Investigation of Doubly Heavy Tetraquark Systems using Lattice QCD

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Physical Motivation (1)

Experimental background

- Experimentally observed states $Z_b(10610)^+$ and $Z_b(10650)^+$
- Mass suggests a bottomonium state $\bar{b}b$ but would be electrically neutral
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Theoretical study

- We study similar but less challenging systems
- Quark content: $\bar{Q}\bar{Q}'qq'$, here: $\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$, $\bar{b}\bar{c}ud$
- In the limit $m_Q \to \infty$ stable tetraquark was shown

[J. Carlson, L. Heller and J. A. Tjon, Phys. Rev. D **37**, 744 (1988)]
[A. V. Manohar and M. B. Wise, Nucl. Phys. B **399**, 17 (1993)]
[E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, no. 20, 202002 (2017)]

Physical Motivation (2)

- **Born-Oppenheimer study** of $\bar{b}\bar{b}ud$, static $\bar{b}$-quarks:
  - Prediction of a bound tetraquark with $I(J^P) = 0(1^+)$ and a binding energy $M_{\bar{b}\bar{b}ud} - (M_B + M_{B^*}) \approx -90\,\mathrm{MeV}$ → Talk by M.Wagner in Session 4B

    [P. Bicudo *et al.* [European Twisted Mass Collaboration], Phys. Rev. D **87**, no. 11, 114511 (2013)]
    [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012)]
    [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, Phys. Rev. D **92**, no. 1, 014507 (2015)]
    [P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D **95**, no. 3, 034502 (2017)]

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- Investigate $\bar{b}\bar{b}ud$ bound state in the $I(J^P) = 0(1^+)$ channel with **Non-Relativistic QCD** i.e. non-static $\bar{b}$-quarks.
  [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, no. 14, 142001 (2017)]
  [P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, no. 3, 034507 (2019)]
  [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D **99**, no. 5, 054505 (2019)]
  [L. Leskovec, S. Meinel, M.P. and M. Wagner, , Phys. Rev. D **100**, no.1, 014503 (2019)]

Lattice Setup

- Use gauge link configuration generated by RBC and UKQCD collaboration
  [Y. Aoki *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **83**, 074508 (2011)]
  [T. Blum *et al.* [RBC and UKQCD Collaborations],Phys. Rev. D **93**, no. 7, 074505 (2016)]

- $2+1$ flavours domain-wall fermions and Iwasaki gauge action
- Five different ensembles which differ in

$$\begin{aligned} &\text{lattice spacing} && a && \approx 0.083\,\text{fm}\ldots 0.114\,\text{fm}, \\ &\text{lattice size} && L && \approx 2.65\,\text{fm}\ldots 5.48\,\text{fm}, \\ &\text{pion mass} && m_\pi && \approx 139\,\text{MeV}\ldots 431\,\text{MeV} \end{aligned}$$

  $\Rightarrow$ explore dependence on L, m_π
- Smeared point-to-all propagators for the up and down quarks
- Utilize all-mode-averaging technique
  [T. Blum, T. Izubuchi and E. Shintani, Phys. Rev. D **88**, no. 9, 094503 (2013)]
  [E. Shintani, R. Arthur, T. Blum, T. Izubuchi, C. Jung and C. Lehner, Phys. Rev. D **91**, no. 11, 114511 (2015)]

Interpolating Operators for $\bar{b}\bar{b}ud$

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    - 2 operators: BB^* and B^*B^* scattering operators

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    - 2 operators: BB^* and B^*B^* scattering operators
- Expectation:
  - Local operators: good overlap to ground state (stable four-quark)
  - Nonlocal operators: sizeable overlap to first excited state (2 meson state)

$\Rightarrow$ Isolate ground state from higher excitations, especially first excited state

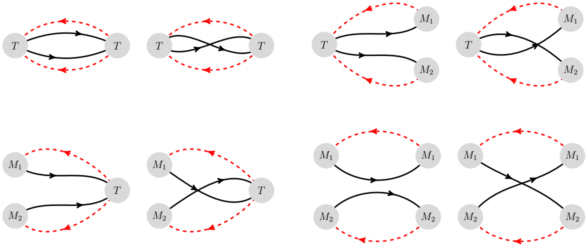
Energy Spectrum for the $\bar{b}\bar{b}ud$ system

