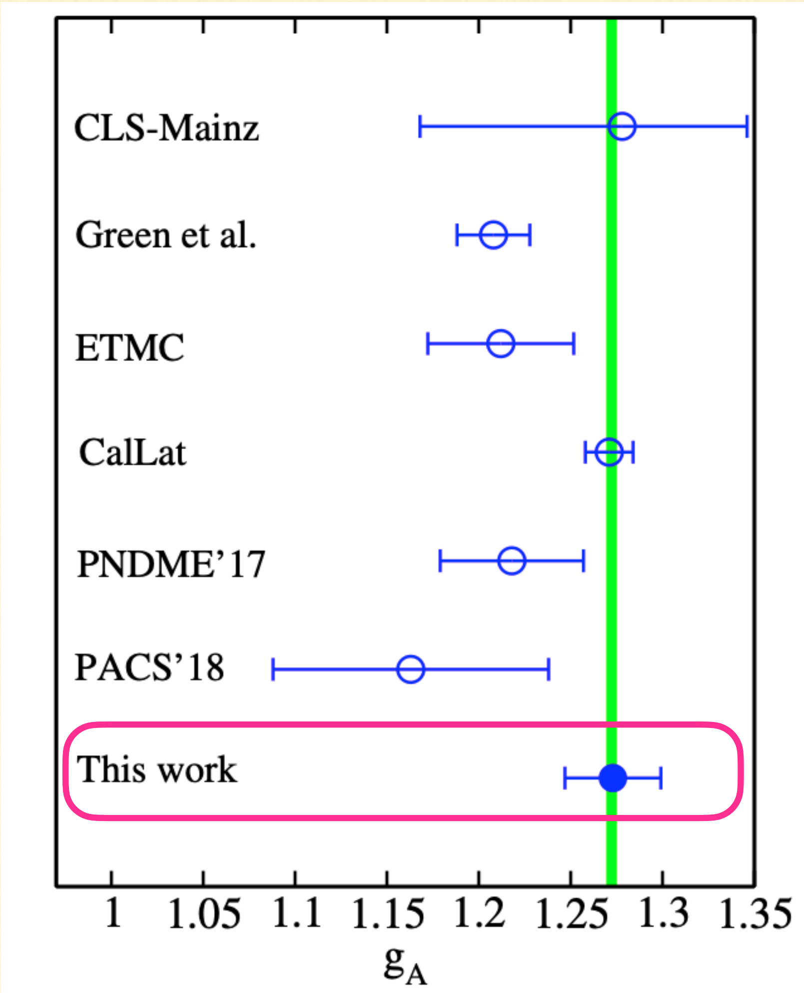
Nucleon structure at physical point in 2+1 flavor lattice QCD

Ryutaro TSUJI for PACS Collaboration

In collaboration with: Y. Aoki, K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, E. Shintani and T. Yamazaki

Mile stone → g_A



Axial charge g_A is studied as a mile stone calculation.

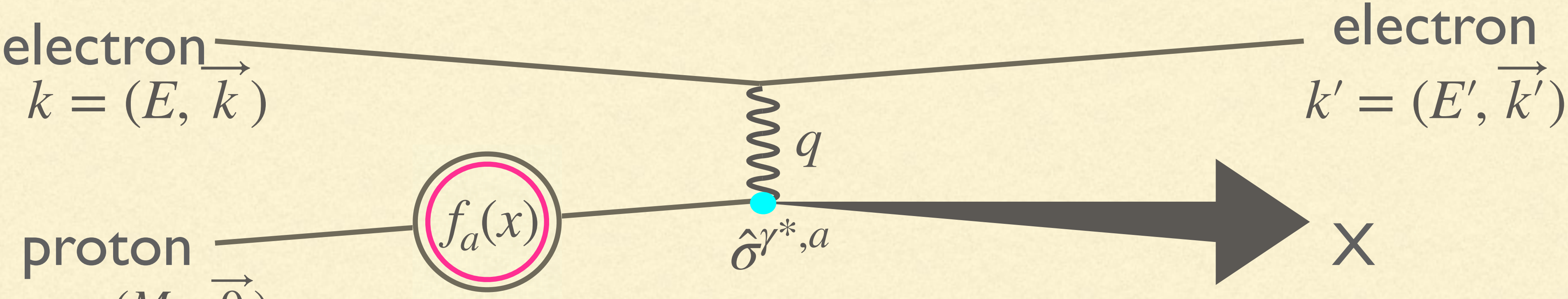
PACS collaboration achieved The 2%-precision of g_A at the physical points on 128^4 lattice.

$$g_A^{\text{PACS}} = 1.273(24)(5)(9)$$

$$g_A^{\text{exp.}} = 1.2756(13)$$

E. Shintani et al., Phys. Rev. **D99,** 014510(2019)

Our next targets → Deep Inelastic Scattering



Parton Distribution function

◉Kinematics

$$Q^2 = -q^2 = -(k - k')^2, \nu = \frac{p \cdot q}{M_N}, x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}$$

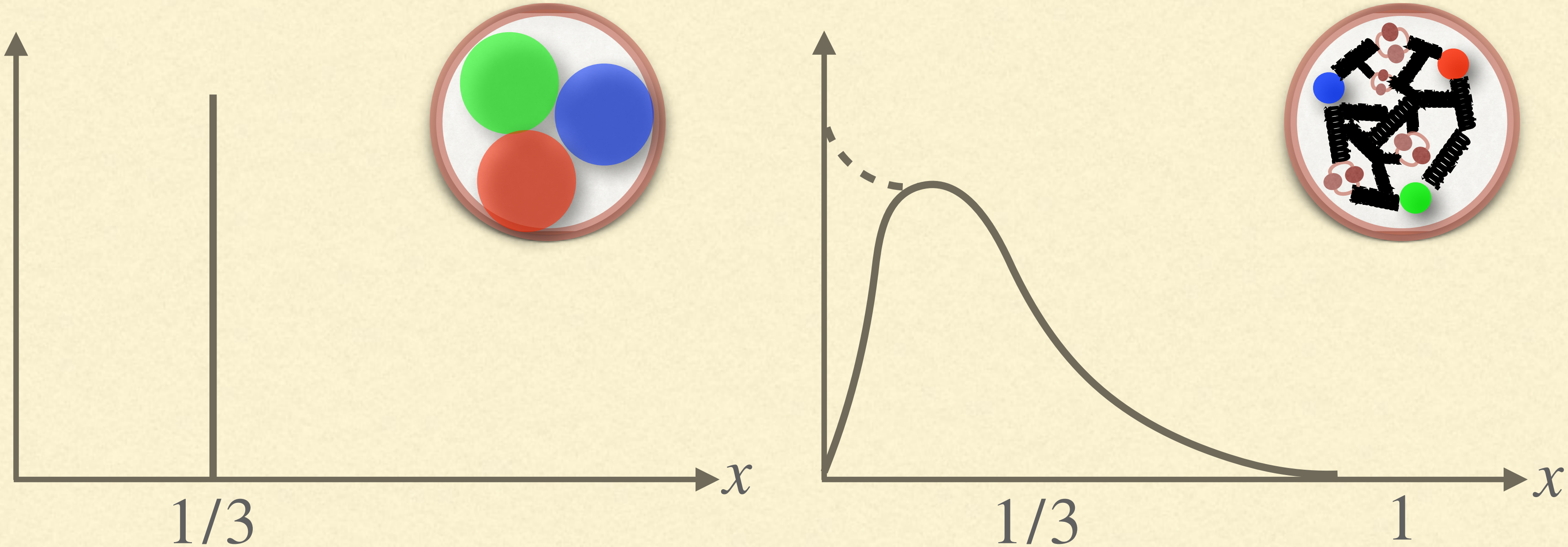
x is regarded as fractional proton momentum carried by quark

◉Patron Distribution functions and Scaling functions

$$F_1(x) = \frac{1}{2}\sum_q e_q^2 f_q(x) \quad \& \quad g_1(x) = \frac{1}{2}\sum_q e_q^2 \left(f_{q+}(x) - f_{q-}(x)\right)$$

Quark momentum fraction

Gluons reduce relative contribution from quarks

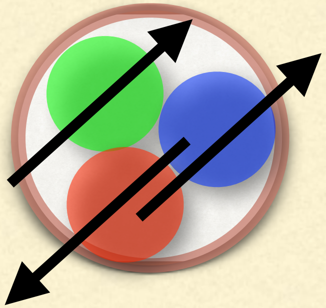


- Quark momentum fraction

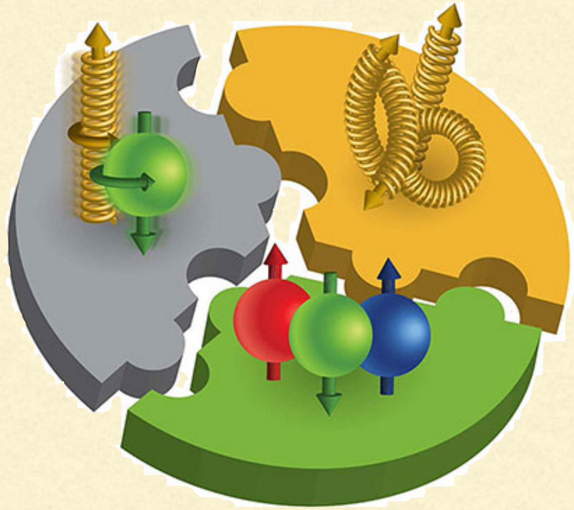
$$\int_0^1 dx\, x F_1(x, Q^2) \to \langle x \rangle_q = \langle p, S\,|\, \bar{q} \left[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3} \sum_k \gamma_k \overleftrightarrow{D}_k \right] |\, p, S \rangle$$

