Long distance contributions to $0\nu 2\beta$ decays in pion sector

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Asia-Pacific Symposium for
Lattice Field Theory (APLAT 2020)
Aug. 4th – Aug. 7th 2020

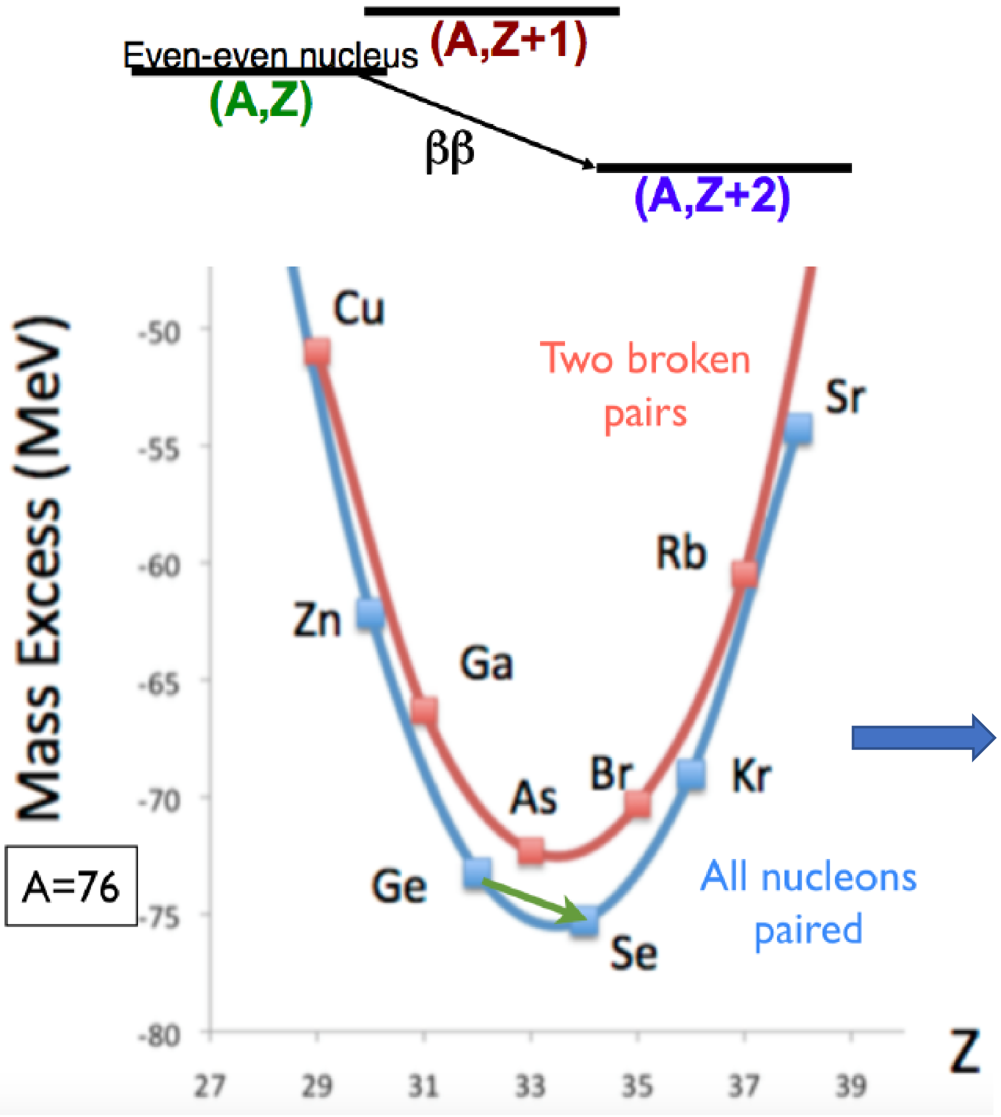
Layout

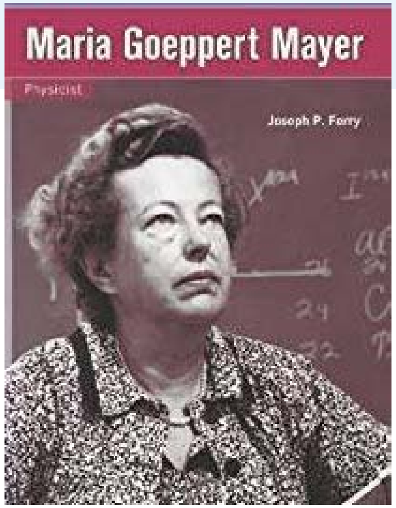
1 Background

2 Lattice calculation of $0\nu 2\beta$ decays

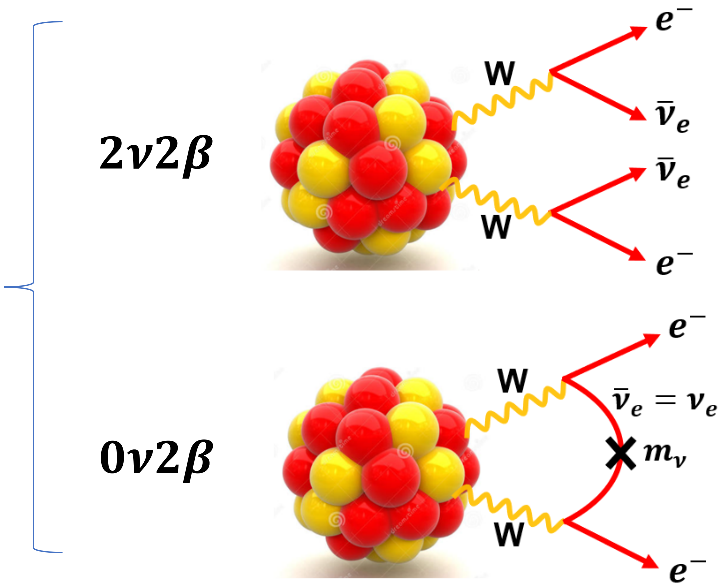
3 Conclusion

1. Background





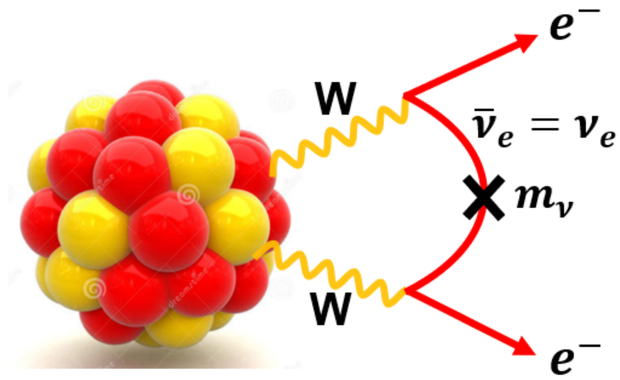
1935, Double beta decay

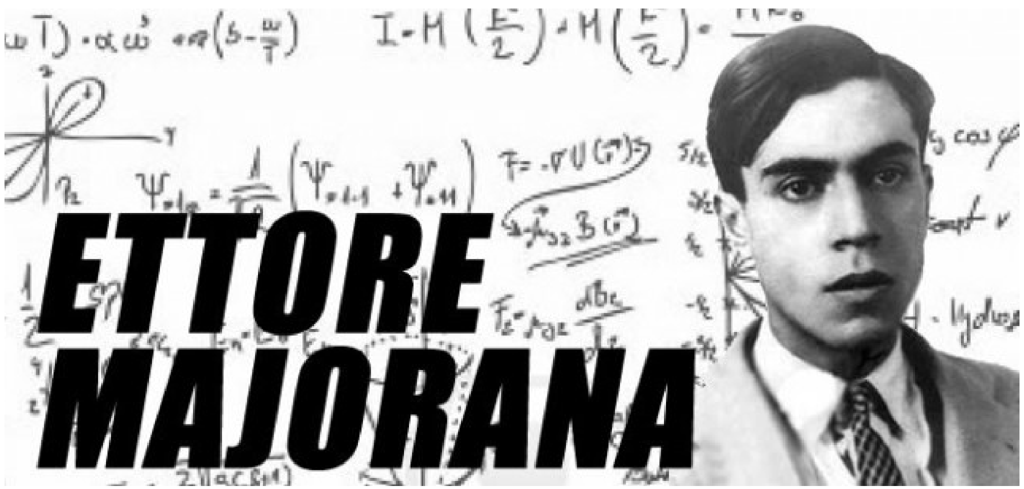


Why are $0\nu2\beta$ decays important?

(1). Understanding Neutrinos:
- Majorana fermions or Dirac fermion?
- Absolute mass scale
- Hierarchy order

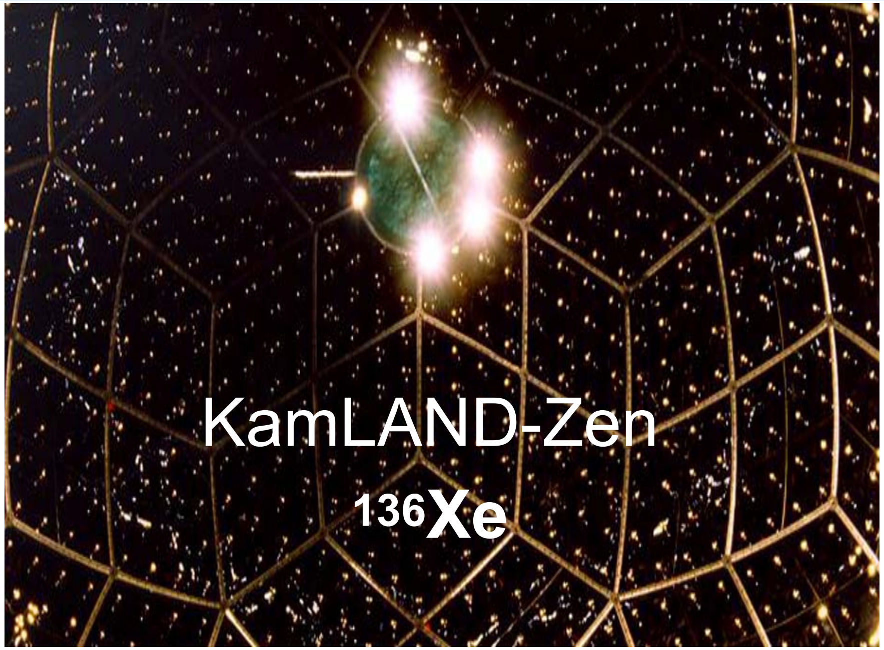
(2). BSM: lepton-number violation (LNV)





1937: $\nu = \bar{\nu}$?

Experiments



Experiments

Collaboration	Element	$T_{1/2}/$ 10^{25} years
Gerda	Ge	>9
MAJORANA	Ge	>2.7
CUPID-0	Se	>0.24
CUORE	Te	>1.5
EXO-200	Xe	>1.8
KamLAND-Zen	Xe	>10.7

1. GERDA Collaboration (M. Agostini et al.) , Science 365 (2019) 1445
2. S. I. Alvis et al., Phys. Rev. C 100, 025501 (2019)
3. O. Azzolini et al., Phys. Rev. Lett. 120, 232502 (2018)
