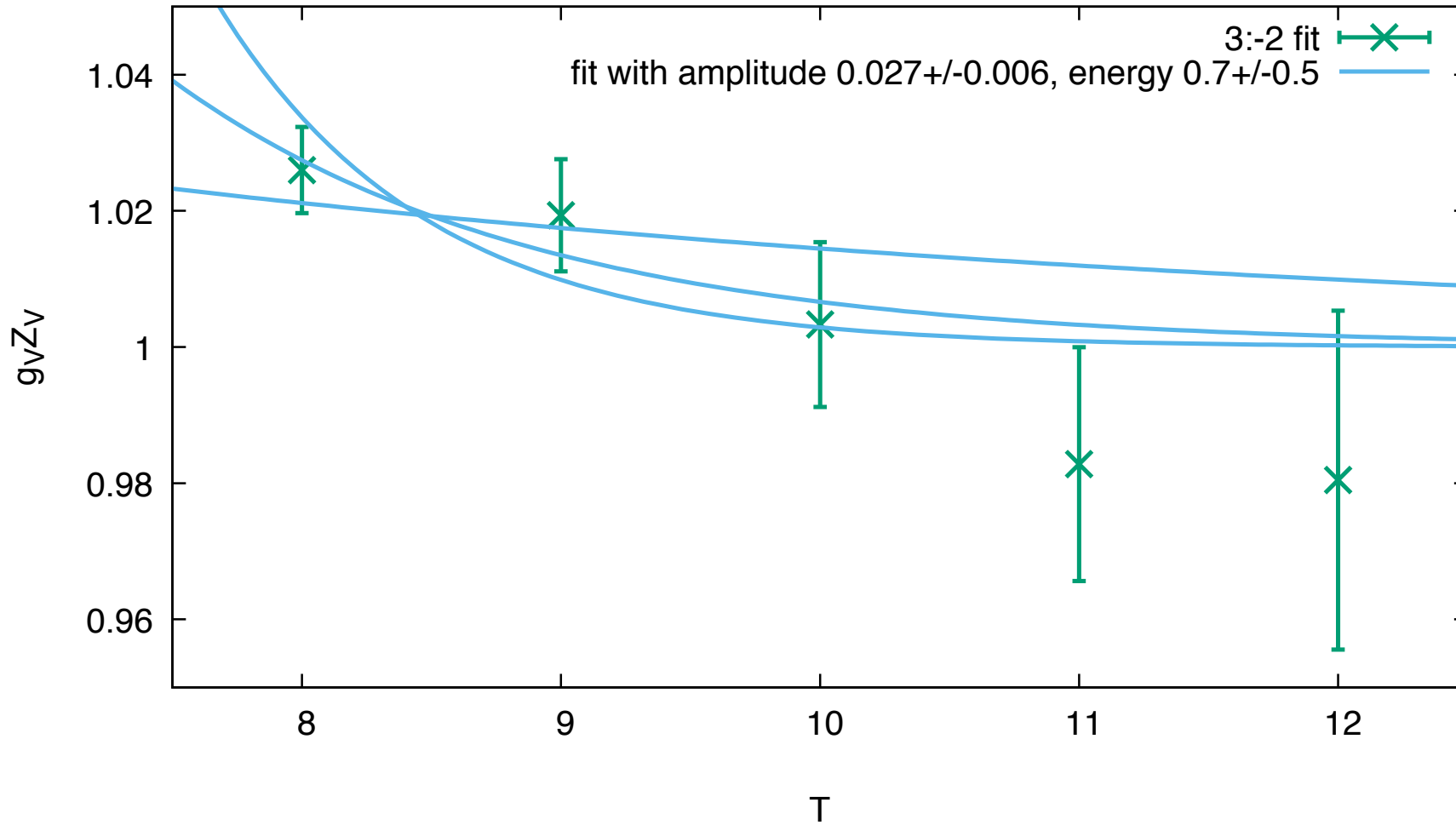
Sub-percent statistical accuracy exposes $O(a^2)$ systematics, at a couple of percent, as expected.

Isvector vector charge, g_V , renormalized with meson-sector $Z_V^{\text{meson}} = 0.71076(25)$:



