Quark composition and color structure of heavy-heavy mesons and tetraquarks

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Outline

- Two parts ...
- ... both are based on lattice QCD static potentials and the Born-Oppenheimer approximation.
- **Part 1**: $\bar{b}\bar{b}qq$ tetraquarks with $I(J^P) = 0(1^+)$.
  - $\bar{b}\bar{b}qq$ / BB potentials.
  - Stable $\bar{b}\bar{b}qq$ tetraquarks.
  - Mesonic molecule versus diquark-antidiquark structure.
- **Part 2**: Bottomonium bound states and resonances with $I = 0$ and $L = 0$.
  [Related to the talk by L. Müller, 04. Aug 16:40]
  - $b\bar{b}/b\bar{b}q\bar{q}$ potentials.
  - Bottomonium bound states and resonances.
  - $b\bar{b}$ versus $b\bar{b}q\bar{q}$ structure.

Part 1: $\bar{b}\bar{b}qq$ tetraquarks with $I(J^P) = 0(1^+)$

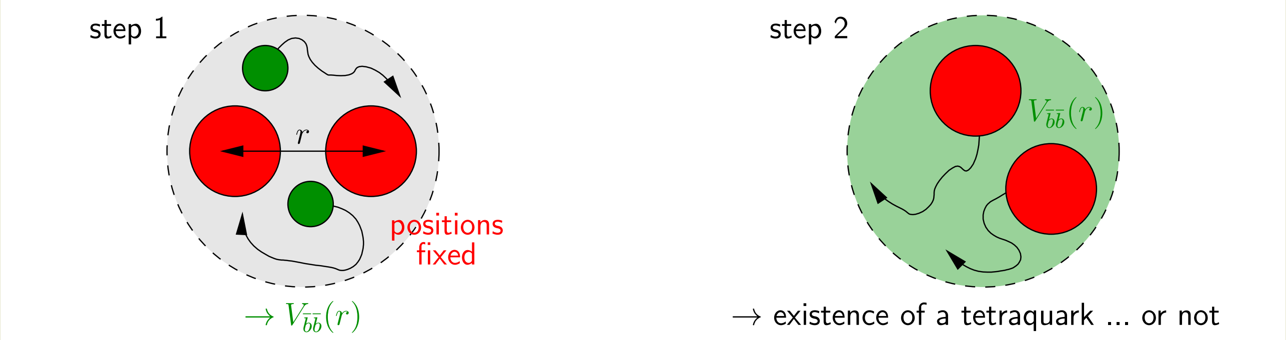
Basic idea: lattice QCD + BO

- Study heavy-heavy-light-light tetraquarks $\bar{b}\bar{b}qq$ in two steps.

  (1) **Compute potentials of two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s, c\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** (→ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

  ((1) + (2) → Born-Oppenheimer approximation).



Previous work on $\bar{b}\bar{b}qq$ tetraquarks

- Lattice QCD static potentials and Born-Oppenheimer approximation.
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [M.W., PoS **LATTICE2010**, 162 (2010) [arXiv:1008.1538]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571]]
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [Z. S. Brown, K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953]]
  [E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]

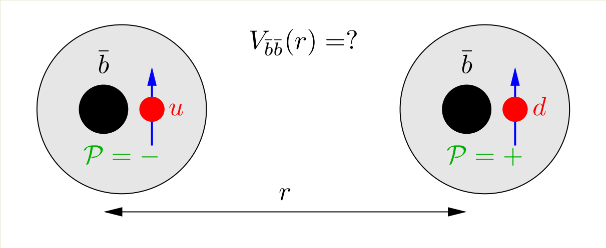
- Full lattice QCD (b quarks with Non Relativistic QCD):
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214 [hep-lat]]]
  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285 [hep-lat]]]
  [L. Leskovec, S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197] [hep-lat]]]