4. C. Alduino et al., Phys. Rev. Lett. 120, 132501 (2018)
5. J. B. Albert et al., Phys. Rev. Lett. 120, 072701 (2018)
6. A. Gando et al., Phys. Rev. Lett. 117, 082503 (2016)

Theoretical study

Progress and Challenges in Neutrinoless Double Beta Decay

ECT* workshop subscription

ECT*, Strada delle Tabarelle, 286, Villazzano, 38123 Trento, Italy

Monday, 15 July 2019 at 08:00 - Friday, 19 July 2019 at 18:00 (CEST)

Summarizes on recent advances:

Connecting very high BSM scale to nuclear scale, through a chain of different theories. (SM-EFT, Lattice QCD, ChEFT, many body theory……)

Lattice QCD plays an important role in this chain.

V. Cirigliano, W. Dekens, J. de Vries,M. L. Graesser, E. Mereghetti(2019) arXiv:1806.02780

The role of Lattice QCD

V. Cirigliano, W. Dekens, J. de Vries,M. L. Graesser, E. Mereghetti(2019) arXiv:1806.02780

Lattice QCD:

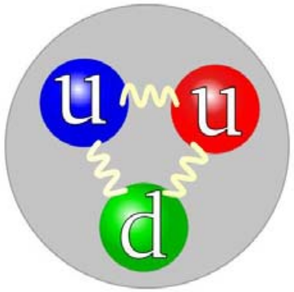
Calculate matrix element $\langle ppee|H_W(x)H_W(0)|nn\rangle$ from first-principle theory (QCD)

ChEFT:

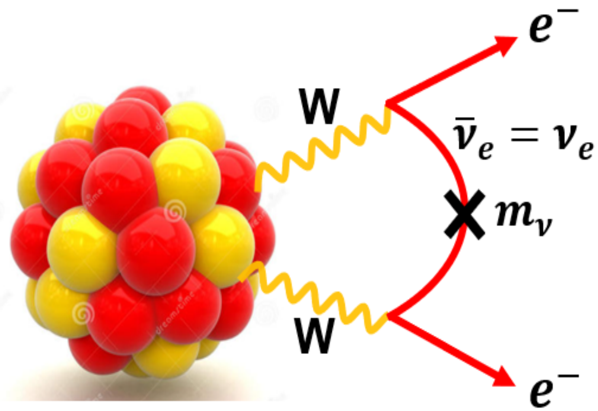
Match ChEFT with LQCD to get low energy constants(LECs), and then derive the two-body nucleon operators.