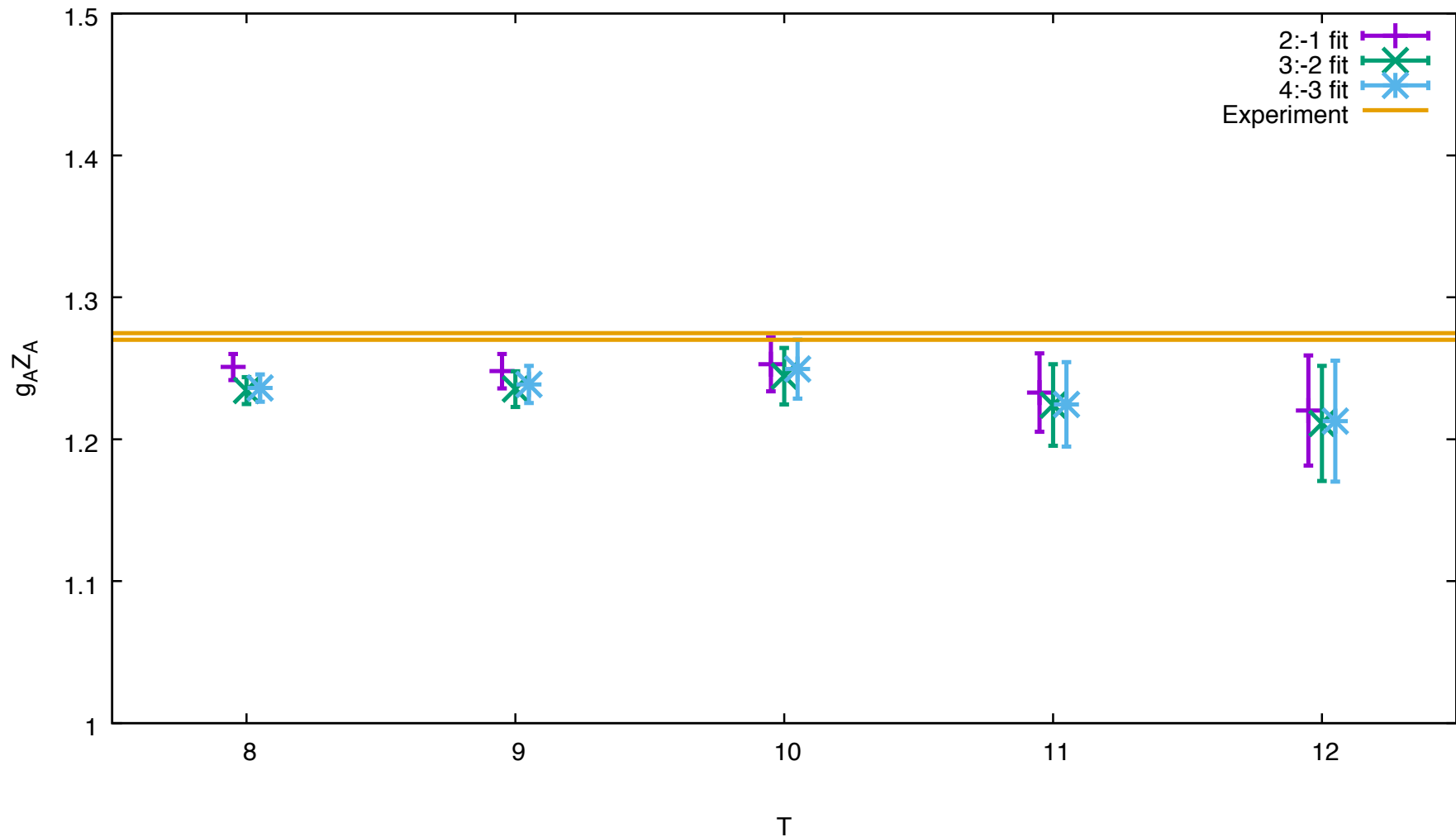
Sub-percent statistical accuracy exposes $O(a^2)$ systematics, at a couple of percent, as expected. We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-11 slope appears steeper than 8-9.

Isovector vector charge, g_V , at $T = 8$ and 9, deviates from unity: possibly $O(a^2)$ mixing with excited states,



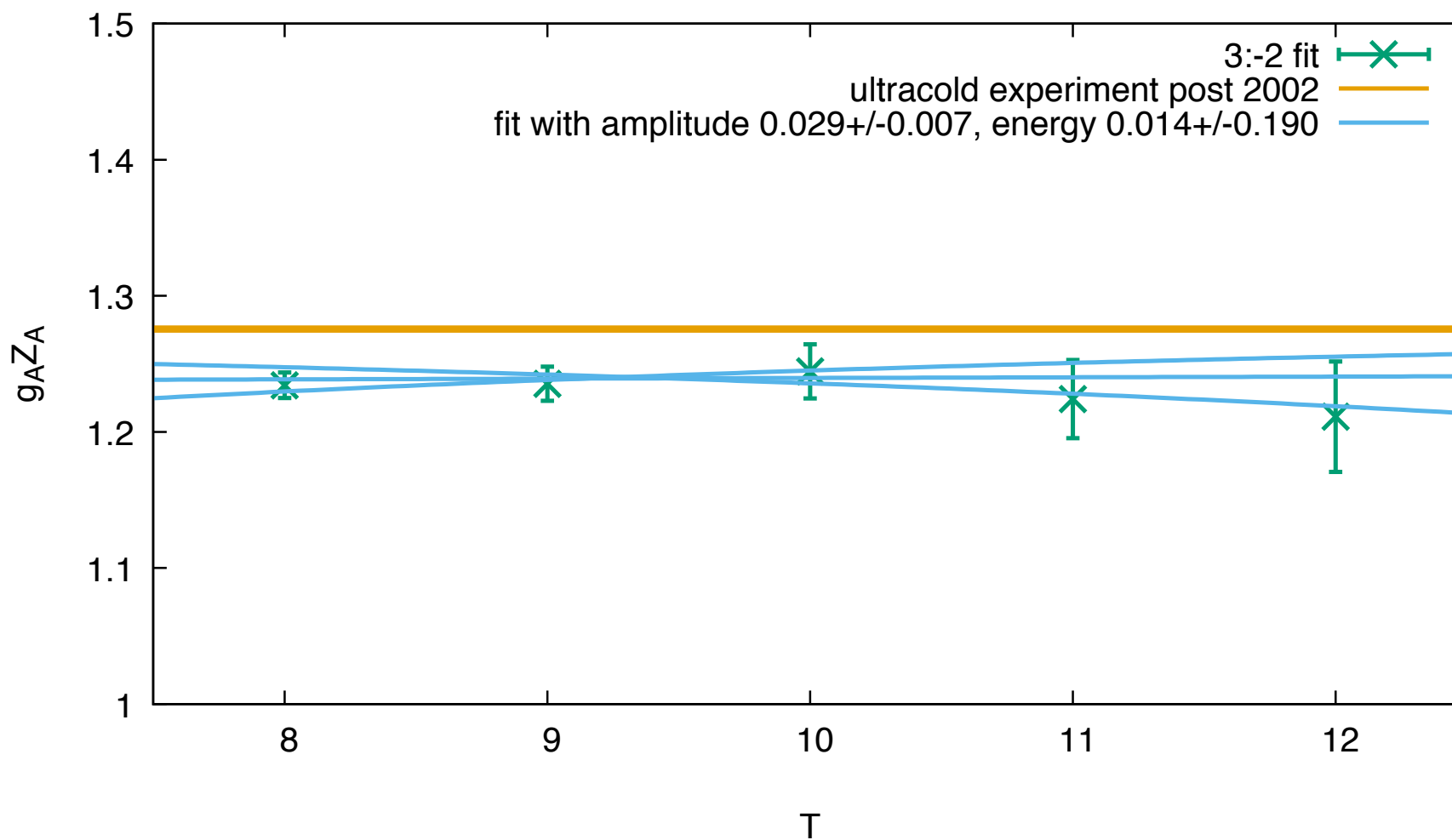
single-excitation fit is not so precise: we need shorter $T = 7$ and 6 calculations for further investigation.