- Due to point-to-all propagators, only 5×3 correlation matrix available (no scattering operator at source)
- Apply multi-exponential matrix fitting: employable also for non-symmetric matrices

$$C_{jk}(t) \approx \sum_{n=0}^{N-1} Z_j^n Z_k^n \mathrm{e}^{-E_n t},$$

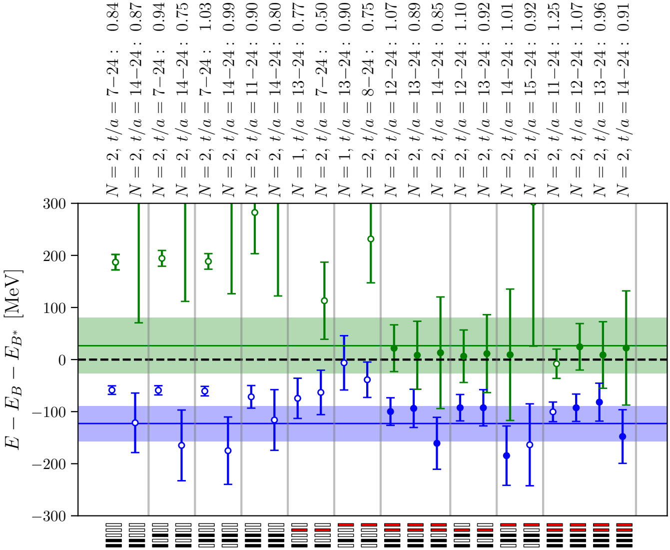
E_n : n-th energy eigenvalue

$Z_j^n = \langle \Omega | \mathcal{O}_j | n \rangle$: overlap factor



Schematic representation of Wick contractions for different correlation matrix elements

Fit Results for Different Operator Bases

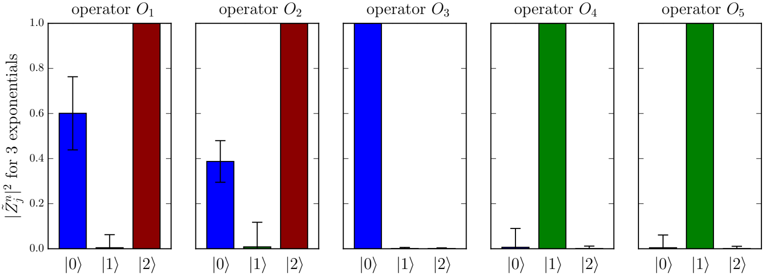


Results for the lowest two $\bar{b}\bar{b}ud$ energy levels relative to the BB^* threshold. Black box: local operator included. Red box: scattering operator included.

Overlap Factors

For fixed j: Z_j^n indicates relative importance of energy eigenstates $|n\rangle$

$$\mathcal{O}_j^\dagger|\Omega\rangle = \sum_{n=0}^{\infty} |n\rangle\langle n|\mathcal{O}_j^\dagger|\Omega\rangle = \sum_{n=0}^{\infty} Z_j^n |n\rangle.$$

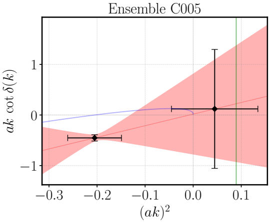


The normalized overlap factors $|\tilde{Z}_j^n|^2 = \frac{|Z_j^n|^2}{\max_m(|Z_j^m|^2)}$ as determined on ensemble C005.

Scattering Analysis

- Relate *finite volume* energy spectrum E_n to *infinite volume* scattering amplitude for 2 energy levels in T_1^+ irrep
- Use Lüscher's formula and scattering momenta k_n^2 to determine phase shift
- Apply effective-range-expansion (ERE)

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4).$$



Plot of the effective-range-expansion for C005. Blue curve: $ak\cot(\delta(k)) + |ak|$. Vertical green line: Inelastic B^*B^* threshold

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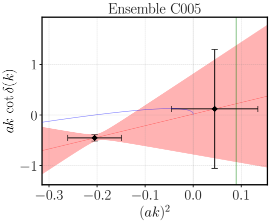

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- Search bound state pole of scattering amplitude below threshold at

$$\cot \delta_0(k_{\mathrm{BS}}) = i, \quad \text{so:} \quad -|k_{\mathrm{BS}}| = \frac{1}{a_0} - \frac{1}{2} r_0 |k_{\mathrm{BS}}|^2$$

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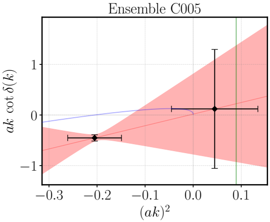

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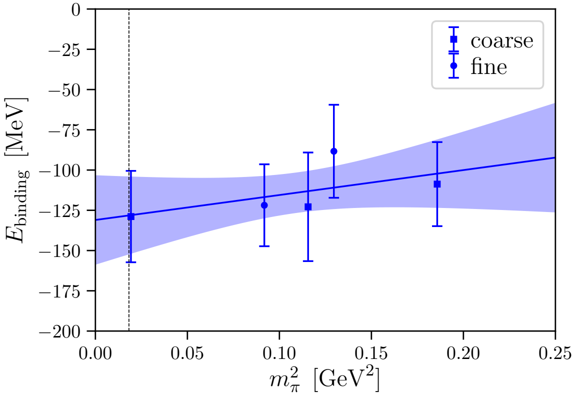
- Results essentially identical to the finite-volume energy levels
- Confirmation that ground state is stable tetraquark.