Quark helicity fraction

Quarks' spin are NOT enough to describe the proton spin



$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$$



→ Nucleon has spin 1/2

$$\rightarrow \frac{\text{Quarks' contribution}}{\text{Nucleon's spin}} = ?$$

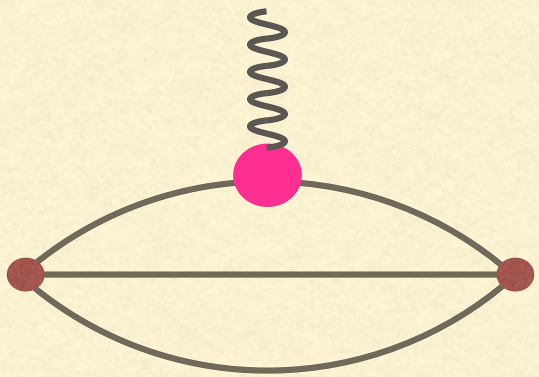
- Quark helicity fraction

$$\int_0^1 dx x g_1(x, Q^2) \rightarrow \langle x \rangle_{\Delta q} = i \langle p, S | \bar{q} \gamma_5 \left[\gamma_3 \overleftrightarrow{D}_4 + \gamma_4 \overleftrightarrow{D}_3 \right] | p, S \rangle$$

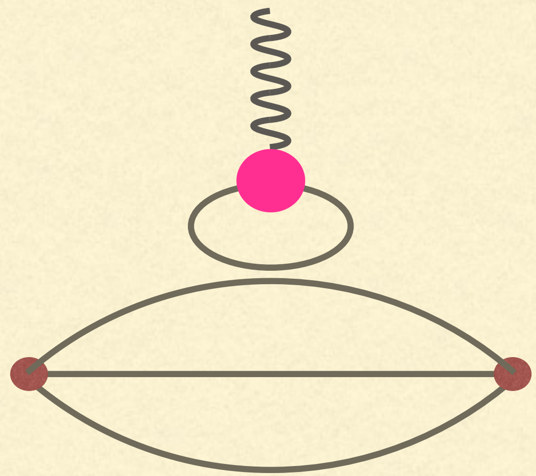
https://www.bnl.gov/eic/

Iso-vector quantities : $O_\Gamma = u\Gamma d$ under the iso-spin symmetry

$\langle N | O(t') | N \rangle$ are composed of two types of contributions.



Connected contribution

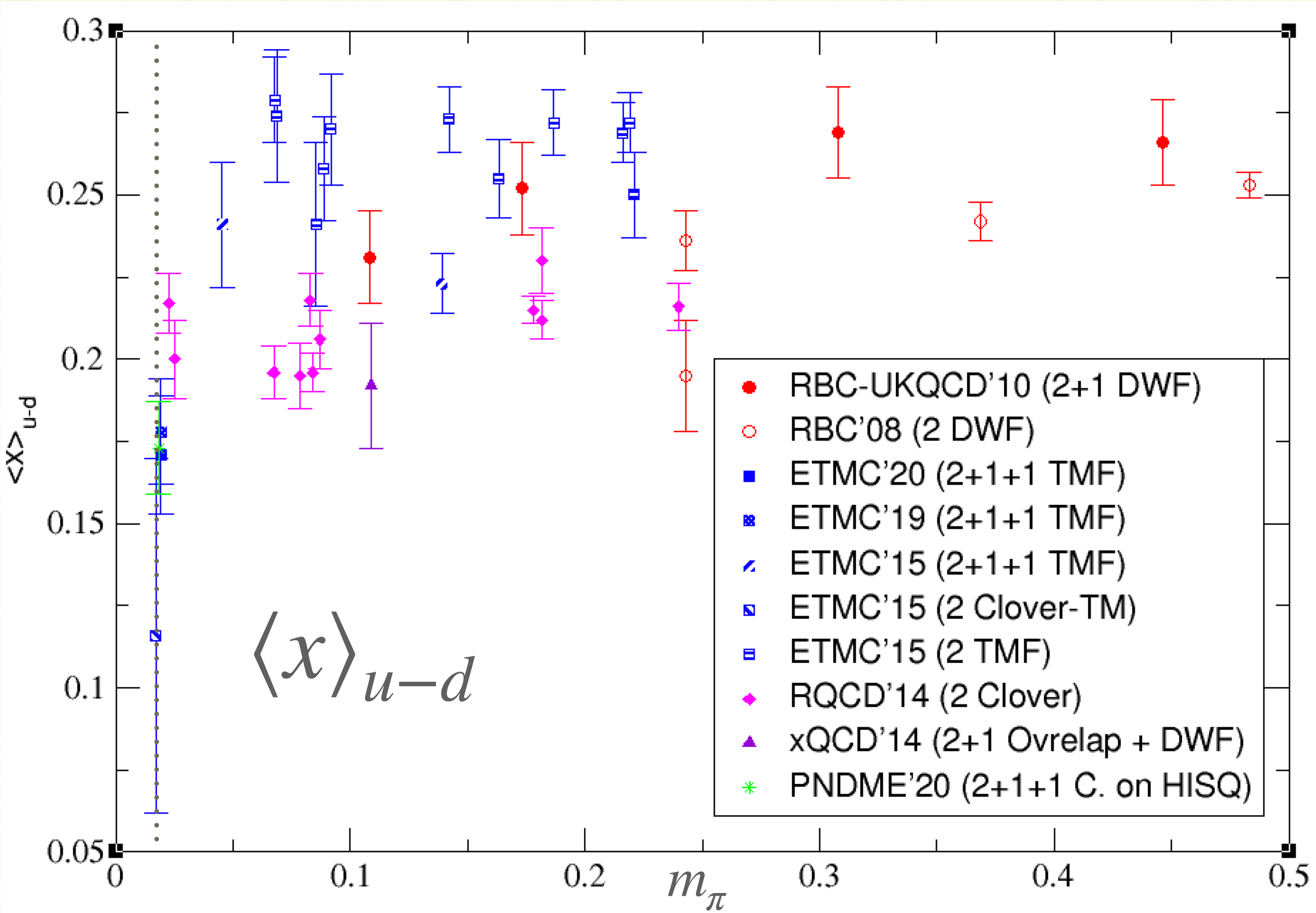


Disconnected contribution
= High computational costs

Iso-vector dose NOT suffer from disconnected contribution.

Overview

Santanu Mondal et al., arXiv:2005.13779v1 (2020)
Huey-Wen Lin et al., arXiv:1711.07916v3 (2018)
Y.-B. Yang et al., Phys. Rev. **D93,** 034503(2016)
A. Adbel-Rehim et al., Phys. Rev. **D92,** 039904(2015)
G.S. Bali et al. Phys. Rev. **D90,** 074501(2014)
Y. Aoki et al., Phys. Rev. **D82,** 014501(2010)
H.-W. Lin et al., Phys. Rev. **D78,** 014505(2008)



