Lattice calculation of GPDs and twist-3 PDFs of the proton

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Collaborators & Projects

1. Generalized parton distributions

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2. Twist-3 parton distributions

Shohini Bhattacharya[4], Krzysztof Cichy[3], Martha Constantinou[4], Andreas Metz[4], Fernanda Steffens[6]

University of Cyprus[1], The Cyprus Institute[2], Adam Mickiewicz University in Poznań[3], Temple University[4], DESY-Zeuthen[5], University of Bonn[6]

Outline

1. Generalized parton distributions
2. Twist-3 parton distribution functions
3. Conclusions

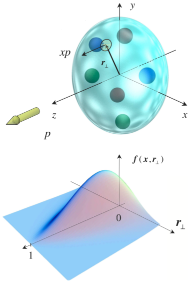
Outline

Generalized parton distributions (GPDs)

D. Muller, A. Radyushkin, X. Ji (1994-1997)

- GPDs provide a unifying picture for a set of fundamental quantities of hadronic structure:
  - form factors and PDFs
  - longitudinal structure and transverse distribution of partons

    $\Rightarrow$ What is the momentum and spatial distributions of quarks?
  - angular momentum of quarks and gluons



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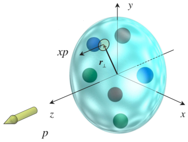
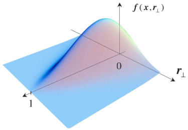
- GPDs experimentally accessed in exclusive processes, e.g. DVCS and DVMP, but:

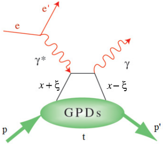
  [Belitskyand Radyushkin, 2005, Kumericki et al., 2016]
  - not directly related to cross sections
  - data are limited

  $\Rightarrow$ GPDs mostly unknown so far

- GPDs are part of the physics program of EIC, HERMES, COMPASS, Jlab







DVCS: $ep \to ep'\gamma$

GPDs as light-cone correlation functions

- For a given quark q (unpolarized hadron)

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_f|\bar{\psi}(0)\gamma^+ e^{-ig\int_0^{z^-} dx' A^+}\psi(z^-)|P_i\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(P_f)\gamma^+ u(P_i) + E^q(x,\xi,t)\,\bar{u}(P_f)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(P_i)\right]$$

2 GPDs: objects of interest

Three-variables:

1. x: quark momentum fraction
2. $t = \Delta^2 = (P_f - P_i)^2$: momentum transfer squared
3. $\xi = -\frac{\Delta^+}{2P^+}$: skewness

Properties:

- In the forward limit ($P_i = P_f$), GPDs reduce to parton densities: $H^q(x,0,0) = q(x)$
- Elastic form factors are moments of GPDs, e.g. $\int_{-1}^{+1} dx H^q(x,\xi,t) = F_1(t)$

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Light-cone dominance: need of sophisticated methods on the lattice!

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Access GPDs through the quasi-distribution approach [X.Ji, Phys. Rev. Lett.110(2013) 262002]

- GPDs computed through purely space-like correlation functions

$$\tilde{F}^q(x,\tilde{\xi},t,P_3) = \frac{1}{2}\int \frac{dz}{2\pi} e^{-ixP_3 z}\langle N(P_f)|\bar{\psi}(0)\gamma_0 W(0,z)\psi(z)|N(P_i)\rangle = \frac{\bar{u}(P_f)}{2P_0}\left[\tilde{H}\gamma_0 + \tilde{E}\frac{i\sigma^{0\mu}\Delta_\mu}{2m_N}\right]u(P_i)$$

Quasi-GPDs — Fourier transform — matrix elements of fast moving nucleons

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Variables:

1. $W(z)$: Wilson line of length z
2. $P = \frac{P_i+P_f}{2}$: average momentum boost
3. $t = \Delta^2 = -Q^2$
4. $\tilde{\xi} = -\frac{Q_3}{2P_3}$: quasi-skewness $\tilde{\xi} = \xi + \mathcal{O}(1/P_3^2)$