Fit of the pion-mass dependence of E_{binding}. The vertical dashed line indicates the physical pion mass.

$$E_{\text{binding}}(m_{\pi,\text{phys}}) = (-128 \pm 24 \pm 10)\,\text{MeV}$$

$$m_{\text{tetraquark}}(m_{\pi,\text{phys}}) = (10476 \pm 24 \pm 10)\,\text{MeV}$$

Expectations for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$

Subsequent promising candidates have heavier light or lighter heavy quarks:

- $\bar{b}\bar{b}us$:
  - Similar quantum numbers to $\bar{b}\bar{b}ud$: $I(J^P) = \frac{1}{2}(1^+)$
  - Previous studies predict a bound state in this channel
- $\bar{b}\bar{c}ud$:
  - Due to different heavy quark structure: 2 promising channels: $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$
  - Supposed to have either a weakly bound state or no binding

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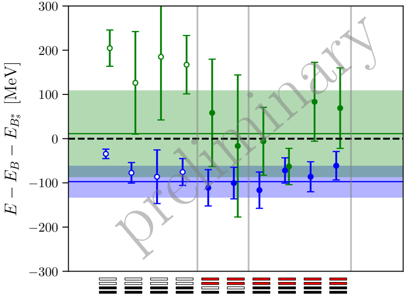
Interpolating Operators

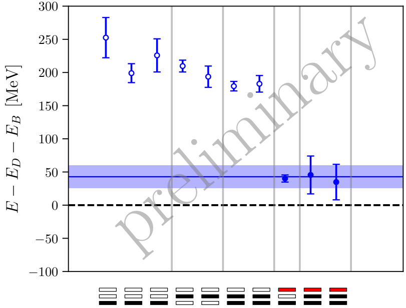
- Local operators: mesonic structure and diquark-antidiquark structure
- Nonlocal operators: relevant scattering states near threshold

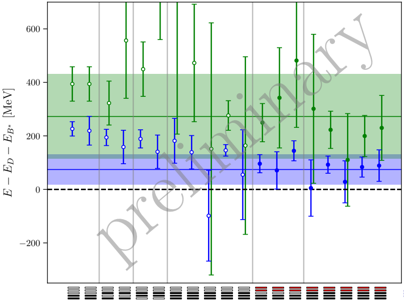
Preliminary Results



- Strong indication of bound state in $\bar{b}\bar{b}us$, stable after Lüscher analysis
- No evidence for bound state in $\bar{b}\bar{c}ud$

top left: $\bar{b}\bar{b}us$. **bottom left:** $\bar{b}\bar{c}ud$, $J = 0$. **bottom right:** $\bar{b}\bar{c}ud$, $J = 1$.





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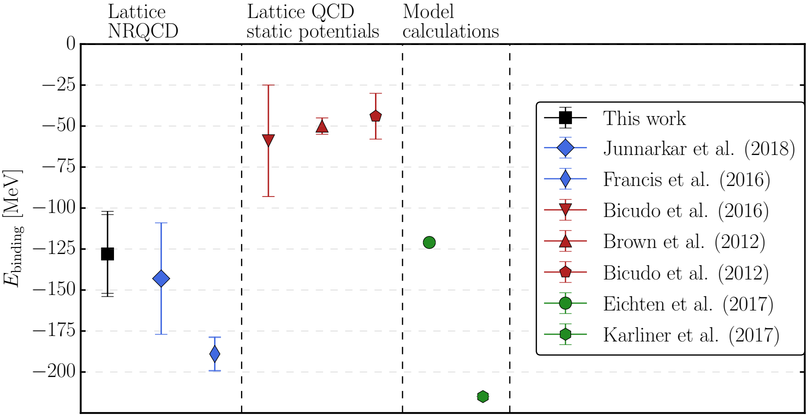
Thank You for Your Attention!

Comparison of Different Results for $\bar{b}\bar{b}ud$



Comparison of $\bar{b}\bar{b}ud$ tetraquark binding energies with $I(J^P) = 0(1^+)$ (black: this work; blue: lattice NRQCD; red: lattice QCD computations of static $\bar{b}\bar{b}$ potentials and solving the Schrödinger equation; green: effective field theories and potential models).