Color confinement due to violation of non-Abelian Bianchi identity

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2. A new idea of QCD monopoles $J_\mu = k_\mu$
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T. Suzuki, arXiv:1402.1294 (2014),
T. Suzuki et al, P.R. D80, 054504 (2009),
T. Suzuki, K.Ishiguro, V.Bornyakov, P.R. D97, 034501, 099905(erratum) (2018),
T. Suzuki, P.R. D97, 034509 (2018)
K. Ishiguro and A. Hiraguchi, Talks at APLAT 2020

1. Introduction

Color confinement problem not yet solved.

Almost half a century history !!!

1. 1963: Quark model (Gell-Mann and Zweig): fractionally charged quarks are searched, but not observed.
2. 1974-75: Idea of dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed.
3. 1981: 'tHooft idea of monopole in QCD: A partial gauge-fixing $SU(3) \to U(1) \times U(1)$ and Abelian projection: Monopoles appear as a topological object. Numerical data supporting this idea are shown especially on the basis of maximally Abelian gauge. But this idea has a serious problem of gauge dependence.

The key point is to find a gauge-independent color magnetic quantity, a magnetic monopole in QCD without any additional assumption.

2. A complete new idea of magnetic monopoles in QCD

Note the Jacobi identities:

$$\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0,$$

where $D_\mu \equiv \partial_\mu - igA_\mu$. Calculate explicitly:

$$\begin{aligned}
[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\
&= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\
&= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma]
\end{aligned}$$

$[\partial_\rho, \partial_\sigma]$ can not be neglected in general!!

$D_\nu G^*_{\mu\nu} = 0 \rightarrow$ Non-Abelian Bianchi identity (NABI):

$$f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2$$

$\partial_\nu f^*_{\mu\nu} = 0 \rightarrow$ Abelian-like Bianchi identity:

Jacobi identity + $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$

$$
\begin{aligned}
\Longrightarrow D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} \\
&= -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma] A_\nu = \partial_\nu f^*_{\mu\nu} \\
J_\mu = \frac{1}{2} J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu} &= \partial_\nu f^*_{\mu\nu} = \frac{1}{2} k^a_\mu \sigma^a = k_\mu
\end{aligned}
$$

$k^a_\mu \neq 0 \rightarrow$ color magnetic Abelian-like monopole: $\partial_\mu k_\mu = 0$
$J^a_\mu \neq 0 \rightarrow$ Violation of NABI

Color magnetic monopoles= Violation of non-Abelian Bianchi identity (VNABI) :Reference C. Bonati et al,, P.R.D81, 085022 (2010)

$$
\begin{gathered}
[\partial_\rho, \partial_\sigma] A_\nu \neq 0 \\
\Downarrow
\end{gathered}
$$

Line singularities existing in gauge fields $A_\mu(x)$ themselves!!! are the origin of the QCD monopoles and $N^2 - 1$ monopoles exist in $SU(N)$.

3. Lattice studies of the new QCD magnetic monopoles

Consider one-colored monopole $k^1(s,\mu)$ among three $k^a(s,\mu)$ ($a = 1 \sim 3$ in $SU(2)$) or eight ($a = 1 \sim 8$ in $SU(3)$) monopoles defined following DeGrand-Toussait.

Lattice monopole is not gauge-invariant. But Elitzer's theorem says that gauge-invariant contents, if exist, can be extracted by Monte-Carlo average of gauge-variant quantities.
S. Elitzur, P.R. D12 (1975) 3978.

Lattice monopole after Abelian projection

Maximize $R = \sum_{s,\mu} Re\,\mathrm{Tr}\, e^{i\theta_1(s,\mu)\lambda_1} U^\dagger(s,\mu)$

$$\Downarrow$$

$$\begin{aligned}
\theta_1(s,\mu) &= \tan^{-1}\frac{Im(U_{12}(s,\mu)+U_{21}(s,\mu))}{Re(U_{11}(s,\mu)+U_{22}(s,\mu))} \\
\theta_1(s,\mu\nu) &= \partial_\mu\theta_1(s,\nu)-\partial_\nu\theta_1(s,\mu) \\
&= \bar{\theta}_1(s,\mu\nu)+2\pi n_1(s,\mu\nu) \quad (|\bar{\theta}_1(s,\mu\nu)|<\pi) \\
k^1(s,\mu) &= \frac{1}{2\pi}\partial_\nu\bar{\theta}_1(s,\mu\nu)
\end{aligned}$$