- Other approches: quark models, effective field theories, QCD sum rules ...
  [M. Karliner, J. L. Rosner, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666]]
  [E. J. Eichten, C. Quigg, Phys. Rev. Lett. **119**, 202002 (2017) [arXiv:1707.09575]]
  [Z. G. Wang, Acta Phys. Polon. B **49**, 1781 (2018) [arXiv:1708.04545]]
  [W. Park, S. Noh, S. H. Lee, Acta Phys. Polon. B **50**, 1151-1157 (2019) [arXiv:1809.05257]]
  [B. Wang, Z. W. Liu, X. Liu, Phys. Rev. D **99**, 036007 (2019) [arXiv:1812.04457]]
  [M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie, L. S. Geng, Phys. Rev. D **99**, 094018 (2019) [arXiv:1902.03044]]

$\bar{b}\bar{b}qq$ / BB potentials (1)

- At large $\bar{b}\bar{b}$ separation r, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of qq on
  - the "light" quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^* (parity).

$\rightarrow$ Many different channels: attractive as well as repulsive, different asymptotic values ...

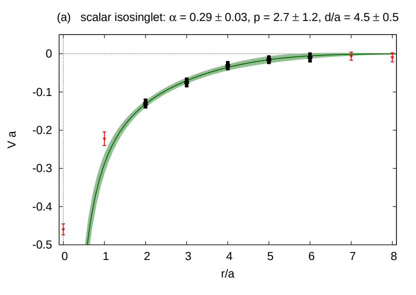


$\bar{b}\bar{b}qq$ / BB potentials (2)

- To determine potentials, compute temporal correlation functions of operators

$$\mathcal{O}_{BB} = \Big(C\Gamma\Big)_{AB}\Big(C\tilde{\Gamma}\Big)_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big).$$

- The most attractive potential of a $B^{(*)}B^*$ meson pair has $(I, |j_z|, P, P_x) = (0, 0, +, -)$:
  - $q^{(1)}q^{(2)} = ud - du$, $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,(1-\gamma_0)\gamma_5\}$.
  - $\tilde{\Gamma} \in \{(1+\gamma_0)\gamma_5\,,\,(1+\gamma_0)\gamma_j\}$ (irrelevant).



- Parameterize lattice results by

$$V_{\bar{b}\bar{b}}(r) = -\frac{\alpha}{r}\exp\bigg(-\Big(\frac{r}{d}\Big)^p\bigg) + V_0$$

(1-gluon exchange at small r; color screening at large r with $p = 2$ from quark models).
[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]

Stable $\bar{b}\bar{b}qq$ tetraquarks

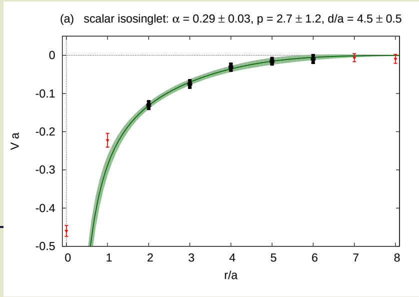
- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / BB potentials,

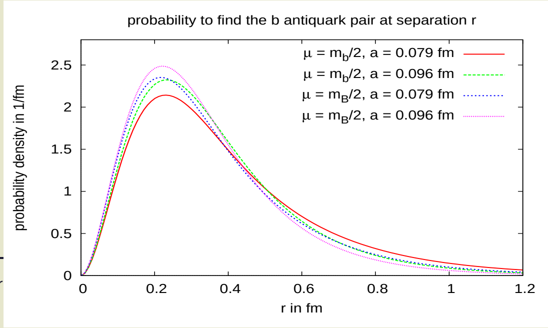
$$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\right)\psi(\mathbf{r}) \quad = \quad E\psi(\mathbf{r}) \quad , \quad \mu \quad = \quad m_b/2.$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}\bar{b}qq$ tetraquarks.

- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $-E = 90^{+43}_{-36}$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.

