Nuclear Science Computing Center at CCNU

Gell-Mann-Oakes-Renner relation in external magnetic fields at zero temperature

Xiao-Dan Wang

Central China Normal University

bases on arXiv:2008.00493, in collaboration with

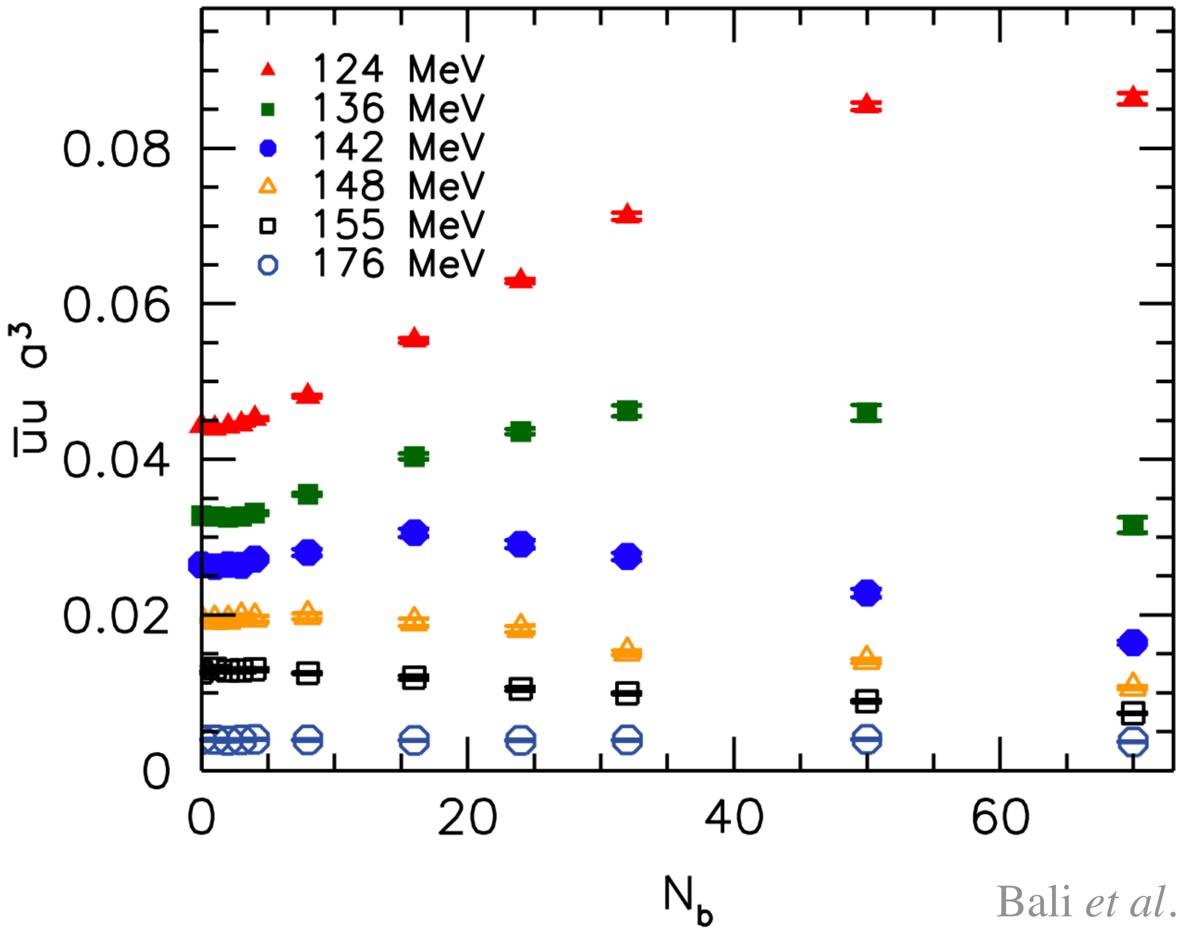
Heng-Tong Ding, Sheng-Tai Li, Akio Tomiya, Yu Zhang

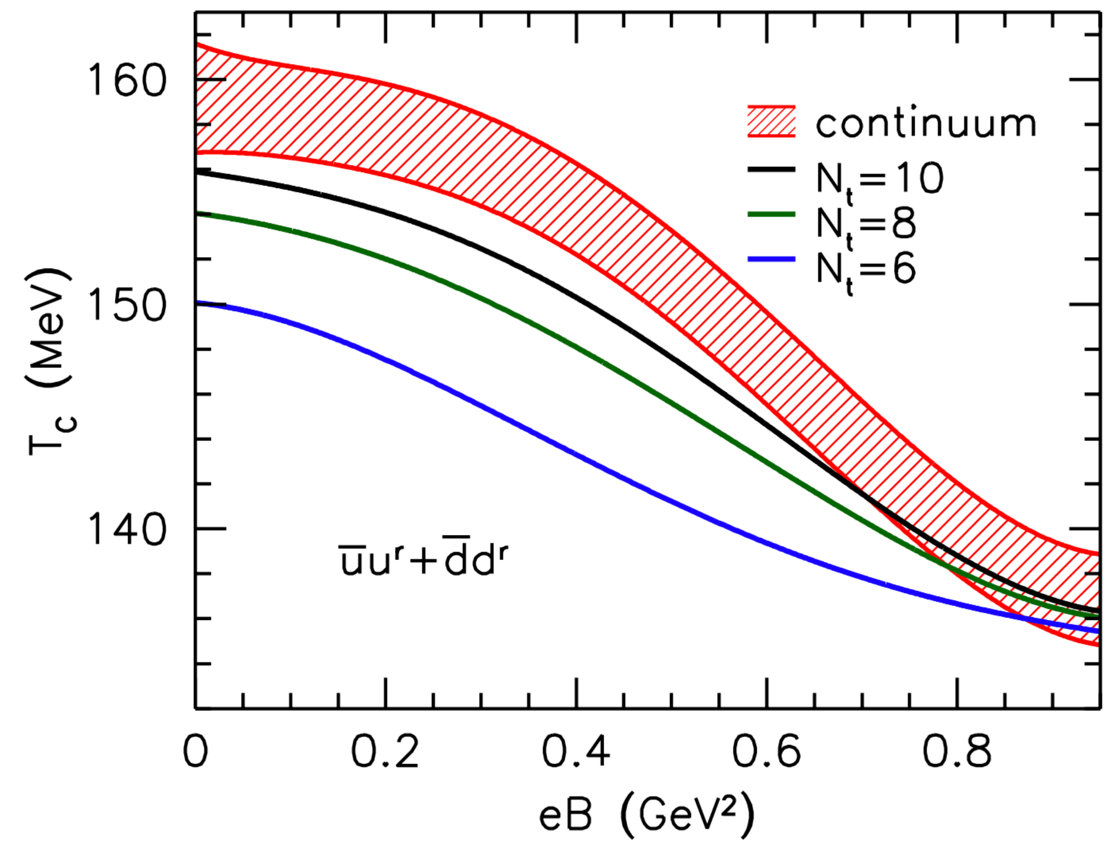
Asia-Pacific Symposium for Lattice Field Theory 2020
August 4-7. 2020

Outline

- Motivation and Introduction
- Lattice Setup
- Results
- Summary

Motivation





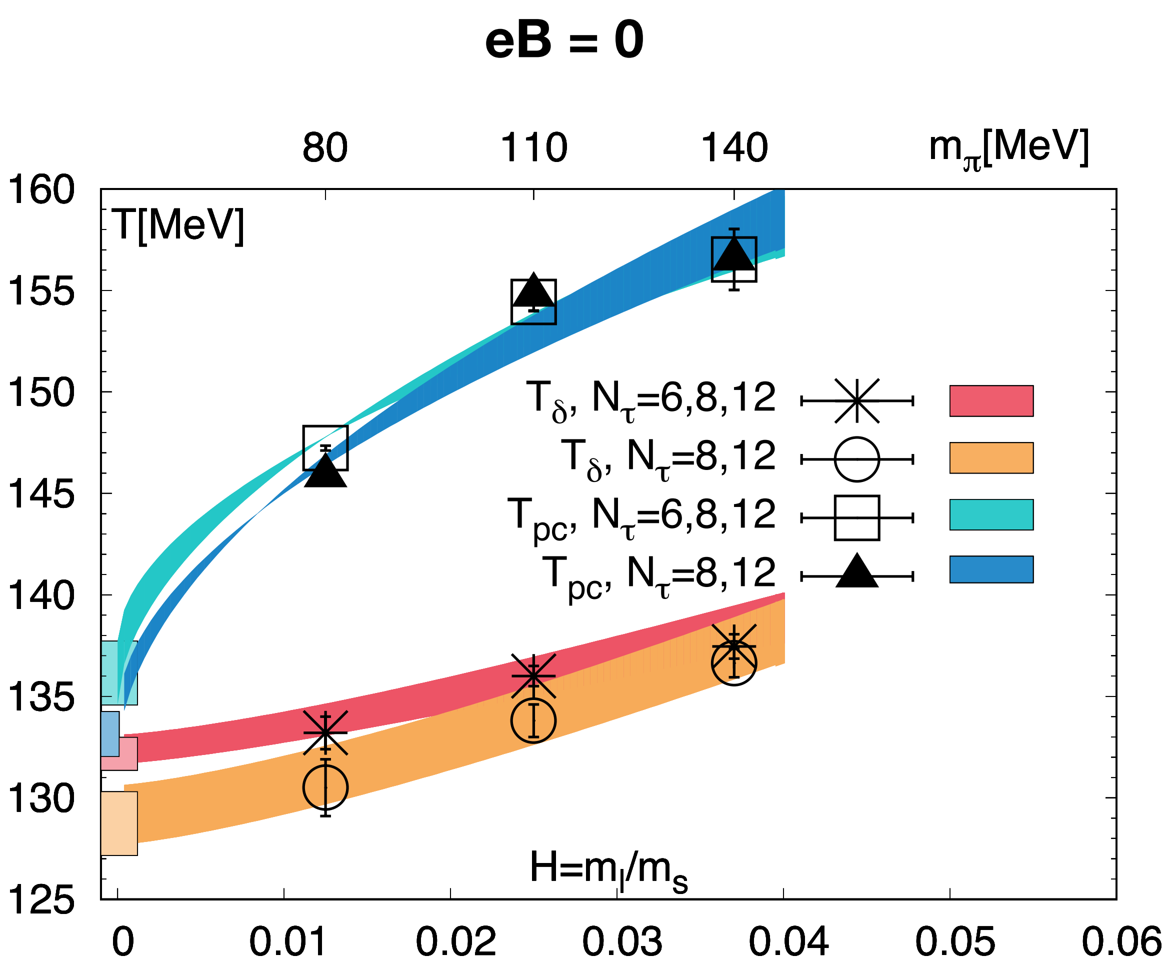
Bali *et al.*, JHEP 02, 044

- Chiral condensate was found decreasing with magnetic field near T_{pc}
- T_{pc} was found decreasing with magnetic field

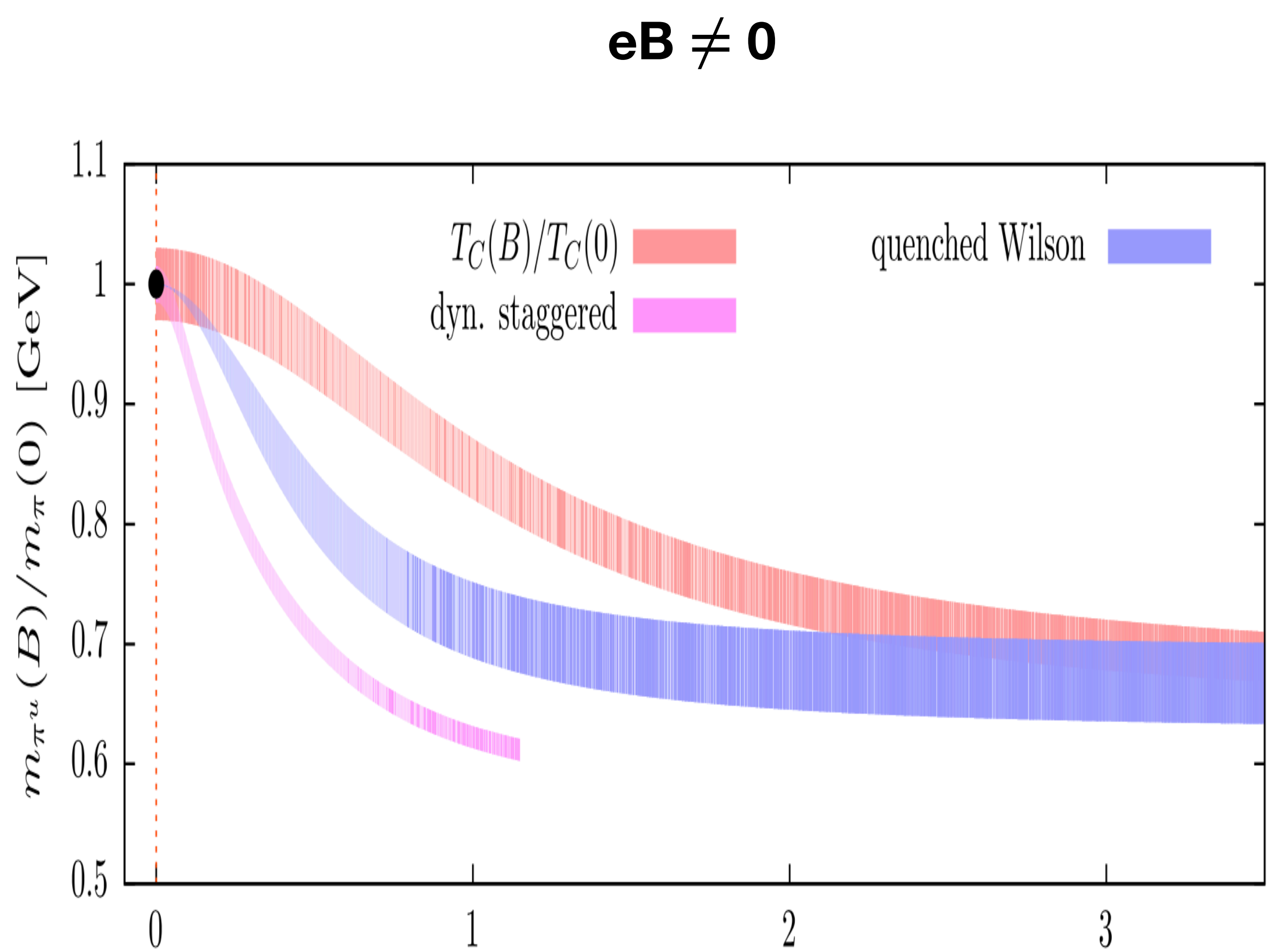
The connection between Tpc and chiral condensate is non-trivial