Isovector axialvector charge, g_A , renormalized with meson-sector $Z_A^{\text{meson}} = 0.71191(5)$:



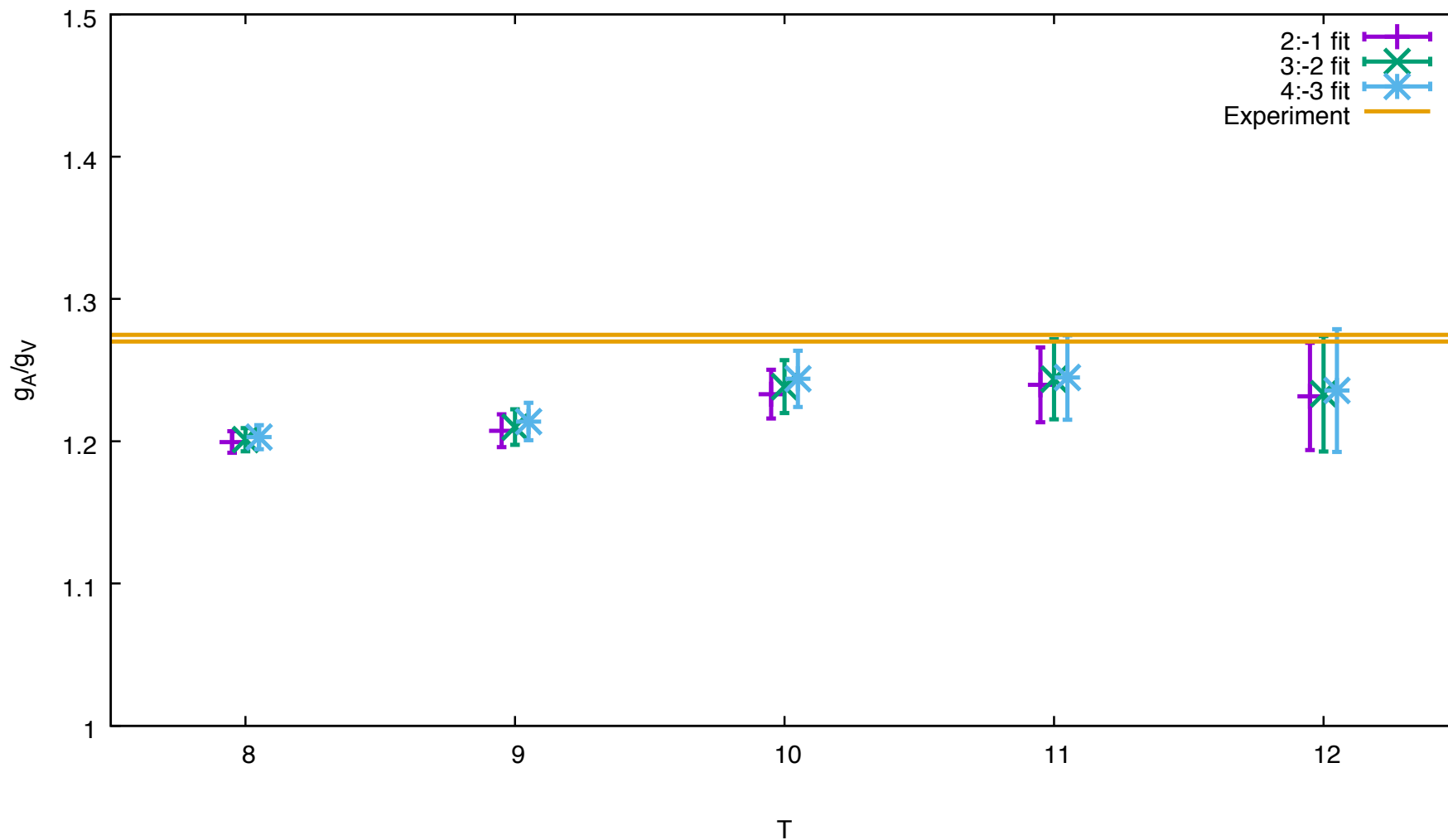
Undershoots the experiment by a few statistical errors without dependence on source-sink separation, T .
 Percent-level statistical accuracy, but not quite in agreement with g_A/g_V in the following either.

Isovector axialvector charge, g_A , renormalized with Z_A^{meson} , undershoots the experiment by a few percent.