Many body theory:

Use two body operators as input for a many-body nuclear matrix element.



Quark scale (QCD)

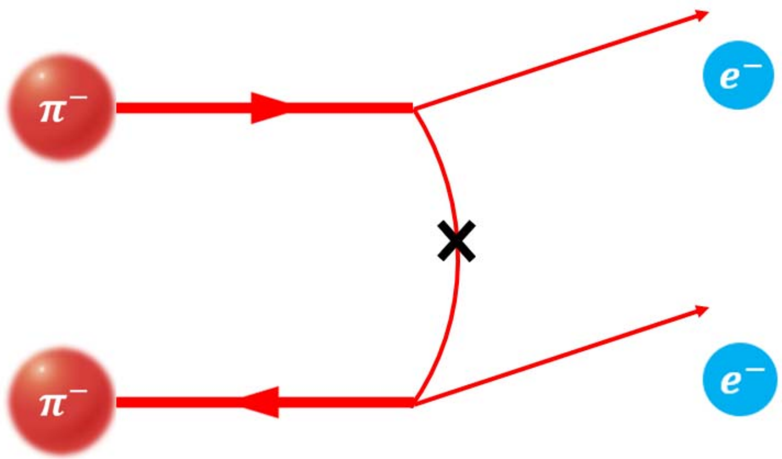


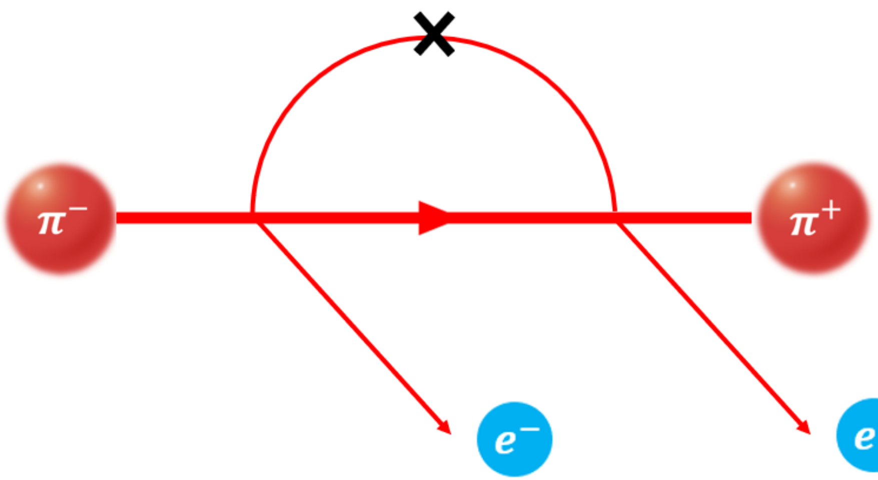
Nuclear scale

2 LQCD calculation: $0\nu2\beta$ decays in pion sector

$0\nu2\beta$ decays in pion sector

- $\pi\pi \to ee$
  (X. Feng, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001, arXiv:1809.10511)
- $\pi^- \to \pi^+ ee$
  (X. Tuo, X. Feng, L. Jin, PRD100 (2019) 094511, arXiv:1909.13525)





Master formula ($\pi^- \to \pi^+ ee$)

Decay amplitude:

$$\mathcal{A} = \langle F, e_1, e_2 | \mathcal{H}_{\text{eff}} | I \rangle.$$

Divided into hadronic part and leptonic part:

$$\mathcal{H}_{\text{eff}} = H_{\mu\nu}(x) L_{\mu\nu}(x),$$

Master formular:

$$\mathcal{A} = -2T_{lept} \int d^4x \, H(x) S_0(x)$$

$H(x) = \langle \pi^+ | T\{J_{\mu L}(x) J_{\mu L}(0)\} | \pi^- \rangle$ → Lattice QCD

$S_0(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{iqx}}{q^2}$ → Massless propagator

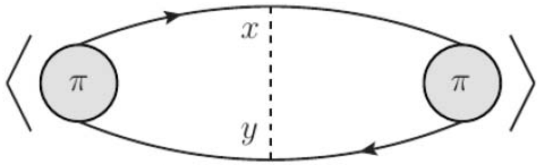
T_{lept} → Leptonic factor

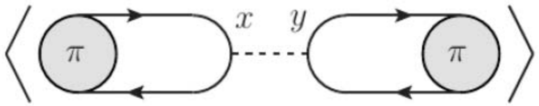
Master formular

$$\mathcal{A} = -2T_{lept}\int d^4x\, H(x)S_0(x)$$

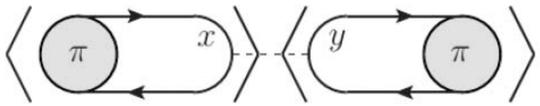
$H(x)$ can be calculated on lattice:

$$\mathcal{H}(x-y) = \frac{C(t_f,x,y,t_i) - C_0(t_f,x,y,t_i)}{C_\pi(t_f,t_i)}$$





Type 1 **Type 2**



Vacuum

Subtraction term of vacuum is the LO term (trivial) in ChEFT:

$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1 + \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + 6 + \frac{5}{6}g_\nu^{\pi\pi}(\mu)\right)$$

vacuum contribution

(Cirigliano, Dekens, Mereghetti, Walker-Loud, PRC97 (2018) 065501)

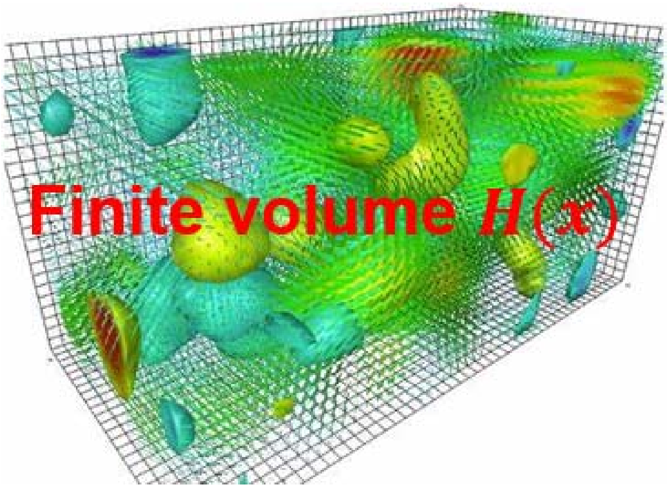
Major difficulty: Finite volume effects

$$\mathcal{A} = -2T_{lept} \int d^4x \, H(x) S_0(x)$$

$H(x)$: From finite volume lattice

$S_0(x)$: Long distance propagator is far beyond the range of lattice.