D'Elia *et al.*, Phys. Rev. D98, 054509. Endrodi et al., JHEP 07, 007. Bonati et al., Phys. Rev. D94, 094007

Motivation



HTD, P. Hegde O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 062002
H.-T. Ding .arXiv:2002.11957



Bali *et al.*, PHYS. REV. D 97, 034505

- **Is neutral pion still a Goldstone boson at eB≠0 ?**
- If assume pion is still Goldstone boson, the mass reduction of pion explains the reduction of T_{pc}

Introduction to GMOR relation

- $(m_u + m_d)\ \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2\, f_{\pi^0}^2\ M_{\pi^0}^2 (1 - \delta_{\pi^0})$

M. Gell-Mannet al, Phys. Rev. 175, 2195
Jamin et al, Phys. Lett. B 538, 71
Bordes et al, JHEP 05, 064
Bordes et al, JHEP 10, 102

- $(m_s + m_d)\ \left(\langle\bar{\psi}\psi\rangle_s + \langle\bar{\psi}\psi\rangle_d\right) = 2\, f_K^2\ M_K^2\ (1 - \delta_K)$

Gasser et al. Nucl. Phys. B 250, 465

- GMOR relation has been confirmed on lattice in the vacuum without magnetic field

Boucaud et al, Phys. Lett. B650, 304

- The GMOR relation for neutral pion valid in chiral limit in chiral perturbation theory in :
  - Low temperature with zero magnetic field — J. Gasser and H. Leutwyler , Phys. Lett. B184, 83
  - Weak magnetic field at zero temperature — I. A. Shushpanov and A. V. Smilga, Phys. Lett. B402, 351
  - Weak magnetic field at low temperature — N. O. Agasian and I. A. Shushpanov, JHEP 10, 006

Lattice Setup

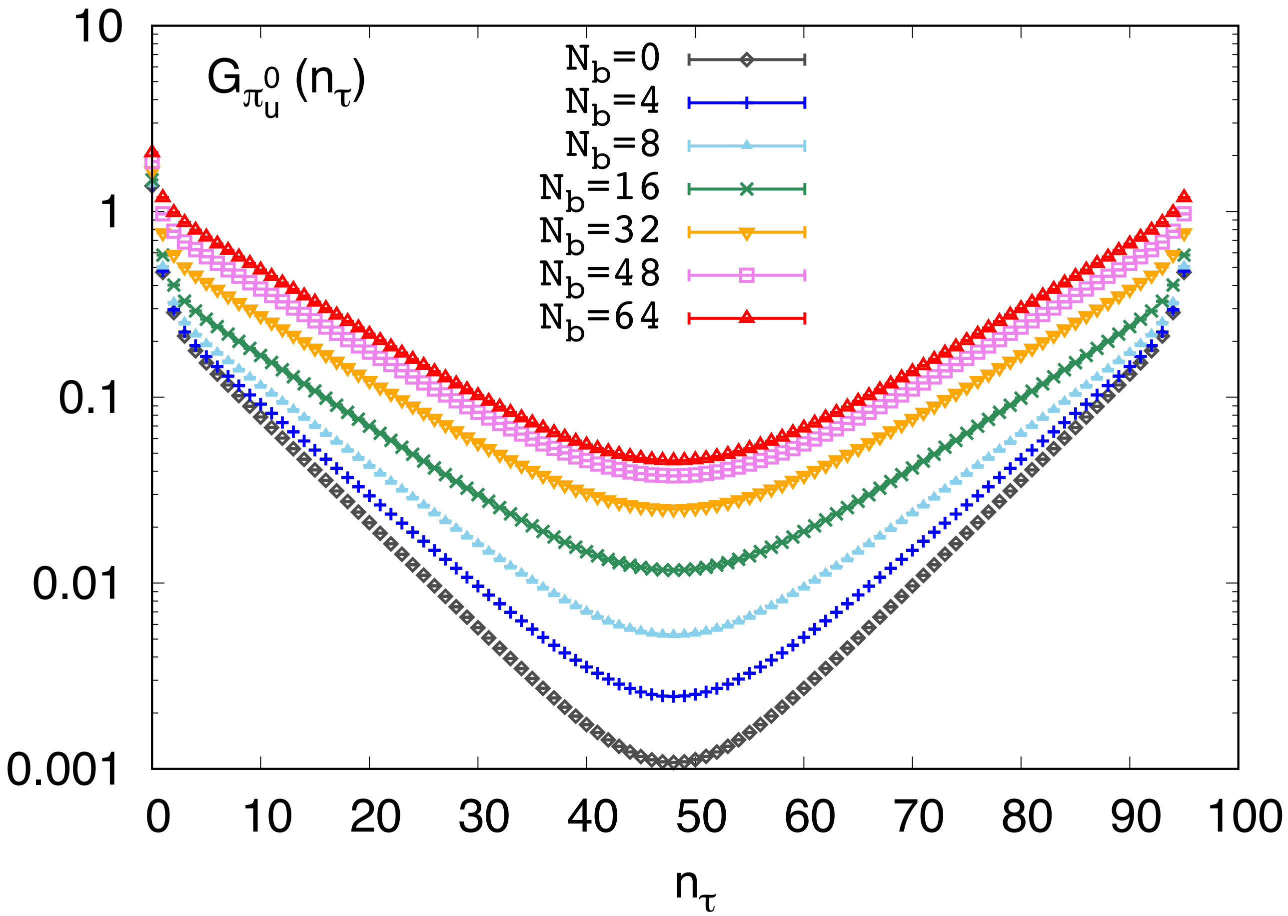
arXiv:2008.00493

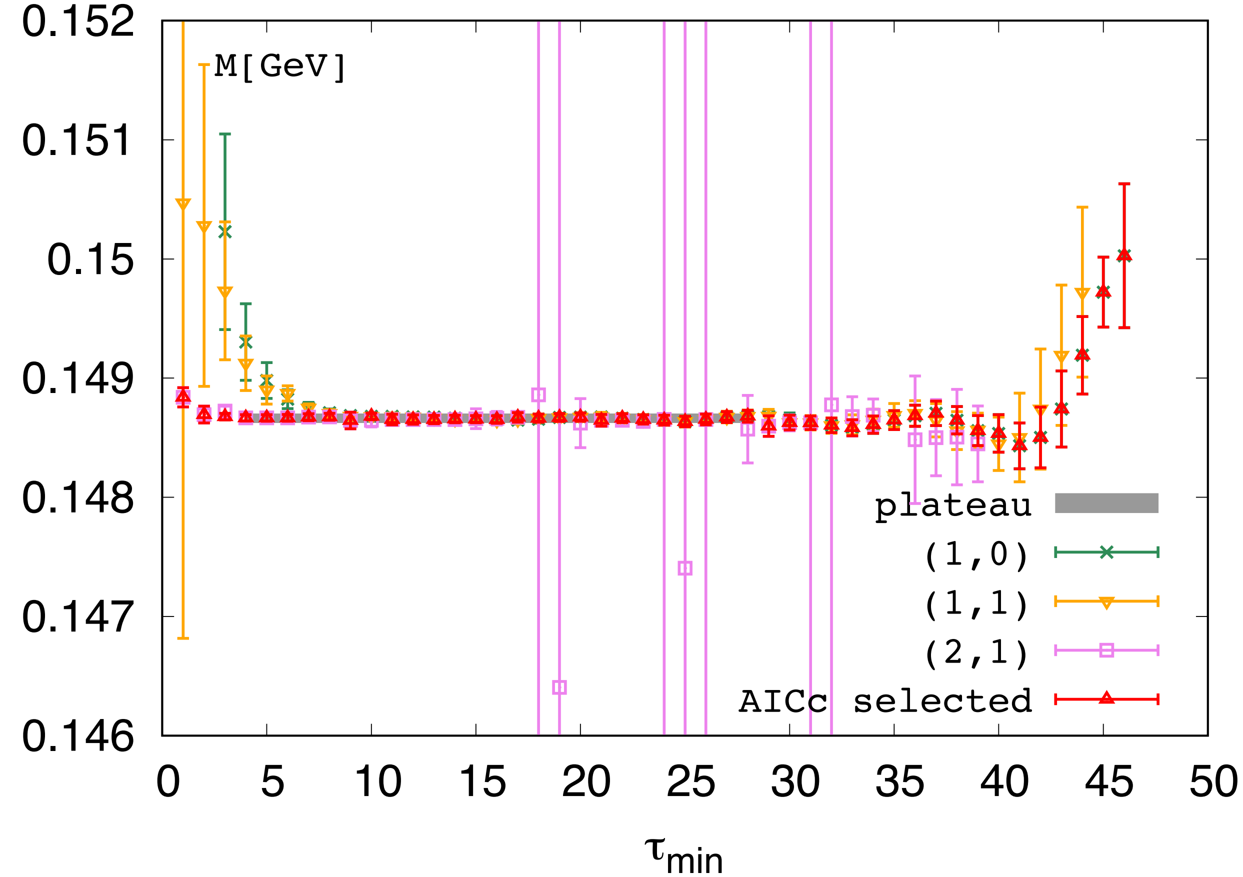
- (2+1) flavor Dynamical HISQ fermion at T=0
- Lattice size: $32^3 \times 96$, a = 0.117 fm
- Our simulation tuned to $M_\pi = 220$ MeV, while $f_\pi = 96.93(2)$ MeV, $f_K = 112.50(2)$ MeV, $f_K / f_\pi = 1.1606(3)$

Flag's review 2019: f_π = 92.1(6) MeV, f_K = 110.1(5) MeV, f_K / f_π = 1.1917(37)

- Magnetic field was set along z direction and quantized as $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$
  - $N_b = 0, 1, 2, 3, 4, 6, 8, 10, 12, 16, 20, 24, 32, 40, 48, 64$
  - $0 < |eB| \lesssim 3.35\ \text{GeV}^2$ ($\sim 70\ M_\pi^2$)

Correlators and Meson mass





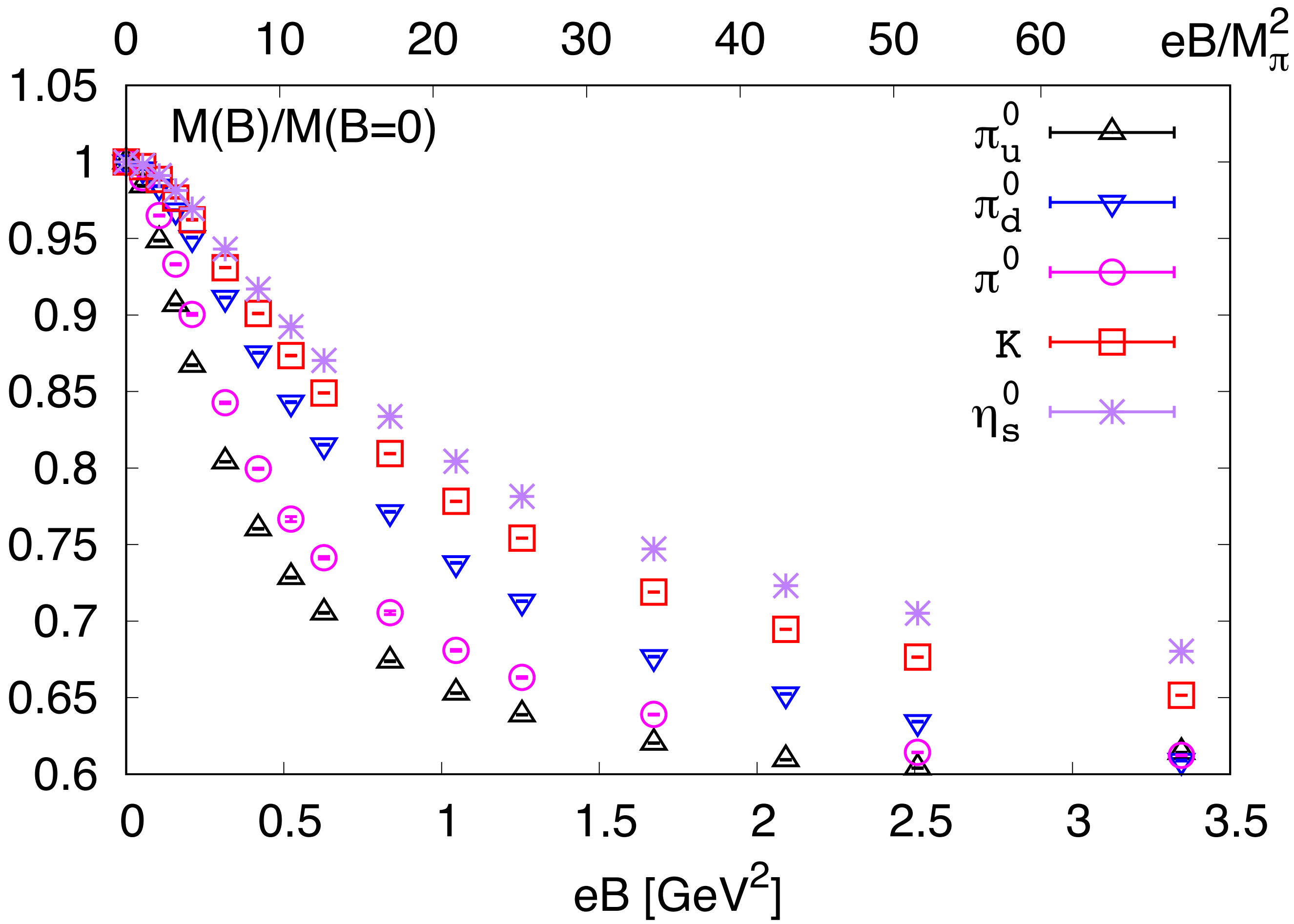
- Wall sources have been used to improve the signal