Pefect monopole dominance about the string tension

Evaluate

$$V_{\mathrm{mon}}(R) = -\frac{1}{aN_t}\ln\langle P_{\mathrm{mon}}(0)P^*_{\mathrm{mon}}(R)\rangle \ .$$

$$P_{\mathrm{A}} = \exp[i\sum_{k=0}^{N_t-1}\theta_1(s+k\hat{4},4)] = P_{\mathrm{ph}}\cdot P_{\mathrm{mon}} \ ,$$

$$P_{\mathrm{ph}} = \exp\{-i\sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_{\nu}\bar{\Theta}_1(s',\nu 4)\} \ ,$$

$$P_{\mathrm{mon}} = \exp\{-2\pi i\sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_{\nu}n_1(s',\nu 4)\}$$

Take average over $4000 \sim 7000$ (in $SU(2)$) and more than 60000 (in $SU(3)$ thermalized vacua and their random gauge-transformed vacua 4000 times for each one.

Table 1: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials V_{NA}, V_{A}, V_{mon} and V_{ph}.

$SU(2)$	nconf=5000	ngf=1000	$\beta = 2.53$		
$36^3 \times 6$	σa^2	c	μa	FR(R/a)	χ^2/N_{df}
V_{NA}	0.072(3)	0.48(9)	0.53(3)	4.6 - 12.1	1.03
V_{A}	0.073(2)	0.47(6)	1.10(2)	4.3 - 11.2	1.03
V_{mon}	0.073(3)	0.46(7)	1.43(3)	4.0 - 11.8	1.01
V_{ph}	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4 - 11.5	1.03
$SU(3)$	nconf=60000	ngf=4000	$\beta = 5.6$		
$24^3 \times 4$	σa^2	c	μa	FR(R/a)	χ^2/N_{df}
V_{NA}	0.193(4)	0.422(3)	1.146(20)	1-7	0.992
V_{A}	0.184(15)	0.458(97)	2.912(80)	1-8	1.10
V_{mon}	0.188(16)	0.453(99)	2.906(82)	1-8	0.967
V_{ph}	$-0.0014(2)$	0.073(5)	1.521(3)	1 - 11	0.997

Even when we are restricted to only one-color component, perfect Abelian and monopole dominance are obtained.
In $SU(2)$ case, the scaling and volume independence are also checked.

For details and other applications, listen to the next talks by my collaborators, Dr Ishiguro and Mr. Hiraguchi.

4. Existence of the continuum limit

Does the continuum limit of $k^a(s,\mu)$ exist?

Study the monopole density in the continuum limit in pure SU2 QCD.

The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, various techniques smoothing the vacuum are introduced.

1. Tadpole improved action:
48^4 at $\beta = 3.0 \sim 3.9$ and 24^4 at $\beta = 3.0 \sim 3.7$

2. Introduction of various smooth gauge-fixings
1) Maximal center gauge(MCG): Maximization of $R = \sum_{s,\mu}(\mathrm{Tr}U(s,\mu))^2$ SU(2) $\rightarrow$ Z(2)

2) Direct Laplacian center gauge (DLCG)

3) Maximal Abelian Wilson loop gauge (AWL): Maximization of
$R = \sum_{s,\mu\neq\nu}\sum_a(cos(\theta^a_{\mu\nu}(s))$

4) Maximal Abelian and $U(1)$ Landau gauge (MAU1):

3. The blockspin transformation of monopoles

the blockspin transformation of monopoles

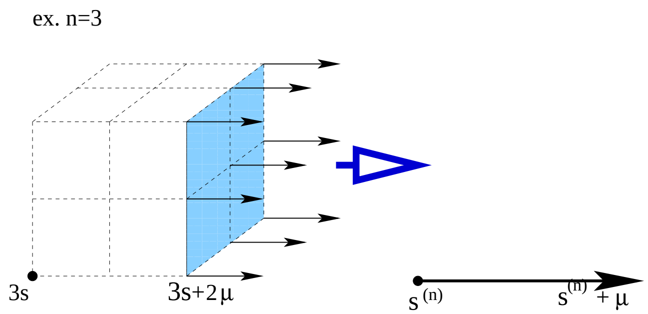


Figure 1: Blockspin definition of monopoles:
T.L. Ivanenko et al., Phys. Lett. **B252**, (1990) 631