→ Big FV effects $\sim O(\frac{1}{L^n})$



$\neq$

Traditional method: QED_L

Z. Davoudi, M. Savage Phys. Rev. D 90, 054503 (2014)

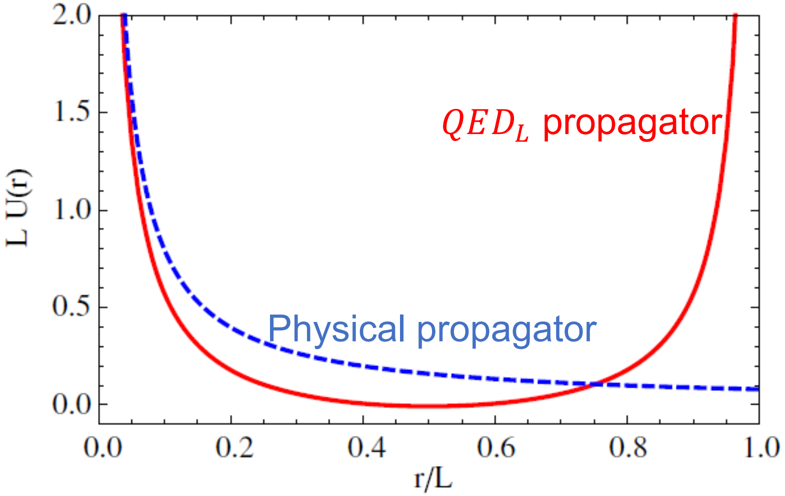
Use propagator without zero mode:

$$S_{lat}(x) = \frac{1}{VT}\sum_{p_0}\sum_{\vec{p}\neq\vec{0}} \hat{S}_{lat}(p)e^{ipx} \text{ with } \hat{S}_{lat}(p) = \frac{1}{\sum_i \hat{p}_i^2}$$

Then calculate the correction term analytically:

$$A = A_{QED_L} + \delta_{QED_L}(L) = -\frac{1}{F_\pi^2}\int_{VT} d^4x\, H_{lat}(x)S_{lat}(x) + \delta_{QED_L}(L),$$

$S_{lat}(x)$ and $S_0(x)$ are very different:

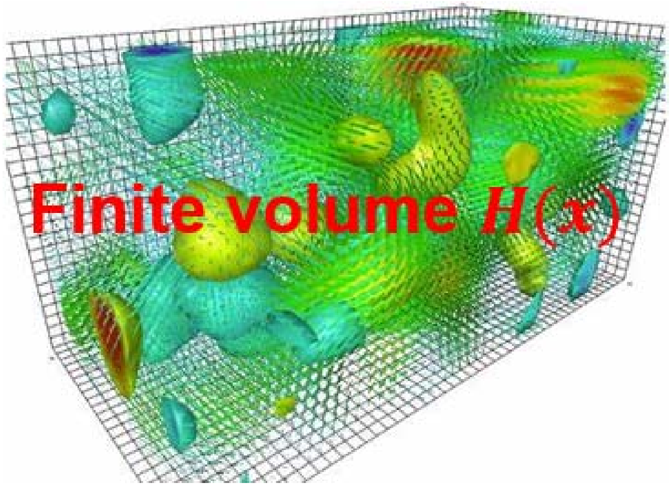


$$\delta_{QED_L}(L) \sim O(\frac{1}{L^n})$$

New method IVR: Infinite volume reconstruction

$$\mathcal{A} = -2T_{lept} \int d^4x \, H(x) S_0(x)$$

FV effects come from hadronic function:



$$\neq$$

So we do not change leptonic propagator,

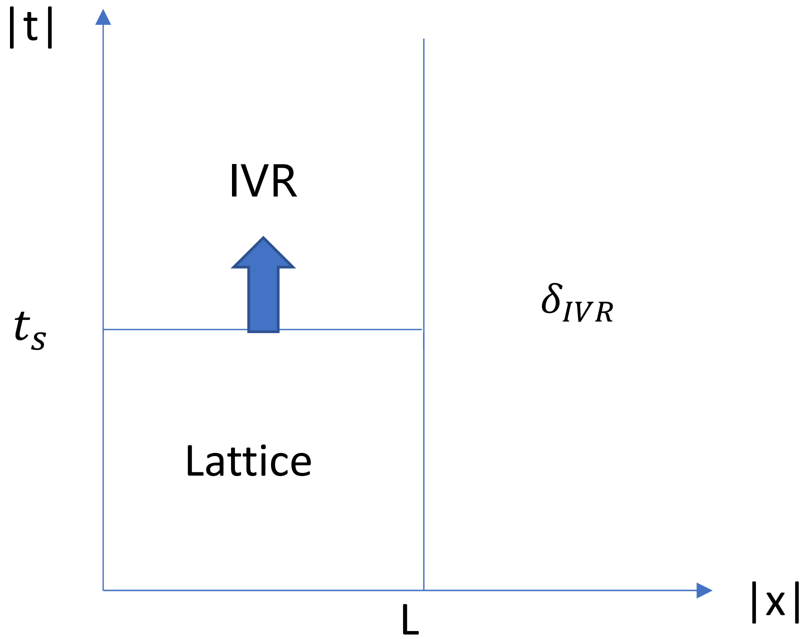
instead we correct hadronic function.

New method IVR: Infinite volume reconstruction

IVR method：

Divide spacetime into two regions: inside the box and outside the box.