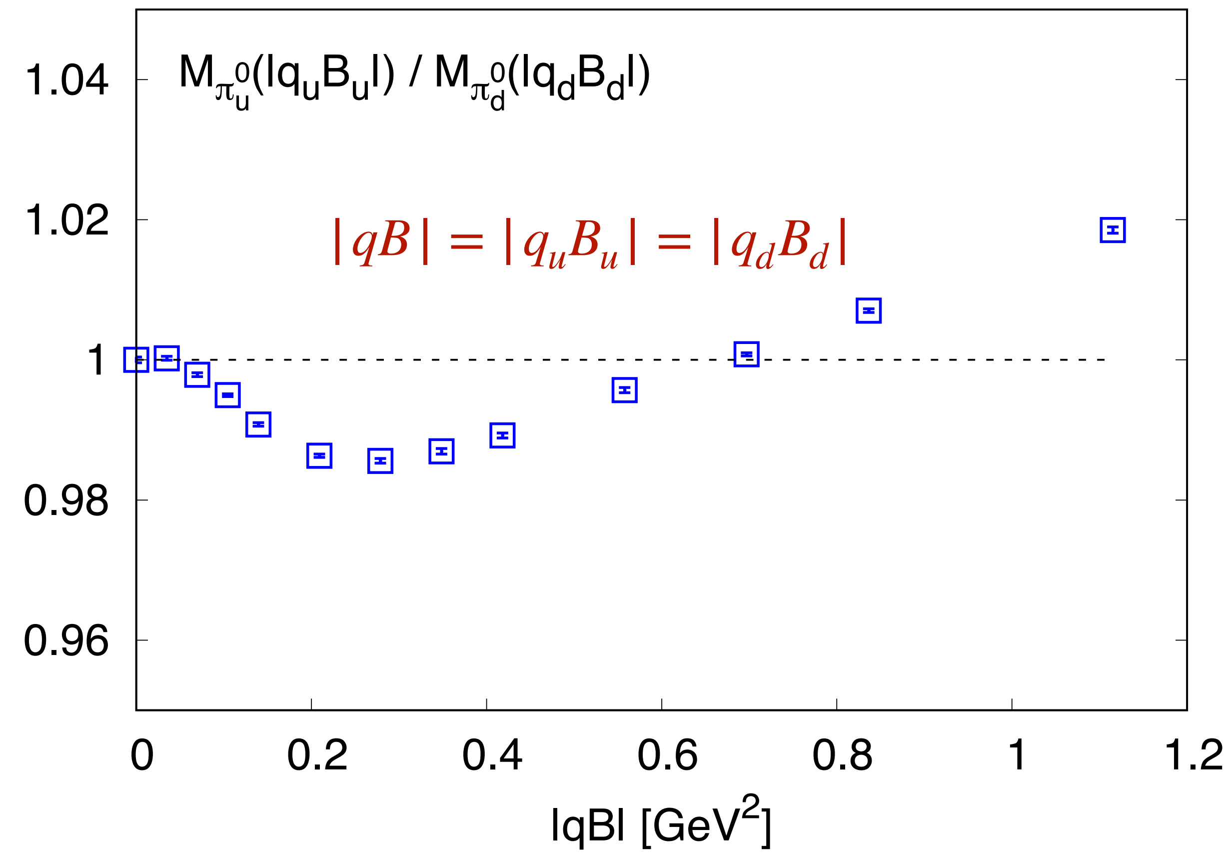
  Reduction of $\delta G/G$: single point source $\xrightarrow{6}$ single wall source $\xrightarrow{\sqrt{\#\ of\ sources}}$ multiple wall sources

- $$G(n_\tau) = \sum_{i=1}^{N_{nosc}} A_{nosc,i} \exp\left(-M_{nosc,i}\, n_\tau\right) - (-1)^{n_\tau} \sum_{i=0}^{N_{osc}} A_{osc,i} \exp\left(-M_{osc,i}\, n_\tau\right)$$

- $$\text{AICc} = 2k - \ln(\hat{L}) + \frac{2k^2 + 2k}{n - k - 1}$$ H. Akaike 1997, J. E. Cavanaugh 1997

Mass of Neutral Pseudo-scalar Meson



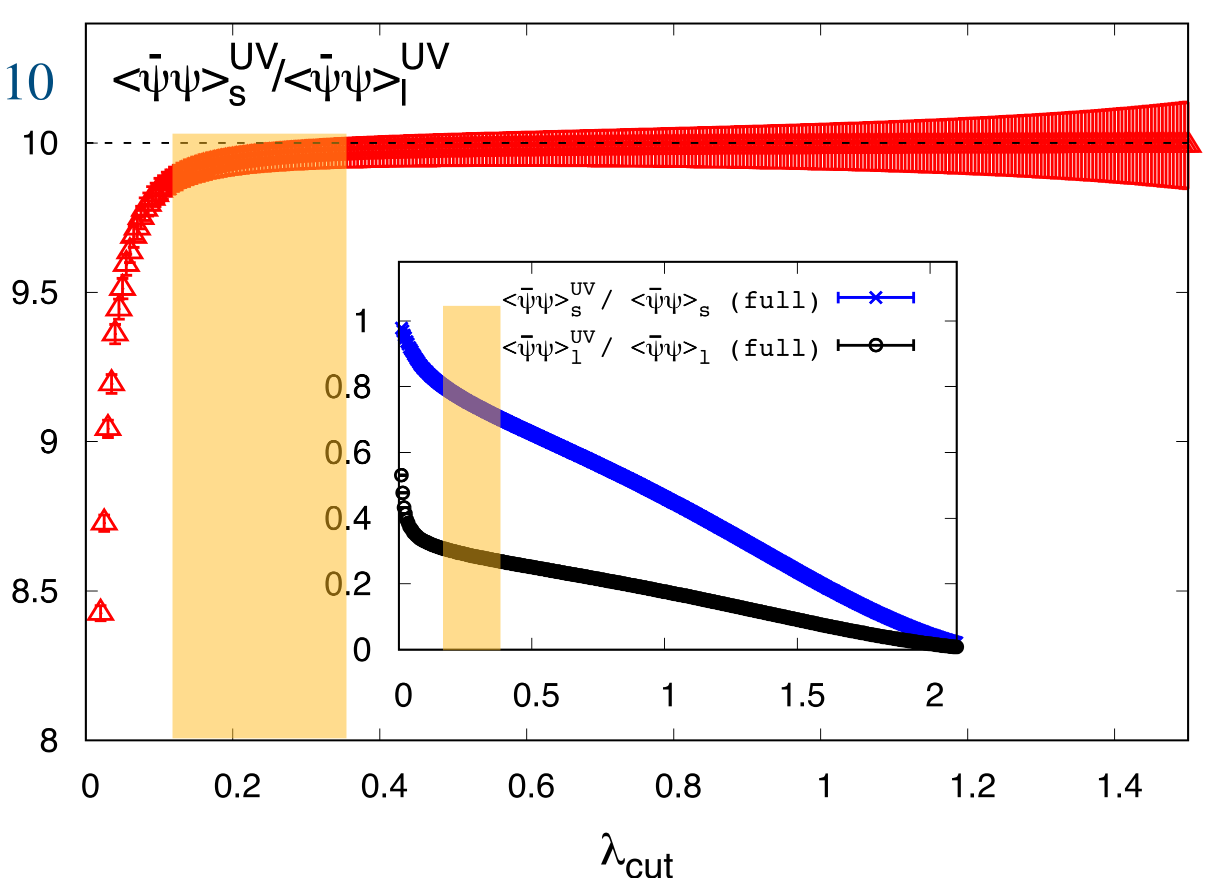


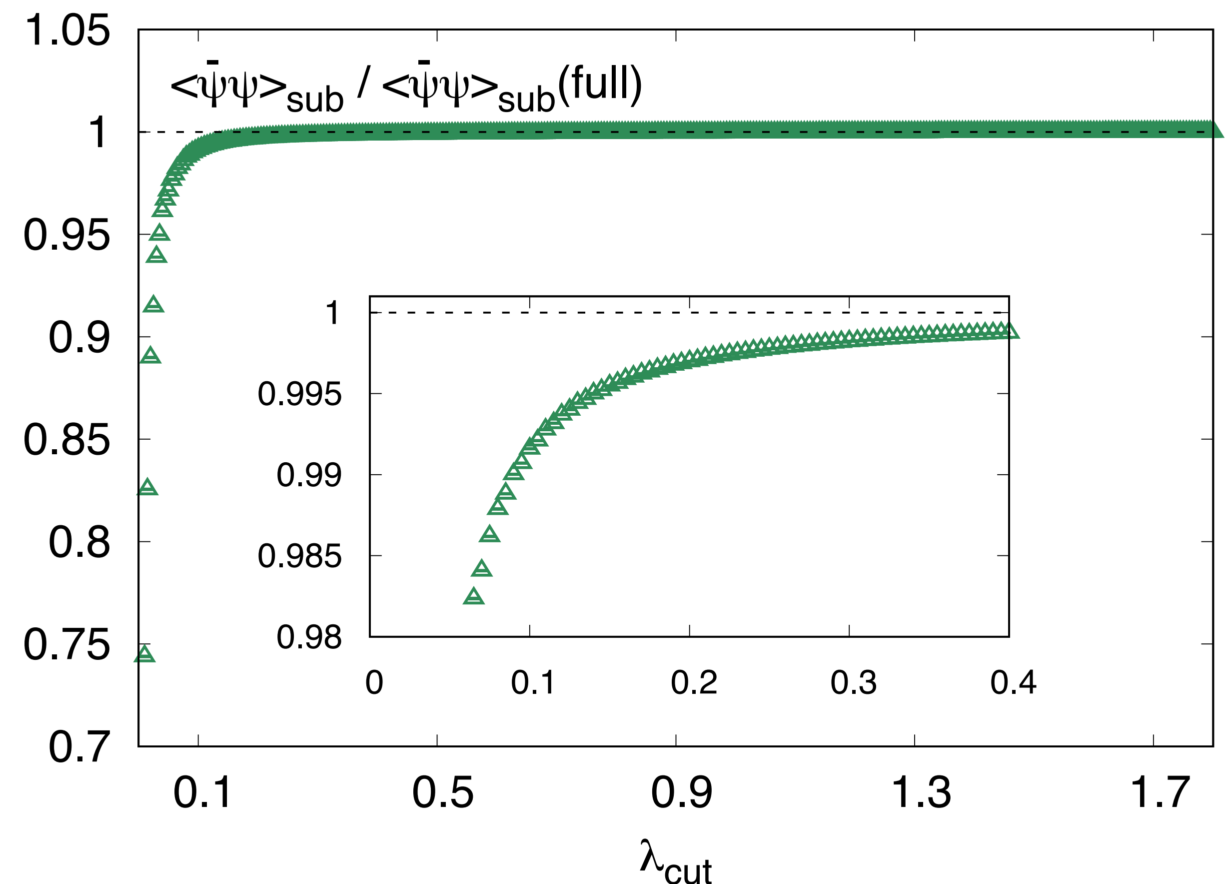
- Neutral PS mesons' masses decrease as eB grows and saturate at large eB
- Lighter mesons are more affected by magnetic field
- Neutral PS mesons have quite large (30~40%) mass reduction
- **qB scaling:**

  electric charge of quark multiplied by B affects the behavior of quantities

UV-divergence of Chiral Condensates

★ $m_s/m_l = 10$





- **Complete Dirac eigenvalue spectrum was obtained and used to estimate UV-divergence part of chiral condensate.**

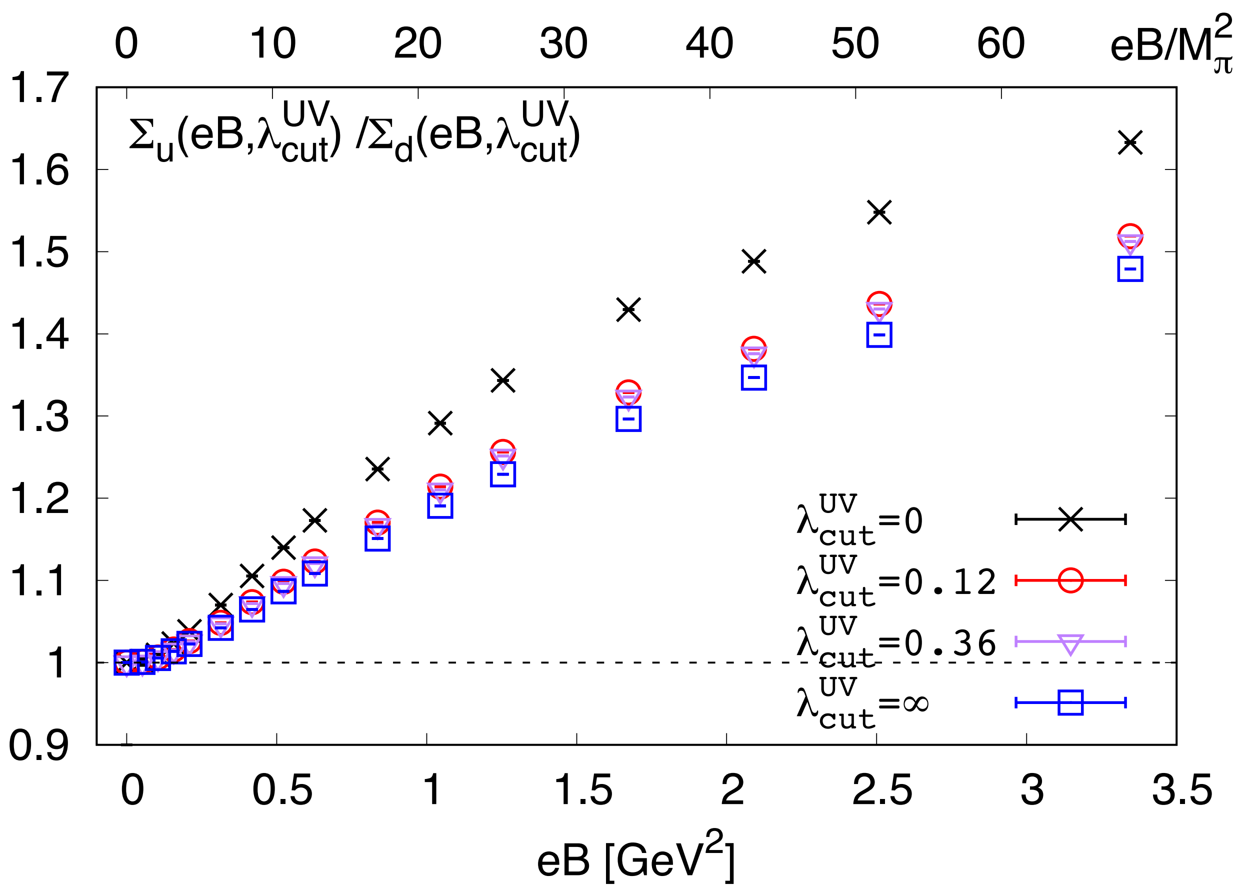
Yu Zhang et al, POS Lattice 2019. L. Giusti and M. Luscher ,JHEP 03, 013