- No further bound states.
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]





Structure of the $\bar{b}\bar{b}qq$ tetraquark (1)

- Now consider two operators, which generate the same quantum numbers:
  - **Meson-meson operator**:

$$\mathcal{O}_1 \;=\; \mathcal{O}_{BB} \;=\; \Big(C\Gamma_{BB}\Big)_{AB}\Big(C\tilde{\Gamma}\Big)_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big).$$

  - **Diquark-antidiquark operator**:

$$\mathcal{O}_2 \;=\; \mathcal{O}_{dD} \;=\; \Big(C\Gamma_{dD}\Big)_{AB}\Big(C\tilde{\Gamma}\Big)_{CD}\Big(\epsilon^{abc}q_A^{b,(1)}(0)q_B^{c,(2)}(0)\Big)$$
$$\Big(\epsilon^{ade}\Big(\bar{Q}(-\mathbf{r}/2)U(-\mathbf{r}/2;0)\Big)^d_C\Big(\bar{Q}(+\mathbf{r}/2)U(+\mathbf{r}/2;0)\Big)^e_D\Big).$$

$\Gamma_{BB} = \Gamma_{dD} = (1+\gamma_0)\gamma_5$, $\tilde{\Gamma} = (1+\gamma_0)\gamma_j$ and $q^{(1)}q^{(2)} = ud - du$.

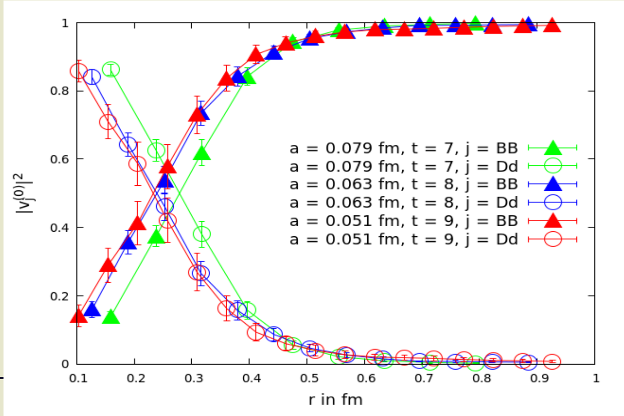
- Compute the 2×2 correlation matrix $C_{jk}(t) = \langle\Omega|\mathcal{O}_j^\dagger(t)\mathcal{O}_k(0)|\Omega\rangle$.
- Solve the generalized eigenvalue problem $C(t)\mathbf{v}_m(t,t_0) = \lambda_m(t,t_0)C(t_0)\mathbf{v}_m(t,t_0)$.
  - Effective mass: $V_{\bar{b}\bar{b}}^{\text{effective}}(r,t,t_0) = -\Big(\ln(\lambda_0(t+a,t_0)) - \ln(\lambda_0(t,t_0))\Big)/a$.
  - $\mathbf{v}_0(t,t_0)$ provides information about the structure of the four-quark system,

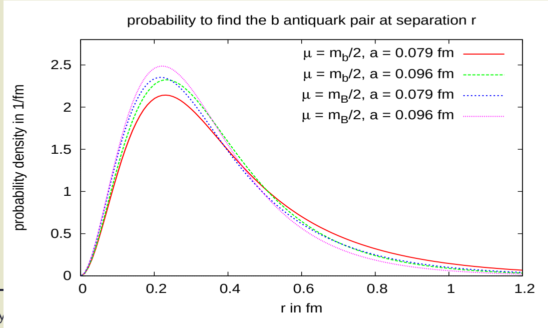
$$|\bar{b}\bar{b}qq;r\rangle \;\approx\; \sum_j v_0^j(t,t_0)\mathcal{O}_j^\dagger|\Omega\rangle$$

($\approx$ denotes expansion in the "$\mathcal{O}_{BB}$ $\mathcal{O}_{dD}$ subspace")

Structure of the $\bar{b}\bar{b}qq$ tetraquark (2)

