Partial deconfinement for some bosonic matrix models

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Introduction

Deconfinement transition

- It takes place in QCD (and several gauge theories) at finite temperature.
    - The property from non-perturbative effects
- Closely related to the **black hole formation/evaporation** via holography.
    - Schwarzschild black hole with negative specific heat appears
      →What is the counterpart in gauge theory side?

Partial Deconfinement

Around the critical temperature, the physical degrees of freedom those behave as in confined/deconfined phases are **coexisting in internal space (color space).**

[Hanada, Ishiki & HW, (2018)/Hanada, Jevicki, Peng & Wintergerst, (2019)/Hanada & Robinson, (2019)]

Confinement at large N ⟷ Bose-Einstein Condensate

Permutation symmetry of N identical boson system is gauge symmetry.

[Hanada, Shimada & Wintergerst, (2019)]

Motivation & Strategy

Demonstration to separate the color d.o.f. contained in matters into confined/deconfined sectors.

c.f.) In gauge sector, the distribution of the Polyakov line phases

$$\rho^{(P)}(\theta) = \left(1 - \frac{M}{N}\right)\rho^{(P)}_{\text{con}}(\theta) + \frac{M}{N}\rho^{(P)}_{\text{dec}}(\theta; M) = \left(1 - \frac{M}{N}\right)\cdot\frac{1}{2\pi} + \frac{M}{N}\rho^{(P)}_{\text{dec}}(\theta; M)$$

- Strong evidence the partially-deconfined phase exists in nonperturbative regime.

- **Gauged Gaussian matrix model**
  - A very simple toy-model but the deconfinement take place.
  - We can estimate many physical quantities analytically.

→ The benchmark test for our methods, before Yang-Mills matrix model.

- **Yang-Mills matrix model**
  - Dimensional reduced model of the Yang-Mills theory in d dimensions
  - Bosonic part of BFSS model ((0+1)dim. Super Yang-Mills theory)

Gauged Gaussian matrix model

[Hanada, Jevicki, Peng & Wintergerst, (2019)]

$$S = N \int_0^\beta \mathrm{d}t \, \mathrm{Tr} \left\{ \frac{1}{2}(D_t X_I)^2 + \frac{1}{2} X_I^2 \right\}$$

X_I : $N \times N$ Hermitian matrices

$I = 1,2,\cdots,d \qquad D_t X_I = \partial_t X_I - \mathrm{i}\left[A_t, X_I\right]$

At critical temperature $T_c = 1/\ln d$,

- The Polyakov loop $P = \frac{1}{N}\mathrm{Tr}\,\mathscr{P}\mathrm{e}^{i\int \mathrm{d}t A_t}$ can take any value in [0,1/2] without changing F.

  In particular, in the large N limit,

  with static diagonal gauge.

$$|P| = \frac{1}{N}\sum_{j=1}^{N} \mathrm{e}^{i\theta_j} = \int_{-\pi}^{\pi} \mathrm{d}\theta \rho^{(P)}(\theta)\, \mathrm{e}^{\mathrm{i}\theta} = \frac{M}{2N}$$

$$A_t = \frac{1}{\beta}\,\mathrm{diag}\left(\theta_1, \cdots, \theta_N\right)$$

$|P|$ can take value in $0 \le |P| \le 1/2$ ⟺ M can take value in $0 \le M \le N$ /

$M = 0$: Hagedorn transition

$M = N$: GWW transition

- For the Polyakov line phases

$$\rho^{(P)}(\theta) = \left(1 - \frac{M}{N}\right) \cdot \rho^{(P)}_{\mathrm{con}}(\theta) + \frac{M}{N} \cdot \rho^{(P)}_{\mathrm{dec}}(\theta) = \left(1 - \frac{M}{N}\right) \cdot \frac{1}{2\pi} + \frac{M}{N} \cdot \frac{1}{2\pi}(1 + \cos\theta)$$

Demonstration 1: distribution of x

In path integral formalism, for the scalar fields

$$\langle E(T_c)\rangle = \left\langle \frac{N}{\beta}\int \mathrm{d}t\, \mathrm{Tr}\, X_I^2 \Big|_{T=T_c} \right\rangle = \frac{d}{2}(N^2 - M^2) + \left(\frac{d}{2}+\frac{1}{4}\right)M^2$$

$$= dN^2\langle x^2\rangle = d(N^2 - M^2)\langle x^2\rangle_{\mathrm{con}} + dM^2\langle x^2\rangle_{\mathrm{dec}}$$

$$x \equiv \left\{\sqrt{N}X_{I,ii}, \sqrt{2N}\mathrm{Re}X_{I,ij}, \sqrt{2N}\mathrm{Im}X_{I,ij}\right\}$$

$(i > j)$

→ The variances are

$$\langle x^2\rangle_{\mathrm{con}} = \int \mathrm{d}x\; x^2 \rho^{(X)}_{\mathrm{con}}(x) = \frac{1}{2}, \qquad \langle x^2\rangle_{\mathrm{dec}} = \int \mathrm{d}x\; x^2 \rho^{(X)}_{\mathrm{dec}}(x) = \frac{1}{d}\left(\frac{d}{2}+\frac{1}{4}\right) = \frac{5}{8}$$

