Strategy of the Heavy quark Operator Product Expansion for the Pion's LCDA

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with
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Roadmap to the Pion's LCDA

HOPE:

$$U^{\mu\nu}(p,q) = \frac{2f_\pi \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_n C_n(\tilde{Q}^2) a_n(\mu^2) \omega^n$$

↓

Study kinematics to enhance signal

↓

Numerically simulate matrix element

↓

Perform fit and extract $\langle \xi^2 \rangle$

This Talk

Next Talk: Anthony Grebe

The Light Cone Distribution Amplitude

- $\phi_M(x, \mu^2)$ is the transition amplitude for converting meson into quark antiquark pair.
- Process independent: Used to predict
  - Pion electromagnetic form factor, pion transition form factor, $B \to \pi\pi$, two photon processes.
- Can decompose in terms of Mellin moments:

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_M(x, \mu^2) \tag{1}$$

- All non-perturbative information is found in the $\langle \xi^n \rangle$.
- Thus full knowledge of moments allows one to reconstruct the full LCDA.

Options on the Market (Not complete list!)

- OPE: Moments of LCDA may be calculated via local matrix elements.
  - G. S. Bali et al., JHEP 08, 065 (2019).
  - V. M. Braun, et al., Phys. Rev. D 92, no.1, 014504 (2015).
- More recently, quasi-PDF and pseudo-PDF: Obtain full x dependence.
  - X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
  - A. V. Radyushkin, Phys. Rev. D 96, no.3, 034025 (2017).
  - J. H. Zhang, et al., Phys. Rev. D 95, no.9, 094514 (2017).
- Operator Product Expansion
  - V. Braun and D. Müller, Eur. Phys. J. C 55, 349-361 (2008).
    - We pursue Heavy quark Operator Product Expansion (HOPE).
    - W. Detmold and C. J. D. Lin, Phys. Rev. D 73, 014501 (2006).
- Other approaches have previously extracted $\langle \xi^2 \rangle$. We aim to show our approach may also be used to extract $\langle \xi^2 \rangle$.

Operator Product Expansion

- Wilson: Expand a non-local operator as the sum of local operators

$$T\{A(z/2)B(-z/2)\} = \sum_n C_W^{(n)}(z^2) z_{\mu_1} \ldots z_{\mu_n} \mathcal{O}_n^{\mu_1 \cdots \mu_n} \tag{2}$$

- Example of factorization.
- Relegate short distance physics to $C_W^{(n)}(z^2)$: perturbatively calculable.
- Non-pert. physics stored in moments $\langle \xi^n \rangle (\mu^2)$:

$$\langle 0 | \mathcal{O}_n^{\mu_1 \cdots \mu_n} | \pi(\mathbf{p}) \rangle = \langle \xi^n \rangle (\mu^2) [p^{\mu_1} \ldots p^{\mu_n} - \mathrm{Tr}] \tag{3}$$

- OPE allows one to reconstruct full LCDA!

Heavy quark Operator Product Expansion

- Aim to study the matrix element $U^{\mu\nu} = (T^{\mu\nu} - T^{\nu\mu})/2$ where

$$T^{\mu\nu}(p,q) = \int d^4 z e^{iq\cdot z} \langle 0| T\{J^\mu(z/2) J^\nu(-z/2)\} |\pi(\mathbf{p})\rangle \quad (4)$$

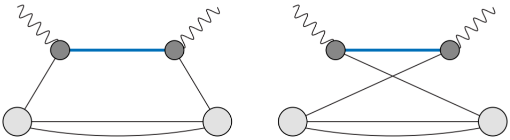
- Replace current J^μ with heavy-light current:

$$J^\mu_Q(x) = \overline{q}(x)\gamma^\mu\gamma_5\psi(x) + \overline{\psi}(x)\gamma^\mu\gamma_5 q(x) \quad (5)$$

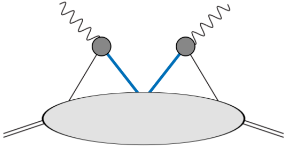
- $\psi(x)$ is a fictitious quenched heavy quark species.
- Heavy quark helps to suppresses long range correlations.
- Moments $\langle \xi^n \rangle (\mu^2)$ unchanged.

Advantages of Heavy Quark Approach

- Use of heavy quark has a number of advantages:
- Theoretical
  - Quenching removes some higher twist contributions (cat's ears diagram)
  - Heavy quark mass provides additional contribution to hard scale $\tilde{Q}^2 = -q^2 - m_\psi^2$: Since higher twist contributions arise from eg $1/\tilde{Q}$, these are suppressed.
  - Heavy quark mass allows for simple analytic continuation since we can choose kinematics to work in the unphysical region (more on this later).