- $r \lesssim 0.25\,\text{fm}$: Diquark-antidiquark structure preferred.
- $r \gtrsim 0.25\,\text{fm}$: Meson-meson structure preferred.
- Maximum of the probability distribution for r at around $0.25\,\text{fm}$.
  $\rightarrow$ Tetraquark is a superposition of
  ... a diquark-antidiquark pair ($\approx 30 \ldots 40\%$) at small r ...
  ... a meson meson pair ($\approx 60 \ldots 70\%$) at large r.
  [S. Velten, Master of Science thesis, Goethe University Frankfurt (2020)]
- Result stable with respect to a variation of the lattice spacing,
  $a = 0.079\,\text{fm}\,,\ 0.063\,\text{fm}\,,\ 0.051\,\text{fm}.$





Part 2: Bottomonium bound states and resonances with $I = 0$ and $L = 0$

Bottomonium: introduction

- Now bottomonium with $I = 0$, i.e. $\bar{b}b$ and/or $\bar{b}b\bar{q}q$ (with $\bar{q}q = (\bar{u}u + \bar{d}d)/\sqrt{2}$).
- $J^{PC} = 1^{--}$ states:
  - $\Upsilon_b(1S)$, $\Upsilon_b(2S)$, $\Upsilon_b(3S)$, $\Upsilon_b(4S)$, $\Upsilon_b(10860)$ have masses compatible with quark model calculations; the last two are resonances have transitions to lower bottomonium with much higher rates than expected.
  - Recently observed resonance $\Upsilon_b(10750)$ in excess compared to the quark model spectrum.
    [R. Mizuk *et al.* [Belle], JHEP **10**, 220 (2019) [arXiv:1905.05521]]

$\rightarrow$ Large $\bar{B}^{(*)}B^{(*)}$ admixture(s) ...? D wave state(s) ...? Exotic structure(s), e.g. hybrid ...?
[C. Meng, K. T. Chao, Phys. Rev. D **77**, 074003 (2008) [arXiv:0712.3595]]
[Y. A. Simonov, A. I. Veselov, Phys. Lett. B **671**, 55-59 (2009) [arXiv:0805.4499]]
[M. B. Voloshin, Phys. Rev. D **85**, 034024 (2012) [arXiv:1201.1222]]
[Q. Li, M. S. Liu, Q. F. L, L. C. Gui, X. H. Zhong, Eur. Phys. J. C **80**, no. 1, 59 (2020) [arXiv:1905.10344]]
[W. H. Liang, N. Ikeno, E. Oset, Phys. Lett. B **803**, 135340 (2020) [arXiv:1912.03053]]
[J. F. Giron, R. F. Lebed, Phys. Rev. D **102**, no. 1, 014036 (2020) [arXiv:2005.07100]]
[Z. G. Wang, Chin. Phys. C **43**, no. 12, 123102 (2019) [arXiv:1905.06610 [hep-ph]]]
[B. Chen, A. Zhang, J. He, Phys. Rev. D **101**, no. 1, 014020 (2020) [arXiv:1910.06065]]
[A. Ali, L. Maiani, A. Y. Parkhomenko, W. Wang, Phys. Lett. B **802**, 135217 (2020) [arXiv:1910.07671]]
[N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo, C. Z. Yuan, [arXiv:1907.07583]]

Bottomonium: Schrödinger equation

- One can derive a 2×2 radial Schrödinger equation (boundary conditions: plane incident wave, as appropriate for scattering and the study of resonances)

$$\left(-\frac{1}{2}\begin{pmatrix} 1/\mu_Q & 0 \\ 0 & 1/\mu_M \end{pmatrix}\partial_r^2 + \frac{1}{2r^2}\begin{pmatrix} 0 & 0 \\ 0 & 2/\mu_M \end{pmatrix} + V_0(r) + 2m_M - E\right)\begin{pmatrix} u(r) \\ \chi(r) \end{pmatrix} = $$

$$= -\begin{pmatrix} V_{\text{mix}}(r) \\ V_{\bar{M}M,\parallel}(r) \end{pmatrix} kr j_1(kr) \quad (1)$$

$$V_0(r) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r) \\ V_{\text{mix}}(r) & V_{\bar{M}M,\parallel}(r) \end{pmatrix}$$

for two channels,

  - a quarkonium channel (upper component), $\bar{Q}Q$ (with $Q \equiv b$), with orbital angular momentum $L = 0$,
  - a heavy-light meson-meson channel (lower component), $\bar{M}M$ (with $M = \bar{Q}q \equiv B^{(*)}$).