Overview

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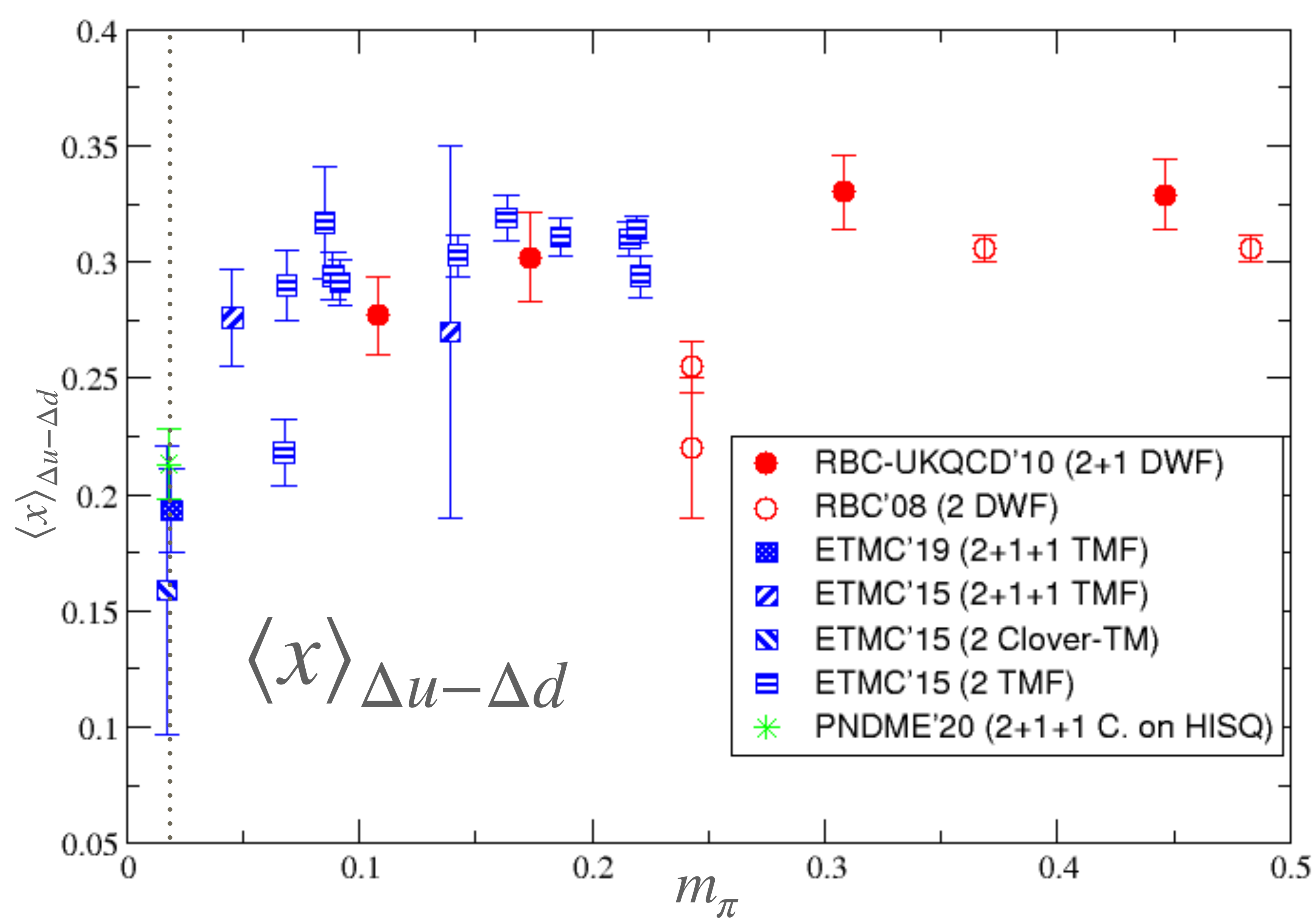
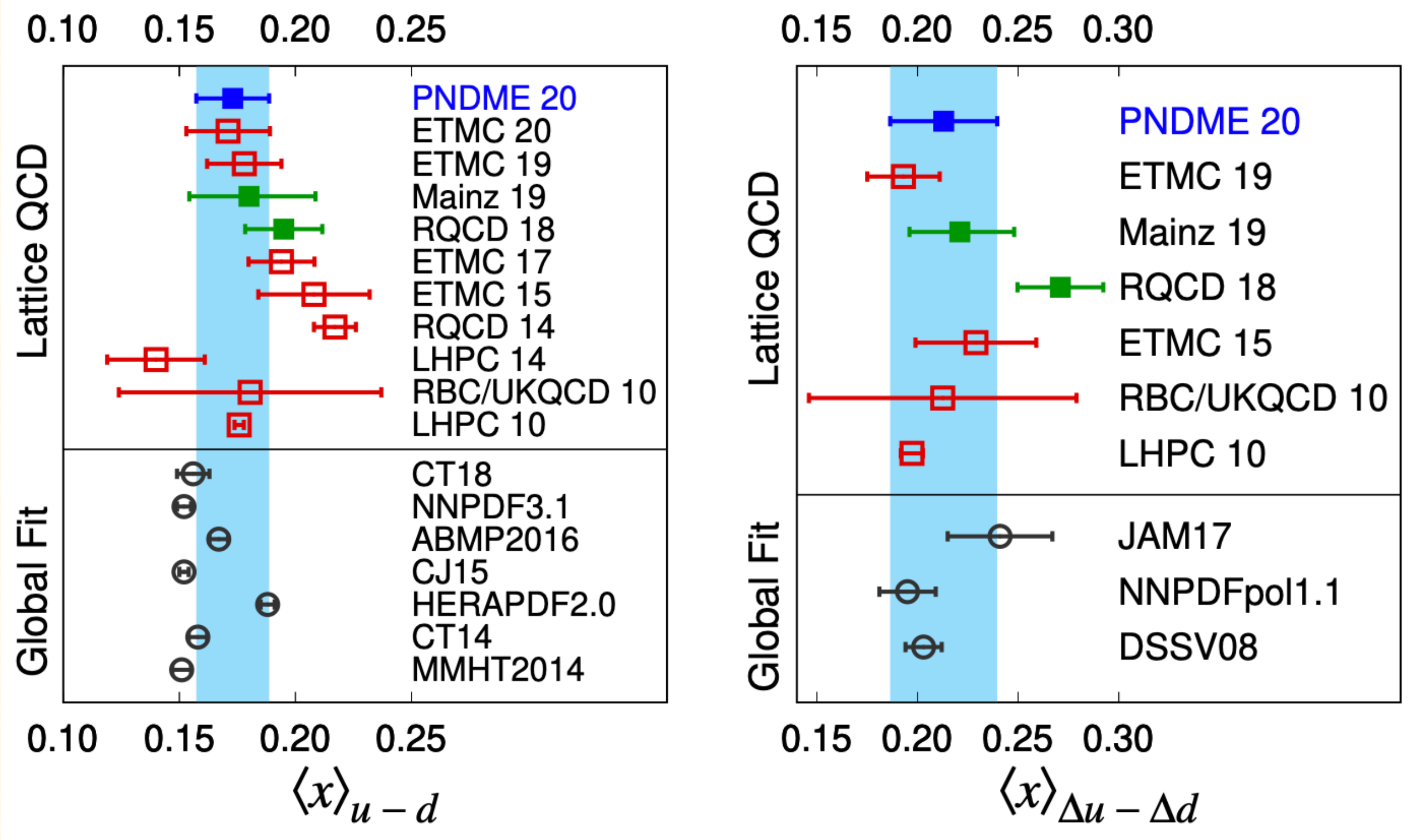


Comparison at the physical point → NOT yet succeeded

Santanu Mondal et al., arXiv:2005.13779v1 (2020)



Fewer systematic errors in PACS

- ~~Finite Vol. effect~~
- ~~Chiral extrapolation~~
- Excited state contamination
- Discretization error

e.t.c.

PACS collaboration members

PACS = Processor Array for Continuum Simulation

N. Ishizuka, N. Ukita, Y. Kuramashi, E. Shintani, Y. Taniguchi, T. Yamazaki, T. Yoshie	Tsukuba U.
Y. Nakayama	Kyoto U.
Y. Aoki, Y. Nakamura	RIKEN-CCS
K.-I. Ishikawa	Hiroshima U.
S. Sasaki, R. Tsuji	Tohoku U.

Nucleon Structure Project

Nucleon structure studies with PACS10 configuration [1][2]

	P.R. **D99**, 014510(2019)	Running
Lattice size	128^4 [1]	160^4 [2]
Spacial volume	$\sim (10.8\ \mathrm{fm})^3$	$\sim (10.3\ \mathrm{fm})^3$
Pion mass	135 MeV	135 MeV
$\lvert t_{sink}-t_{src}\rvert/a$	10, 12, 14, 16	16
Lattice spacing	~ 0.084 fm	~ 0.064 fm

[1] E. Shintani et al., Phys. Rev. **D99,** 014510(2019)

[2] E. Shintani and Y.Kuramashi, Phys.Rev. **D100,** 034517(2019)

The stout-smeared $O(a)$ improved Wilson fermions and Iwasaki gauge action.

In this talk, we present

① $|t_{sink}-t_{src}|/a$ dependence on $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u-\Delta d}$

→ Check for excited state contamination on 128^4 lattice.

② Bare result of g_A and ratio : g_A/g_V

→ A benchmark calculation for 160^4 lattice.

③ Ratio : $\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u-\Delta d}$

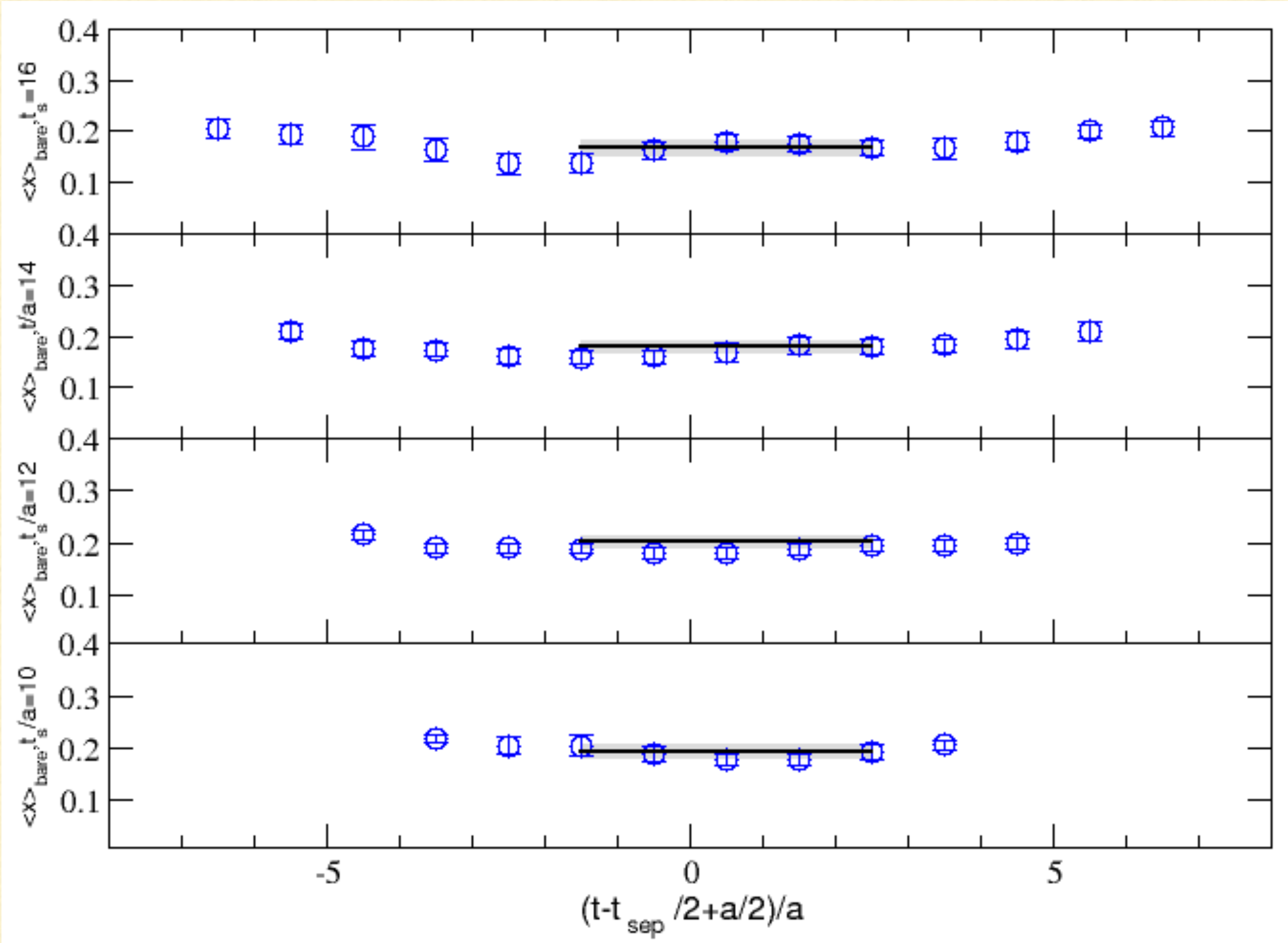
→ Direct comparison with experimental data without renormalization factors.