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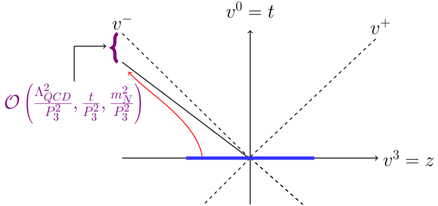

- For sufficiently large P_3, quasi-GPDs are matched onto GPDs within LaMET framework

$$\tilde{F}^q = \int_{-1}^{1}\frac{dy}{|y|}\, \mathcal{C}_\Gamma\, F^q(x,\xi,t,P_3,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_3^2}, \frac{t}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$$

↳ matching kernel

$\mathcal{C}_\Gamma$ computed to 1-loop level in RI/MOM scheme [Y-S. Liu et al., Phys.Rev. D100 (2019) no.3, 034006]

Lattice setup

Gauge ensemble

- Configurations of $N_f = 2 + 1 + 1$ flavors & clover term [ETMC collaboration]

Ensemble	N_f	$L^3 \times T$	lattice spacing a	m_π	$m_\pi L$
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Nucleon momenta

- Breit frame:

$$\vec{P}_i = P_3 \hat{z} - \vec{Q}/2, \quad \vec{P}_f = P_3 \hat{z} + \vec{Q}/2$$

(GPDs defined in the Breit frame)

P_3 [GeV]	$\vec{P}_i \times \frac{L}{2\pi}$	$\vec{P}_f \times \frac{L}{2\pi}$	Q^2 [GeV^2]	ξ	N_{meas}
0.83	(0,-1,2)	(0,1,2)	0.69	0	4152
1.25	(0,-1,3)	(0,1,3)	0.69	0	35136
1.67	(0,-1,4)	(0,1,4)	0.69	0	112192

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Classes chosen such that:

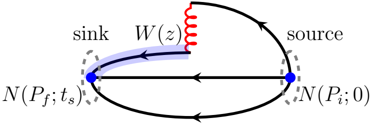
1. P_3-dependence can be investigated
2. different GPDs can be disentangled $(H(x,t,\xi), E(x,t,\xi), \tilde{H}(x,\xi,t), \ldots)$

PRELIMINARY RESULTS: finalyze analysis at $\xi \neq 0$

- For different classes of momenta we compute

$$\mathcal{M}(z,P_3,\vec{Q},\xi,\mathcal{P}) = \langle N(P_3\hat{z}+\vec{Q}/2)|\bar{\psi}(0)\tau_3\Gamma W(0,z)\psi(z)|N(P_3\hat{z}-\vec{Q}/2)\rangle$$

  - ⋆ $\Gamma=\gamma_0,\gamma_5\gamma_j,\ldots$ gives access to a specific GPD
  - ⋆ $\mathcal{P}$ is a parity projector used in the three-point functions



Every matrix element, $\mathcal{M}$, extracted from the ratio

$$R_{\mathcal{O}}(\mathcal{P},\vec{P}_f,\vec{P}_i;t,t_{\text{ins}}) = \frac{C^{3pt}_{\mathcal{O}}(\mathcal{P},\vec{P}_f,\vec{P}_i;t,t_{\text{ins}})}{C^{2pt}(\vec{P}_f;t)} \times \sqrt{\frac{C^{2pt}(\vec{P}_i;t-t_{\text{ins}})C^{2pt}(\vec{P}_f;t_{\text{ins}})C^{2pt}(\vec{P}_f;t)}{C^{2pt}(\vec{P}_f;t-t_{\text{ins}})C^{2pt}(\vec{P}_i;t_{\text{ins}})C^{2pt}(\vec{P}_i;t)}}$$

Lattice methods:

- Sequential inversions through the sink ($t_s = 12a \simeq 1.13$ fm)
- Momentum smearing [G. Bali et al., Phys.Rev.D 93 (2016) 9, 094515]

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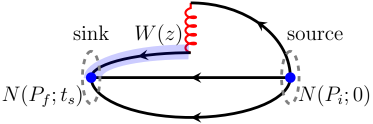