Excitation consistent with 0: this deficit appears independent of excited state contamination.

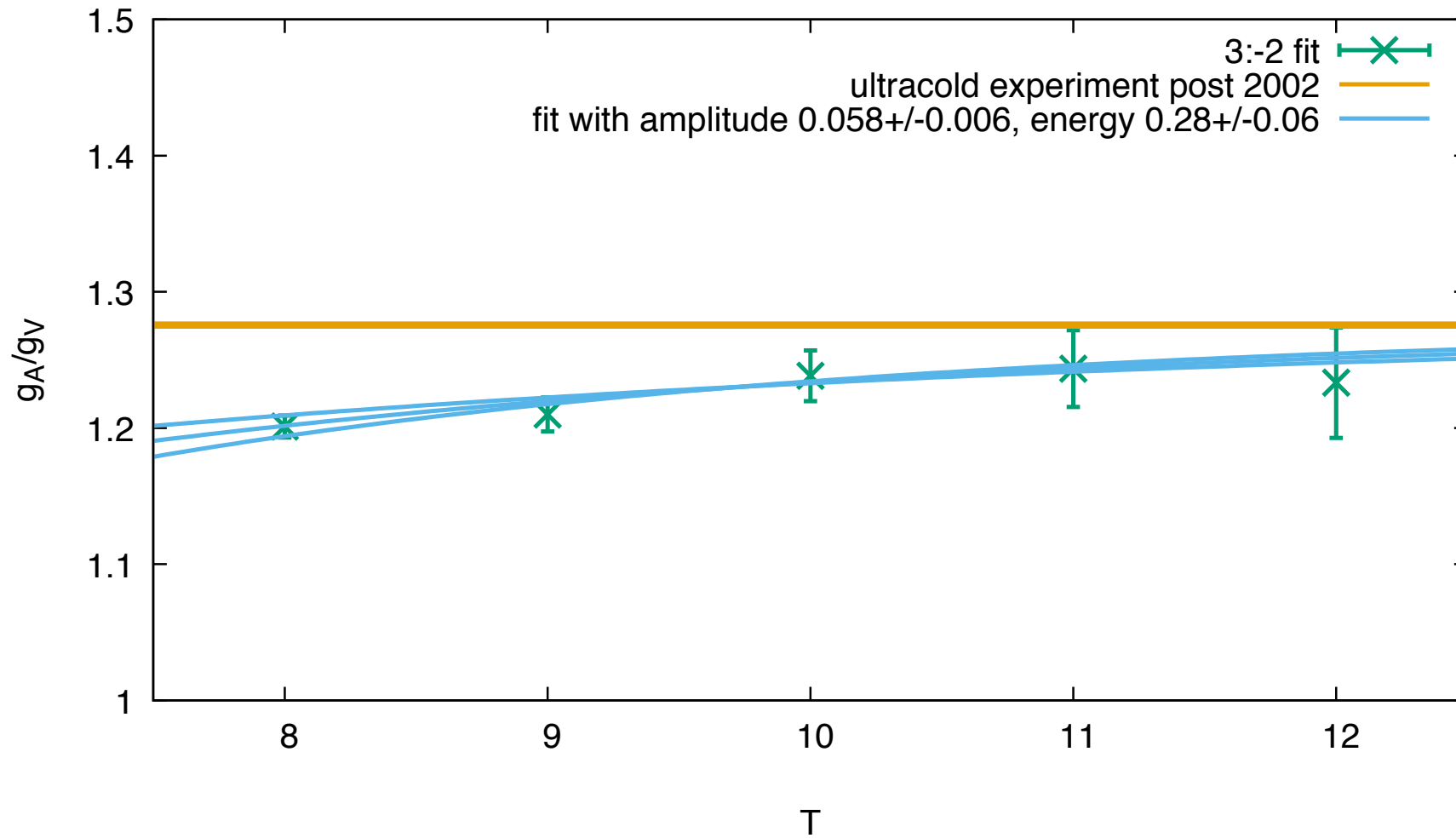
Isovector axialvector to vector charge ratio, g_A/g_V :



Undershoots the experiment by several times the statistical error, so rather different from $g_A Z_A$ in the above.