→ Toward the continuum limit, using both results obtained from 128^4 and 160^4 lattices.

|tsink-tsrc| dependence on $\langle x \rangle_{u-d}$ from 128^4 lattice

|tsink-tsrc|/a	$\langle x \rangle_q^{bare}$
16	0.168(14)
14	0.179(10)
12	0.203(10)
10	0.194(12)

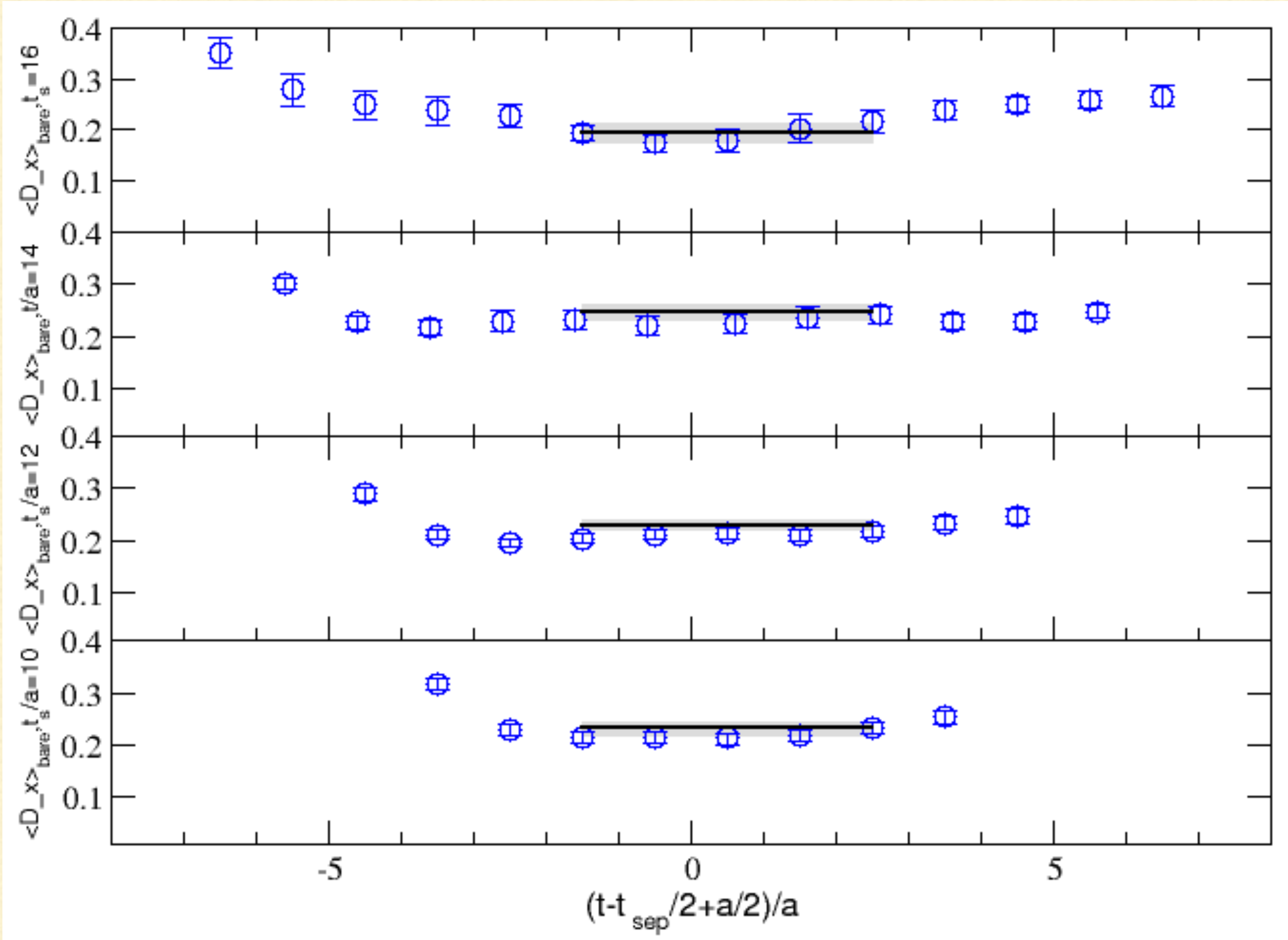
\|tsink-tsrc\|/a $\geq$ 14



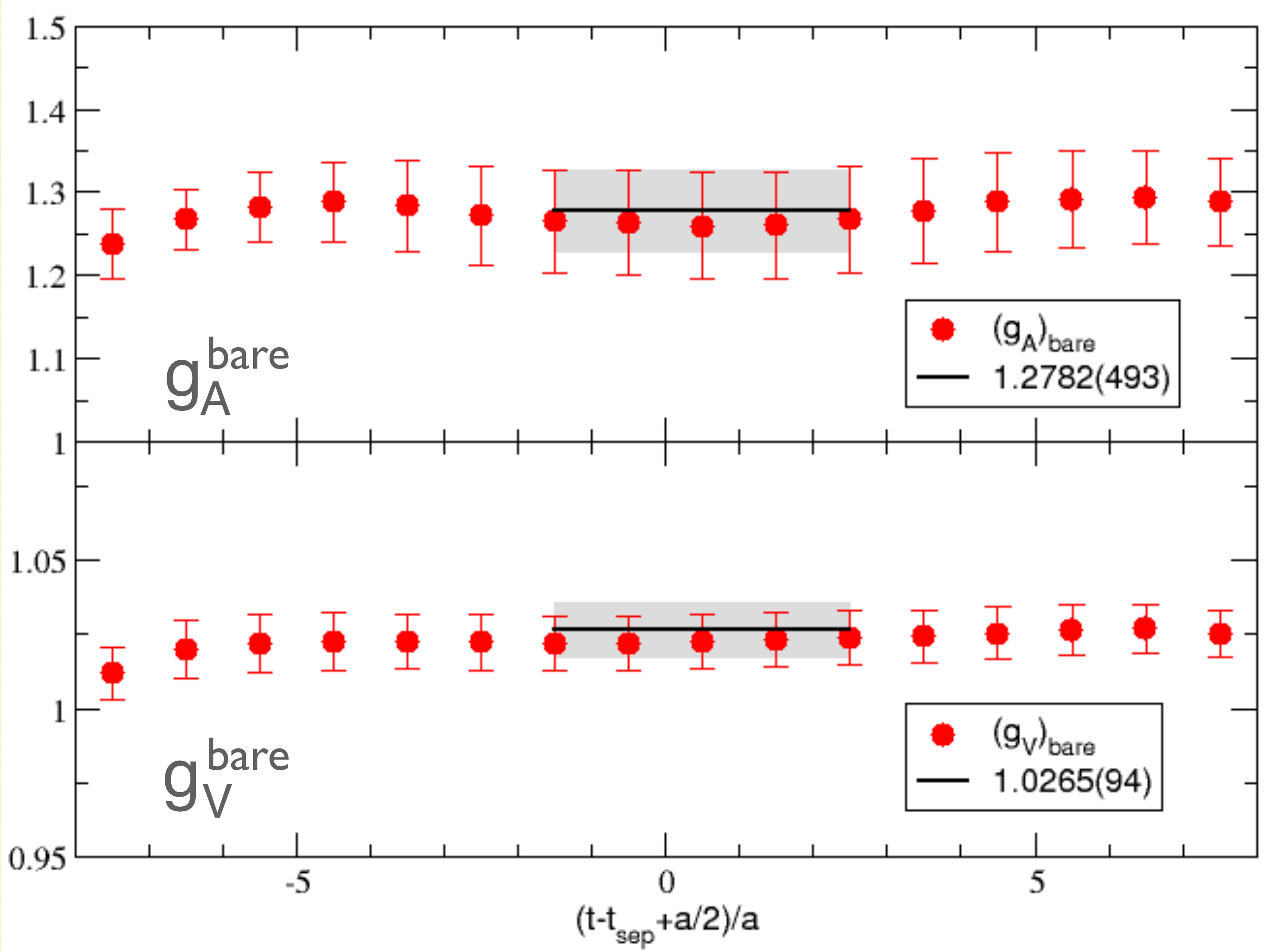
|tsink-tsrc| dependence on $\langle x \rangle_{\Delta u - \Delta d}$ from 128^4 lattice