At critical temperature,

$$\rho^{(\mathrm{X})}(x;M) = \left(1 - \left(\frac{M}{N}\right)^2\right)\rho^{(X)}_{\mathrm{con}}(x) + \left(\frac{M}{N}\right)^2 \rho^{(X)}_{\mathrm{dec}}(x)$$

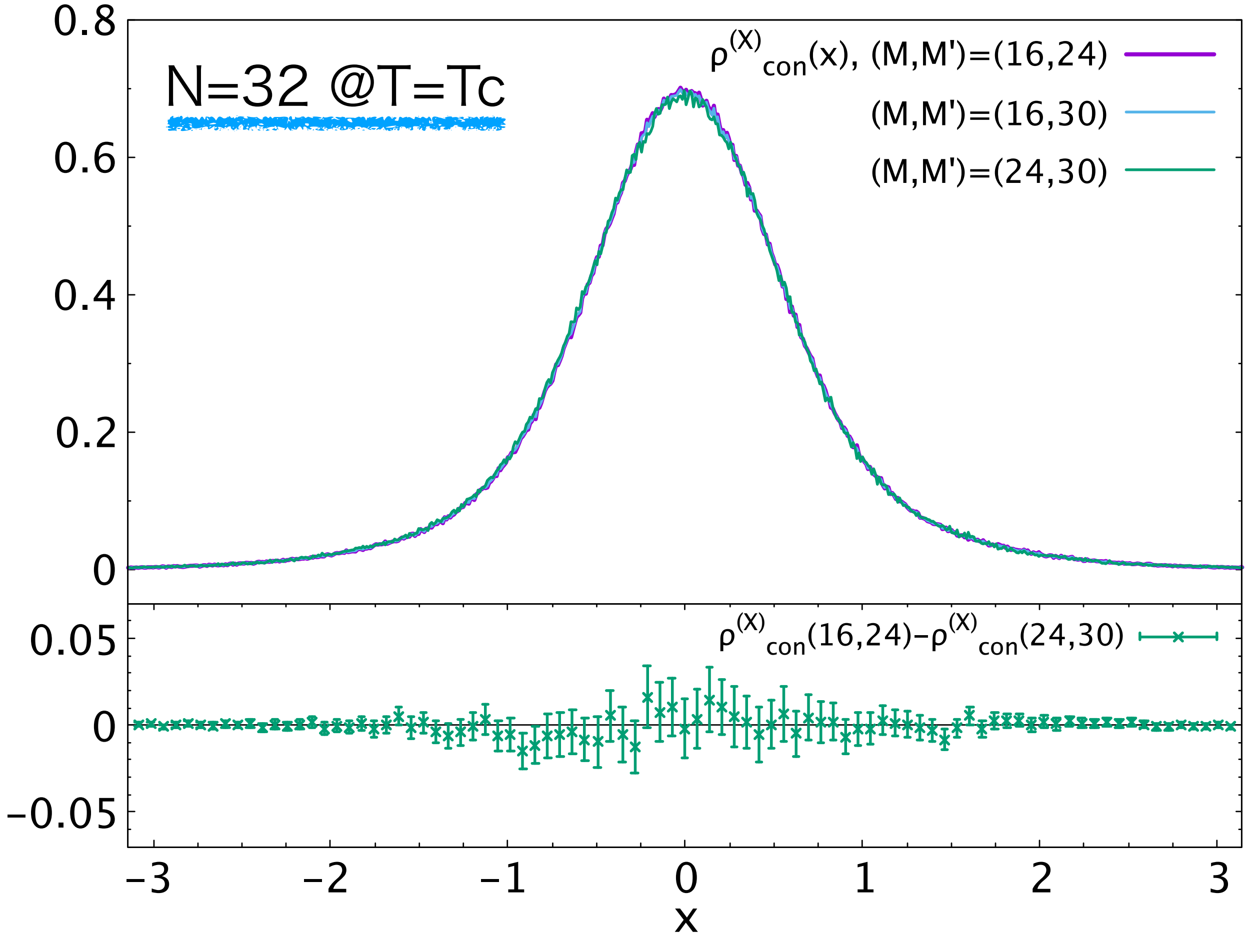
By choosing M, M' as $M = 2NP$