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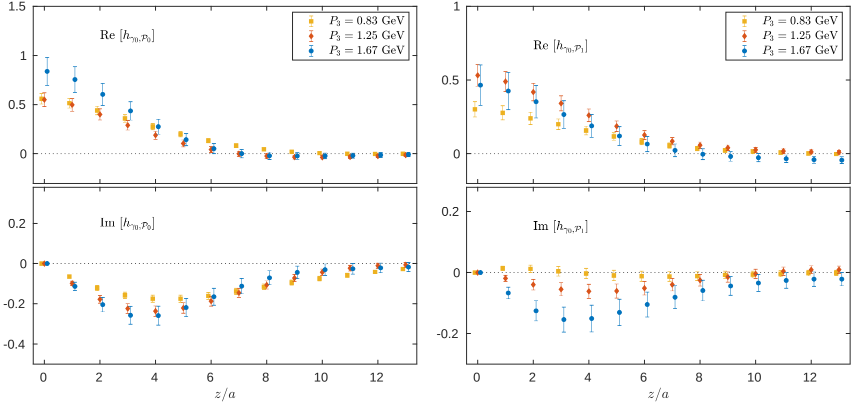
Multiple matrix elements needed to disentangle different GPDs

Bare matrix elements for unpolarized GPDs

- To disentangle $H(x, \xi, t)$ and $E(x, \xi, t)$ two matrix elements are needed

$-t = Q^2 = 0.69$ GeV ^{2}, $\xi = 0$



Projectors:

$$\mathcal{P}_0 = \frac{(1+\gamma_0)}{4}$$

$$\mathcal{P}_1 = \frac{(1+\gamma_0)}{4}\gamma_5\gamma_1$$

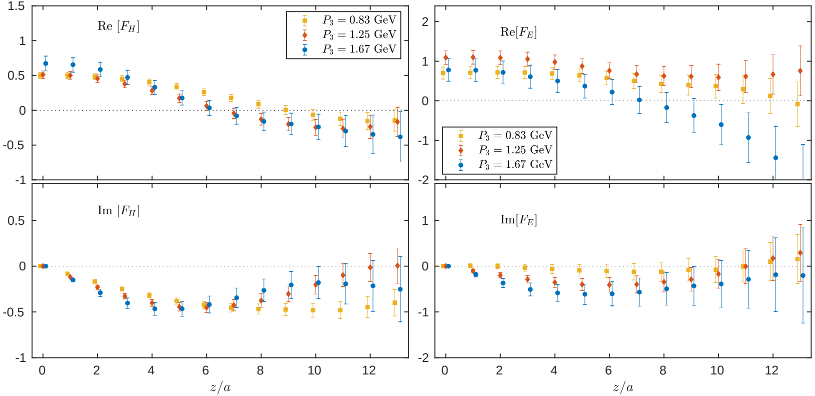
- Both matrix elements contribute (real parts have the same magnitude)

Disentangling F_H and F_E

- $F_E(z;\xi;t)$ and $F_H(z;\xi;t)$ extracted through a decomposition

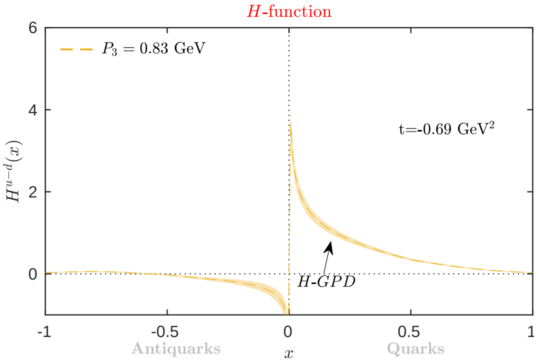
$$F_H(z,\xi,t) = \mathcal{K}_H(P_i,P_f,\Gamma_0)\mathcal{M}(\gamma_0,\bar{\Gamma}_0) + \mathcal{K}'_H(P_i,P_f,\Gamma_1)\mathcal{M}(\gamma_0,\bar{\Gamma}_1)$$
$$F_E(z,\xi,t) = \mathcal{K}'_E(P_i,P_f,\Gamma_0)\,\mathcal{M}(\gamma_0,\bar{\Gamma}_0) + \mathcal{K}'_E(P_i,P_f,\Gamma_1)\mathcal{M}(\gamma_0,\bar{\Gamma}_1)$$