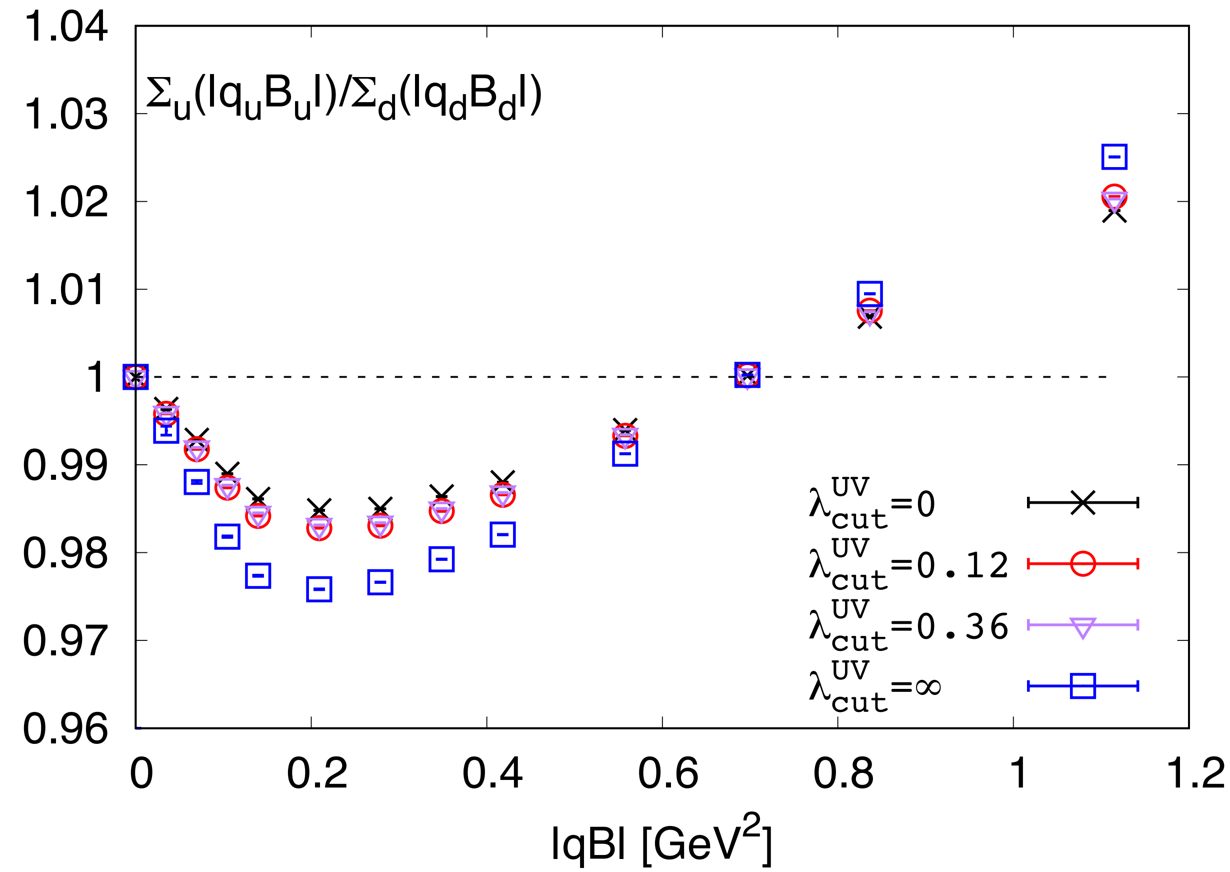
G. Cossu, PTEP 2016, 093B06 . Z. Fodor, PoS LATTICE2015, 310

- $\langle\bar{\psi}\psi\rangle_{sub} \equiv \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s = \int_0^\infty \frac{2m_l(m_s^2 - m_l^2)\rho(\lambda)}{(\lambda^2+m_l^2)(\lambda^2+m_s^2)}\mathrm{d}\lambda$
- $\langle\bar{\psi}\psi\rangle_{l,s} = \int_0^\infty \frac{2\,m_{l,s}\,\rho(\lambda)}{\lambda^2+m_{l,s}^2}\mathrm{d}\lambda \;,\quad \langle\bar{\psi}\psi\rangle_{l,s}^{\mathrm{UV}} = \int_{\lambda_{cut}^{\mathrm{UV}}}^\infty \frac{2\,m_{l,s}\,\rho(\lambda)}{\lambda^2+m_{l,s}^2}\mathrm{d}\lambda$

λ_{cut}^{UV}	$\langle\bar{\psi}\psi\rangle_l^{UV}/\langle\bar{\psi}\psi\rangle_l(full)$	$\langle\bar{\psi}\psi\rangle_s^{UV}/\langle\bar{\psi}\psi\rangle_s(full)$
0.12	32%	83%
0.24	29%	76%
0.36	27%	71%

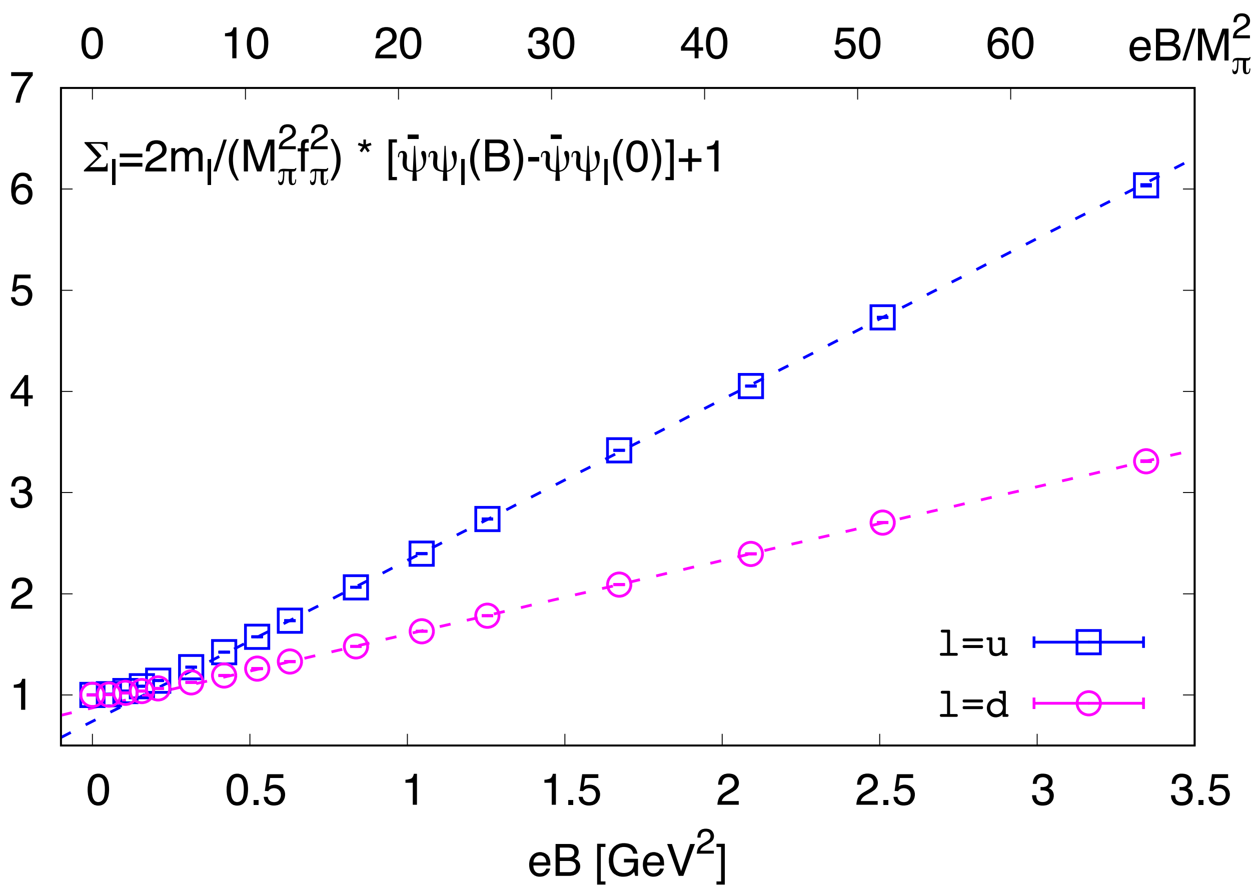
Chiral Condensates

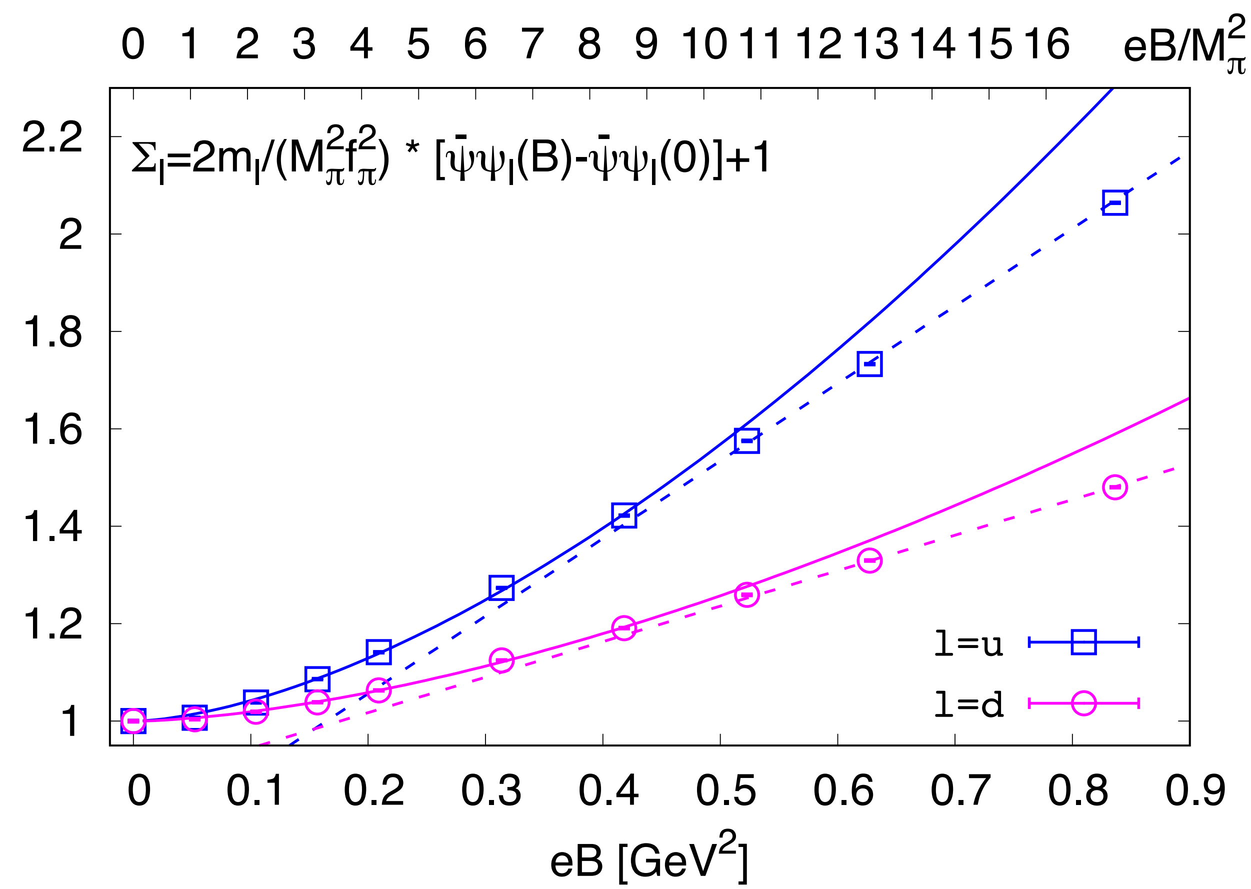




- $$\Sigma_l(B, \lambda_{cut}^{UV}) = \frac{2m_l}{M_\pi^2 f_\pi^2}\left(\langle\bar\psi\psi\rangle_l(B) - \langle\bar\psi\psi\rangle_l^{UV}(B=0,\ \lambda_{cut}^{UV})\right) + 1$$
- qB scaling also holds true for chiral condensate