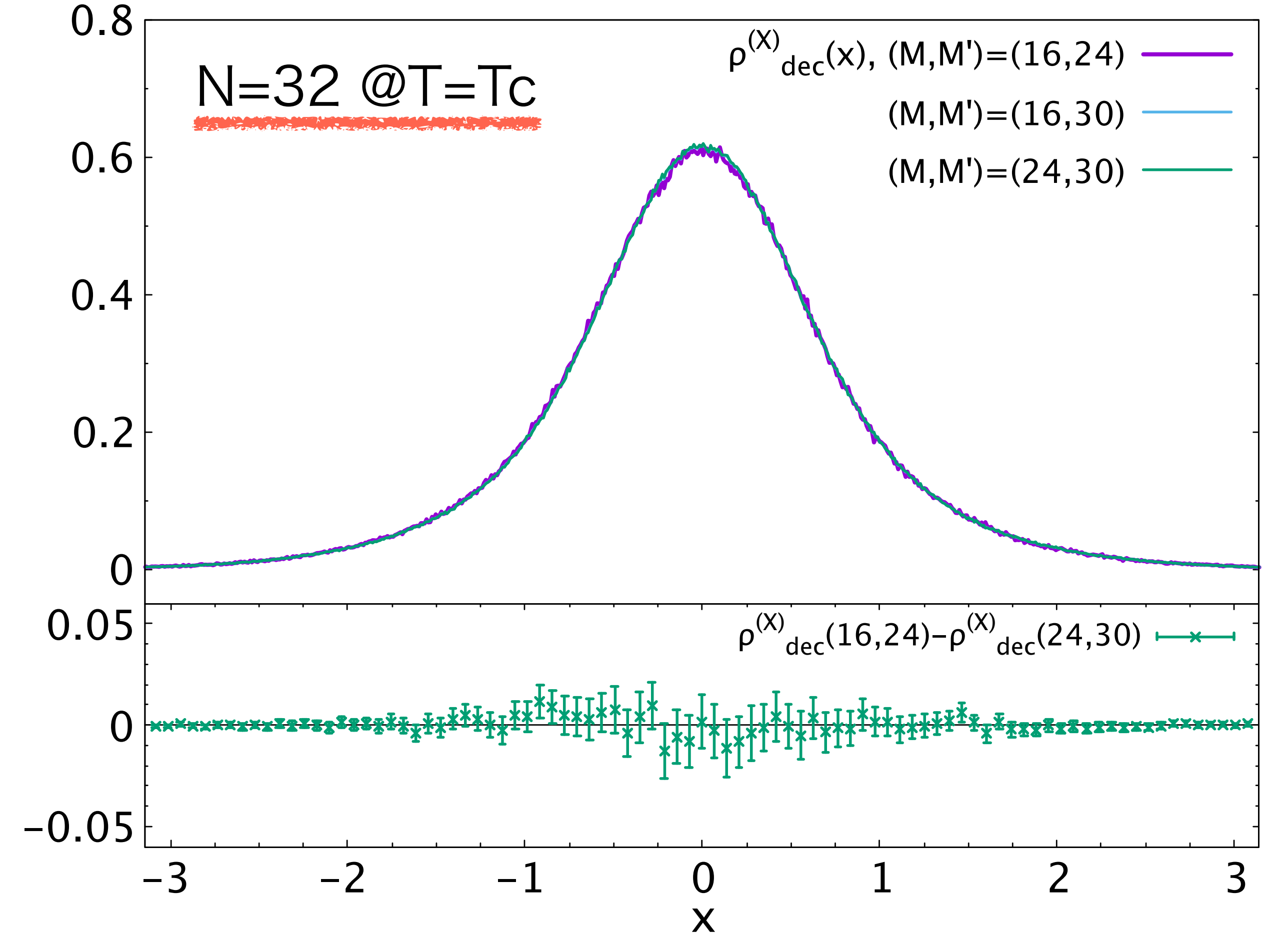
$$(b-a)\rho^{(X)}_{\mathrm{con}}(x) = b\rho^{(\mathrm{X})}(x;a) - a\rho^{(X)}(x;b)$$

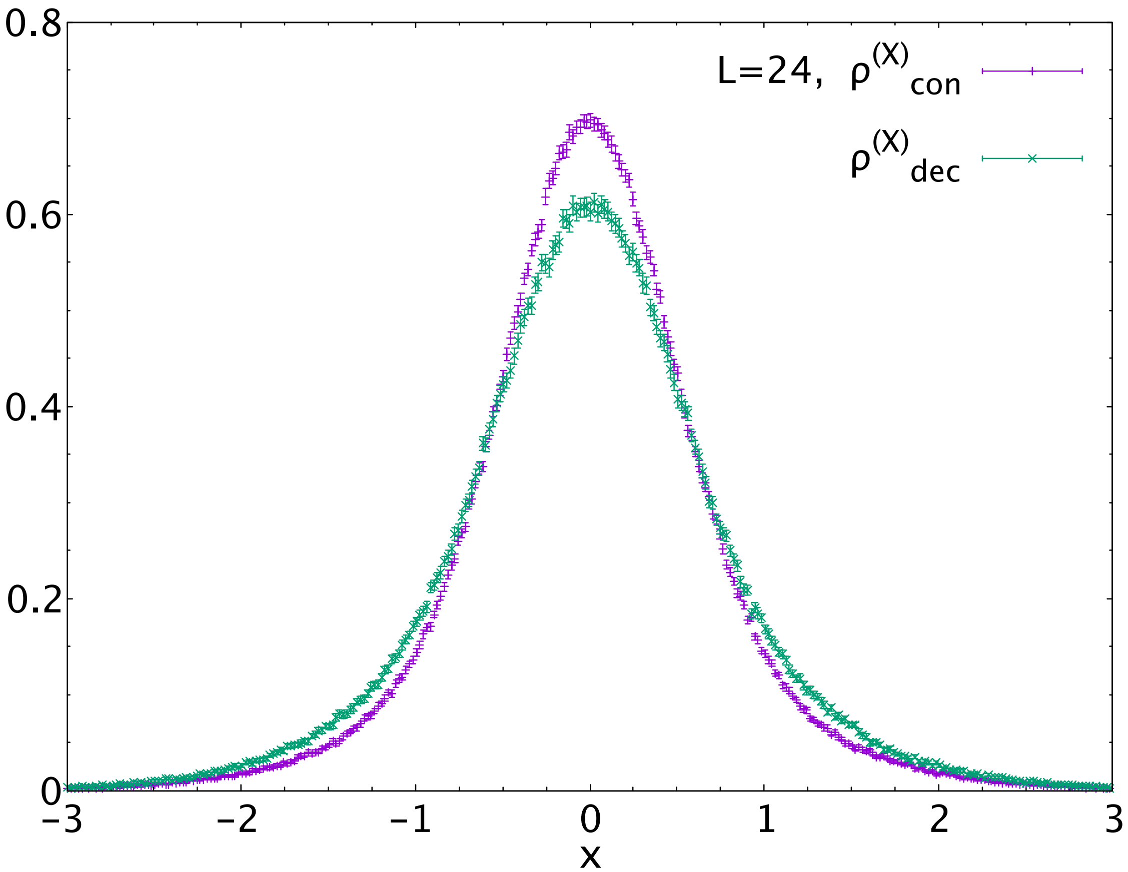
$$(a-b)\rho^{(X)}_{\mathrm{dec}}(x) = (1-b)\rho^{(X)}(x;a) - (1-a)\rho^{(X)}(x;b)$$