|tsink-tsrc|/a	$\langle x \rangle^{bare}_{\Delta q}$
16	*0.196(18)
14	0.247(15)
12	0.230(10)
10	*0.232(11)

|tsink-tsrc|/a $\geq$ 12



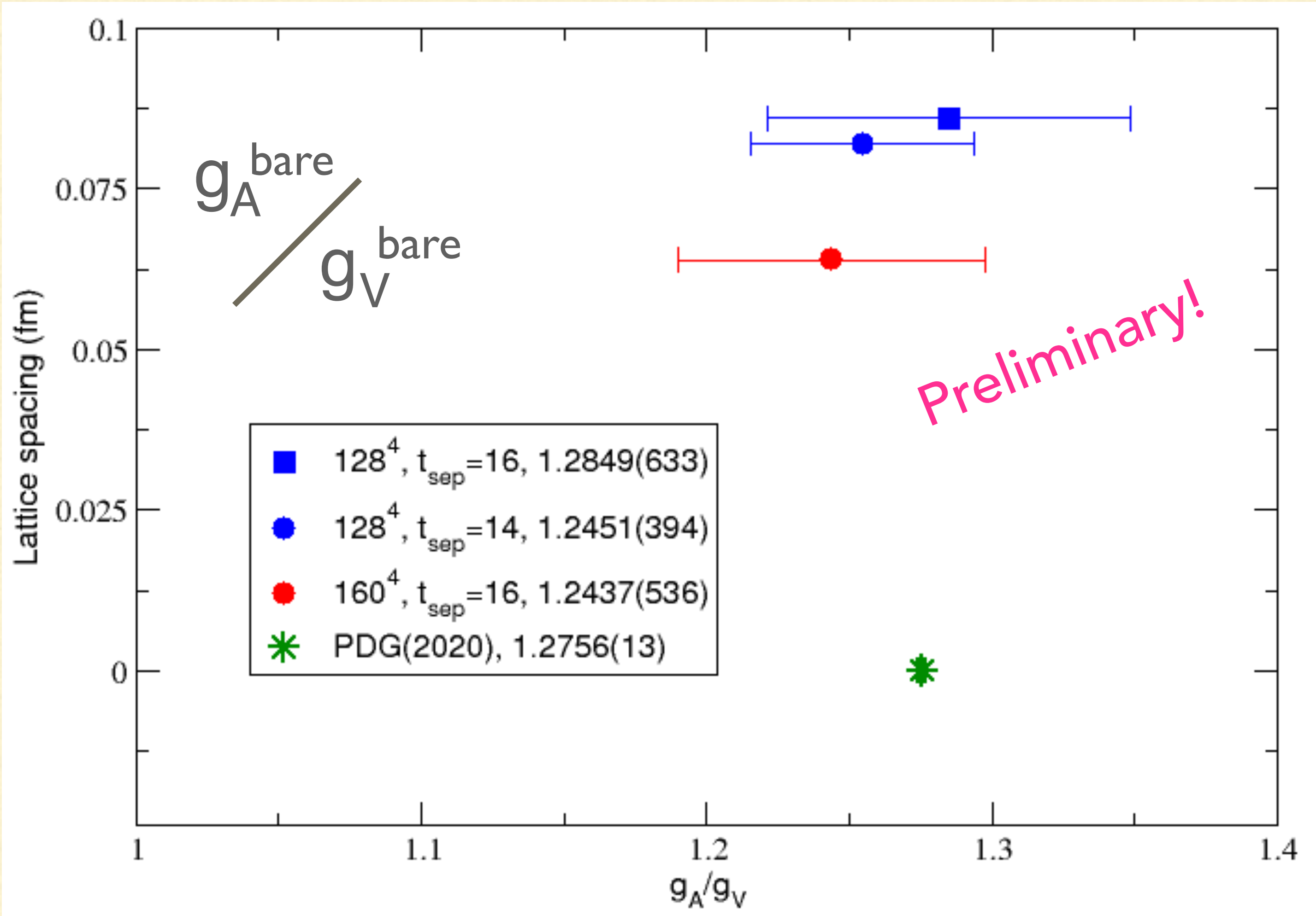
Bare result of g_A and g_V from 160^4 lattice



Good plateau behavior

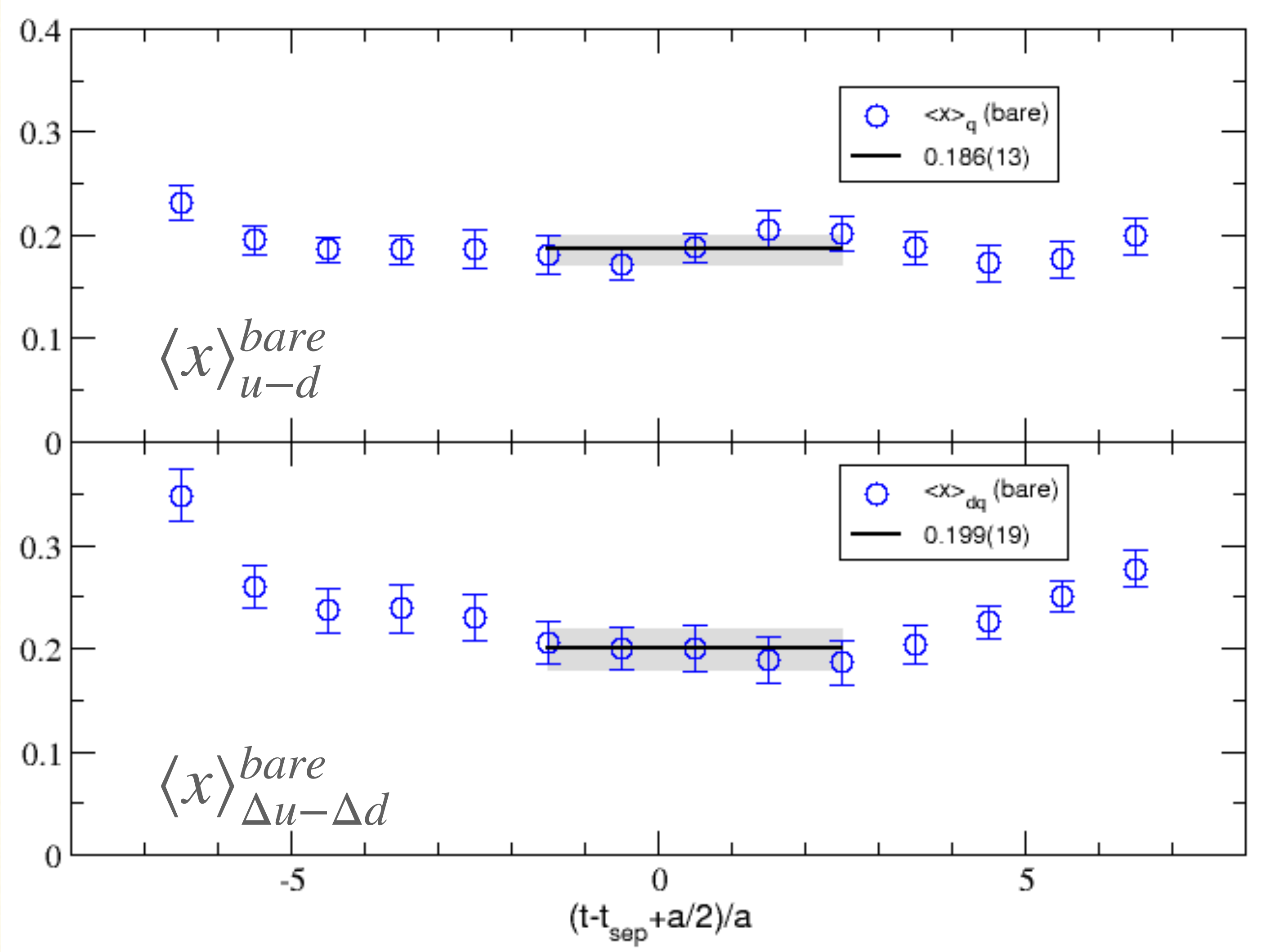
Ratio : g_A / g_V

$|$tsink-tsrc$|/a \gtrsim 14$ on 128^4 $\leftrightarrow$ $|$tsink-tsrc$|/a \gtrsim 16$ on 160^4