Using ground state dominance, $H(x)$ outside the box can be reconstructed by lattice data.



$$\mathcal{A} = -2T_{lept} \int d^4x\, H(x) S_0(x)$$

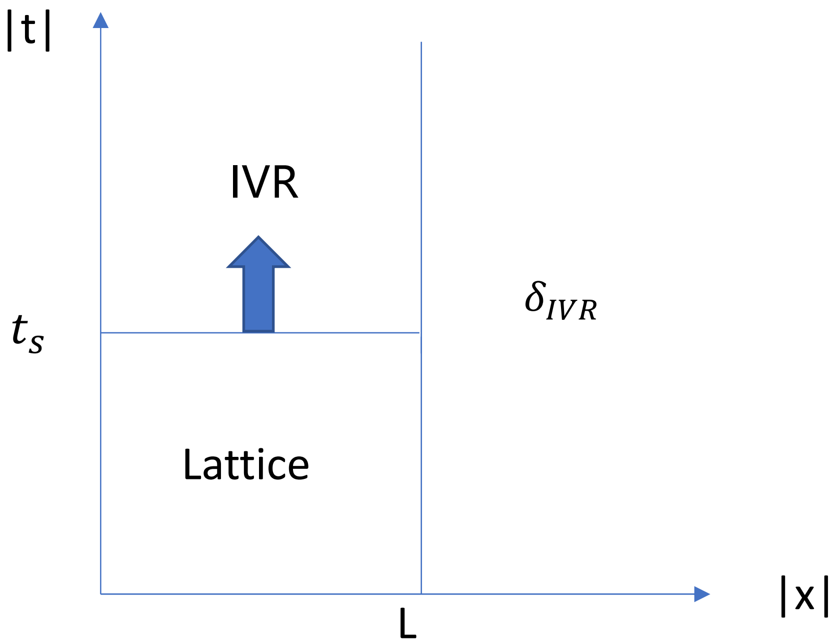
X. Feng, L. Jin, PRD100 (2019) 094509, arXiv:1809.10511

New method IVR: Infinite volume reconstruction

$$A_{IVR} = \int_{|t|<t_s} d^4x\, H(x)S(x) + \int_{|t|>t_s} d^4x\, H(x)S(x)$$

Lattice data

Ground state dominance:
$H(t > t_s)$ can be reconstructed
by $H(t = t_s)$



$$\mathcal{A} = -2T_{lept} \int d^4x\, H(x)S_0(x)$$

New method IVR: Infinite volume reconstruction

Advantages of IVR:

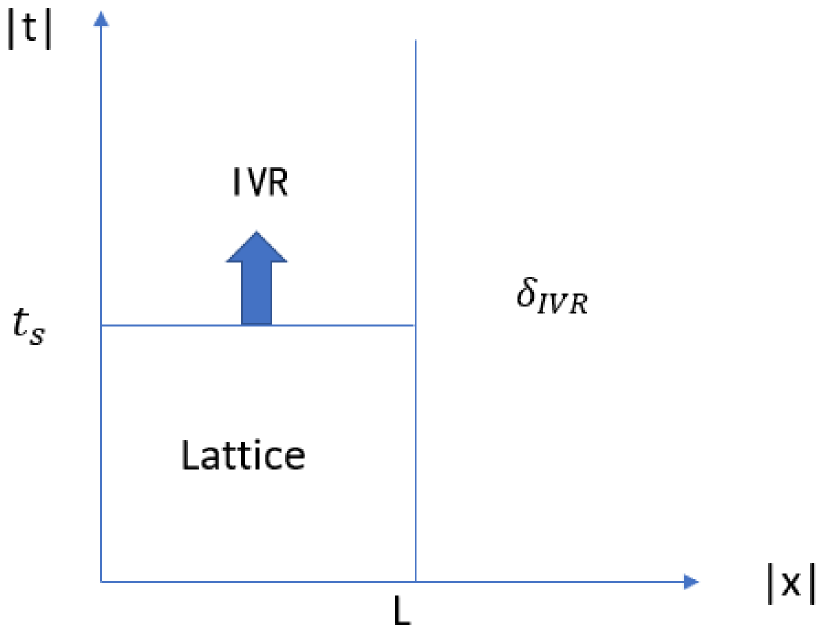
1.Smaller FV effects

$$A = A_{IVR} + \delta_{IVR}(L)$$

$$\delta_{IVR}(L) \sim O(e^{-mL}) \quad \ll \quad \delta_{QED_L}(L) \sim O(\frac{1}{L^n})$$

2. $\delta_{IVR}(L)$ can be determined by π form factor easily.

$$\delta_{IVR}(L) \approx \delta^{\pi}_{IVR}(L) = A^{\pi}_{IVR} - A^{\pi}_{IVR}(L)$$