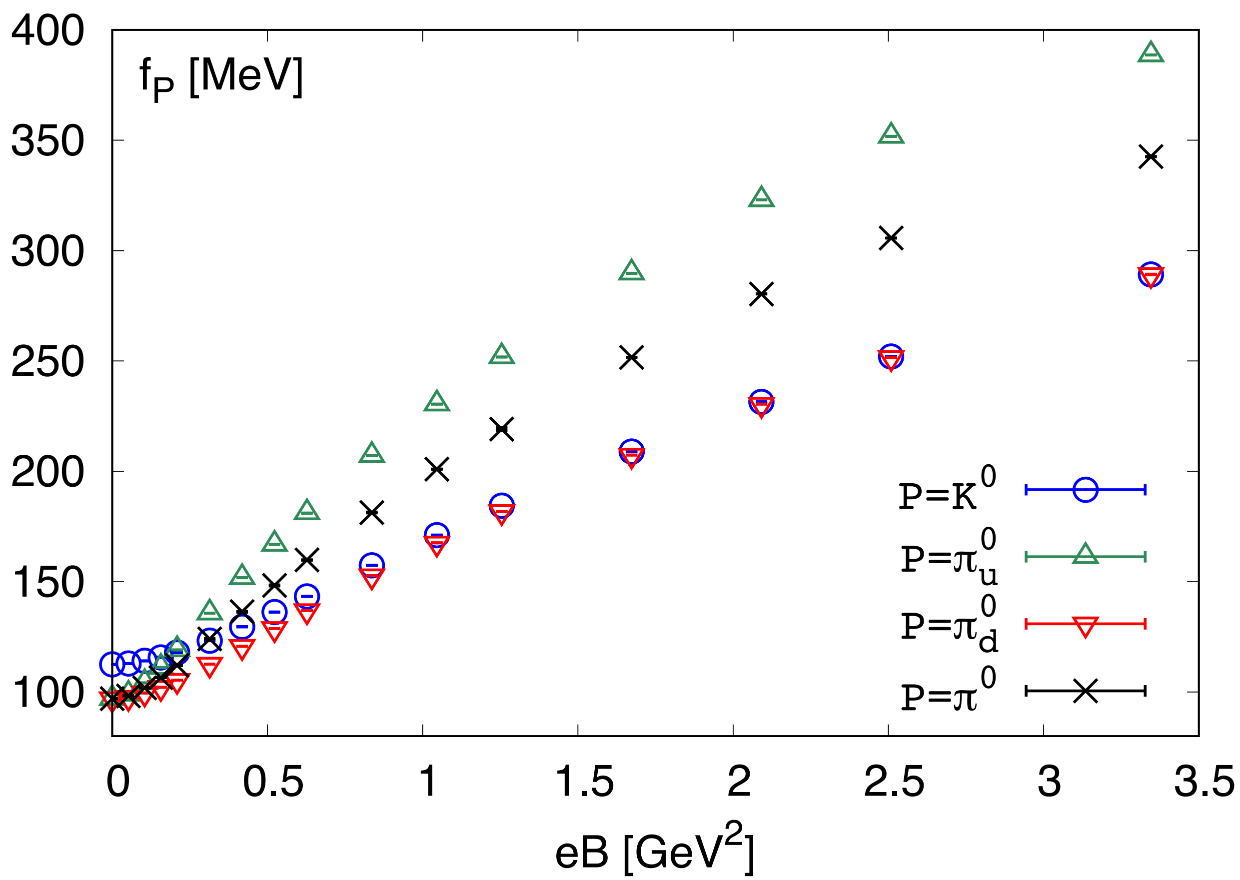
Chiral Condensates

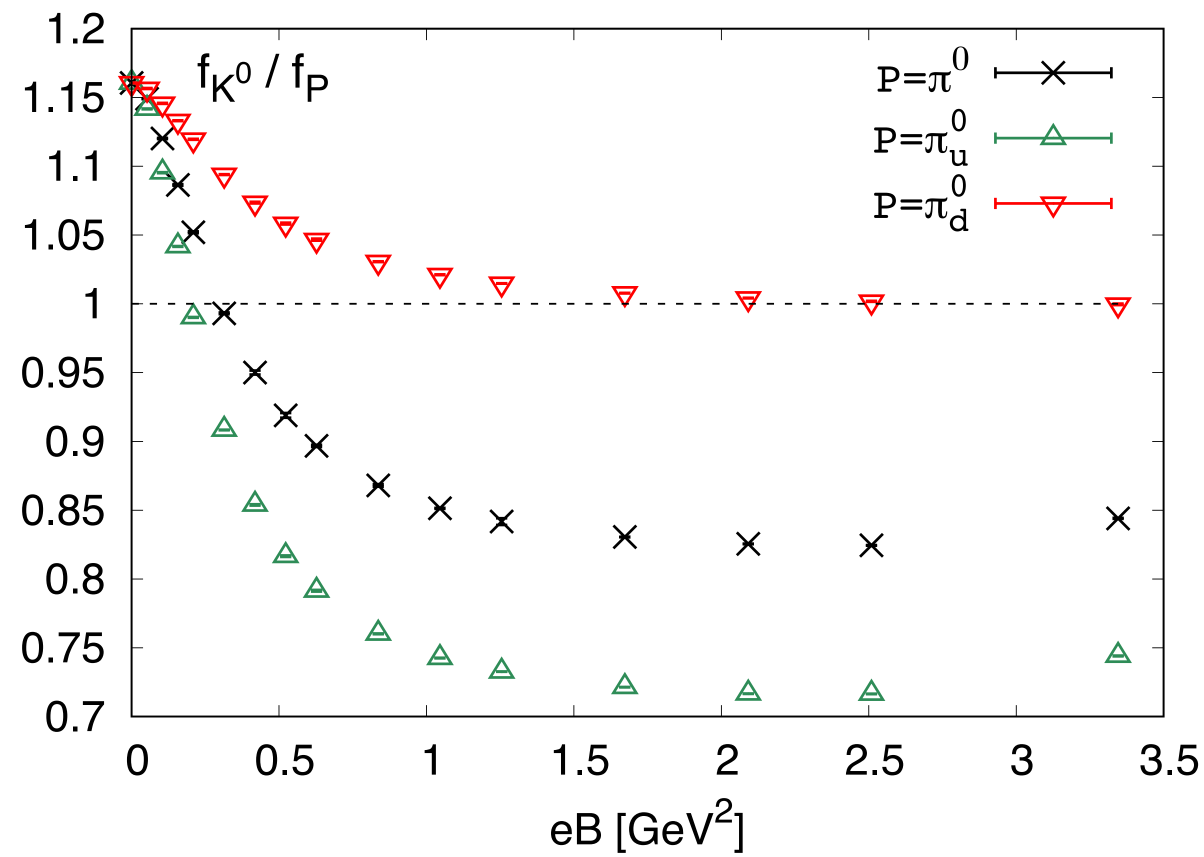




- Chiral condensates increase as eB grows
- **Two-parameter fits:**
  - In large eB ∈ [0.5, 3.5] GeV^2, Σ_l is almost linear in eB (dashed line)
  - In small eB ∈ [0, 0.5] GeV^2 , Σ_l can be described with $h(eB)^\gamma + 1$ (solid line)

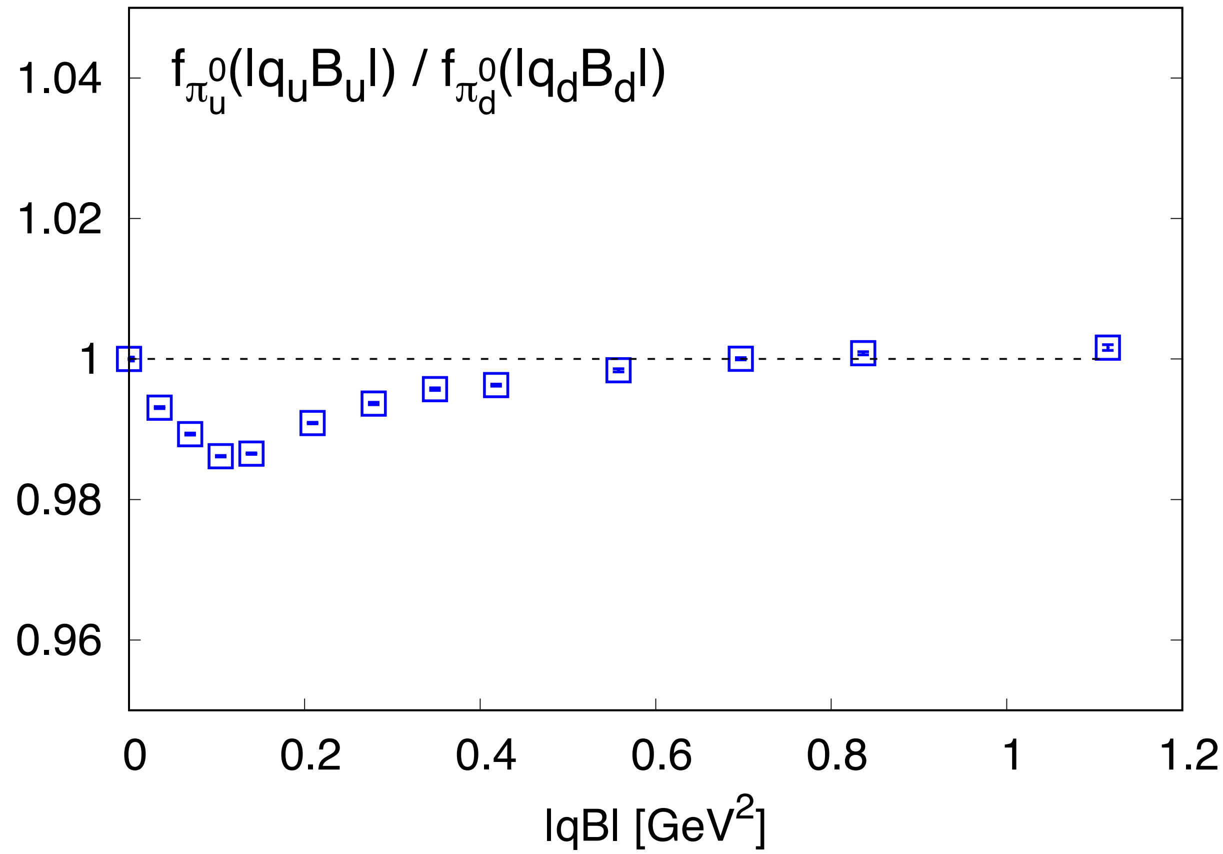
Decay Constants

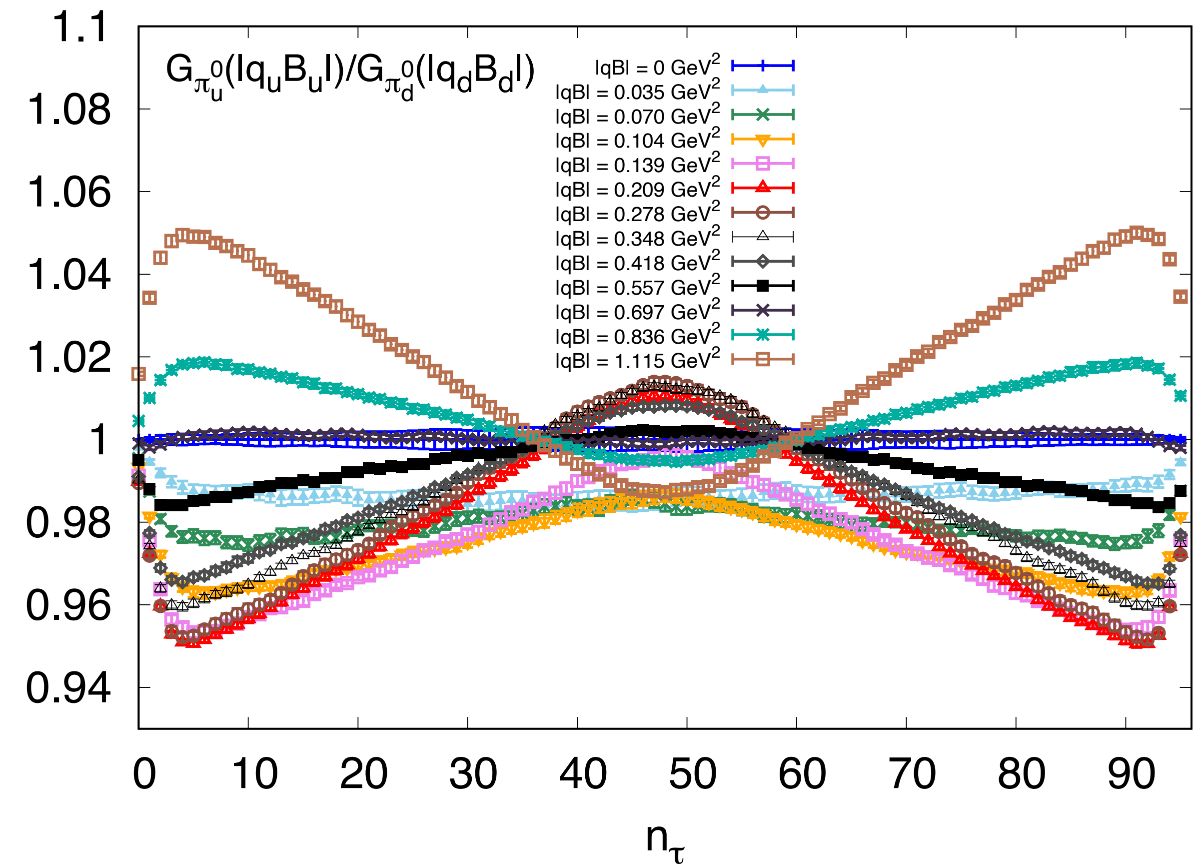




- Neutral pion and kaon decay constants increase as eB grows
- f_{K^0} / f_P decrease as eB increases in eB $\in$ [0,1.5] GeV^2
- f_{K^0} / f_P saturate in [1.5, 2.5] GeV^2