$a \equiv (M/N)^2$

$b \equiv (M'/N)^2$







- The shape is determined independently of M, M'

  (up to finite-size effect which is small there)

- The values of variances matches $\approx 1/2$ and $\approx 5/8$.

- This is not Gaussian distribution with above variances.

Demonstration 2: correlation to θ

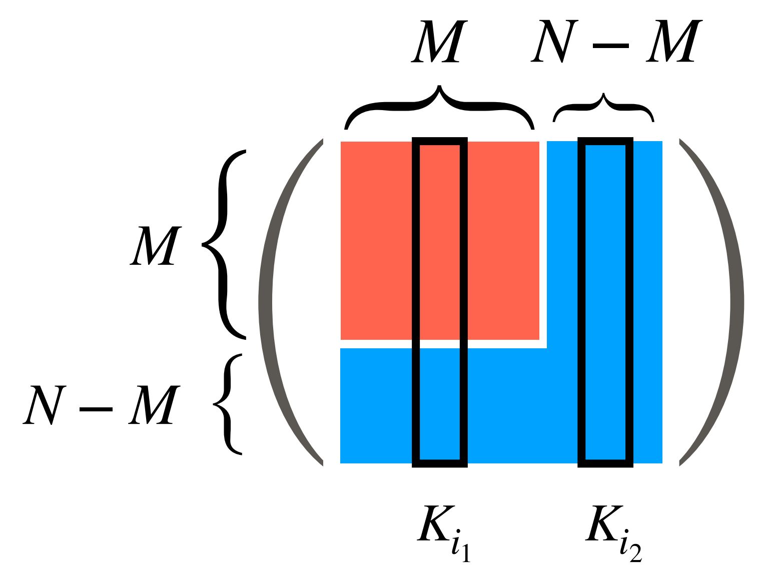
We define R ($= E$ for Gaussian matrix model)

$$R \equiv \frac{N}{\beta}\sum_I \int \mathrm{d}t\, \mathrm{Tr}\, X_I^2 \;= M^2 r_1 + (N^2 - M^2) r_0$$

This is valid at large N and we can also see it numerically.

We also define

$$K_i \equiv \frac{1}{\beta}\int \mathrm{d}t \sum_{I,j} \left| X_{I,ij} \right|^2$$



If partial deconfinement takes place,

$$\langle R \rangle = \left\langle \textstyle\sum_i K_i \right\rangle = M \cdot \langle K_i \rangle_{\mathrm{dec}} + (N - M) \cdot \langle K_i \rangle_{\mathrm{con}}$$

$$\langle K_i \rangle_{\mathrm{con}} = r_0 = \frac{d}{2}, \quad \langle K_i \rangle_{\mathrm{dec}} = \left(1 - \frac{M}{N}\right) r_0 + \frac{M}{N} r_1 = \frac{d}{2} + \frac{M}{4N}$$