$\mathcal{K},\mathcal{K}'$: kinematic factors



- F_E noisier than F_H (E-GPD subleading compared to H-GPD)

x-dependence of GPDs



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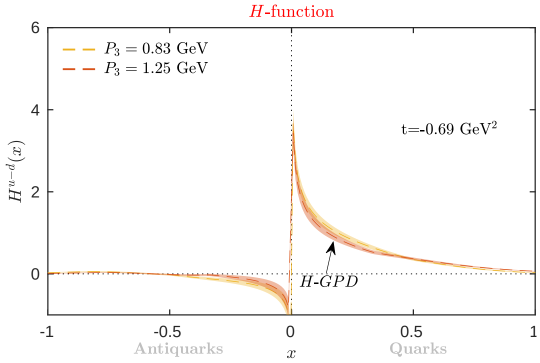

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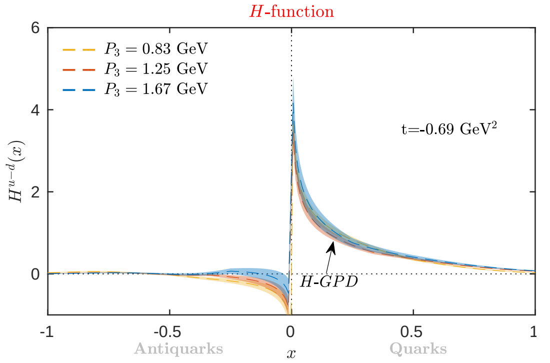

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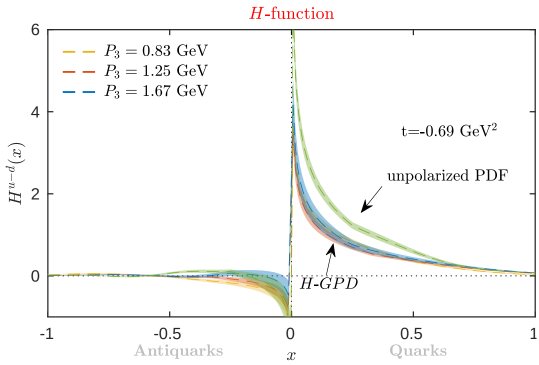
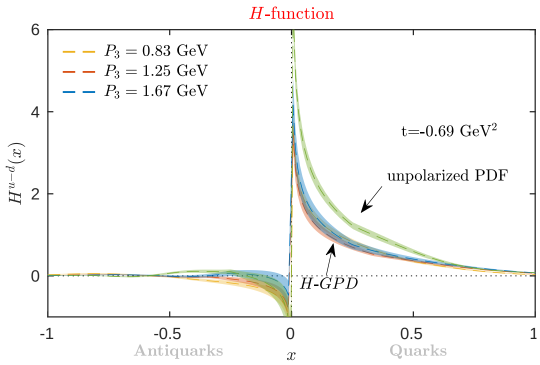
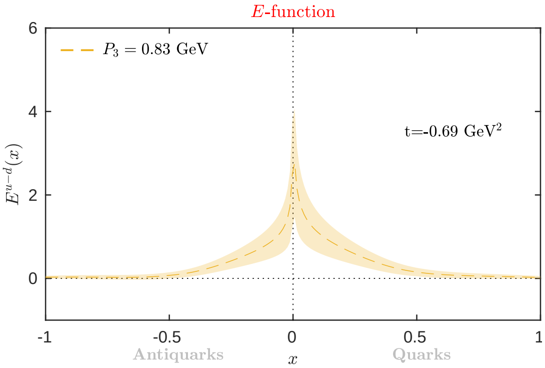


