Baryons in the Gross-Neveu model in 1+1 dimensions at finite number of flavors

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Introduction

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- Phase diagram of QCD at intermediate densities is largely unknown.
- Several toy models for QCD have inhomogeneous phases (chiral condensate is a function of space) in the $N_\mathrm{f} \to \infty$ limit / mean-field approximation.
- We focus on the Gross-Neveu (GN) Model in 1+1 dimensions at finite number of flavors.
- The main interests are whether the inhomogeneous phase still occurs, the structure of the phase diagram and the role of baryons.
- This talk is about our two recent papers:
  - [J. Lenz, L. Pannullo, M. Wagner, B. Wellegehausen, A. Wipf, Phys. Rev. D 101, 094512 (2020), arXiv:[2004.00295]]
  - [J. Lenz, L. Pannullo, M. Wagner, B. Wellegehausen, A. Wipf, (2020) arXiv:[2007.08382]]

Gross-Neveu Model

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- The Gross-Neveu Model is a toy model with crude similarities to QCD:
  - The fermions interact by a 4-point fermion interaction.
  - A discrete chiral symmetry is realized in the action.
  - This symmetry can be spontaneously broken.
- Euclidean action of the Gross-Neveu model:

$$S_{\mathrm{E}} = \int \mathrm{d}^2 x \left(\bar{\psi}_f \left(\partial\!\!\!/ + \gamma_0 \, \mu \right) \psi_f - \frac{\lambda}{2N_{\mathrm{f}}} \left(\bar{\psi}_f \, \psi_f \right)^2 \right).$$

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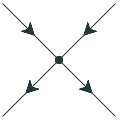

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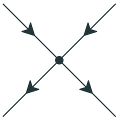


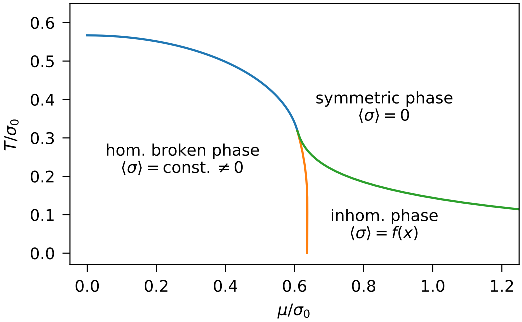
- Hubbard-Stratonovich transformation:

$$Z = \mathcal{N} \int \mathrm{D}\psi_f \, \mathrm{D}\bar{\psi}_f \, \mathrm{D}\sigma \, \exp \left[- \int \mathrm{d}^2 x \left(\bar{\psi}_f \left(\partial\!\!\!/ + \gamma_0 \, \mu + \sigma \right) \psi_f + \frac{N_{\mathrm{f}}}{2\lambda} \, \sigma^2 \right) \right].$$

- $\langle \bar{\psi}(x) \psi(x) \rangle = -\frac{N_{\mathrm{f}}}{\lambda} \langle \sigma(x) \rangle \rightarrow$ from now on refer to σ as chiral condensate

Rich and interesting phase diagram in the limit of $N_\mathrm{f} \to \infty$
(suppresses all fluctuations of σ, i.e. only the minimum of the effective action contributes)

Phase diagram of the Gross-Neveu model:



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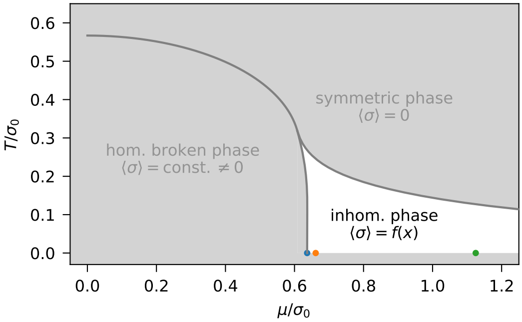
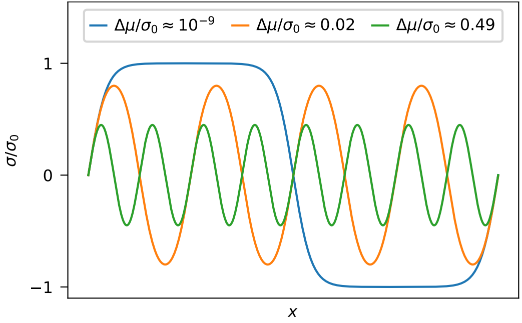


The chiral condensate for $T = 0$ and different μ:



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Baryons in the $N_\mathrm{f} \to \infty$ limit

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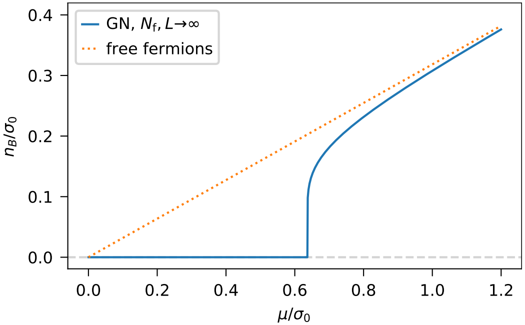
- Baryons align with the condensate in fixed spatial separation.
  $\to$ The inhomogeneous phase is interpreted as a crystal of baryons.
- Baryon number matches the number of cycles of the condensate.

