Investigating the Anomalous Magnetic Moment of the Muon on the Lattice

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Background

Muon anomalous magnetic moment $(g-2)$ is an extremely sensitive test of the SM

Experimentally measured to high accuracy 0.54 ppm

$$11659209.1(5.4)(3.3) \times 10^{-10}$$

Calculated with SM to high accuracy 0.51 ppm

$$11659182.04(3.56) \times 10^{-10}$$

Note the $3-4\sigma$ discrepancy

Background

Two new experiments aim to improve experimental result

- Fermilab E989 [Started March 2018, first results expected soon]
- J-PARC E34 [Expected start early 2024]

Aim to improve experimental results fourfold

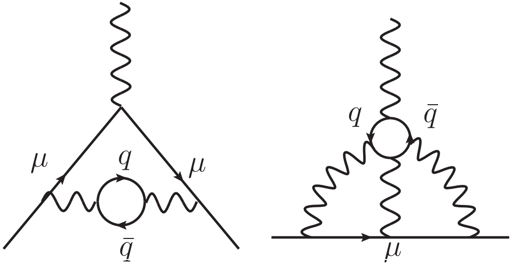
Standard Model calculations needs similar improvements

SM Calculation

Current best estimate

$$a_\mu^{SM} = (\text{QED} \pm 0.008 + \text{EW} \pm 0.10 + \text{Hadronic} \pm 5.02) \times 10^{-10}$$

- QED and EW terms are well constrained
- Largest uncertainty come from Hadronic term
- We are interested in the leading order HVP term



SM Calculation

Current state of calculations

Contribution	Value ($\times 10^{10}$)
QED	11658471.895(8)
Electroweak	15.36(10)
HVP LO	693.26(2.46)
HVP NLO	-9.84(6)
HLbL	10.5(2.6)

Best results for HVP have relied on e^+e^- scattering cross sections.

$$2.5 \times 10^{-10} \text{ uncertainty} \sim 0.3\%$$

However, there is some tension in scattering results.

Lattice

Lattice provides an independent method from first principles

Current lattice results quote uncertainty $\sim 0.6 - 2\%$

Recent results with high precision such as BMW [2002.12347]

As lattice results approachs this level of precision, we need to investigate QED effects

In this study we use fully dynamical QCD+QED

Lattice Setup

Simulate with 6 QCDSF QCD+QED ensembles

- $32^3 \times 64$
- $48^3 \times 96$

Lattice spacing $a = 0.068 fm$

Partially-quenched quark masses

$$260 \leq m_\pi^{q\bar{q}} \leq 770 MeV$$

Non-compact QED with exaggerated coupling

$$\alpha_{QED} = \frac{e^2}{4\pi} \sim 0.1$$

Ensemble	$L^3 \times T$	N_f	$m_{u\bar{u}}$	$m_{d\bar{d}}$	$m_{s\bar{s}}$	$m_{q\bar{q}}^{min} L$	m_{π^+}	m_{K^+}
1	$32^3 \times 64$	2+1	430	405	405	4.4	435	435
2	$32^3 \times 64$	2+1	360	435	435	4.0	415	415
3	$32^3 \times 64$	1+1+1	290	300	585	3.2	320	470
4	$48^3 \times 48$	2+1	430	405	405	6.7	435	435
5	$48^3 \times 48$	2+1	360	435	435	5.9	420	420
6	$48^3 \times 48$	1+1+1	290	300	580	4.8	320	470