- H-GPDs compatible at the three nucleon momenta
- H-GPDs suppressed with respect to PDFs
  (as expected from usual form factors)

x-dependence of GPDs





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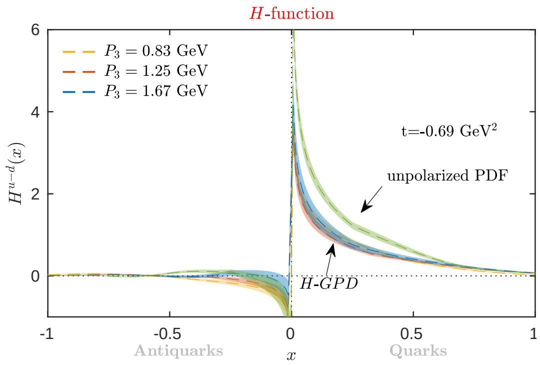

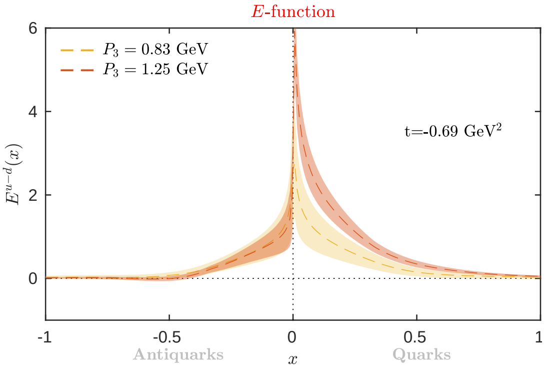

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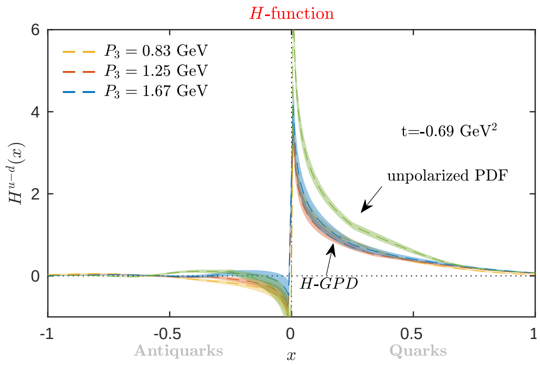

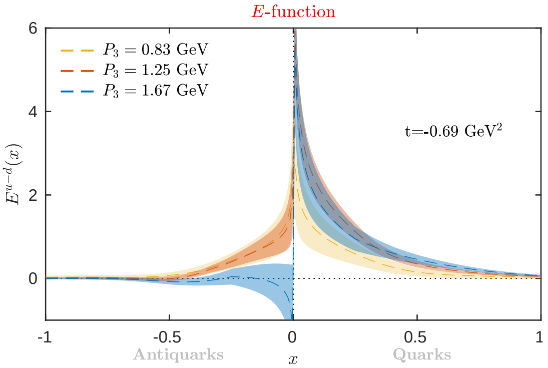

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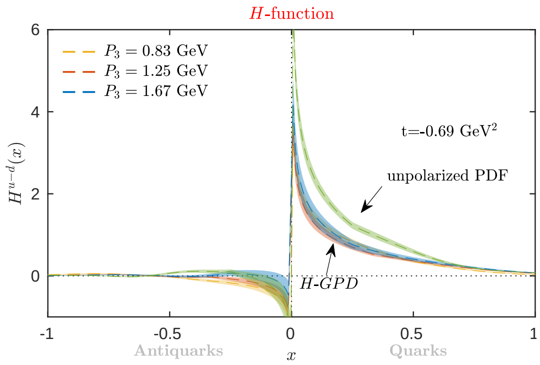

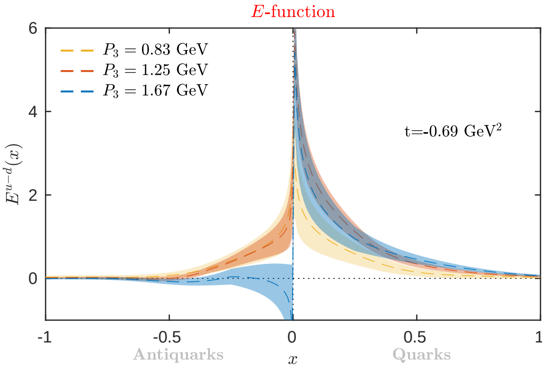

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GPDs results will be compared with experimental data when they become available!