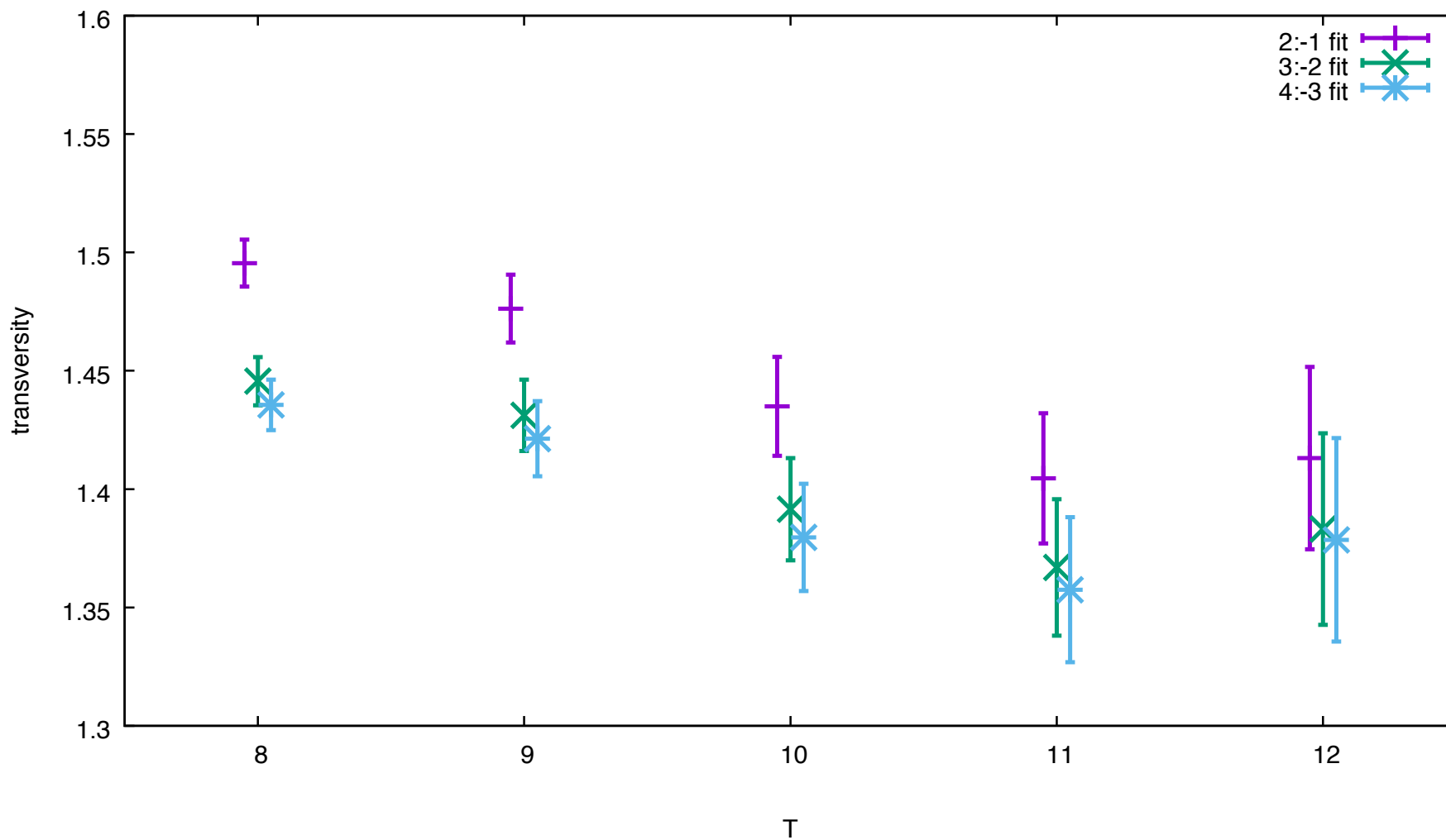
We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-10 slope appears steeper than 8-9.

Isovector axialvector to vector charge ratio, g_A/g_V , undershoots the experiment by several percent.



On top of the g_V T -dependence, a better precision.

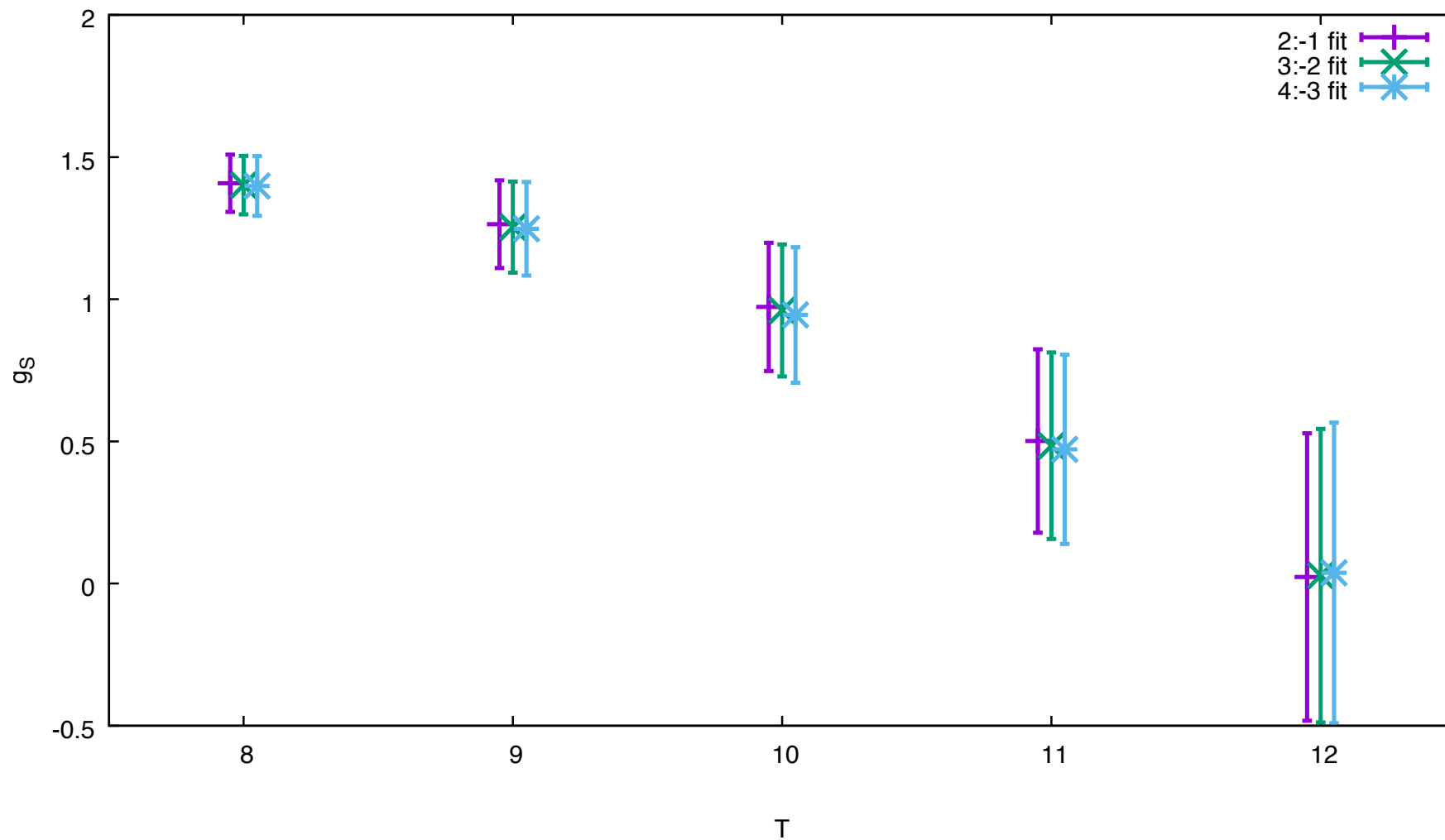
Isvector transversity, bare:



We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-11 slope appears steeper than 8-9.

We are yet to work out the renormalization, Z_T .

Isovector scalar coupling, g_S , bare:



We know the renormalization, $Z_S = 1/Z_m$.

We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-11 slope appears steeper than 8-9.

Validation of lattice QCD:

As of Lattice 2017, with similar quark mass and lattice cuts off, and small volumes,

- Calculations with overlap-fermion valence quarks on RBC+UKQCD DWF ensembles: $\sim 1.2^{26}$,
- Wilson-fermion unitary calculations now agree too once $O(a)$ systematics is removed:
 - PACS, $1.16(8)^{27}$,
 - QCDSF $\sim 1.1^{28}$,
- and even a Wilson valence on HISQ, PNDME²⁹, ~ 1.2 ,
- except the then latest DWF valence³⁰ on HISQ staggered ensembles after an extrapolation.

g_A from different actions “blindedly” agree with deficits once $O(a)$ systematics is removed,

This stayed true, as of Lattice 2018, for raw data with similar quark mass, cuts off, and small volumes,

- only the values are now more refined with better statistical errors,
- and clustering around ~ 1.2 , but **up to $O(a^2)$ systematics**,
 - including DWF-valence/HISQ raw data, which agree with Wilson-valence/HISQ raw,
 - but except the newest, low-statistics Wilson-fermion “PACS10.”

²⁶J. Liang, Y. B. Yang, K. F. Liu, A. Alexandru, T. Draper and R. S. Sufian, arXiv:1612.04388 [hep-lat].

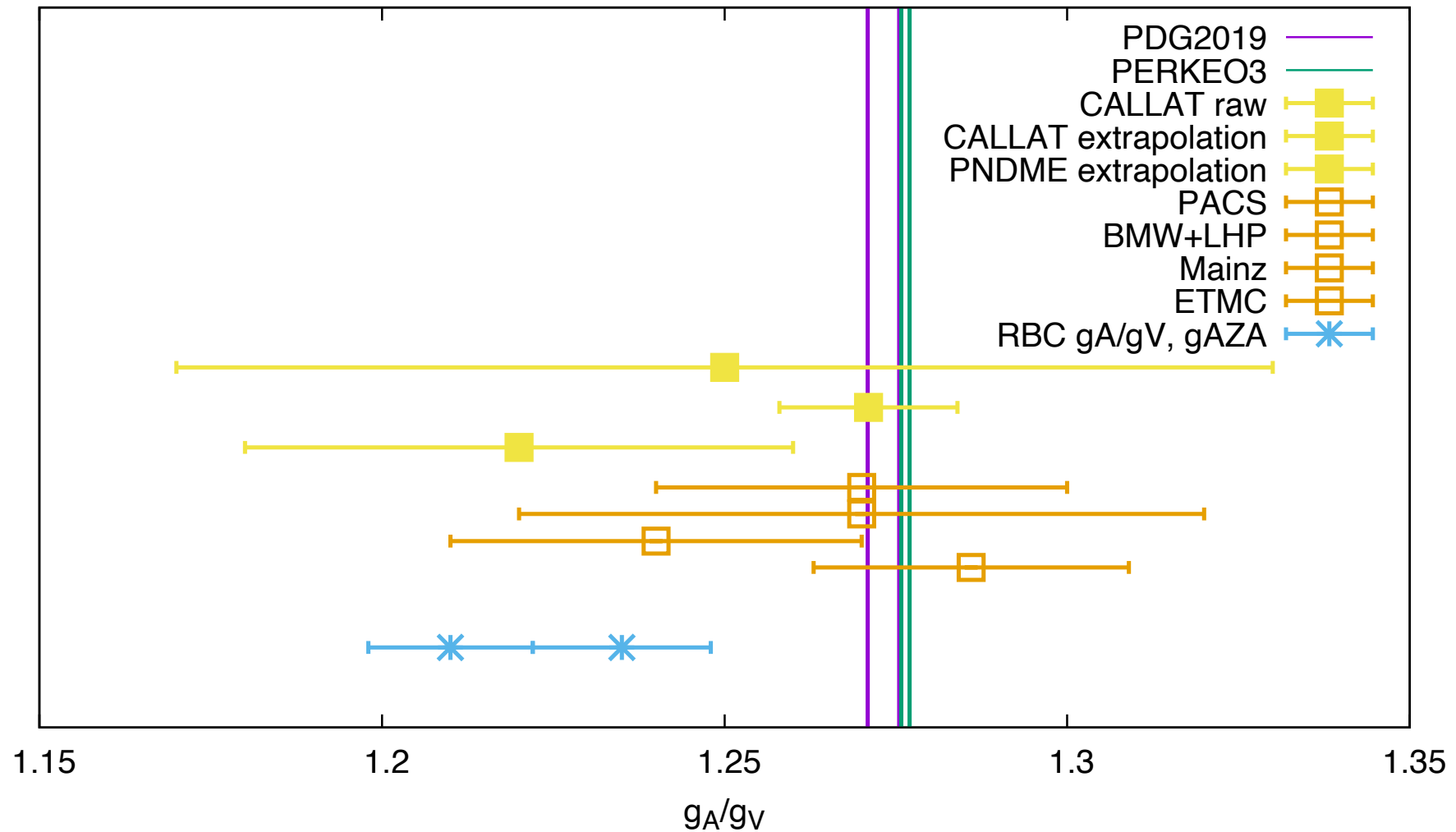
²⁷A parallel talk by Tsukamoto at Lattice 2017, Granada; K. I. Ishikawa *et al.* [PACS Collaboration], Phys. Rev. D **98**, no. 7, 074510 (2018) doi:10.1103/PhysRevD.98.074510 [arXiv:1807.03974 [hep-lat]].