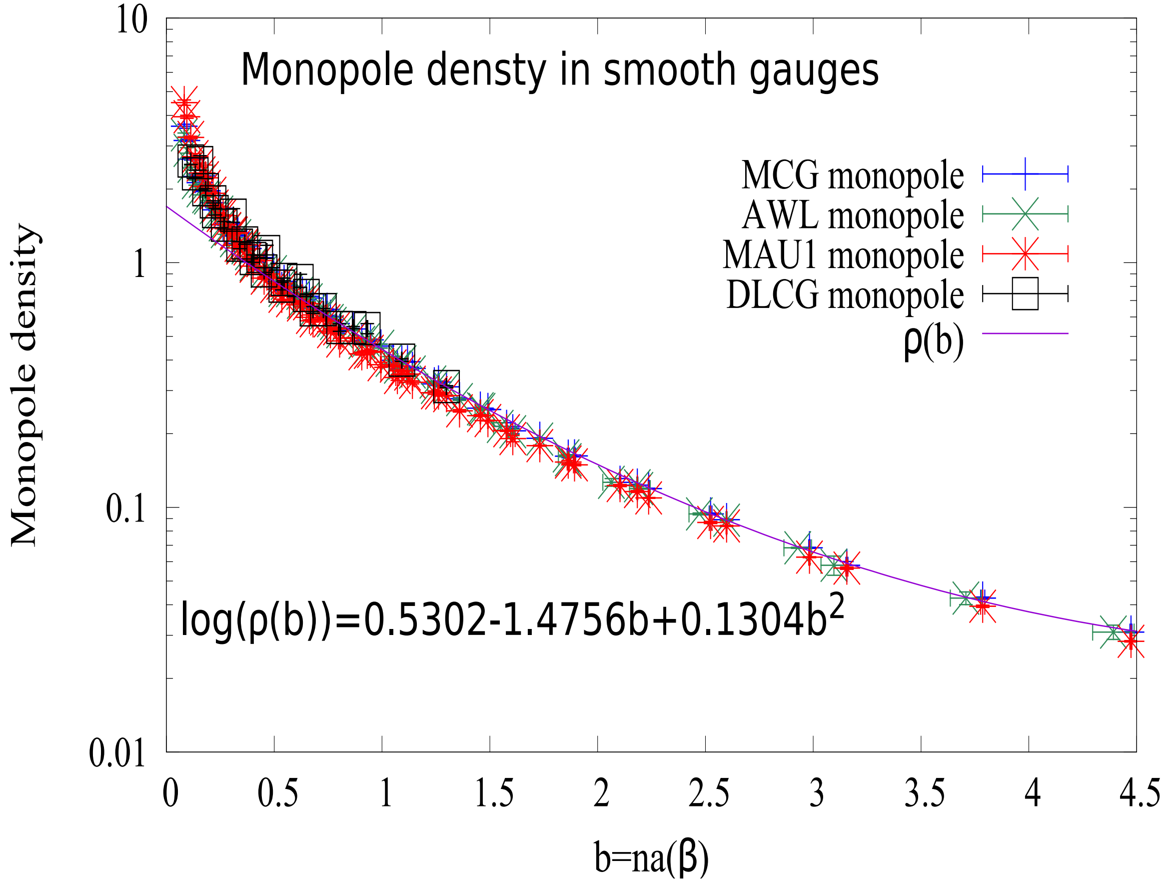
Monopole is defined on a a^3 cube and the n-blocked monopole is defined on a cube with a lattice spacing $b = na$

$$\boldsymbol{k}_\mu^{(n)}(s_n) = \sum_{i,j,l=0}^{n-1} k_\mu(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

Evaluate a gauge-invariant density of the n-blocked monopole:

$$\boldsymbol{\rho} = \frac{\sum_{\mu, s_n} \sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{3} V_n b^3}$$

Figure 2: Comparison of the VNABI (Abelian-like monopoles) densities versus $b = na(\beta)$ in MCG, AWL, DLCG and MAU1 cases.



Summary

1. Clear scaling behaviors are observed up to the 12-step blockspin transformations for $\beta = 3.0 \sim 3.9$. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$. Existence of the continuum limit!

2. When the vacuum becomes smooth enough shown here in MCG, DLCG, AWL, MAU1, the same $\rho(b)$ is obtained. Gauge independence!
   This is naturally expected in the continuum limit.

3. The similar scaling and gauge-independence are observed also with respect to the effective monopole actions under the block-spin transformation. $S(k) = S(a(\beta), n) = F(b = na(\beta))$

7. Future outlook

1. To check if VNABI could explain all mass generation in QCD such as hadron masses is interesting.
2. To construct an effective Abelian dual Higgs model based on VNABI is interesting.
3. Monopole dominance is proved using a huge amount of vacuum configurations. But, to extract them efficiently, it is important to adopt an additional gauge-fixing making the vacuum as smooth as possible and then perform the block-spin transformation in various future studies of monopole effects.
4. The Dirac's monopole as violation of the Abelian Bianchi identity in QED is not found experimentally. Is there any singular effect in other gauge theories?
5. How to formulate a field theory containing line singularities mathematically is not known yet.