The averaged baryon density at $T = 0$:



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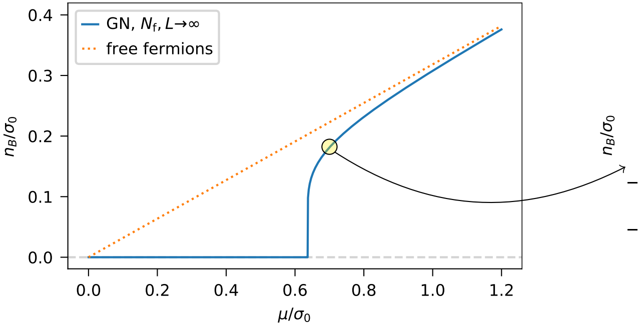
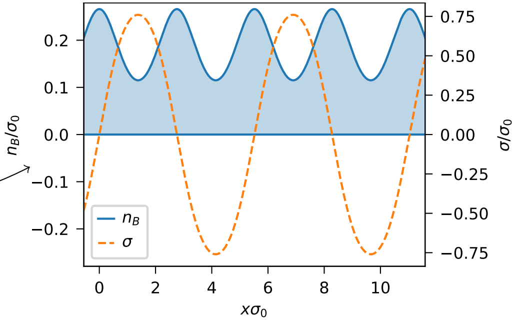


$n_B(x)$ and $\sigma(x)$ for $(\mu/\sigma_0, T/\sigma_0) = (0.7, 0)$:



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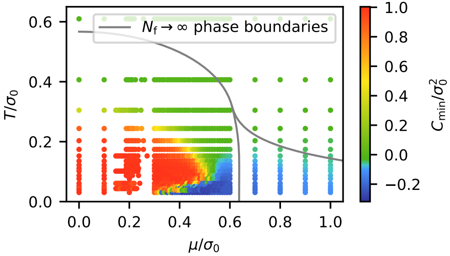

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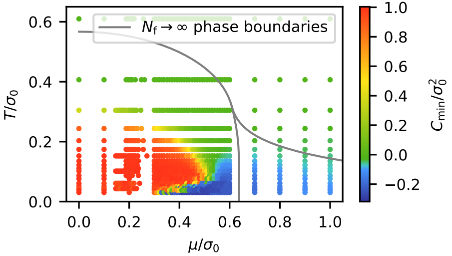

→ Now we turn to the investigation of baryons and their spatial distribution at finite N_f.

Baryons at finite N_f

Correlation observables

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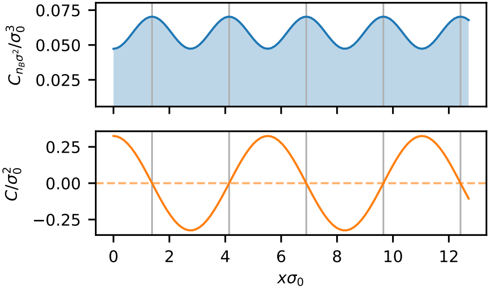
- The HMC algorithm is blind to the breaking of translational symmetry.
  $\rightarrow \langle\sigma(x)\rangle \approx 0$ due to destructive interference
- Use correlation observables that preserve the inhomogeneities instead:

$$C(x) = \left\langle \frac{1}{N_\mathrm{t} N_\mathrm{s}} \sum_{t,y} \sigma(t, y+x)\sigma(t,y) \right\rangle ,$$

$$C_{n_B\sigma^2}(x) = \left\langle \frac{1}{N_\mathrm{t} N_\mathrm{s}} \sum_{t,y} n_B(t, y+x)\, \sigma^2(t,y) \right\rangle .$$

Spatial distribution of the baryon density

C and $C_{n_B\sigma^2}$ for $N_\mathrm{f} \to \infty$ at $(\mu/\sigma_0, T/\sigma_0) = (0.700, 0.038)$:



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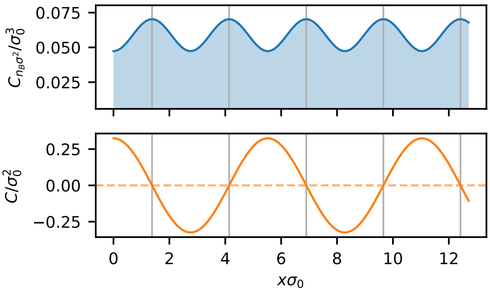
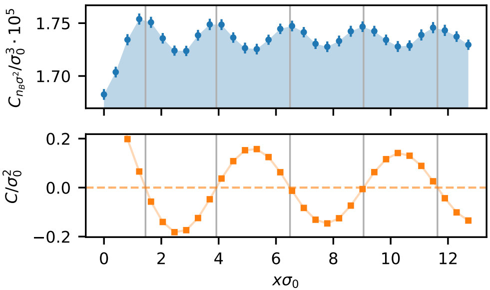


C and $C_{n_B\sigma^2}$ for $N_\mathrm{f} = 8$ at
$(\mu/\sigma_0, T/\sigma_0) = (0.700, 0.038)$:



Correspondence of condensate cycles and baryon number

- $N_\mathrm{f} \to \infty$: Baryonnumber = number of cycles of the oscillation of the condensate
- The dominating frequency of C is equal to the dominating frequency in σ.
- Define observable to extract this:

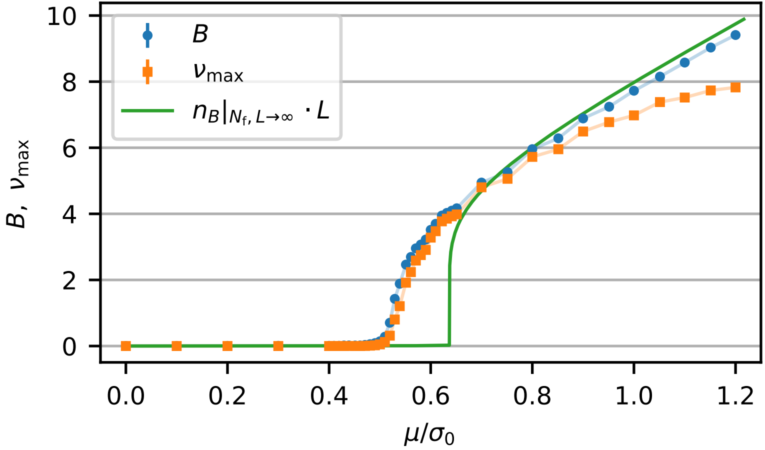
$$k_\mathrm{max} = \arg\max_k \tilde{c}(k).$$

- The number of cycles of the oscillation is then

$$\nu_\mathrm{max} = \frac{L\langle |k_\mathrm{max}| \rangle}{2\pi} \, .$$

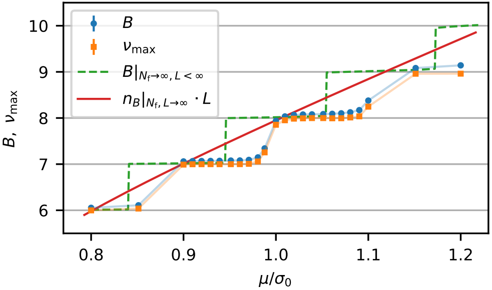
Condensate cycles and baryon number at $T = 0.076$

B and ν_{max} at $T/\sigma_0 = 0.076$:



Condensate cycles and baryon number at $T = 0.038$

B and $\nu_{\max}$ at $T/\sigma_0 = 0.038$:



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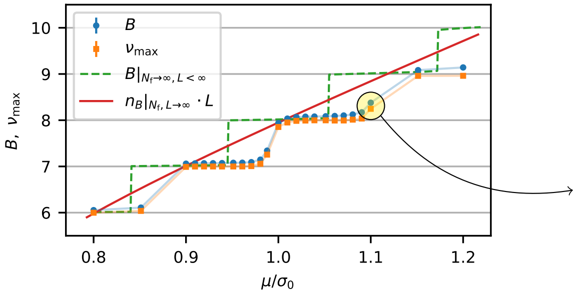
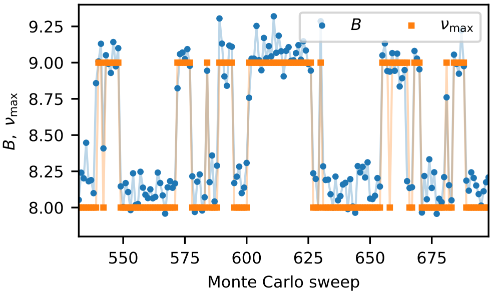


MC history of B and $\nu_{\max}$ at $(\mu/\sigma_0, T/\sigma_0) = (1.100, 0.038)$:



Conclusion

- spatial distribution of the baryons at finite N_f as in $N_f \to \infty$
- $N_f \to \infty$ limit retains important properties of inhomogeneous behavior in finite N_f physics.
- Future steps: apply our techniques to models more relevant to QCD such as the quark-meson model in 3+1 dimensions.