[P. Bicudo, M. Cardoso, N. Cardoso, M.W. [arXiv:1910.04827]].
[Talk by L. Müller, 04. Aug 16:40]

Bottomonium: potentials

- Use lattice QCD to compute the 2×2 potential matrix

$$V_0(r) \quad = \quad \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r) \\ V_{\text{mix}}(r) & V_{\bar{M}M,\parallel}(r) \end{pmatrix}.$$

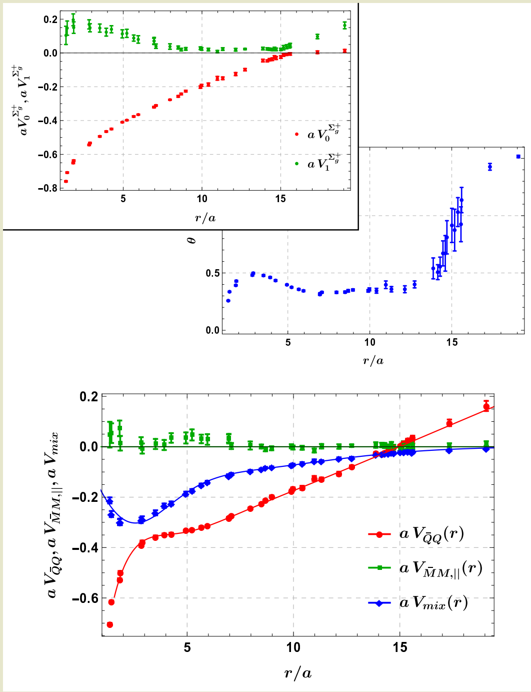
- $V_{\bar{Q}Q}(r)$, $V_{\bar{M}M,\parallel}(r)$, $V_{\text{mix}}(r)$:
  - Lattice computation of string breaking with optimized $\bar{Q}Q$ and $\bar{M}M$ operators:
    $\rightarrow V_0^{\Sigma_g^+}(r)$ (ground state), $V_1^{\Sigma_g^+}(r)$ (first excitation), $\theta(r)$ (mixing angle).

$$V_{\bar{Q}Q}(r) \quad = \quad \cos^2(\theta(r))V_0^{\Sigma_g^+}(r) + \sin^2(\theta(r))V_1^{\Sigma_g^+}(r)$$

$$V_{\bar{M}M,\parallel}(r) \quad = \quad \sin^2(\theta(r))V_0^{\Sigma_g^+}(r) + \cos^2(\theta(r))V_1^{\Sigma_g^+}(r)$$

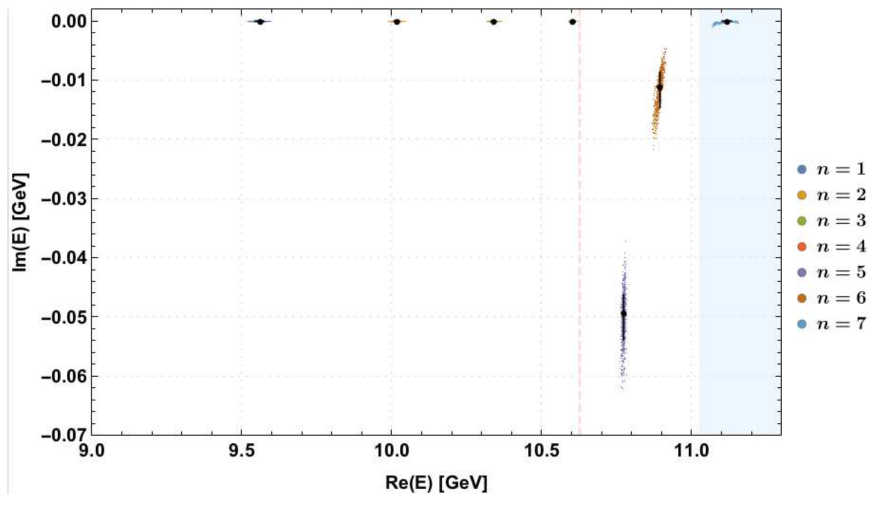
$$V_{\text{mix}}(r) \quad = \quad \cos(\theta(r))\sin(\theta(r))\Big(V_0^{\Sigma_g^+}(r) - V_1^{\Sigma_g^+}(r)\Big).$$