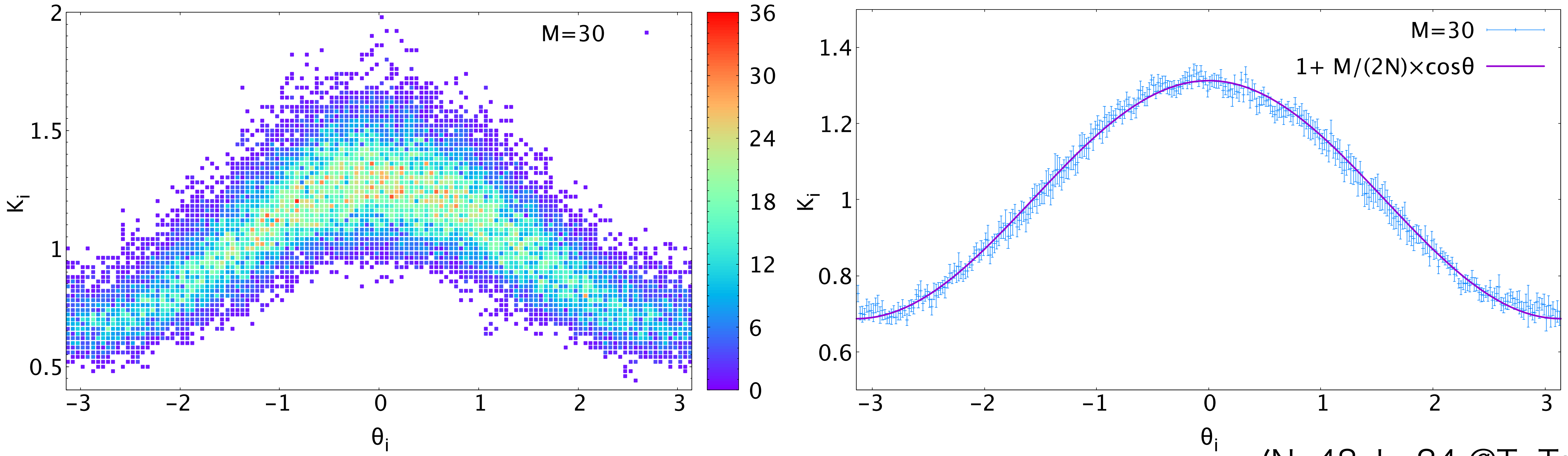
and therefore

$$\left\langle K_i \right\rangle_{\mathrm{p.d.}} = \left(1 - \frac{M}{N}\right) \left\langle K_i \right\rangle_{\mathrm{con}} + \frac{M}{N} \left\langle K_i \right\rangle_{\mathrm{dec}} = \frac{d}{2} + \frac{1}{4}\left(\frac{M}{N}\right)^2$$

How do the correlation between scalars X_I and a gauge field A_t appear?

We observe the following relation,

$$K_i = r_0 + \frac{M}{N} \cdot 2\left(r_1 - r_0\right)\cos\theta_i = 1 + \frac{M}{2N}\cos\theta_i$$



(N=48, L=24 @T=Tc)

- This behavior seems to be consistent with the partial deconfinement;

  Assuming θ_i and K_i distribute in the same manner,

$$\int d\theta\, \rho^{(P)}_{\text{con}} K_i = 1 = \langle K_i \rangle_{\text{con}}, \qquad \int d\theta\, \rho^{(P)}_{\text{dec}}(\theta) K_i = 1 + \frac{M}{4N} = \langle K_i \rangle_{\text{dec}}$$

$$\rho^{(P)}(\theta) = \left(1 - \frac{M}{N}\right)\rho^{(P)}_{\text{con}}(\theta) + \frac{M}{N}\rho^{(P)}_{\text{dec}}(\theta) \Longleftrightarrow \left\langle K_i \right\rangle_{\text{p.d.}} = \left(1 - \frac{M}{N}\right)\left\langle K_i \right\rangle_{\text{con}} + \frac{M}{N}\left\langle K_i \right\rangle_{\text{dec}}$$

Yang-Mills matrix model

$$S = N\int_0^\beta \mathrm{d}t\ \mathrm{Tr}\left\{\frac{1}{2}(D_tX_I)^2 - \frac{1}{4}\left[X_I, X_J\right]^2\right\}$$

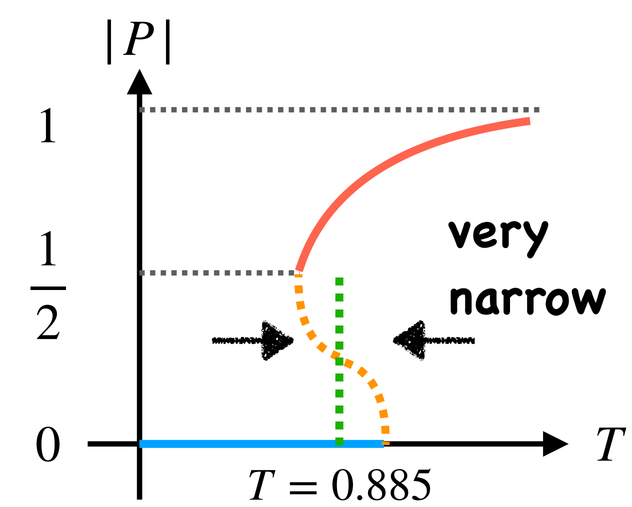
$$I, J = 1,2,\cdots,d = 9$$

$$D_tX_I = \partial_t X_I - \mathrm{i}\left[A_t, X_I\right]$$

Interesting model for $d = 9$ in terms of gauge/gravity duality.

Numerically, [Bergner, Bodendorfer, Hanada, Rinaldi, Schafer & Vranas, (2019)]

