On the ratio between scalar and tensor glueball masses in Yang-Mills theories

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Yang-Mills (YM) theories are a cornerstone of current theoretical physics.

A YM theory can be defined for a compact Lie group $\mathcal{G}$

$$\mathcal{G} = SU(N_c),\ Sp(2N_c),\ SO(N_c),\ G_2, \dots \tag{1}$$

where N_c is the number of colors.

At high energy, perturbative theories of gluons.

At low energy, their dynamics is non-perturbative and they are believed to be confining, independently of $\mathcal{G}$ and N_c:

- The Potential between static color sources:

$$V(R) \sim \sigma R, \qquad R \to \infty \ , \tag{2}$$

  where σ is the string tension.
- Color singlet states called glueballs, labelled by J^{PC} and torelons if the system has finite size.

If confinement is a universal properties of YM theories, Universal properties of the glueball spectrum might help us understand the confining mechanism.

We will argue that the ratio of tensor to scalar glueball masses,

$$R = \frac{m(2^{++})}{m(0^{++})} \ ,$$

captures some universal property of the spectrum of pure YM theories.

In this contribution we will:

1. Enumerate the basic properties of R
2. Discuss the computation of glueball masses in general, report on our estimates for the $Sp(2N_c)$ LGT.
2. Summarize (non-exhaustively) the analytical computation of R.
3. Discuss some ideas related to the universality of R.

An energy momentum tensor (EMT) $T_{\mu\nu}$ can be defined in any Lorentz-invariant theory.

The EMT can be decomposed in its trace and tensor parts, of masses m_0 and m_T, and the ratio

$$R = \frac{m_T}{m_0}$$

is thus:

- well defined and scheme independent.
- computable.
- not explicitly dependent on internal symmetries.

It makes sense to compare R for different theories. In particular, for glueball states of YM theories

$$R = \frac{m(2^{++})}{m(0^{++})} \ ,$$

with different gauge groups and different N_c.

On hypercubic lattice of spacing a we define the *link variable*.

$$U_\mu(x) = \exp\left(\imath \int_x^{x+\hat{\mu}} \mathrm{d}\lambda^\mu \ \tau^A A_\mu^A(\lambda)\right) .$$

We adopt the Wilson action defined as

$$S_\mathrm{W} \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{2N} \Re \, \mathrm{Tr} \, \mathcal{P}_{\mu\nu}(x)\right) ,$$

where $\mathcal{P}_{\mu\nu}(x) \equiv U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)$ is the *plaquette variable* and $\beta = 2N/g_0^2$, g_0 coupling of the YM theory at scale a.

Ensembles of configurations at (β, L) can be obtained by sampling

$$Z(\beta) = \int [\mathcal{D}U] \exp\left(-S_W\left[U\right]\right) ,$$

with a combination of HeatBath (HB) and OverRelaxation (OR) algorithms.

Implementations of the HB and OR algorithms for $Sp(2N_c)$ can be easily obtained from that of $SU(2N_c)$.

The symplectic group $Sp(2N_c)$ can be defined as a subgroup of $SU(2N_c)$,

$$U \in Sp(2N_c) \Longleftrightarrow \left\{ U \ \in \ SU(2N_c) \,/\, \Omega^\dagger U \Omega = U^* \right\} \tag{3}$$

where Ω is the symplectic matrix.

As direct consequences of the definition:

- $Sp(2) \simeq SU(2)$.
- The group is pseudo-real, every IRREP is unitarily equivalent to its complex conjugate. As a consequence, $C = 1$
- The block structure of Ω is inherited by $Sp(22N_c)$ matrices,

$$U = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix} \tag{4}$$

  with

$$\begin{cases} A^\dagger A + B^\dagger B = \mathbb{I}, \\ A^T B = B^T A, \end{cases} \quad A, B \ \in \ \mathbb{C}^{N \times N} \tag{5}$$

- The Cabibbo-Marinari update algorithm can be adpated from the case of $SU(2N_c)$.

In the continuum, the confined spectrum is composed of

- Glueballs, labelled by J^P.
- Torelons of mass $m(L) \sim \sigma L$ for a wrapping direction of size L

On the lattice, glueball states are labelled by IRREPs of the octahedral group, also called *channels*[8]:

$$R^P = A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm} \ .$$

In the continuum limit

$$m(A_1^+)a \to m(0^+), \quad m(E^+)a, m(T_2^+)a \to m(2^+) \ .$$

and, if rotational invariance is to be reestablished, as $a \to 0$ we must have

$$m(E^{\pm})a \to m(T_2^{\pm})a$$

States $|\Psi\rangle$ are obtained as $|\Psi\rangle = O(t)|\Omega\rangle$ with,

$$O(t) = \sum_{\vec{x}} O(\vec{x}, t), \quad O(\vec{x}, t) = \mathrm{Tr}\, U_{\mathcal{C}}(\vec{x}, t) \ , \tag{6}$$

where:

- O is a single trace operator: it creates a single glueball state,
- $\mathcal{C}$ is a closed spacelike lattice path transforming in R^P,
- Glueballs for contractible paths, Torelons for non-contractible paths,
- The sum over space projects on the zero momentum subspace,

The mass $m(R^P)$ is obtained from the large t behaviour of euclidean correlators, where the smallest energy dominates

$$C(t) = \frac{\langle R^P | e^{-\mathcal{H}t} | R^P \rangle}{\langle R^P | R^P \rangle} \sim |c_0|^2 e^{-m(R^P)t} \ , \quad t \gg 1/m(R^P) \ , \tag{7}$$

where $\mathcal{H}$ is the Hamiltonian of the system.

The variational method

We have the problem an exponentially decaying signal-to-noise ratio at large t. Our only hope is to maximize $|c_0|^2$ with a clever choice of operators.

We perform a variational computation, looking for the $v_i \in \mathbb{C}$ in

$$\Phi(t) = \sum_i v_i O_i^{R^P}(t) \ . \tag{8}$$

for which

$$am_{\text{var}}(\tau) = -\frac{1}{\tau} \log \left\{ \min_{v_i \in \mathbb{C}} \frac{\langle \Omega | \Phi(0)^\dagger e^{\mathcal{H}\tau} \Phi(0) | \Omega \rangle}{\langle \Omega | \Phi(0)^\dagger \Phi(0) | \Omega \rangle} \right\}, \tag{9}$$

where τ is a time chosen for minimization.

The creation of the variational basis $\left\{ O_i^{R^P} \right\}$ of operators can be automatized and can also include smeared-blocked operators.

The mass $m(R^P)$ can be obtained from the correlator of the optimal operators at large t,

$$\tilde{C}(t) \sim |\tilde{c}|^2 e^{-m(R^P)t} \ . \tag{10}$$

The spectrum from the lattice

Ensembles of configurations were obtained for each $Sp(2N_c)$ with $N_c = 1, 2, 3, 4$:

- At a coupling β and on a lattice of dimensions $L \times L^3$ with p.b.c in every direction, for various values of (β, L).
- 10000 thermalized configurations were stored for later analysis

For each ensemble, the masses $m_(R^P)$ and σ are determined variationally from a basis of ~ 200 smeared-blocked operators.

The finite volume effects are put under control and the continuum limit is obtained from

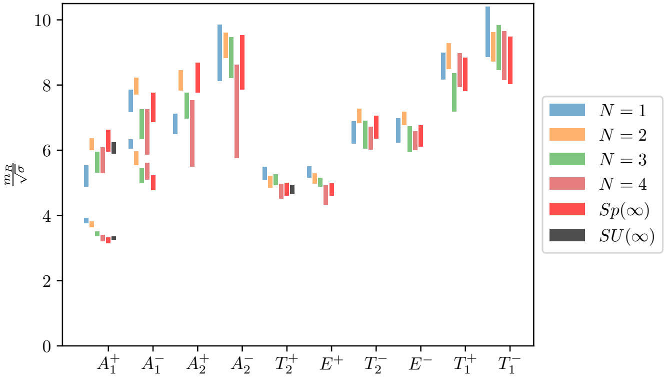
$$\frac{m(R^P)}{\sqrt{\sigma}}(a) = \frac{m(R^P)}{\sqrt{\sigma}}(0) + c_0(R^P)\sigma a^2 \ . \tag{11}$$

The large-N limit can be obtained from[1]

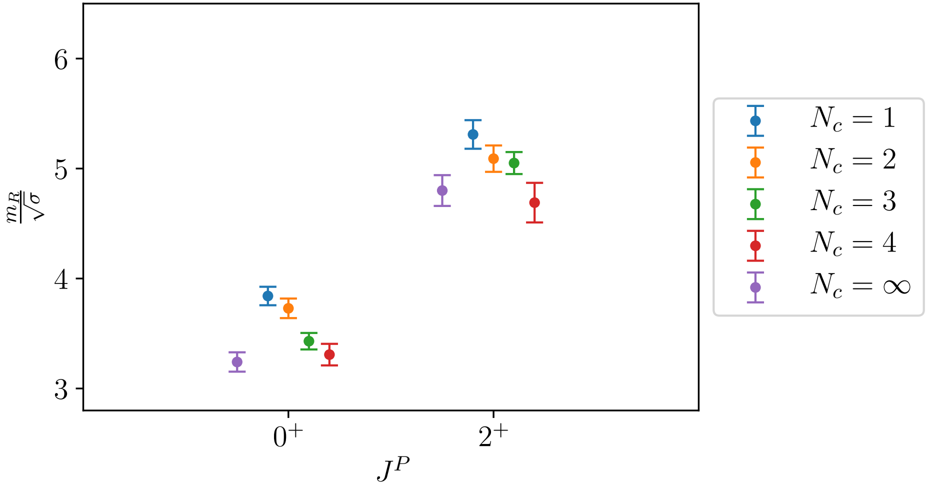
$$\frac{m(R^P)}{\sqrt{\sigma}}(N) = \frac{m(R^P)}{\sqrt{\sigma}}(\infty) + \frac{c_1(R^P)}{N} \ . \tag{12}$$