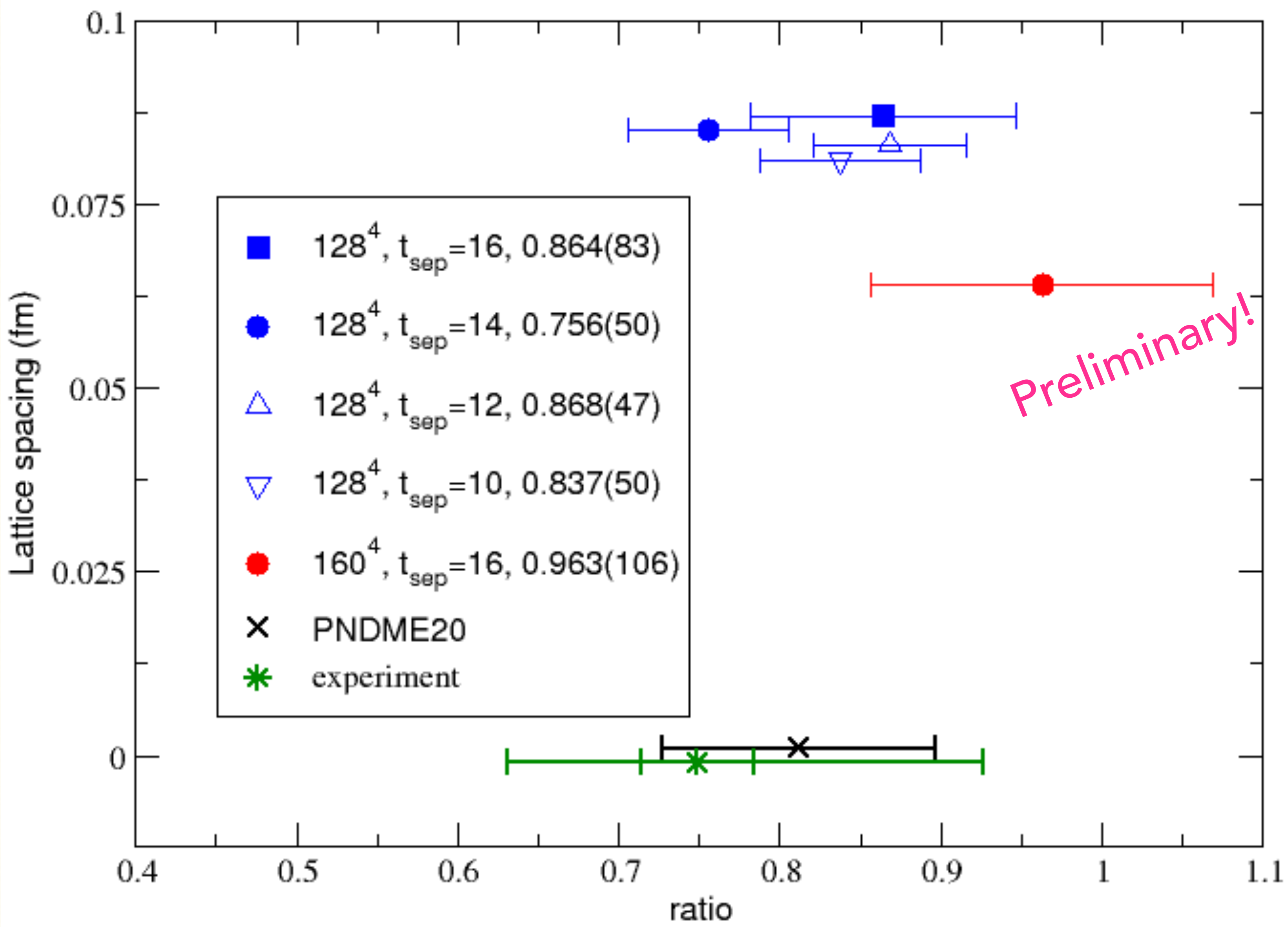
Suggest small discretization errors on g_A

Bare results of $\langle x \rangle_q$ and $\langle x \rangle_{\Delta q}$ from 160^4 lattice



Good plateau behavior

Ratio of $\langle x \rangle_{u-d}$ to $\langle x \rangle_{\Delta u - \Delta d}$



Santanu Mondal et al., arXiv:2005.13779v1 (2020)

Summary

We have studied nucleon structure at the physical point in 2+1 flavor lattice QCD on both 128^4 and 160^4 lattices.

→ Our next targets : $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$

◉ Calculation on 128^4 lattice.

→ |tsink-tsrc|/a $\geq$ 14 for $\langle x \rangle_{u-d}$ is enough.
|tsink-tsrc|/a $\geq$ 12 for $\langle x \rangle_{\Delta u - \Delta d}$

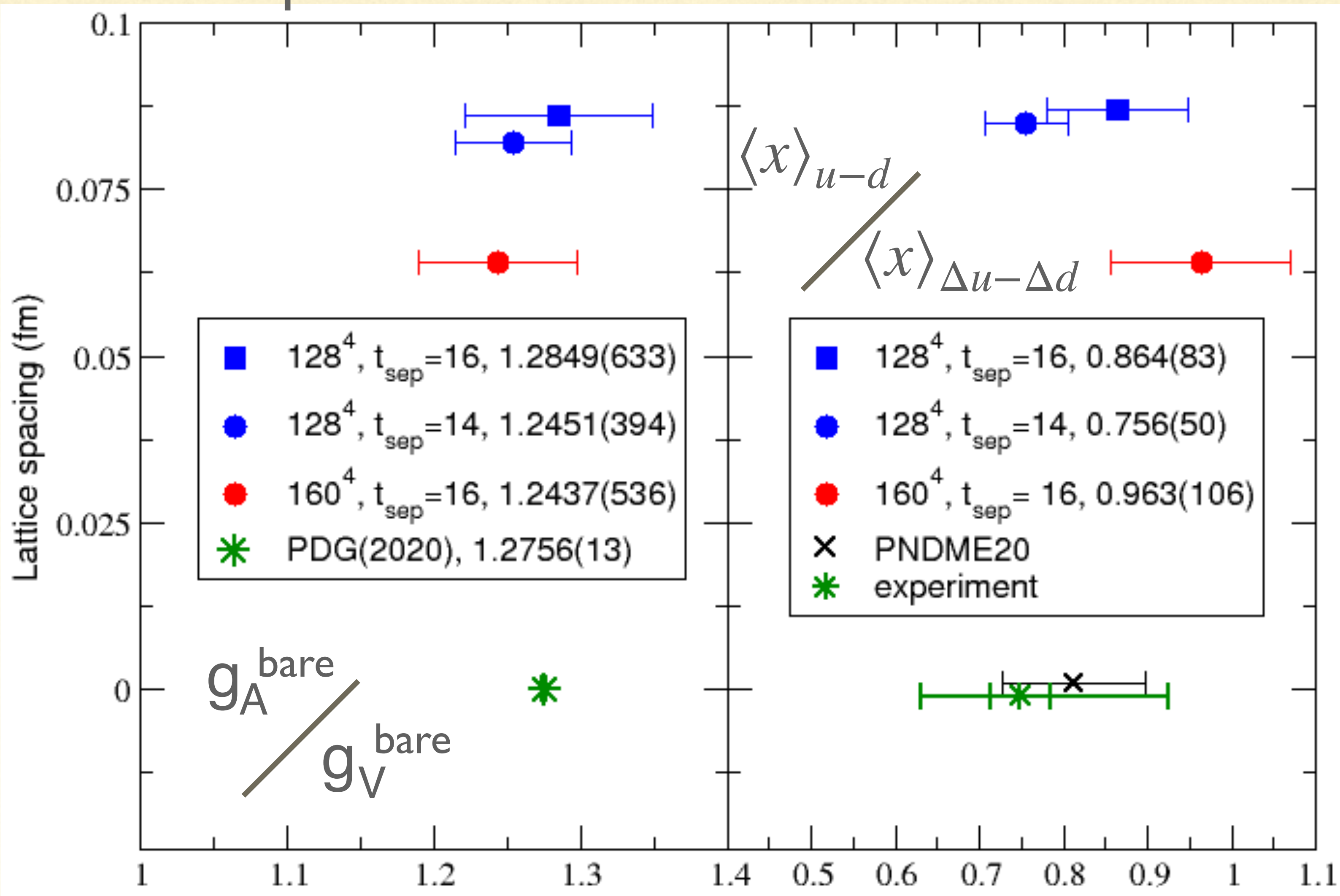
$\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$ are not yet renormalized.

◉ Calculation on 160^4 lattice.

→ Fixed |tsink-tsrc|/a = 16 for g_A ,$\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$
All quantities are not yet renormalized.

- Toward the continuum limit

→ g_A / g_V and $\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$ are consistent with experiments.



BACKUPS

Accuracy required in lattice-QCD

→ Match the accuracy of the 1st moment of unpolarized and polarized PDFs determined from a global fit = a few %.

e.g.) Unpolarized case

	Value	Global analysis	Pert.	α_s	Uncertainty
NNPDF3.1	0.152(3)	✓	✓	✓	✓
CT14	0.158(4)	✓	✓	✓	✓
MMHT2014	0.151(4)	✓	✓	✓	✓
AMBP2016	0.167(4)	✓	×	×	✓
CJ15	0.152(2)	×	×	✓	×
HERAPDF2.0	0.188(3)	×	✓	✓	✓
CT18	0.156(7)	–	–	–	–

H.-W. Lin et al., arXiv:1711.07916v3

Error reduction techniques

- All-Mode-Averaging (AMA)

For original operator $O^{(\mathrm{org})}$, improved one $O^{(\mathrm{imp})}$ is defined as

$$O^{(\mathrm{imp})} = \frac{1}{N_{\mathrm{org}}}\sum_{f\in G}^{N_{\mathrm{org}}}\left(O^{(\mathrm{org})f} - O^{(\mathrm{approx})f}\right) + \frac{1}{N_G}\sum_{g\in G}^{N_G} O^{(\mathrm{approx})g}$$

where G is the lattice symmetry. (e.g. translation symmetry)
$O^{(\mathrm{approx})}$ is relaxed CG solution.

$$O^{(\mathrm{approx})} = O^{(\mathrm{approx})}\left[S_{\mathrm{AM}} = \sum_{i=1}^{N_{\mathrm{eig}}} v_i \frac{1}{\lambda_i} v_i^\dagger + P_n(\lambda)\left(1 - \sum_{i=1}^{N_{\mathrm{eig}}} v_i v_i^\dagger\right)\right]$$