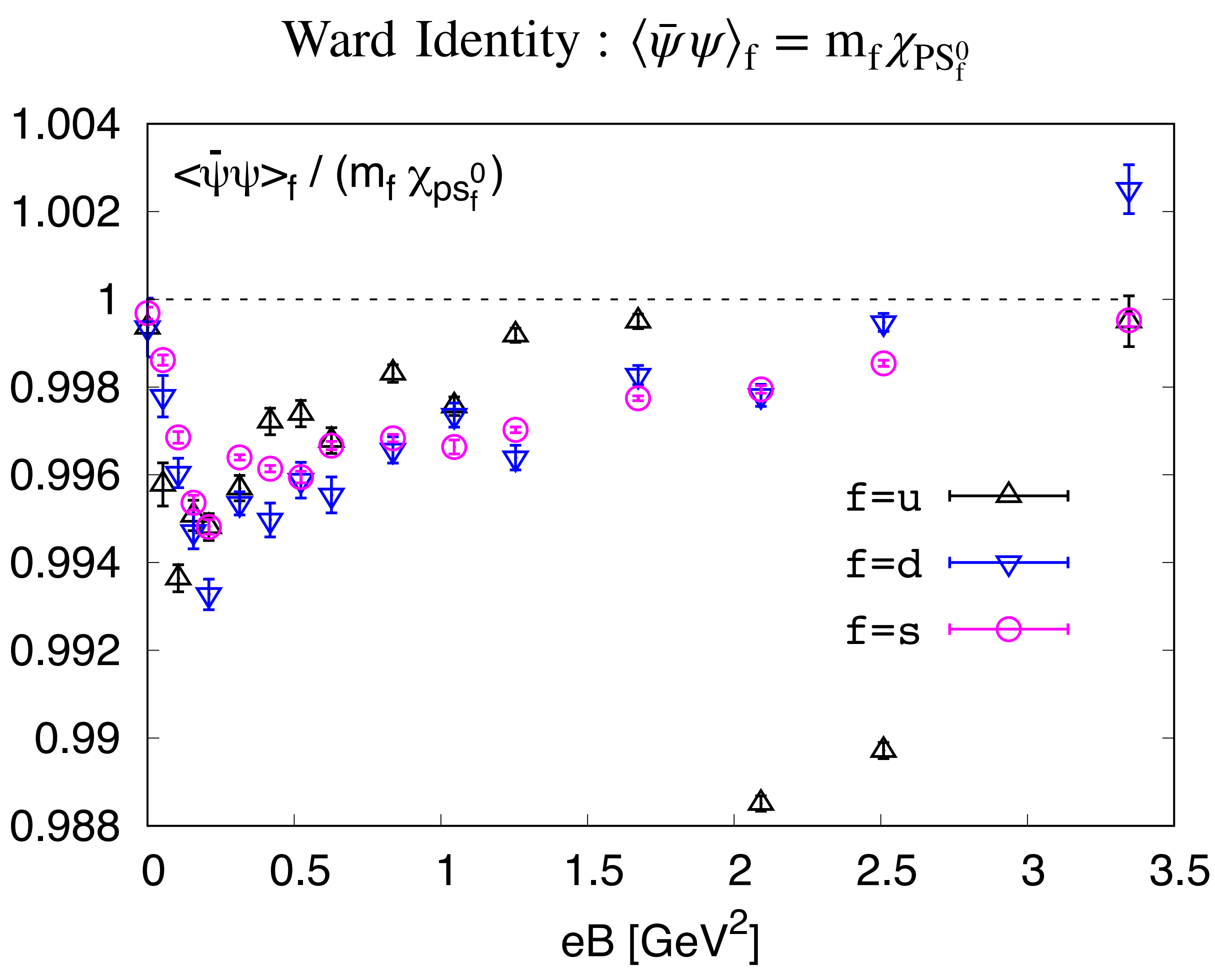
Decay Constants

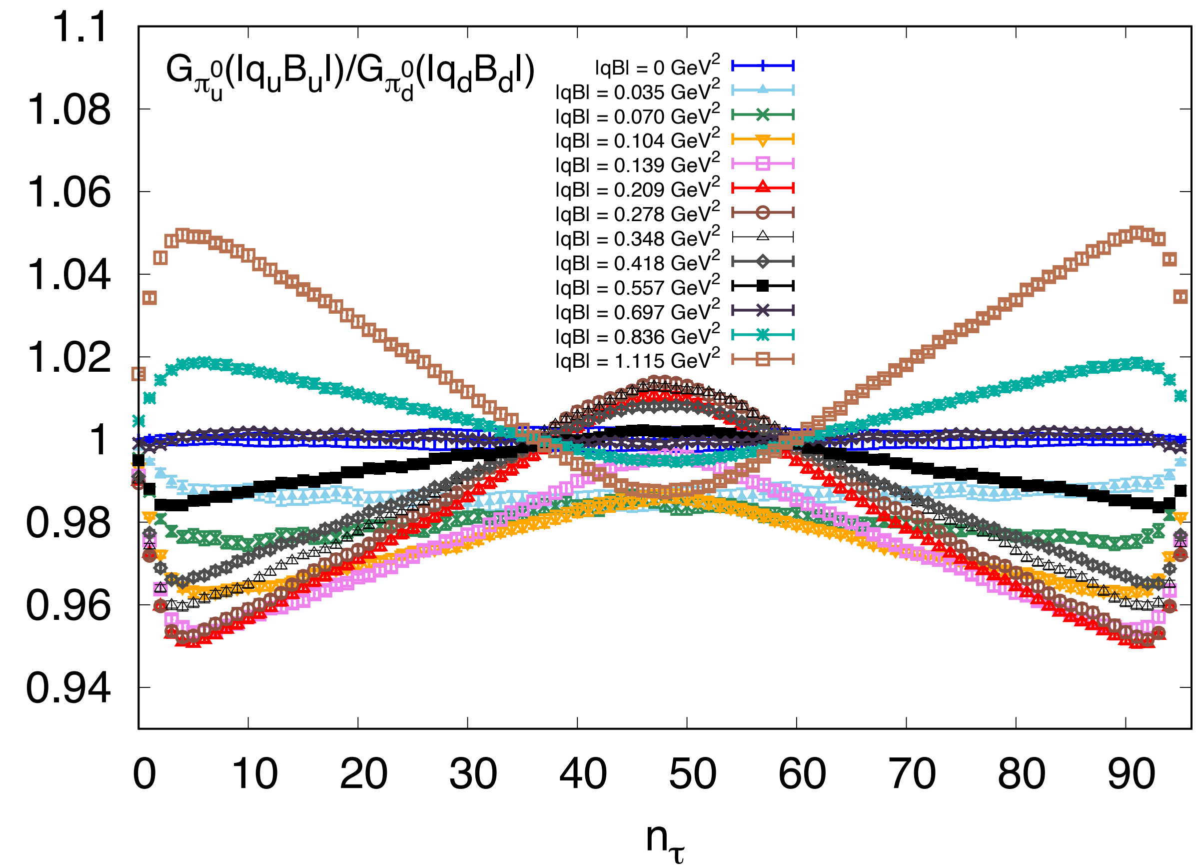




- qB scaling holds for $M_{\pi_u^0}(M_{\pi_d^0})$, $\Sigma_u(\Sigma_d)$, $f_{\pi_u^0}(f_{\pi_d^0})$
- The origin of all is the correlator, $G_{\pi_u^0}\left(\tau, q_uB_u\right)/G_{\pi_d^0}\left(\tau, q_dB_d\right)$ itself holds for qB scaling

Decay Constants

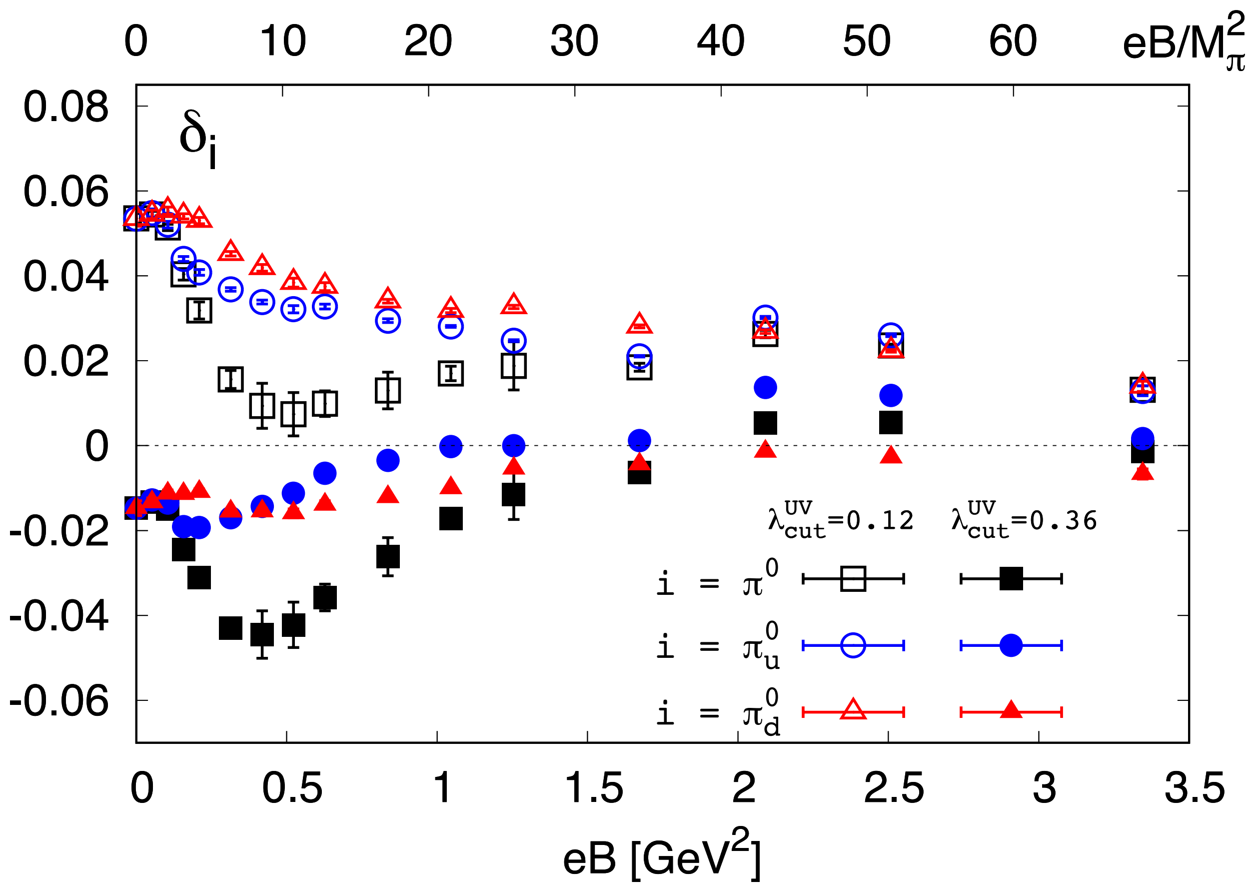




- qB scaling holds for $M_{\pi_u^0}(M_{\pi_d^0})$, $\Sigma_u(\Sigma_d)$, $f_{\pi_u^0}(f_{\pi_d^0})$
- The origin of all is the correlator, $G_{\pi_u^0}\left(\tau, q_u B_u\right) / G_{\pi_d^0}\left(\tau, q_d B_d\right)$ itself holds for qB scaling.

GMOR relation

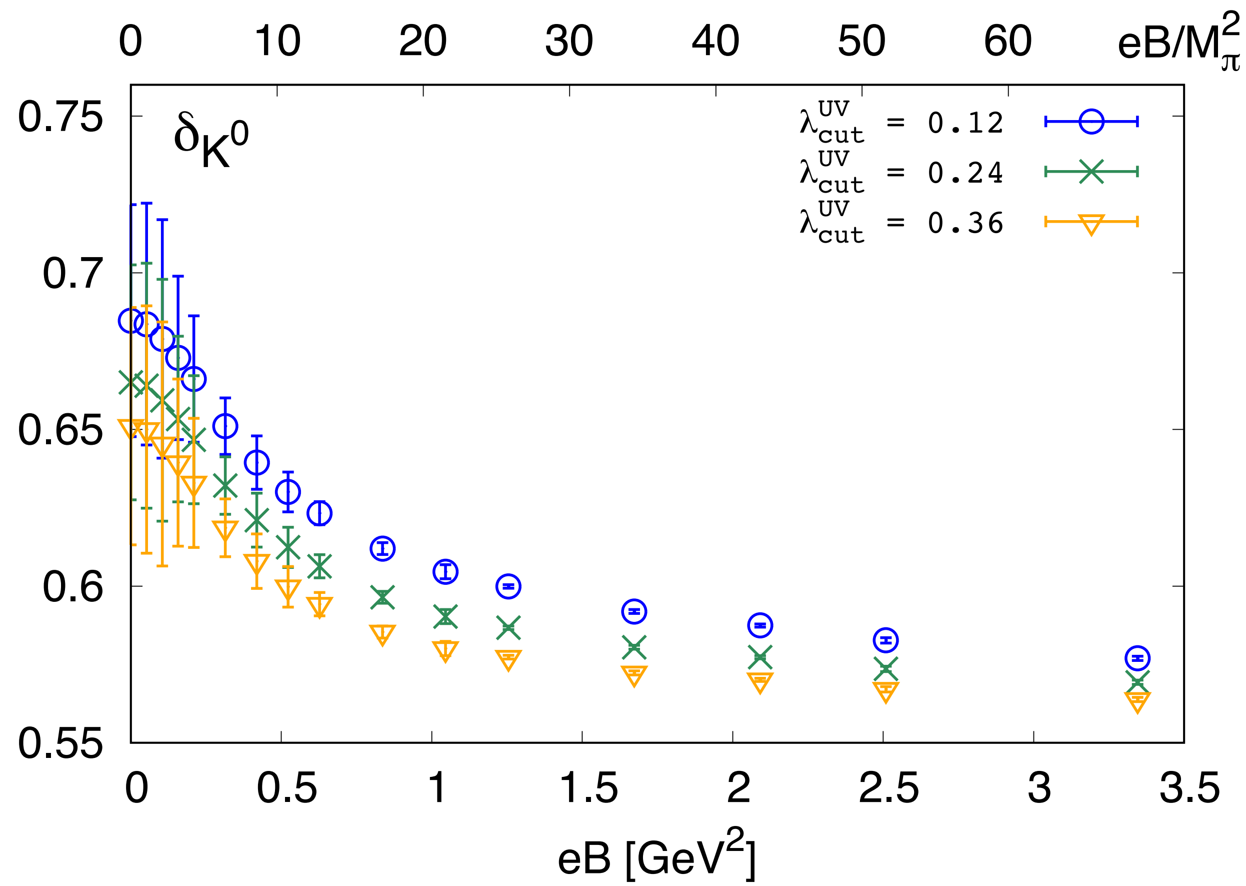
- $4m_u \langle \bar{\psi}\psi \rangle_u = 2f^2_{\pi^0_u} \, M^2_{\pi^0_u} (1 - \delta_{\pi^0_u})$
- $4m_d \langle \bar{\psi}\psi \rangle_d = 2f^2_{\pi^0_d} \, M^2_{\pi^0_d} (1 - \delta_{\pi^0_d})$
- $(m_u + m_d) \, \left(\langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d \right) = 2 \, f^2_{\pi^0} \, M^2_{\pi^0} (1 - \delta_{\pi^0})$
- $(m_s + m_d) \, \left(\langle \bar{\psi}\psi \rangle_s + \langle \bar{\psi}\psi \rangle_d \right) = 2 \, f^2_K \, M^2_K \, (1 - \delta_K)$