  - We use existing results from:
    [G. S. Bali *et al.* [SESAM Collaboration], Phys. Rev. D **71**, 114513 (2005) [hep-lat/0505012]]



Bottomonium: masses, structure (1)

- Determine the scattering amplitude $t_{1\to 0,0}$ from the Schrödinger equation (1) with boundary conditions $u(r) = 0$ and $\chi(r) = it_{1\to 0,0}krh_1^{(1)}(kr)$ for $r \to \infty$.
- Find poles of $t_{1\to 0,0}$ in the complex energy plane to identify bound states and resonances:
  - (Resonance) mass $m = \mathrm{Re}(E)$, decay width $\Gamma = -2\mathrm{Im}(E)$.
  - Four bound states on the real axis ($n = 1, 2, 3, 4$), previous results confirmed.
  - Two resonances, which can decay to $\bar{B}^{(*)}B^{(*)}$ ($n = 5, 6$).
  - Higher resonances not trustworthy, because excited B mesons neglected ($n \geq 7$).



Bottomonium: masses, structure (2)

- Four bound states ($n = 1, 2, 3, 4$), correspond to experimentally observed $\eta_b(1S) \equiv \Upsilon_b(1S)$, $\Upsilon_b(2S)$, $\Upsilon_b(3S)$, $\Upsilon_b(4S)$.
- Two resonances ($n = 5, 6$), close to experimentally observed $\Upsilon_b(10750)$ and $\Upsilon_b(10860)$.

	masses and decay widths from poles of $t_{1\to 0,0}$			quark composition		masses and decay widths from experiment		
n	$m = \mathrm{Re}(E)$ [GeV]	$\mathrm{Im}(E)$ [MeV]	Γ [MeV]	$\%\bar{Q}Q$	$\%\bar{M}M$	name	m [GeV]	Γ [MeV]
1	9.562^{+11}_{-17}	0	–	$0.89^{+0.004}_{-0.005}$	$0.11^{+0.005}_{-0.004}$	$\eta_b(1S)$	9.399(2)	10(5)
						$\Upsilon_b(1S)$	9.460(0)	≈ 0
2	10.018^{+8}_{-10}	0	–	$0.90^{+0.002}_{-0.001}$	$0.10^{+0.001}_{-0.002}$	$\Upsilon_b(2S)$	10.023(0)	≈ 0
3	10.340^{+7}_{-9}	0	–	$0.88^{+0.002}_{-0.002}$	$0.12^{+0.002}_{-0.002}$	$\Upsilon_b(3S)$	10.355(1)	≈ 0
4	10.603^{+5}_{-6}	0	–	$0.70^{+0.025}_{-0.025}$	$0.30^{+0.025}_{-0.025}$	$\Upsilon_b(4S)$	10.579(1)	21(3)
5	10.774^{+4}_{-4}	$-49.3^{+3.0}_{-4.6}$	$98.5^{+9.2}_{-5.9}$	$0.05^{+0.004}_{-0.006}$	$0.95^{+0.006}_{-0.004}$	$\Upsilon_b(10750)$	10.753(7)	36(22)
6	10.895^{+7}_{-10}	$-11.1^{+2.4}_{-3.6}$	$22.2^{+7.1}_{-4.9}$	$0.58^{+0.038}_{-0.042}$	$0.42^{+0.042}_{-0.038}$	$\Upsilon_b(10860)$	10.890(3)	51(7)

- Percentages of $\bar{Q}Q$ and of $\bar{M}M$ present in each of the bound states and resonances:

$$\%\bar{Q}Q = \frac{Q}{Q+M} \quad , \quad \%\bar{M}M = \frac{M}{Q+M} \quad , \quad Q = \int_0^{R_\mathrm{max}} dr \left|u(r)\right|^2 \quad , \quad M = \int_0^{R_\mathrm{max}} dr \left|\chi(r)\right|^2 .$$

($u(r)$, $\chi(r)$: radial wave functions of the $\bar{Q}Q$ and $\bar{M}M$ channels; R_max dependence weak).