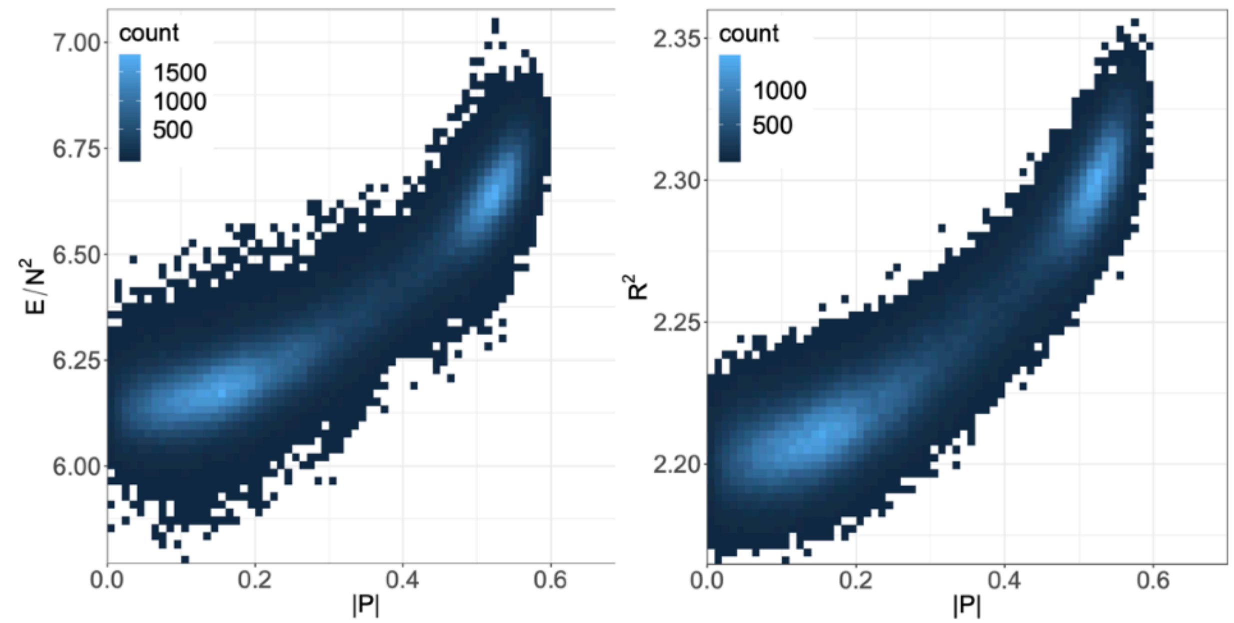
$$\rho^{(P)}(\theta) = \left(1 - \frac{M}{N}\right)\cdot\frac{1}{2\pi} + \frac{M}{N}\cdot\frac{1+\cos\theta}{2\pi} \qquad \text{with} \qquad P = \frac{M}{2N}$$



Also,

$$E = \left\langle -\frac{3N}{4\beta}\sum_{I\neq J}\int \mathrm{d}t\ \mathrm{Tr}\left[X_I, X_J\right]^2\right\rangle$$
$$= \left(N^2 - M^2\right)\varepsilon_0 + M^2\varepsilon_1$$

$$R = \left\langle \frac{N}{\beta}\sum_I \int \mathrm{d}t\ \mathrm{Tr}\,X_I^2\right\rangle$$
$$= (N^2 - M^2)r_0 + M^2 r_1$$



N=64, L=24 @ T=0.885

We apply the same methods
as used in GMM to see two-phase-coexistence.

Two types of configs.

We introduce source terms **for different purposes**.

- "Unconstrained" simulations $(\gamma \gg 1)$

$$\Delta S = \begin{cases} \frac{\gamma}{2}\left(|P| - p_1\right)^2 & (|P| < p_1) \\ \frac{\gamma}{2}\left(|P| - p_2\right)^2 & (|P| > p_2) \end{cases}$$

Collecting the config in which $p_2 < |P| < p_1$ effectively

- Constrained simulations $(\gamma \gg 1, \quad \delta \ll 1)$

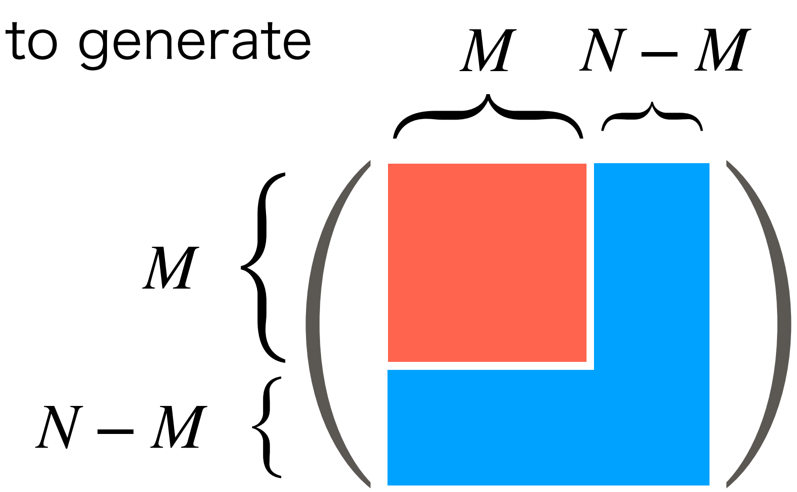
$$\Delta S = \begin{cases} \frac{\gamma}{2}\left(\left|P_M\right| - \frac{1+\delta}{2}\right)^2 & \left(\left|P_M\right| > \frac{1+\delta}{2}\right) \\ \frac{\gamma}{2}\left(\left|P_M\right| - \frac{1-\delta}{2}\right)^2 & \left(\left|P_M\right| < \frac{1-\delta}{2}\right) \\ \frac{\gamma}{2}\left(\left|P_{N-M}\right| - \delta\right)^2 & \left(\left|P_{N-M}\right| > \delta\right) \end{cases}$$

$$P_M = \frac{1}{M}\sum_{j=1}^{M} e^{i\theta_j}, \quad P_{N-M} = \frac{1}{N-M}\sum_{j=M+1}^{N} e^{i\theta_j}$$