$1/\tilde{Q}^2$ (Leading twist)



$1/\tilde{Q}^3$ (Higher twist)
Removed by quenching

HOPE: Final Equation

- Also summed target mass effects: Improve agreement at sub-asymptotic scales.

$$U_{\pi}^{\mu\nu}(p,q) = \frac{2if_{\pi}\epsilon^{\mu\nu\alpha\beta}q_{\alpha}p_{\beta}}{\tilde{Q}^2}\sum_{\text{even}}\frac{\mathcal{C}_n^{(2)}(\eta)}{n+1}\left(\frac{\zeta}{2}\right)^n \underbrace{C_W^{(n)}(\tilde{Q}^2/\mu^2, \tilde{Q}^2/m_{\psi}^2)}_{\text{hard}}\underbrace{\langle\xi^n\rangle}_{\text{soft}} \tag{6}$$

- where $\eta = p\cdot q/\sqrt{p^2q^2}$, $\zeta = \sqrt{p^2q^2}/\tilde{Q}^2$

Wilson Coefficients

Wilson Coefficients

- Heavy quark: have to recalculate these coefficients.
  - Presence of explicit mass scale makes calculation much harder.
- Calculation completed by Yong Zhao; various cross checks have been performed.
- Calculate Compton Scattering operator between free quark states using fixed order PT: expand in powers of α_S.

$$|\pi(\mathbf{p})\rangle \to \left| u(1/2+x_0)p, \lambda_1; \overline{d}(1/2-x_0)p, \lambda_2 \right\rangle \tag{7}$$

- Perform matching by calculating OPE operators between free quark states to same order in α_S.

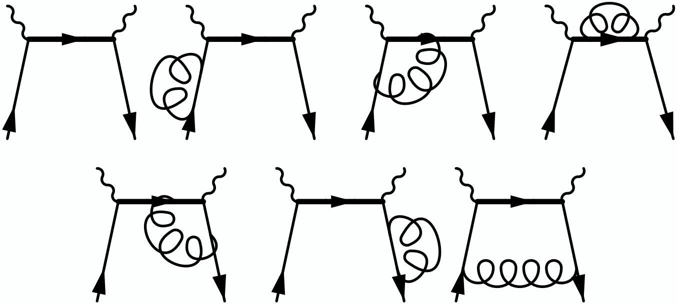


Figure 2: The hard scattering kernel at $\mathcal{O}(\alpha_s)$. One must also sum crossed versions of these diagrams.

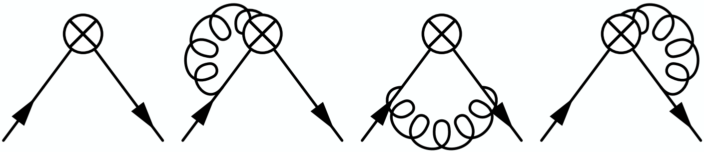


Figure 3: Performing matching to local operators.

Strategy for HOPE Analysis

Kinematic Strategy

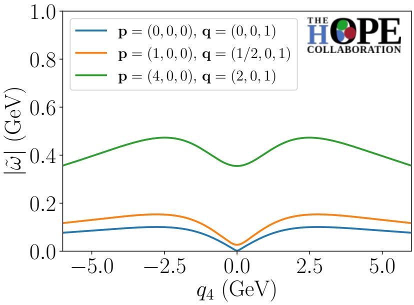
- OPE proportional to

$$U^{\mu\nu}(p,q) \sim \sum_{n=0}^{\infty} \langle \xi^n \rangle \, \omega^n \quad (8)$$

$$\omega = \frac{2p \cdot q}{\tilde{Q}^2} = \frac{1}{x} \quad (9)$$

- Physical region $\omega > 1$, $x < 1$. HOPE valid for $|\omega| < 1$. We wish to enhance our sensitivity to higher moments, so must ensure ω not too small.

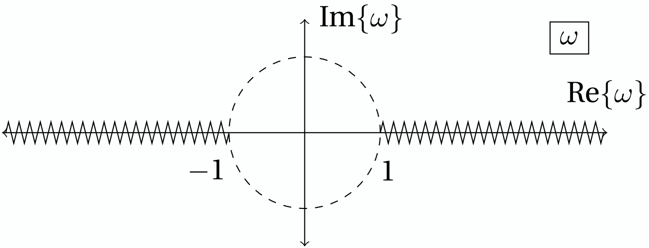
- Increasing $2p \cdot q$ while keeping $\tilde{Q}^2$ fixed lets us enhance the contribution from higher moments.