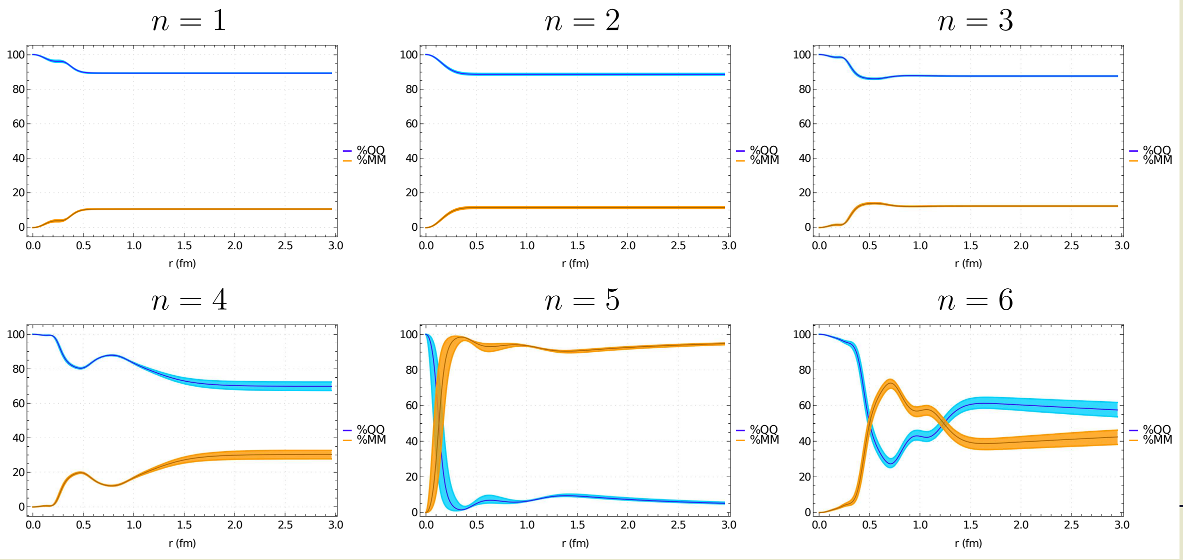
Bottomonium: masses, structure (3)

- Percentages of $\bar{Q}Q$ and of $\bar{M}M$ present in each of the bound states and resonances:

$$\%\bar{Q}Q = \frac{Q}{Q+M} \quad , \quad \%\bar{M}M = \frac{M}{Q+M} \quad , \quad Q = \int_0^{R_{\max}} dr \left|u(r)\right|^2 \quad , \quad M = \int_0^{R_{\max}} dr \left|\chi(r)\right|^2 .$$

($u(r)$, $\chi(r)$: radial wave functions of the $\bar{Q}Q$ and $\bar{M}M$ channels).

- Plots confirm that $R_{\max}$ dependence is weak.



Bottomonium: conclusions

- Bound states $\Upsilon_b(1S)$, $\Upsilon_b(2S)$, $\Upsilon_b(3S)$ are quarkonium states (as expected).
- $\Upsilon_b(4S)$ quarkonium dominated, but with a sizable meson-meson component ($\approx 30\%$).
- The new resonance $\Upsilon_b(10750)$ observed by Belle seems to be an S wave state with a very large meson-meson component ($\approx 95\%$).
- $\Upsilon_b(10860)$ slightly quarkonium dominated, but with an almost comparable meson-meson component ($\approx 42\%$).
- Systematic errors are possibly large, $\mathcal{O}(50\,\text{MeV})$
  important next step is to include heavy spins and the B-B^* mass splitting.