Constrained as

$$\left|P_M\right| \approx \frac{1}{2}, \quad \left|P_{N-M}\right| \approx 0$$

to generate



Demonstration 1: distribution of x

Numerically,

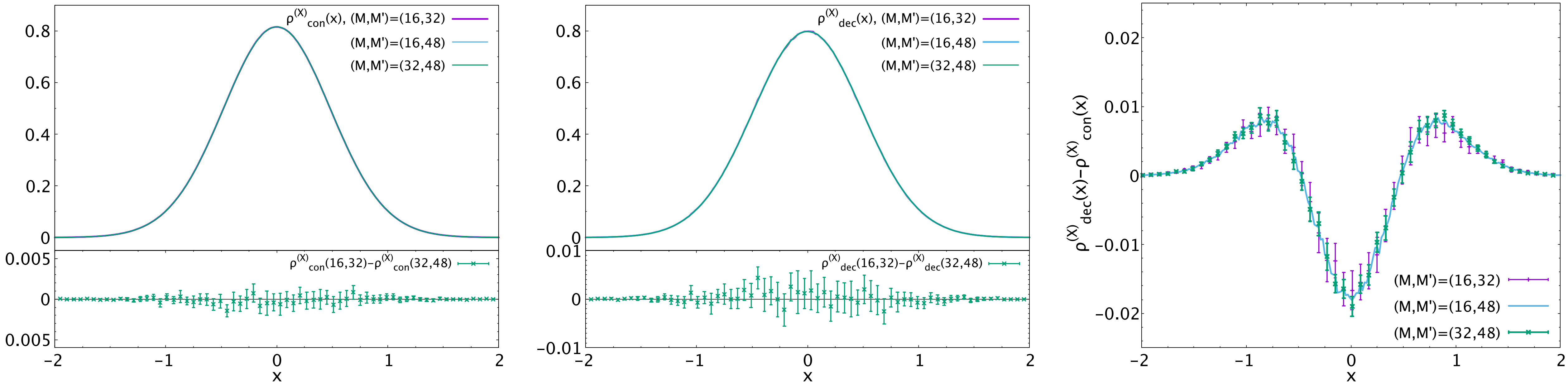
$$R = \left\langle \frac{N}{\beta} \sum_I \int \mathrm{d}t \, \mathrm{Tr}\, X_I^2 \right\rangle = (N^2 - M^2) r_0 + M^2 r_1$$

$$r_0 \simeq 2.20, \quad r_1 \simeq 2.29$$

$$\sigma^2_{\mathrm{con}} \equiv \int \mathrm{d}x x^2 \rho^{(X)}_{\mathrm{con}}(x) = \frac{r_0}{d} \simeq \frac{2.20}{9} \simeq 0.244$$

$$\sigma^2_{\mathrm{dec}} \equiv \int \mathrm{d}x x^2 \rho^{(X)}_{\mathrm{dec}}(x) = \frac{r_1}{d} \simeq \frac{2.29}{9} \simeq 0.254$$

For unconstrained simulations, (same behavior can be seen for const. sim.)



- The difference $\rho^{(X)}_{\mathrm{con}}(x)$ and $\rho^{(X)}_{\mathrm{dec}}(x)$ are greater than the error bars.
- The distributions reproduce the values of variances.