$P_n(\lambda)$ is polynomial approximation of $1/\lambda$

→ Low computational cost
Improved error : $\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}$

Measurements

- 128^4 configuration $\rightarrow \#\ \text{gauge conf.} = 20$

|tsink-tsrc|	N_{org}	N_{G}	# meas.
10	1	128	2,560
12	1	256	5,120
14	2	320	6,400
16	4	512	10,240

- 160^4 configuration $\rightarrow \#\ \text{gauge conf.} = 6 \times 4^*$

|tsink-tsrc|	N_{org}	N_{G}	# meas.
16	1	64	1,536

* Rotations of Lattice

Interpolating operator of Nucleon

The interpolating operator of Nucleon used here are following:

$$N_X(t, \overrightarrow{p}) = \sum_{\overrightarrow{x}\overrightarrow{x_1}\overrightarrow{x_2}\overrightarrow{x_3}} \mathrm{e}^{-i\overrightarrow{p}\cdot\overrightarrow{x}} \varepsilon_{abc} \left[u_a^T(t, \overrightarrow{x_1}) C\gamma_5 d_b(t, \overrightarrow{x_2})\right] u_c(t, \overrightarrow{x_3})$$

$$\times \phi_X(\overrightarrow{x_1} - \overrightarrow{x})\phi_X(\overrightarrow{x_2} - \overrightarrow{x})\phi_X(\overrightarrow{x_3} - \overrightarrow{x})$$

where

$$\phi_X(\overrightarrow{x_i} - \overrightarrow{x}) = \begin{cases} \phi_L(\overrightarrow{x_i} - \overrightarrow{x}) = \delta(\overrightarrow{x_i} - \overrightarrow{x}) & \text{:Local type} \\ \phi_S(\overrightarrow{x_i} - \overrightarrow{x}) = A\exp(-B\,|\,\overrightarrow{x_i} - \overrightarrow{x}\,|) & \text{:exp. smeared} \end{cases}$$

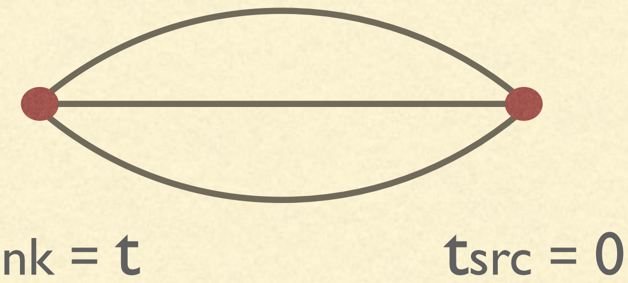
and the parameters described here are $(A, B) = (1.2, 0.16)$

2pt. and 3pt. function

- 2pt. function

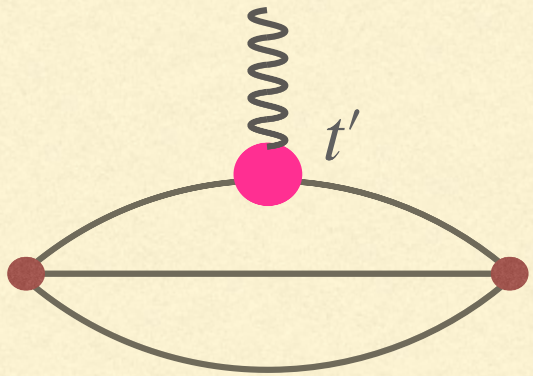
$$\langle N(t)N^\dagger(0)\rangle = \sum_i |\langle 0|N(0)|i\rangle|^2 \mathrm{e}^{-E_i t}$$

$$\underset{t \gg 0}{\rightarrow} |\langle 0|N(0)|N\rangle|^2 \mathrm{e}^{-M_N t}$$



- 3pt. function

$$\langle N(t)\underline{O(t')}N^\dagger\rangle = \sum_{ij} \langle 0|N(0)|i\rangle\langle i|O(0)|j\rangle\langle j|N^\dagger|0\rangle \mathrm{e}^{-E_i t}$$



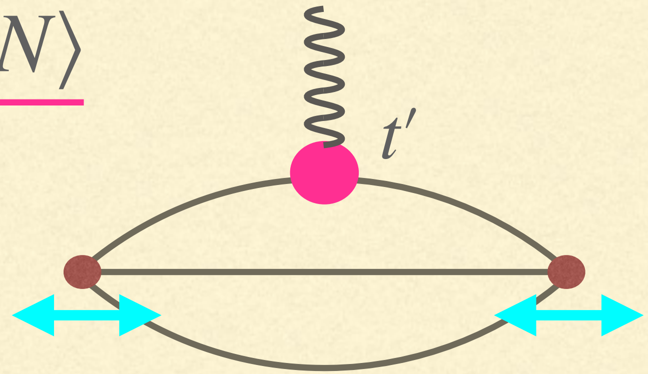
$$\underset{t \gg t' \gg 0}{\rightarrow} |\langle 0|N(0)|N\rangle|^2 \underline{\langle N|O(0)|N\rangle} \mathrm{e}^{-M_N t}$$

Nucleon matrix element

Extracting matrix elements

Nucleon matrix elements can be extracted from the ratio of 3pt. function to 2pt. function

$$\frac{\langle N(t)O(t')N(0)^\dagger\rangle}{\langle N(t)N(0)^\dagger\rangle} \xrightarrow[t \gg t' \gg 0]{} \langle N|O(0)|N\rangle$$



N.B. Excited state contamination

Interpolating operator also creates excited states.

$$\frac{\langle N(t)O(t')N(0)^\dagger\rangle}{\langle N(t)N(0)^\dagger\rangle} \xrightarrow[\require{cancel}\cancel{t \gg t' \gg 0}]{} \langle N|O(0)|N\rangle + A\mathrm{e}^{-E(t'-t)} + \cdots$$

→ Gaze |t_{sink}-t_{src}| independence = confirm no contamination

Chiral symmetry $\rightarrow$ explicitly broken due to $O(a)$ errors

$\rightarrow$ 160^4 results show smaller values

$$i\bar{q}\left[\gamma_3 \overleftrightarrow{D}_4 - \gamma_4 \overleftrightarrow{D}_3\right] q$$

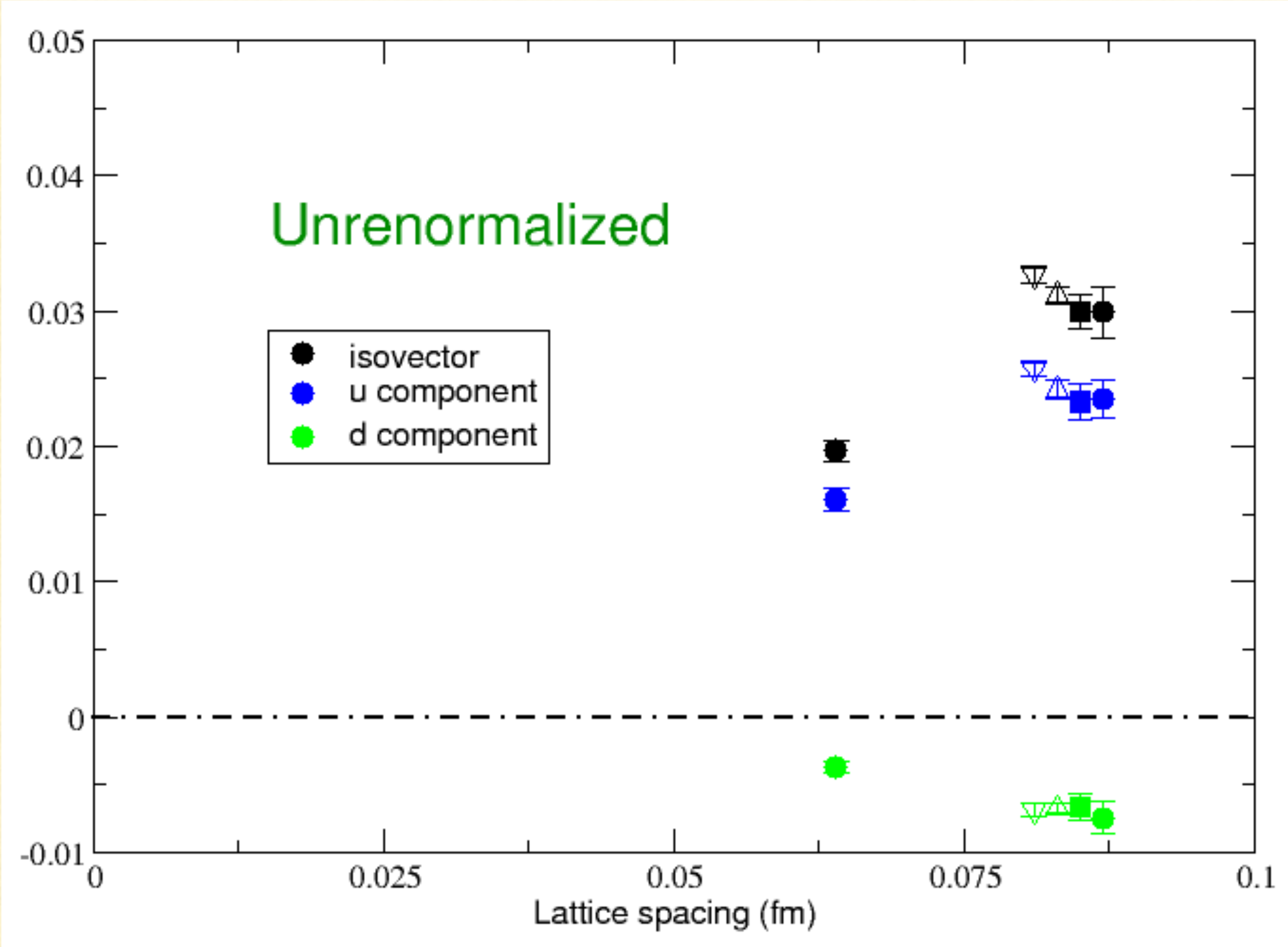
$$\downarrow$$

$$\langle N | O | N \rangle = 0$$

Under chiral sym.