Outline

Beyond twist-2 distributions: twist-3 PDFs

- Some properties of twist-3 PDFs:
  - information about quark-gluon-quark correlations
  - connections with TMDs
  - suppressed as $1/Q$ in relation to twist-2 PDFs in structure functions

Helicity g_T PDF

- x-dependence not known in phenomenology $\Rightarrow$ Lattice QCD?
- can we test the Wandzura-Wilczek (WW) approximation?

[S. Wandzura and F. Wilczek, Phys. Lett.72B, 195, 1977]

$g_T(x)$ may be obtained by:

$$g_T^{WW}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$

g_1 : helicity twist-2

$\Rightarrow$ the study of the WW approximation gives direct information about the importance of twist-3 operators

$g_T^{u-d}(x)$ from the quasi-PDF approach

$g_T^{u-d}(x)$ extracted from:

- Matrix element: $\mathcal{M}_{g_T} = \langle N(P)|\bar{\psi}(0)\gamma_5\gamma_j W(0,z)\psi(z)|N(P)\rangle_\mu$

  $\gamma_j = \gamma_x, \gamma_y, \quad P = (iE, 0, 0, P_3)$

- Fourier transform to momentum space (x)

$$\text{Quasi-}g_T: \quad \widetilde{g}_T(x,\mu,P_3) = 2P_3 \int_{-\infty}^{+\infty} \frac{dz}{4\pi}\, e^{-ixP_3 z}\, \mathcal{M}_{g_T}(P_3,z)$$

  Reconstruction through Backus-Gilbert method **[J.Karpie et al, JHEP 04 (2019) 057]**

- Matching procedure

$$\boxed{g_T(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \widetilde{g}_T\left(\frac{x}{\xi}, \mu, P_3\right)}$$

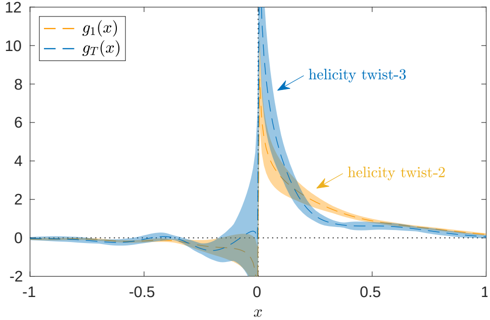
[S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, (2020), arXiv:2005.10939, accepted in PRD]

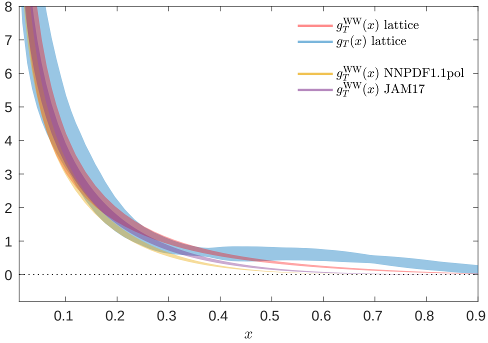
Results for $g_T^{u-d}(x)$

Ensemble: $N_f = 2+1+1$ twisted mass fermions & clover term
$a \simeq 0.093$ fm, V=64×32^3, $m_\pi = 270$ MeV

[S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, (2020), arXiv:2004.04130]

$P_3 = 1.67$ GeV





- Helicity twist-3 is suppressed only for $0.3 < x < 0.5$
- g_T and g_T^{WW} are consistent for a large x-range (but violations of WW approximation can still be at the level of 40% for $x \lesssim 0.4$)

Outline

Summary

Using the quasi-distribution approach, first lattice evaluation of:

- Twist-2 $(u-d)$ GPDs

  $\rightarrow$ twisted mass fermions, $N_f = 2+1+1$ at $M_\pi \simeq 270$ MeV
  $\rightarrow$ laborious calculation (GPDs multi-dimensional quantities - P_3, ξ, t)
  $\rightarrow$ x-dependence of H and E extracted at $\xi = 0$
  $\rightarrow$ statistical errors on $H(x)$ allow qualitative comparison with unpolarized PDFs

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Various systematics need to be addressed:
cutoff effects, finite volume effects, truncation errors in the matching, etc.

Thank you very much for your attention