²⁸J. Dragos *et al.*, Phys. Rev. D **94**, no. 7, 074505 (2016) doi:10.1103/PhysRevD.94.074505 [arXiv:1606.03195 [hep-lat]].

²⁹T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H. W. Lin and B. Yoon, Phys. Rev. D **94**, 054508 (2016) [arXiv:1606.07049].

³⁰E. Berkowitz *et al.*, arXiv:1704.01114 [hep-lat]; C. C. Chang *et al.*, Nature **558**, no. 7708, 91 (2018) [arXiv:1805.12130 [hep-lat]].

As of the first half of September, 2019,



Volumes are mostly small, some physical mass, some extrapolations, errors are large except for ours, ...

... 獨酌無相親 舉杯邀明月 ... 醉後各分散 永結無情遊 相期遙雲漢³¹.

³¹from 李白's 月下獨酌; who also sang 黃鶴樓送孟浩然之廣陵“故人西辭黃鶴樓 烟花三月下揚州 孤帆遠影碧空盡 唯見長江天際流”

Nucleon from RBC+UKQCD 2+1f DWF ensemble at physical mass, 48I, as of 2019, with $a^{-1} = 1.730(4)$ GeV, 130 configurations, 2000 eigenvalues, 256/4 AMA samples each, $T = 8-12$:

Nucleon mass: 947(6) MeV.

Vector charge: sub-percent statistical accuracy,

- expected $O(a^2)$ systematics can be fit by single-excitation,
- excitation energy consistent with $m_\pi + 2\pi/La$ or $2m_\pi$ though poorly determined,
- $T = 8$ and 9 alone give excitation energy of $\sim 0.3a^{-1}$.

Axial charge: around-a-percent statistical accuracy,

- $g_A Z_A$ and g_A/g_V both undershoot g_A^{exp} with much smaller statistical errors,
- but do not agree with each other: Systematics is yet to be understood,
- especially isospin breaking, as logarithmic enhancement ($\delta_{u-d} \ln \delta_{u-d}$) is likely, ...

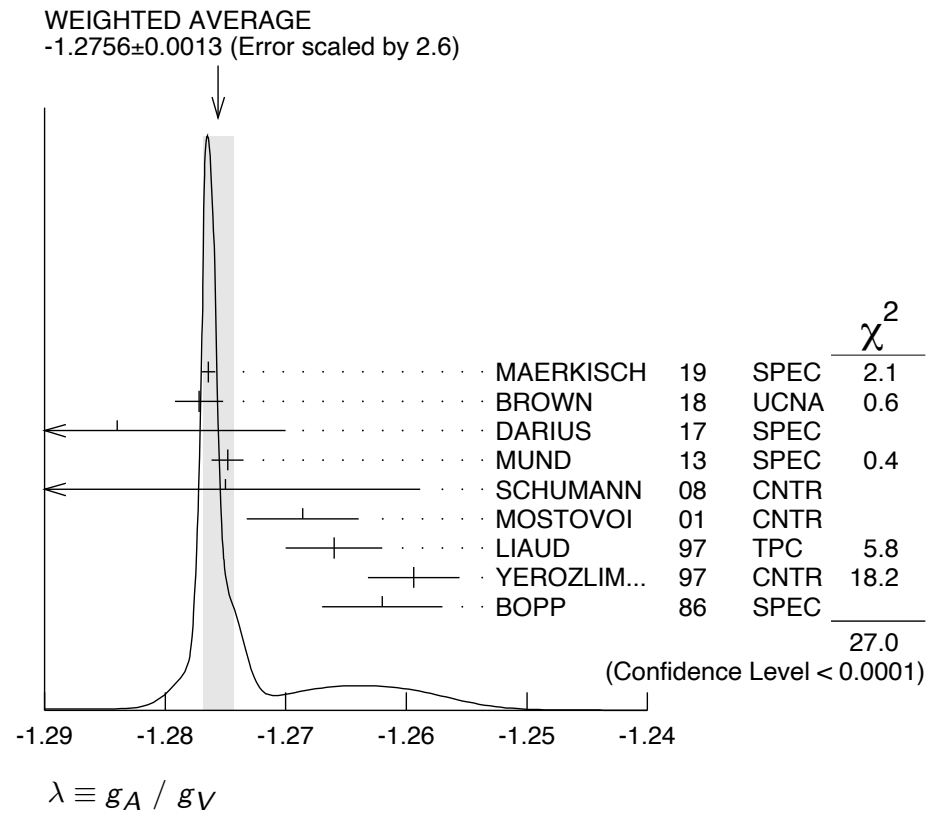
Signals in transversity and scalar couplings with dependence on source-sink separation, T , are seen.