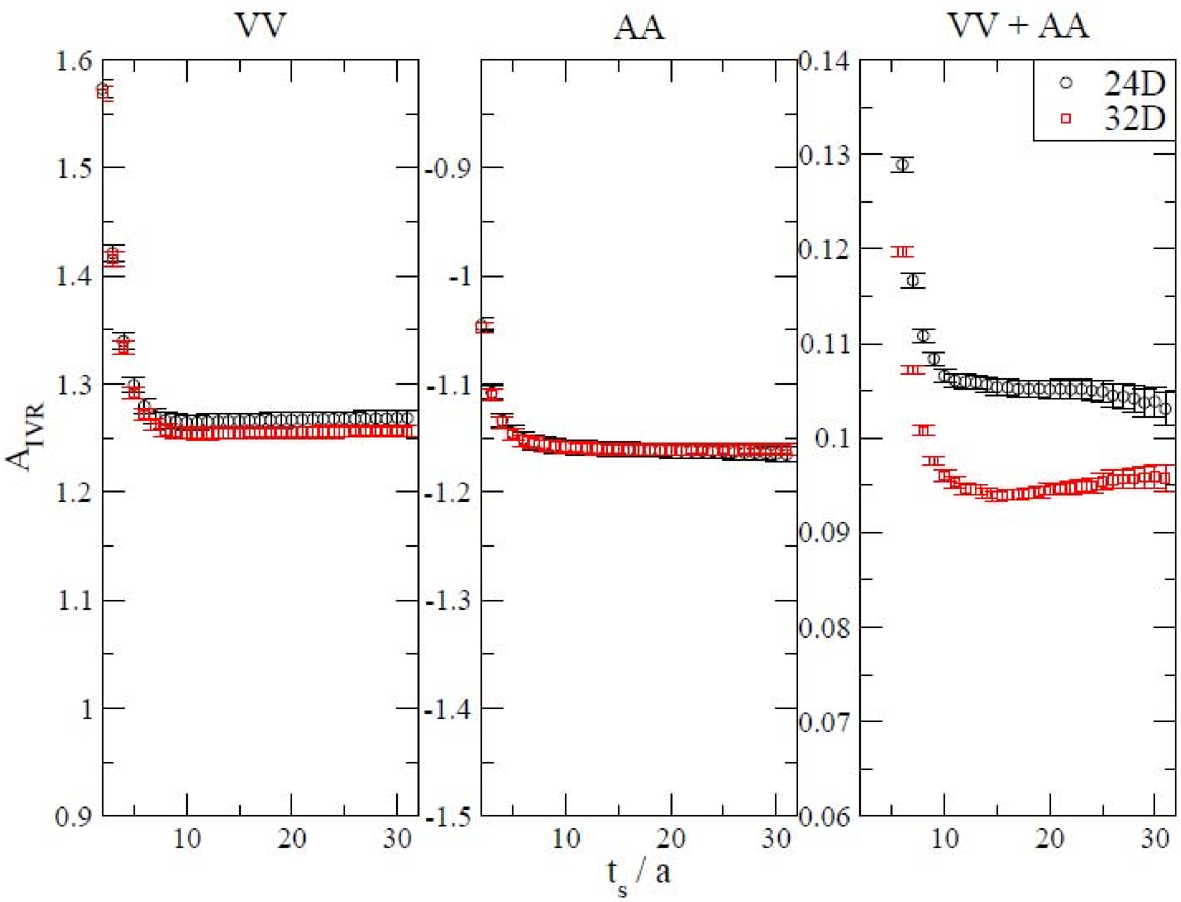


Enhancement of FV effects due to VV+AA

In $\pi^- \to \pi^+ ee$, FV effects is enhanced: cancellation of VV and AA part.

Although $\delta_{IVR}(L) \sim O(e^{-mL})$ and not sizable in VV, it's sizable in VV+AA.

We should also consider $\delta_{IVR}(L)$!



IVR result:
L=24 and L=32

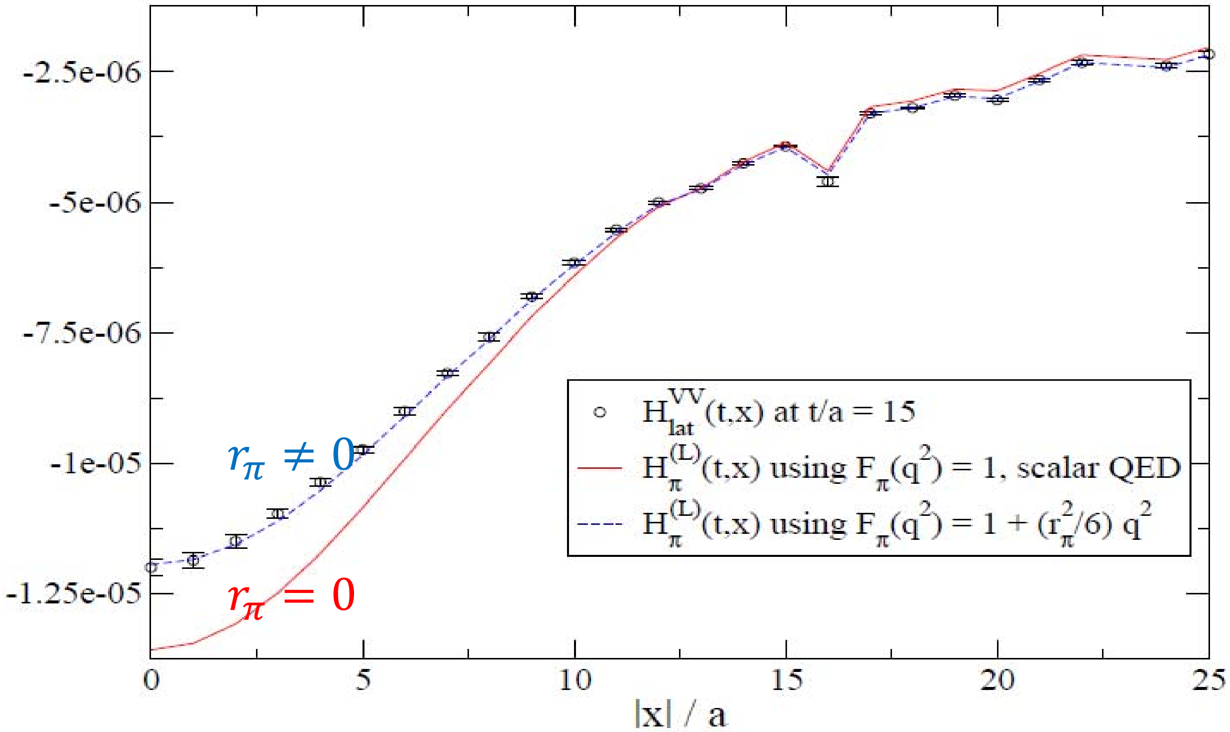
New method IVR: Infinite volume reconstruction

In IVR method, $\delta_{IVR}(L)$ can be determined by π state easily.

Long distance physics dominate FV effects, and can be well described by π state in finite volume.

π state in finite volume L:

$$H_\pi^{(L)}(x) = -\frac{1}{L^3}\sum_{\vec{p}} \frac{1}{2E_{\pi,\vec{p}}} m_\pi (m_\pi + E_{\pi,\vec{p}}) \left[F_\pi(q^2)\right]^2 e^{i\vec{p}\cdot\vec{x}} e^{-(E_{\pi,\vec{p}}-m_\pi)|t|}.$$