However,

$$\langle N | O | N \rangle \neq 0$$



Ratio

In the chiral and continuum limit , we expect

$$Z_A = Z_V$$

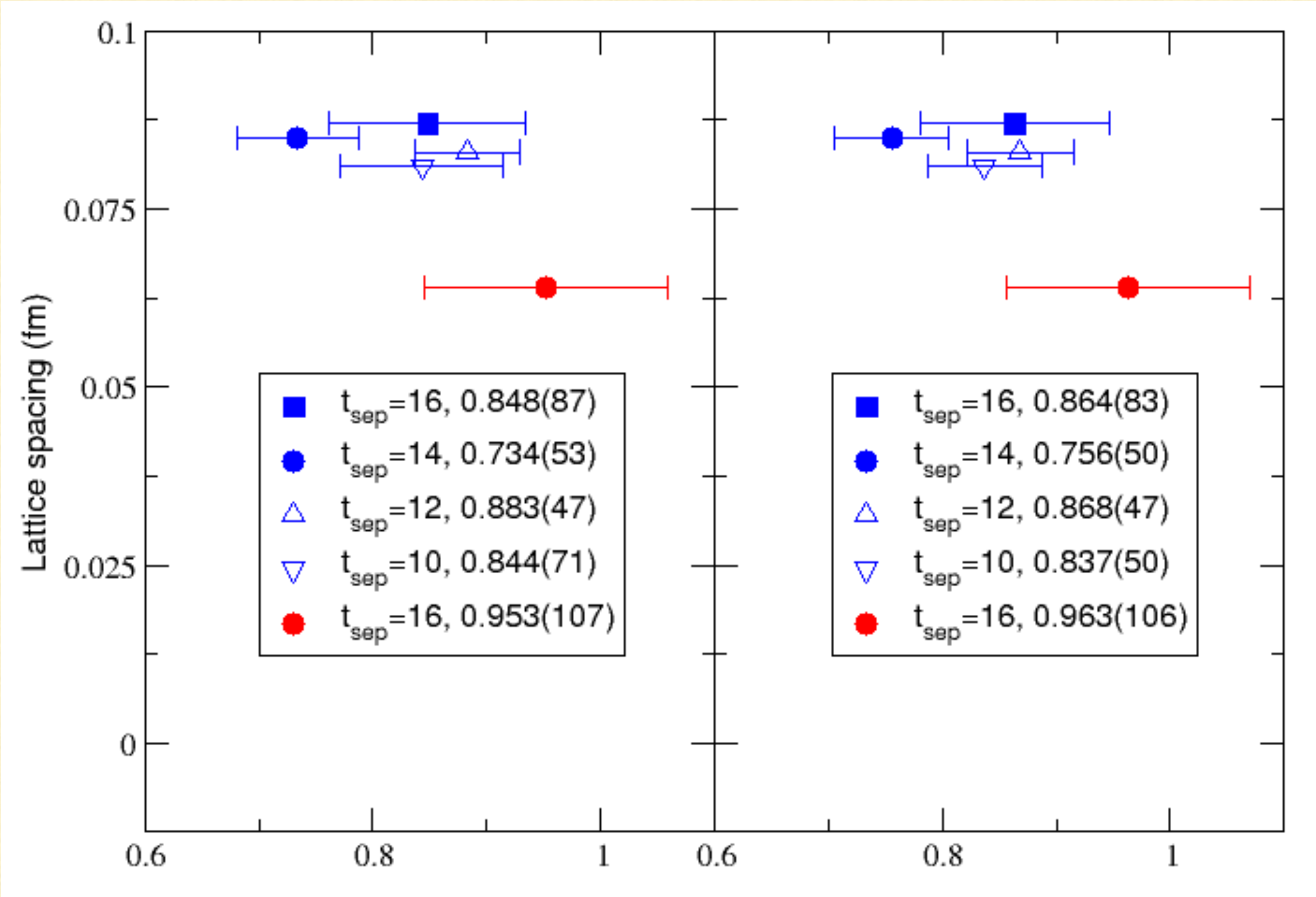
→ The following ratio is the renormalization independent quantity

$$\frac{g_A}{g_V} = \frac{Z_A g_A^{bare}}{Z_V g_V^{bare}} = \frac{g_A^{bare}}{g_V^{bare}}$$

This is similar for twist2 case as $Z_{\langle x \rangle_q} = Z_{\langle x \rangle_{\Delta q}}$

$$\rightarrow \quad \frac{\langle x \rangle_q}{\langle x \rangle_{\Delta q}} = \frac{Z_{\langle x \rangle_q} \langle x \rangle_q^{bare}}{Z_{\langle x \rangle_{\Delta q}} \langle x \rangle_{\Delta q}^{bare}} = \frac{\langle x \rangle_q^{bare}}{\langle x \rangle_{\Delta q}^{bare}}$$

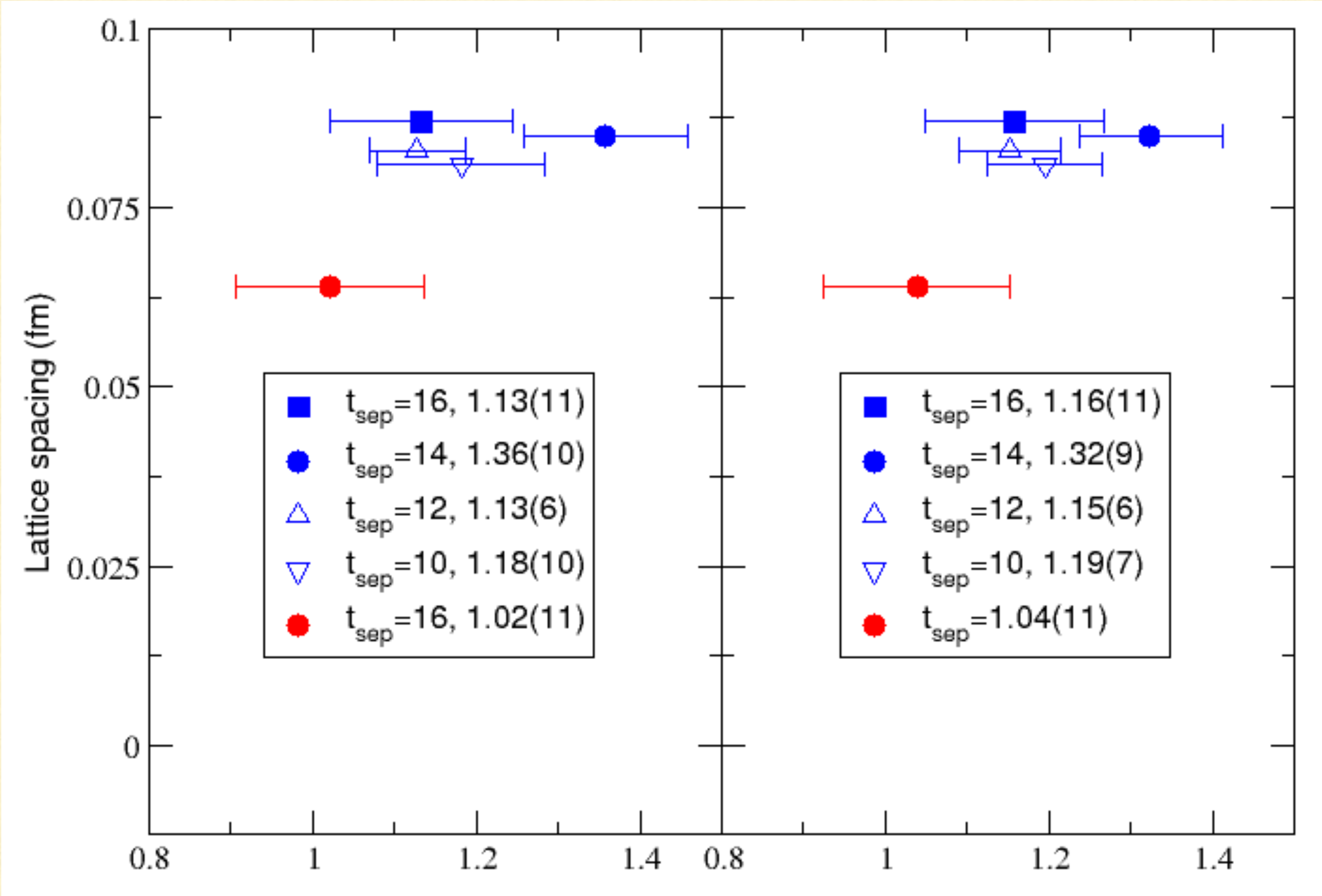
$\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$



Before FIT

After FIT

$\langle x \rangle_{\Delta u - \Delta d} / \langle x \rangle_{u-d}$



Before FIT

After FIT