Analytic Continuation

- Physical pion has $p_4 = iE_\pi(\mathbf{p})$, we choose q_4 real.
- In general, this choice leads to complex ω:

$$\omega = \frac{2p \cdot q}{\tilde{Q}^2} = \frac{2\mathbf{p} \cdot \mathbf{q}}{q_4^2 + \mathbf{q}^2 + m_Q^2} + \frac{2iE_\pi q_4}{q_4^2 + \mathbf{q}^2 + m_Q^2} \tag{10}$$

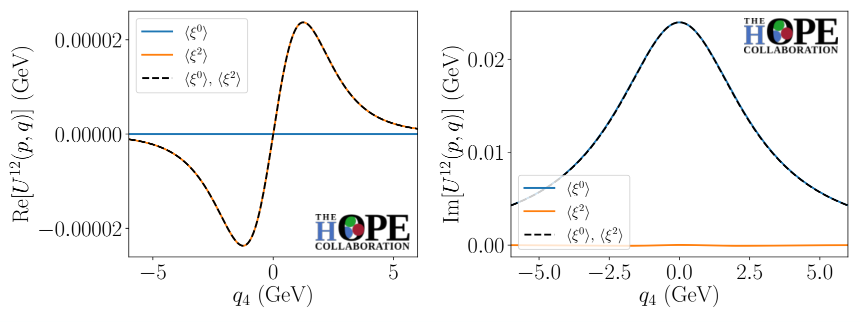


- Complex ω ensures we avoid physical region of amplitude. No complication from on-shell states propagating between currents.

Kinematic Strategy

- Split matrix element into real and imaginary parts
- In certain kinematics, Re part has no $\langle \xi^0 \rangle$ contribution: starts at $\langle \xi^2 \rangle$.

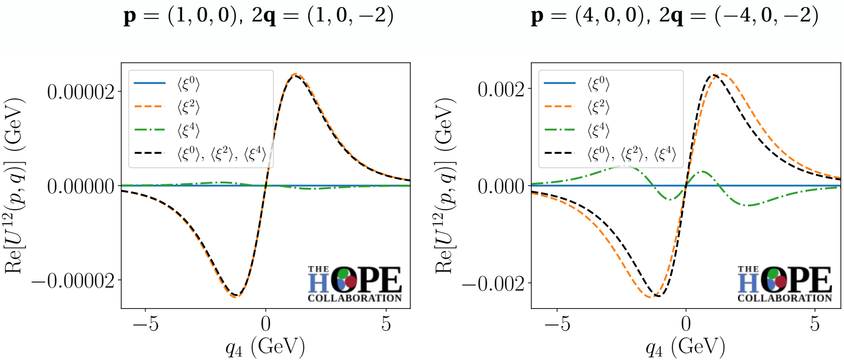
$\mathbf{p} = (1, 0, 0)$, $2\mathbf{q} = (1, 0, -2)$



- Allows extraction of $\langle \xi^2 \rangle$ at very low momentum.

Can We Access Higher Moments?

- To access moments beyond the $\langle \xi^2 \rangle$, we must boost the hadronic state.
- Required to extract $\langle \xi^4 \rangle$ from this approach.



Conclusion

- Pion light cone distribution amplitude important for exclusive measurements at high energies: process independent
- Discussed the Heavy quark Operator Product Expansion approach to extract the second Mellin Moment.
  - Require Wilson Coefficients to accurately extract value.
- Higher momentum will allow us to extract higher moments.

Spare Slides

Equations

$$C_\pi(x_E) = \langle 0|\, T\{\mathcal{O}_{\pi^+}(x_E)\mathcal{O}^\dagger_{\pi^+}(0)\}\, |0\rangle \tag{11}$$

$$C_3^{\mu\nu}(x_E, y_E) = \langle 0|\, T\{J^\mu(x_E)J^\nu(y_E)\mathcal{O}^\dagger_\pi(0)\}\, |0\rangle \tag{12}$$

$$T^{\mu\nu}(p_E, q_E) = \int dY_4 e^{-iY_4\cdot q_4} \frac{C_3^{\mu\nu}(x_4, \mathbf{p}_1, y_4, \mathbf{p}_2)}{C_\pi((x_4+y_4)/2), \mathbf{p}_2+\mathbf{p}_2)} \sqrt{Z_\pi(\mathbf{p}_1+\mathbf{p}_2)} \tag{13}$$

where we identify

$$p_E = (iE_\pi(\mathbf{p_1}+\mathbf{p}_2), \mathbf{p}_1+\mathbf{p}_2) \tag{14}$$

$$q_E = (q_4, (\mathbf{p}_1-\mathbf{p}_2)/2) \tag{15}$$

3-Point Function Calculation

- Utilize a sequential source: Fix momentum insertion at $\mathbf{p}_e$