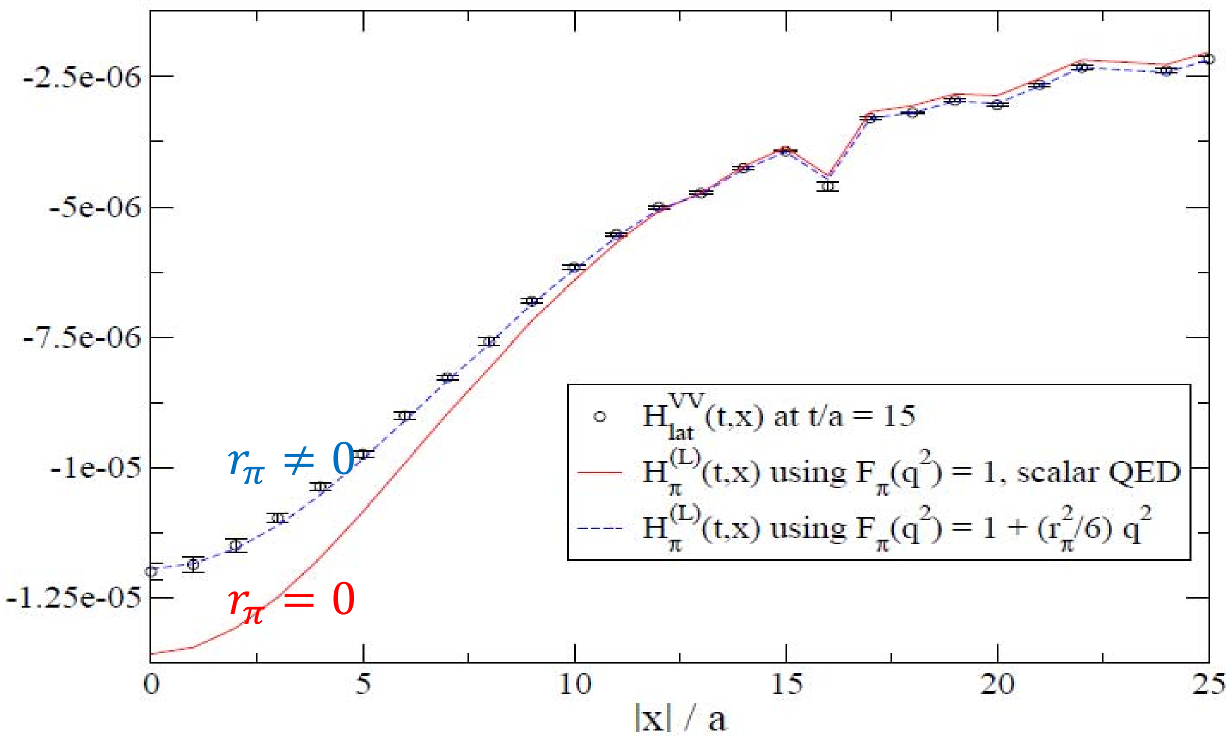
Lattice data
vs.
π state in finite volume:

consistent

New method IVR: Infinite volume reconstruction

In IVR method, $\delta_{IVR}(L)$ can be determined by π state analytically:

$$\delta_{IVR}(L) \approx \delta_{IVR}^{\pi}(L) = \underbrace{A_{IVR}^{\pi}(L \to \infty) - A_{IVR}^{\pi}(L)}_{\text{Analytical calculation}}$$



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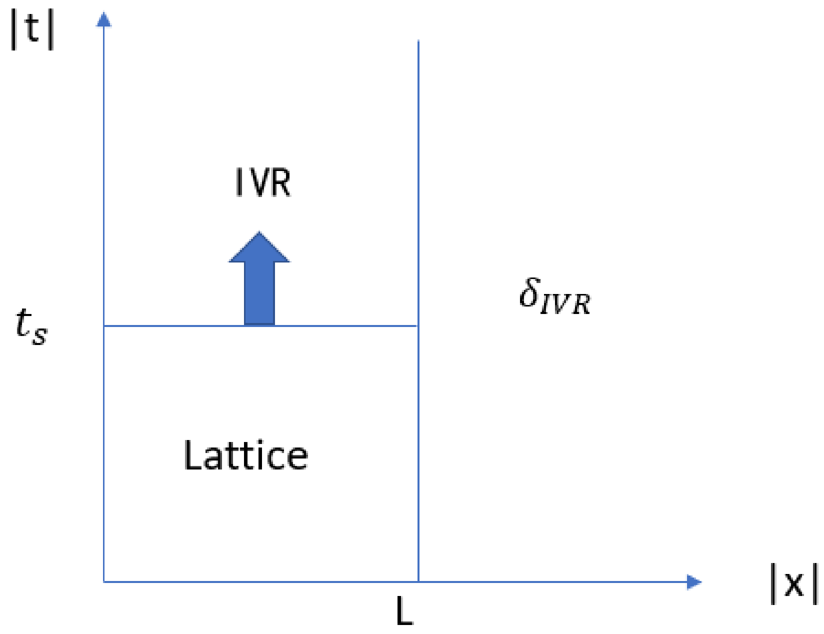
Analytical calculation