χ_{PT} : $\delta_\pi = 6.2 \; \pm \; 1.6 \, \%$

M. Jamin. Phys. Lett. B 538, 71

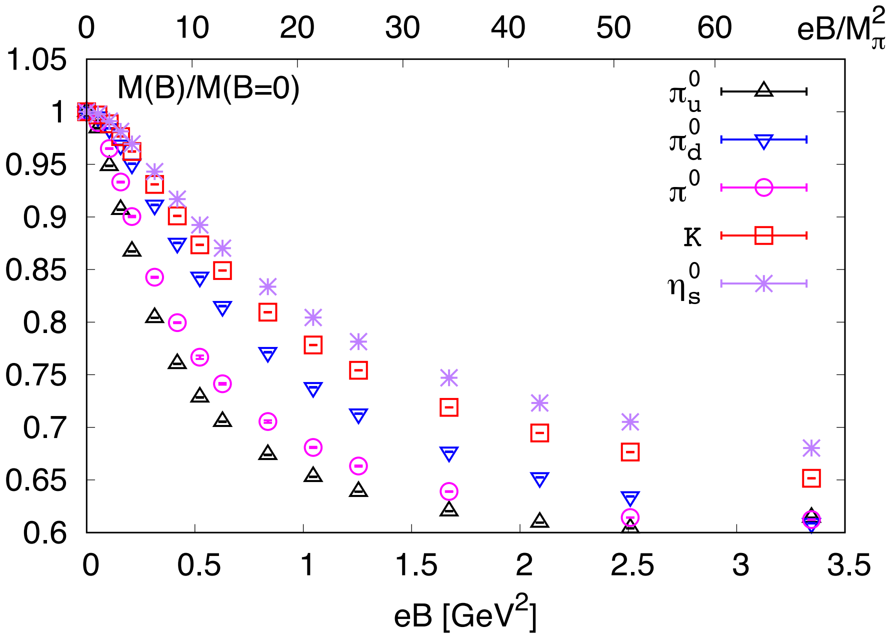
J. Bordes et al. JHEP 05, 064

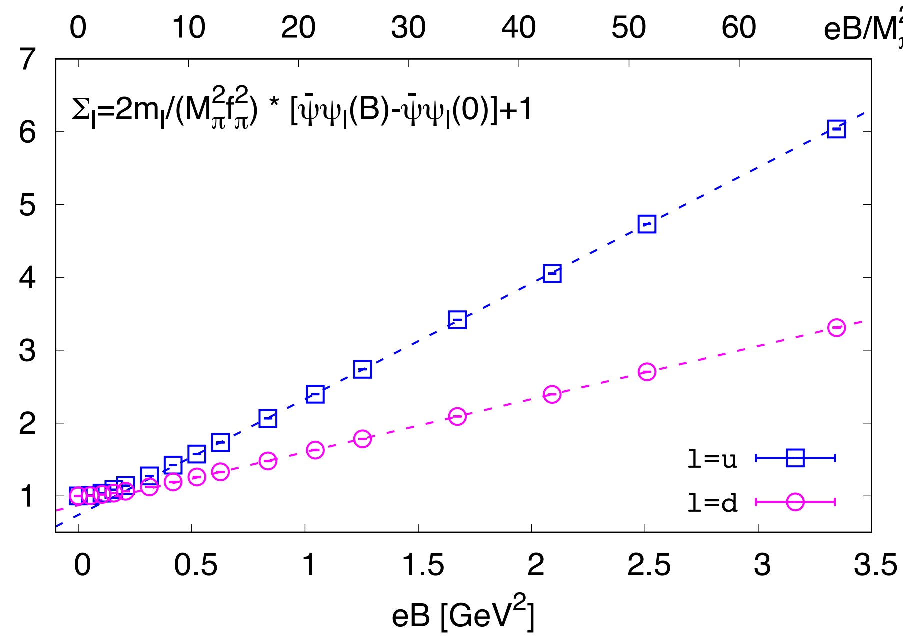


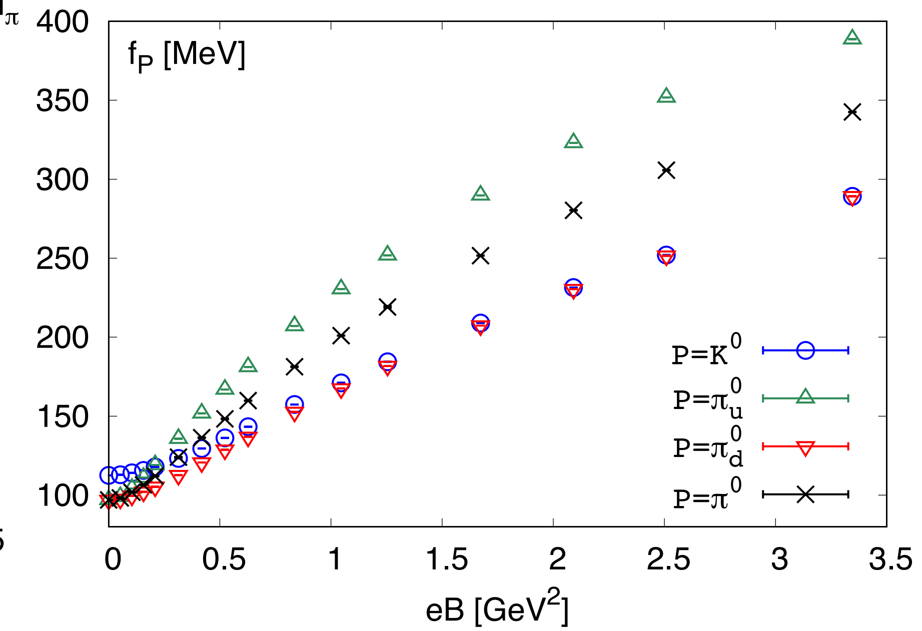
χ_{PT} : $\delta_K = 55 \; \pm \; 5 \, \%$

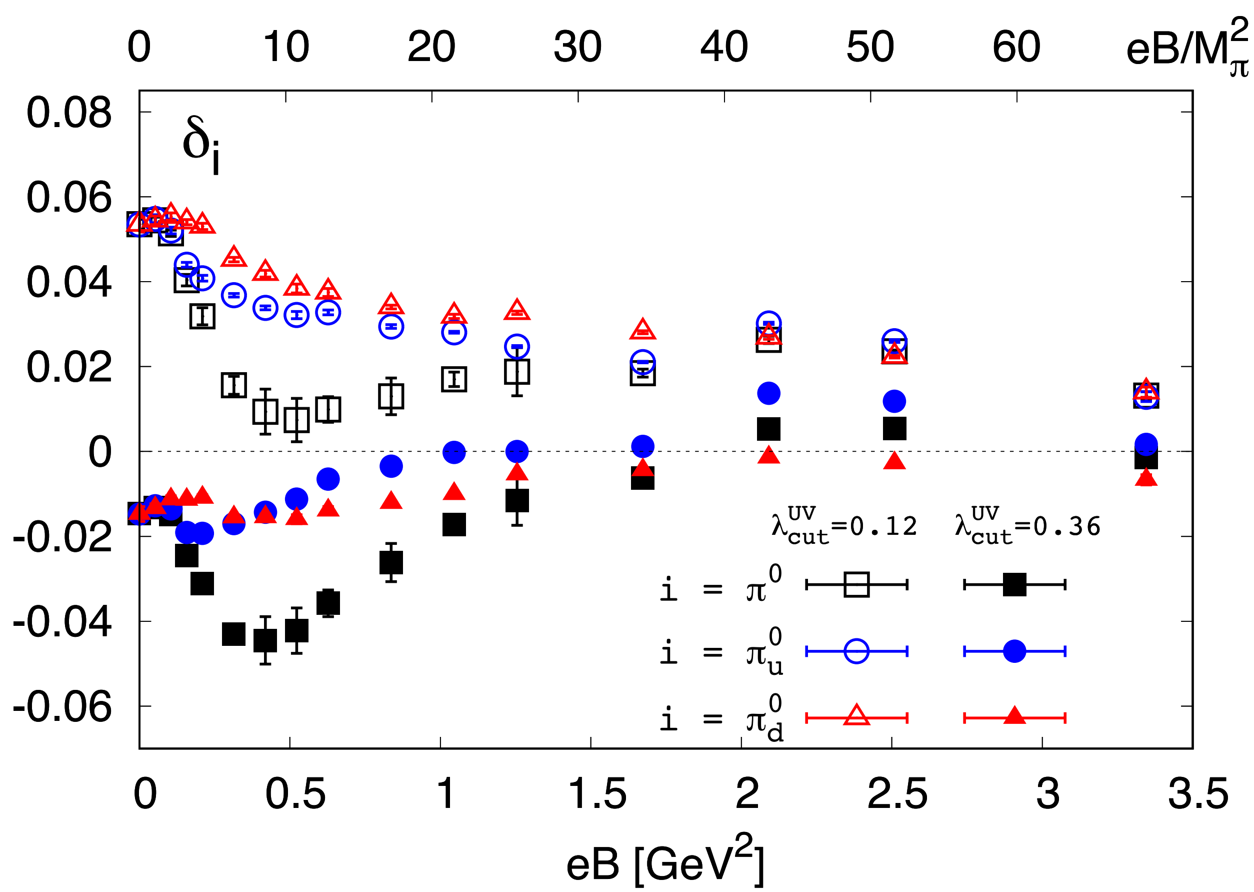
J. Bordes et al. JHEP 10, 102

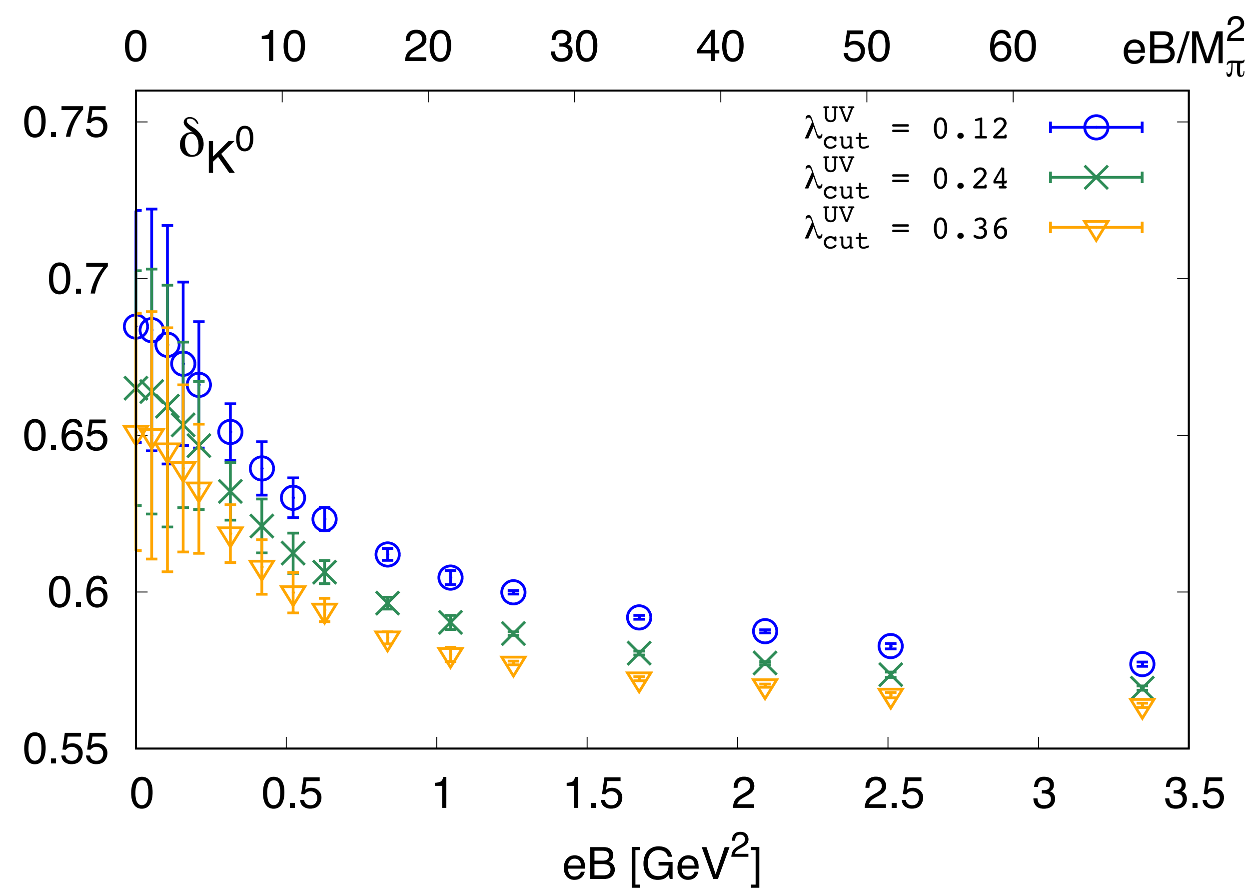
Summary


M(B)/M(B=0)
π_u^0
π_d^0
π^0
K
η_s^0
eB [GeV^2]
eB/M$_\pi^2$

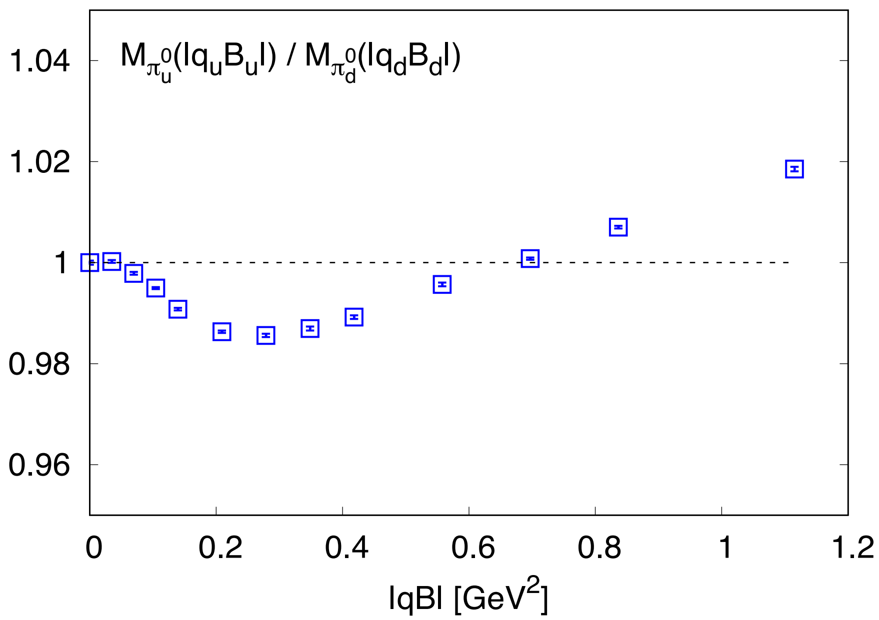

$\Sigma_l=2m_l/(M_\pi^2 f_\pi^2) * [\bar\psi\psi_l(B)-\bar\psi\psi_l(0)]+1$
l=u
l=d
eB [GeV^2]
eB/M$_\pi^2$