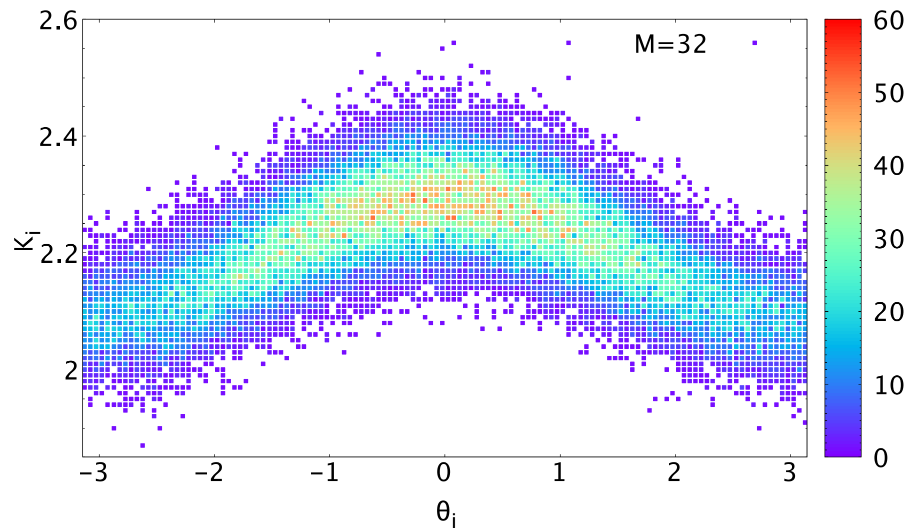
Demonstration 2: correlation to θ

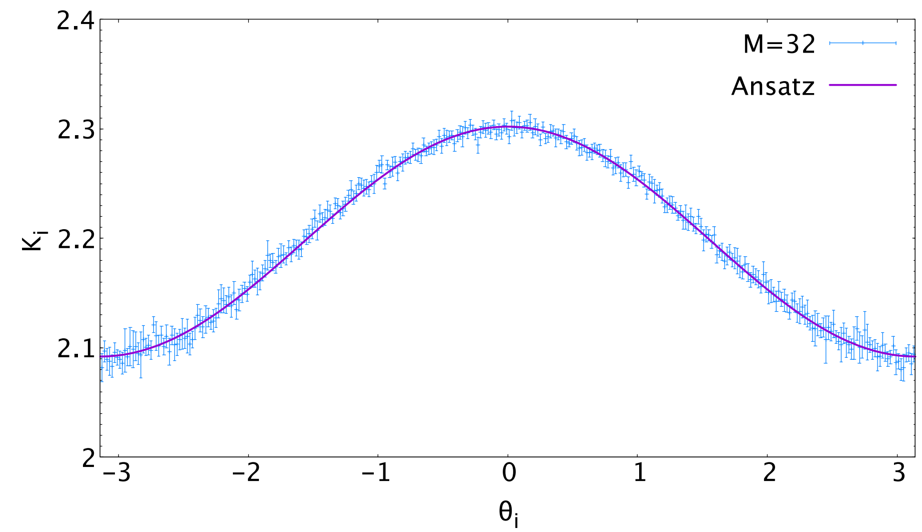
$$K_i = \left\langle \frac{1}{\beta} \int \mathrm{d}t \sum_{I,j} \left| X_{I,ij} \right|^2 \right\rangle$$

$$K_i = r_0 + \frac{M}{N} \cdot 2\left(r_1 - r_0\right) \cos\theta_i \quad \text{M-indep. ansatz}$$

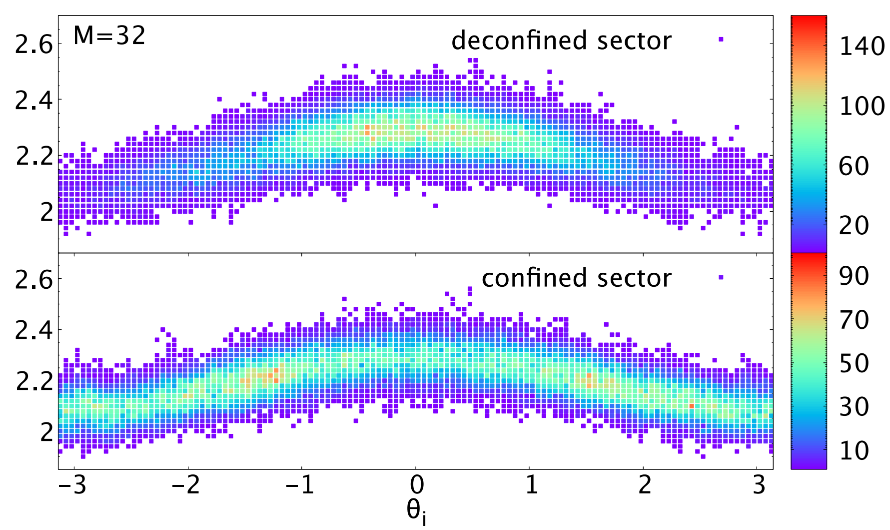
$$K_i = \langle K_i \rangle_{\mathrm{con}} + 2\left(\langle K_i \rangle_{\mathrm{dec}} - \langle K_i \rangle_{\mathrm{con}}\right) \cos\theta_i \quad \text{M-dep. ansatz}$$

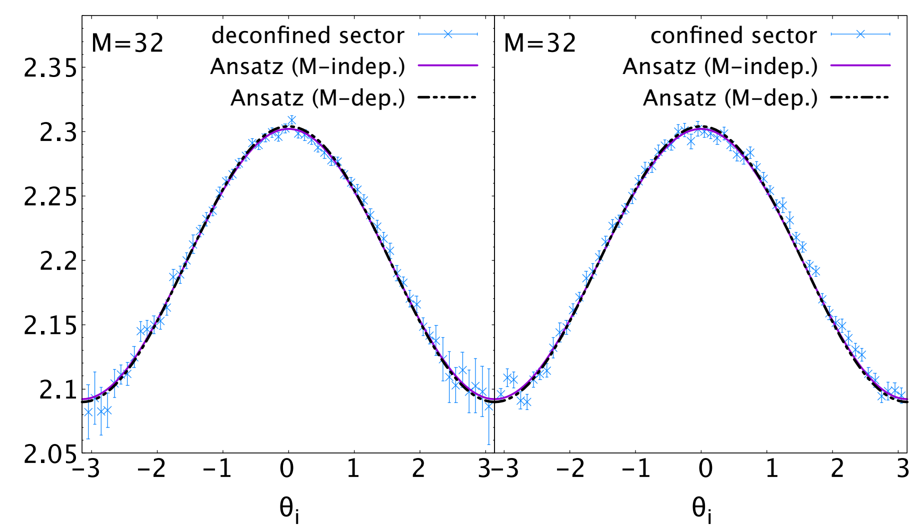
For unconstrained simulations,





For constrained simulations, we can analyze each sector separately;





Summary & Discussion

- Partial deconfinement takes place
  in several large N gauge theories at finite temperature.
  - The phase is regarded as the coexistence of confinement/deconfinement.
- We constructed the way to separate the color d.o.f. of scalar fields
  into confined/deconfined sectors
  and apply it to two bosonic matrix models.
  - The distributions of the components of the scalar fields
  - The correlation between the scalar fields and the Polyakov line phases

→ **Totally supporting the properties of the two-phase-coexistence.**

Future works,

- How about the case with strong hysteresis?
- Introduction of fermions and relation to the chiral symmetry breaking
- Continuum limit extrapolation or treating finite N corrections.
- Application to QCD (large-N_c/-N_f theories)
  c.f.) enhanced flavor symmetry [Denissenya, Glozman & Lang, (2014)]