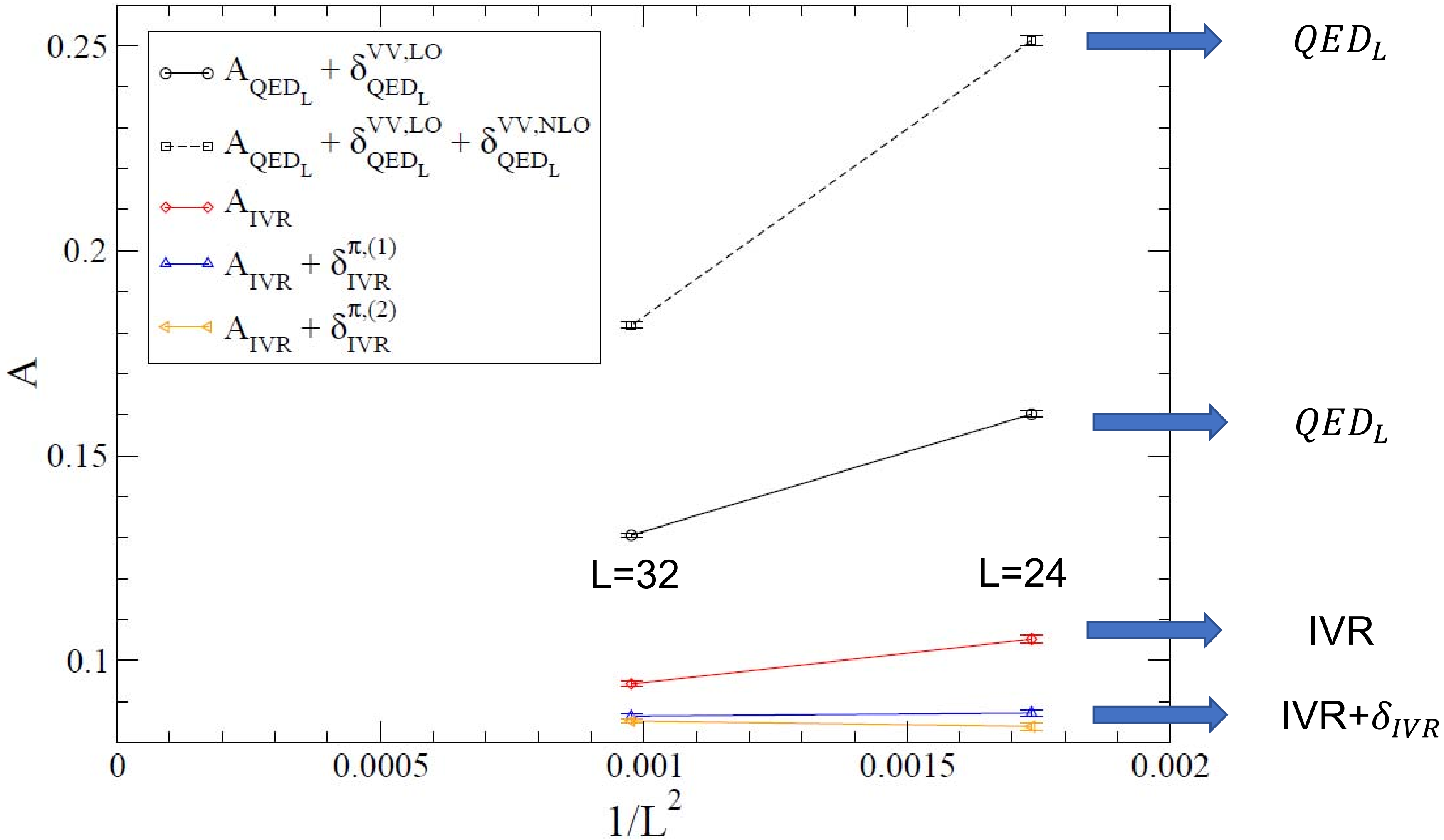


Correct hadronic function:

1. IVR
2. IVR+δ_{IVR}

How does it work?

Compare FV effects of different methods

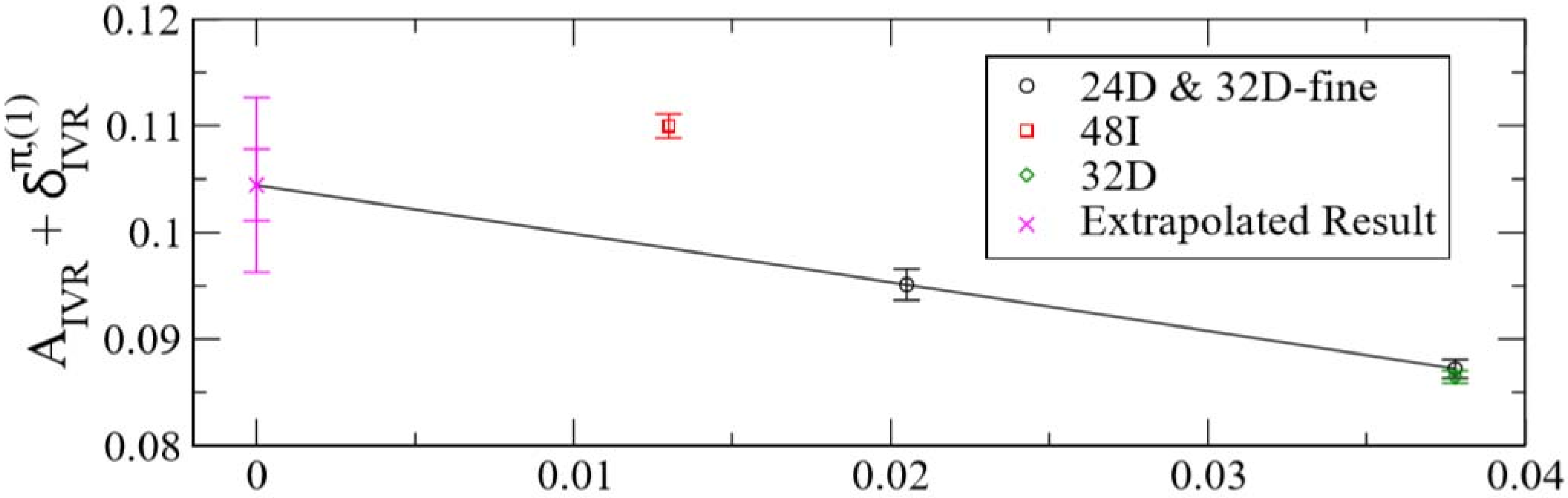


Lattice setup

Ensemble	m_π [MeV]	$L^3 \times T$	a^{-1} [GeV]	N_{conf}	$m_\pi L$	$\Delta T/a$
24D	142	$24^3 \times 64$	1.015	91	3.3	8
32D	142	$32^3 \times 64$	1.015	56	4.5	8
32D-fine	143	$32^3 \times 64$	1.378	24	3.3	10
48I	139	$48^3 \times 96$	1.73	34	3.9	12

Domain Wall fermion, Iwasaki+DSDR action, RBC-UKQCD (24D, 32D, 32D-fine)

Iwasaki action (48I)



a

$$A = 0.1045(34)(50)_L(55)_a$$

Matching with ChEFT

1. $\pi\pi \to ee$ ChEFT:
(Cirigliano, Dekens, Mereghetti, Walker-Loud, PRC97 (2018) 065501)

$$\frac{\mathcal{A}(\pi^-\pi^- \to ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + \frac{7}{2} + \frac{\pi^2}{4} + \frac{5}{6}g_\nu^{\pi\pi}(\mu)\right)$$

Lattice QCD（QED_L， only statistical error）：
(X. Feng, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001, arXiv:1809.10511)

$$\frac{\mathcal{A}(\pi\pi \to ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 0.910(3) \quad \Rightarrow \quad g_\nu^{\pi\pi}(m_\rho) = -12.0(3)$$

2. $\pi^- \to \pi^+ ee$ ChEFT:

$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1 + \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + 6 + \frac{5}{6}g_\nu^{\pi\pi}(\mu)\right)$$

Lattice QCD（IVR， statistical error + systematic error）：
(X. Tuo, X. Feng, L. Jin, PRD100 (2019) 094511, arXiv:1909.13525)

$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1.105(3)(7) \quad \Rightarrow \quad g_\nu^{\pi\pi}(m_\rho) = -10.9(3)(7)$$

3 Conclusion

1.New method：we use IVR+δ_{IVR} to correct FV effects. This method can be easily generalized to other problems.

2.In pion sector, we present a first calculation of $0\nu 2\beta$ decay from Lattice QCD；From two different processes, we get coincident value of $g_{\nu}^{\pi\pi}(m_{\rho})$

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