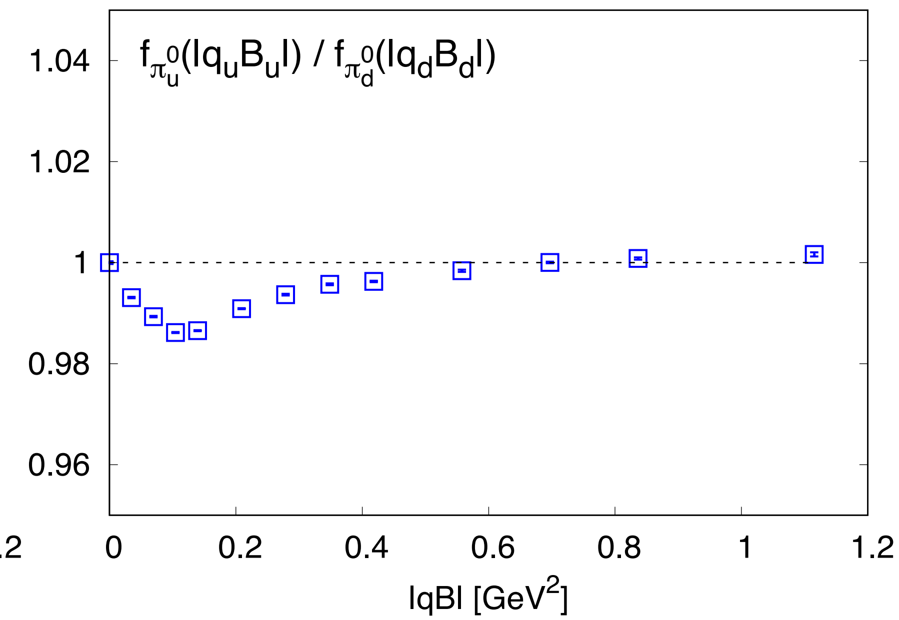

f$_P$ [MeV]
P=K^0
P=π_u^0
P=π_d^0
P=π^0
eB [GeV^2]

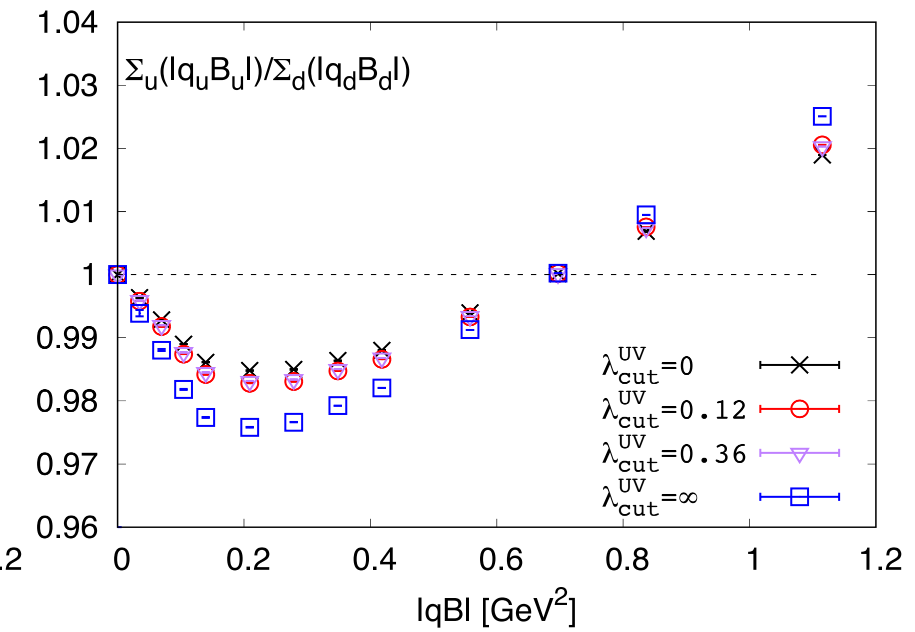

δ_i
λ_{cut}^{UV}=0.12
λ_{cut}^{UV}=0.36
i = π^0
i = π_u^0
i = π_d^0
eB [GeV^2]
eB/M$_\pi^2$

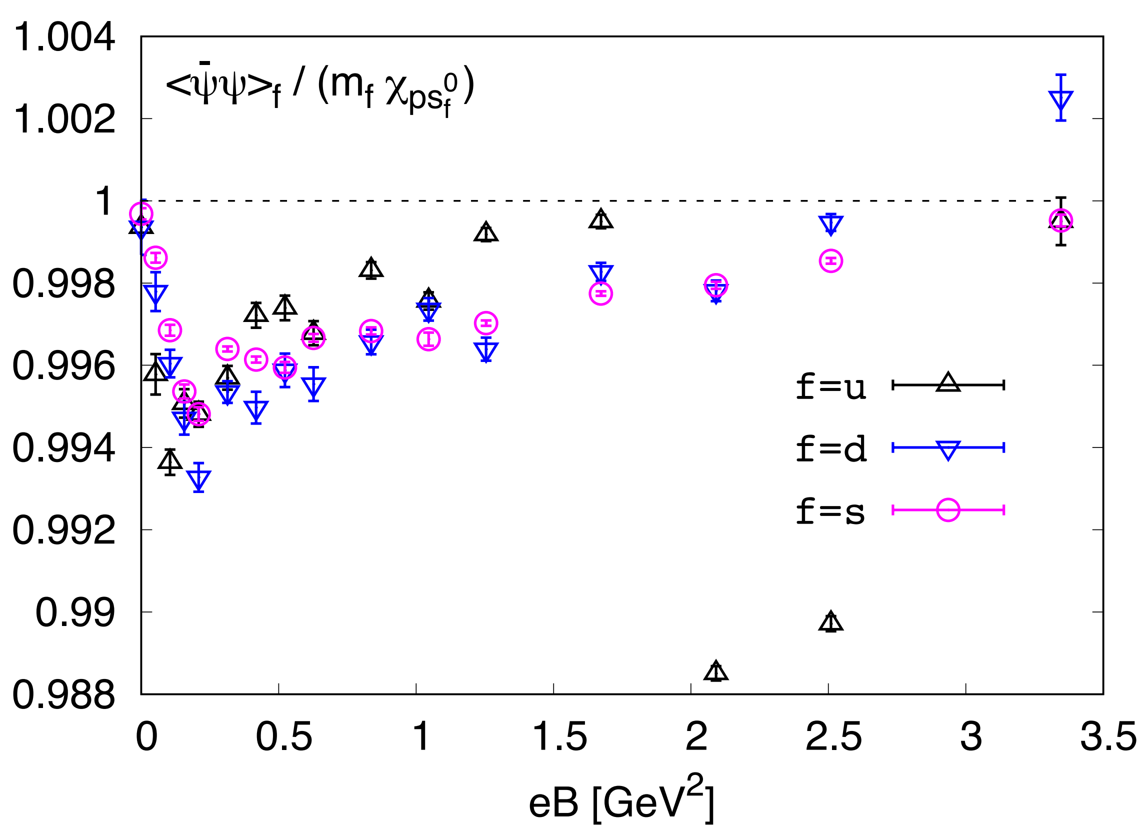

δ_{K^0}
λ_{cut}^{UV} = 0.12
λ_{cut}^{UV} = 0.24
λ_{cut}^{UV} = 0.36
eB [GeV^2]
eB/M$_\pi^2$

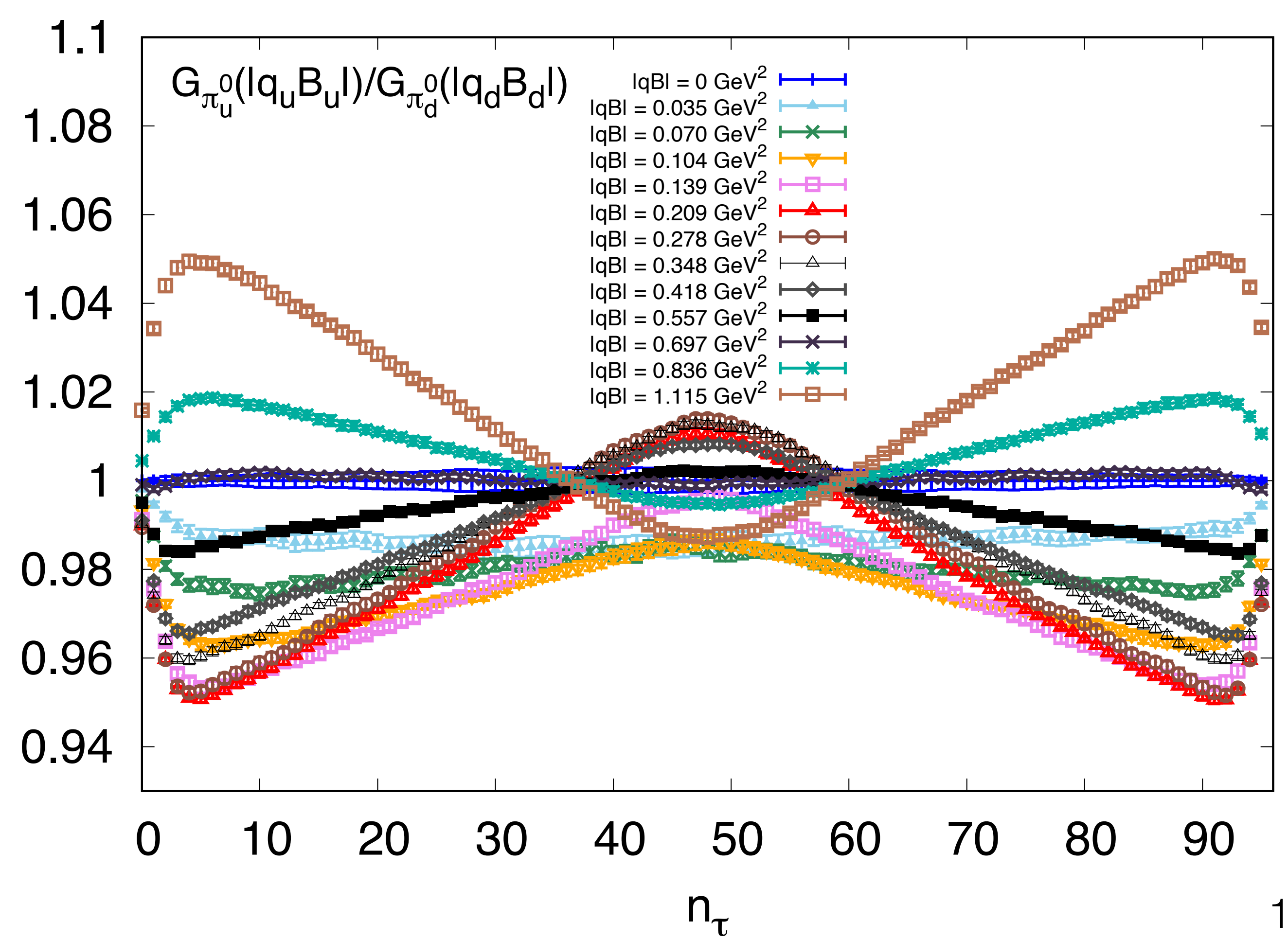
Summary


$M_{\pi_u^0}(|q_uB_u|) / M_{\pi_d^0}(|q_dB_d|)$
|qB| [GeV^2]


$f_{\pi_u^0}(|q_uB_u|) / f_{\pi_d^0}(|q_dB_d|)$
|qB| [GeV^2]


$\Sigma_u(|q_uB_u|)/\Sigma_d(|q_dB_d|)$
$\lambda^{UV}_{cut}=0$
$\lambda^{UV}_{cut}=0.12$
$\lambda^{UV}_{cut}=0.36$
$\lambda^{UV}_{cut}=\infty$
|qB| [GeV^2]


$<\bar{\psi}\psi>_f / (m_f \chi_{ps_f^0})$
f=u
f=d
f=s
eB [GeV^2]


$G_{\pi_u^0}(|q_uB_u|)/G_{\pi_d^0}(|q_dB_d|)$
|qB| = 0 GeV^2
|qB| = 0.035 GeV^2
|qB| = 0.070 GeV^2
|qB| = 0.104 GeV^2
|qB| = 0.139 GeV^2
|qB| = 0.209 GeV^2
|qB| = 0.278 GeV^2
|qB| = 0.348 GeV^2
|qB| = 0.418 GeV^2
|qB| = 0.557 GeV^2
|qB| = 0.697 GeV^2
|qB| = 0.836 GeV^2
|qB| = 1.115 GeV^2
n_τ