We are losing the signals as early as $T = 10$, or 1.1 fm:

- steeper slopes at later T ,
- so we are yet to understand $O(a^2)$ or excited-state systematics.

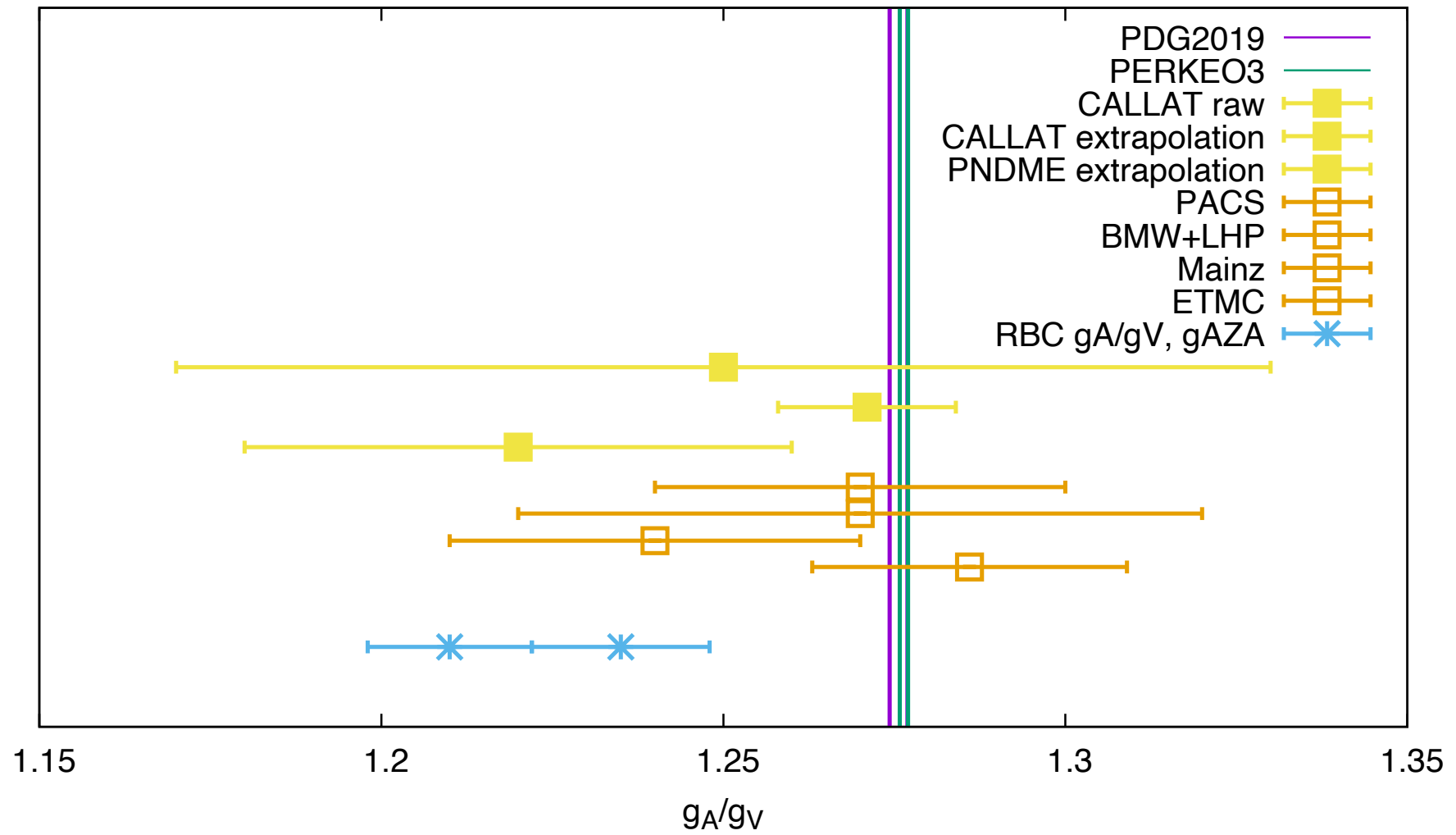
Shorter T such as 7 and 6 would help, as well as another coupling such as in a finer 64I or coarser IDs.

This year: Particle Data Group finally dropped cold beam experiment ³²: $g_A/g_V = 1.2756(13)$.



³²P.A. Zyla et al. (Particle Data Group), to be published in Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

This year: lattice calculations are yet to advance further,



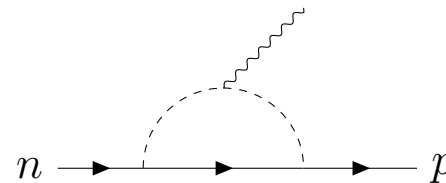
RBC+UKQCD DWF ensembles at physical mass, future plans as of 2020:

Immediate plan: we are finishing up on ID32 form factors and structure functions.

Short term: we are improving the statistics so we know better about excited-state and other systematics.

Mid term: isospin breaking,

- both u-d mass difference, $\delta_{u-d} \log \delta_{u-d}$,



- and EM.

Longer term: finer lattice spacing, $a^{-1} \geq 3$ GeV,

- to unquench charm, and eventually bottom.