The results can be compared with $SU(N_c)$, $SO(N_c)$ at $D = 3+1$ and $D = 2+1$ already present in the litterature.

[1] Note that this is different from $SU(N_c)$!



- $SU(\infty)$ and $Sp(\infty)$ yield compatible masses in the lightest channels.
- $m(0^+)$ is obtained from the A_1^+ channel.
- $m(E^\pm)$ and $m(T_2^\pm)$ are compatible.
- $m(2^+)$ is obtained as the weighted average of $m(E^+)$ and $m(T_2^+)$.



$\frac{m_R}{\sqrt{\sigma}}$
6
5
4
3
0^+
2^+
J^P
$N_c = 1$
$N_c = 2$
$N_c = 3$
$N_c = 4$
$N_c = \infty$

Analytical results can be obtained in the strict $N_c = \infty$ limit.

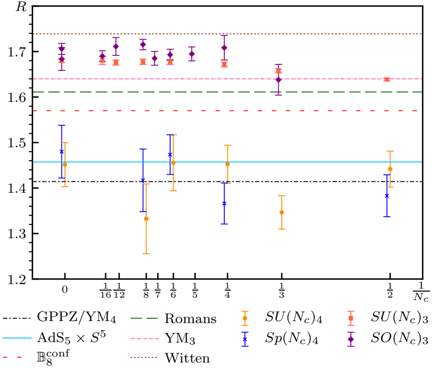
In $D = 3 + 1$[4, 2, 3, 11, 10],

Model	R
GPPZ/YM_4	$\sqrt{2}$
$AdS_5 \times S^5$	1.46
Witten	1.74
Romans	1.61

While in $D = 2 + 1$[5, 7],

Model	R
$\mathbb{B}_8^{conf}$	1.57
YM_3	1.64

The value of R



- A remarkable agreement between $GPPZ/YM_4$, $AdS_5 \times S^5$ and the $SU(N_c)$, $Sp(2N_c)$ lattice data[9, 6, 1].
- Circle reductions of Romans supergravity and the Witten model predict a value of R which is not compatible with lattice data.
- At $D = 2 + 1$ R is slightly underestimated with respect to lattice data.

It is natural to start from universal principles:

1. Scale invariance: If the scalar glueball and dilaton are identified,

$$m_D^2 = \beta f_D^2 \tag{13}$$

   where m_D is the dilaton mass, f_D its decay constant, and β a non-universal constant.

2. Perturbative Unitarity: The dilaton scattering amplitude is

$$\mathcal{M}(2\sigma \to 2\sigma) \sim \frac{-1}{(\alpha f_D)^4}(s^2 + t^2 + u^2)\ , \text{ for } E_{\rm CM} \gg m_D \tag{14}$$

   where α is a non-universal dimensionless constant. At high energy, unitarity is lost. It can be recovered with the exchange of a spin-2 massive field of mass m_T,

$$m_T^2 = \alpha \kappa^2 f_D^4\ . \tag{15}$$

   where κ is the universal coupling of the spin-2 field to the dilaton EMT.

If the spin-2 massive field is identified with the tensor glueball,

$$R^2 = \frac{m_T^2}{m_0^2} = \kappa f_D^2\ \frac{\alpha}{\beta} \tag{16}$$

where α and β are non-universal, in principle computable in the EFT. Lattice data shows the non-universalities cancel.

Conclusion

- R is well defined and computable for a large set of theories both on the lattice and analytically.
- In Lattice data obtained at $D = 3 + 1$ and $D = 2 + 1$ for the $Sp(2N_c)$ LGT shows no dependence of R on N_c.
- Lattice measurements taken from the litterature are supportive of the idea that R only depends on the dimensionanility of space-time.
- Comparison with analytical calculations performed at $N_c = \infty$ gives mixed results: remarkable agreement in many cases but incompatibilities in two cases. This non exhaustive list of results must be enlarged.

Thank you for your attention!

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