Lattice Setup

Use the novel QCDSF mass tuning

Keep average quark mass fixed at physical value

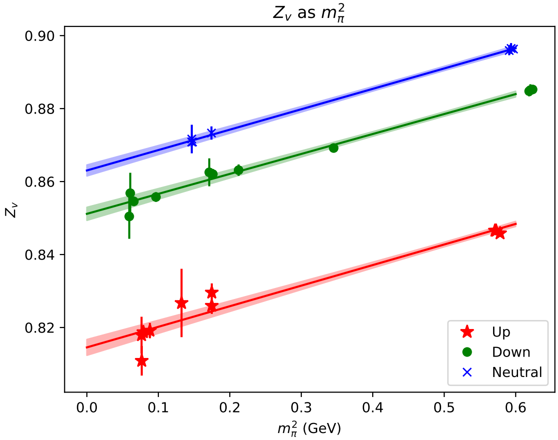
$$\bar{m}^{phys} = \frac{1}{3}\left(m_{up} + m_{down} + m_{strange}\right)$$

QED inclusion effects quark mass

Use 'Dashen Scheme', define $SU(3)_{sym}$ via $m_\pi^{u\bar{u}} = m_\pi^{d\bar{d}} = m_\pi^{n\bar{n}}$

- Neutral, $q = 0$ $m_\pi^{n\bar{n}} = 408(3)MeV$
- Down, $q = -\frac{1}{3}$ $m_\pi^{d\bar{d}} = 409(1)MeV$
- Up, $q = \frac{2}{3}$ $m_\pi^{u\bar{u}} = 407(3)MeV$

Vector Current Renormalisation



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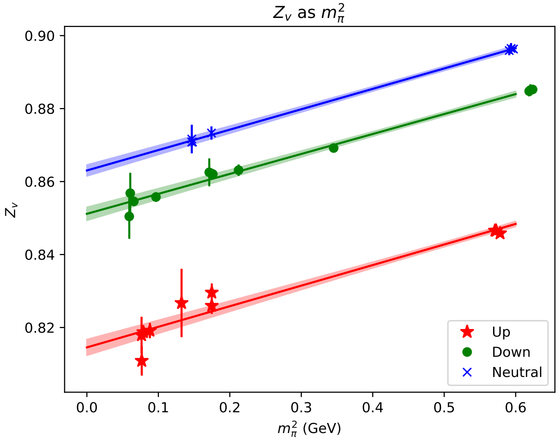
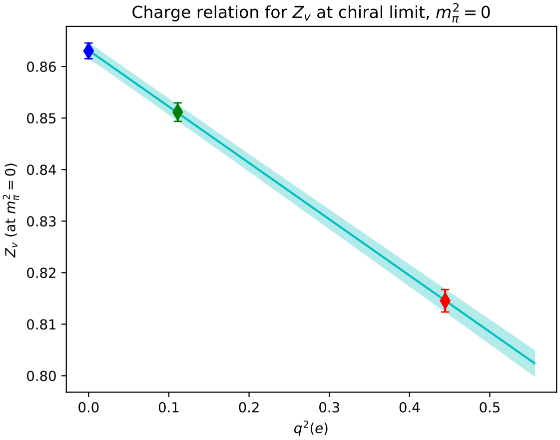




Accessing a_μ^{HVP}

Time-momentum representation

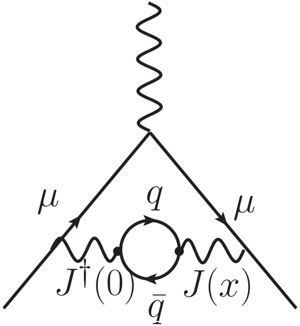
$$a_\mu^{HVP} = 4\alpha^2 \int_0^\infty dt G(t) \tilde{K}\left(t; m_\mu^2\right)$$

$G(t)$ is the vector-vector 2-pt function

$$G(t) = \frac{1}{3} \sum \int d^3x \left\langle J(x) J^\dagger(0) \right\rangle$$

$\tilde{K}$ is known kernel
[Bernecker-Meyer arXiv:1107.4388]

Note: requires long-time integral.



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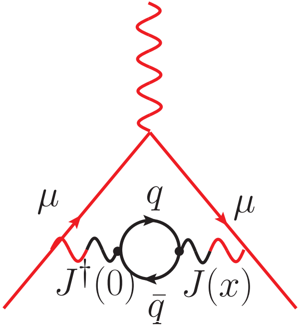

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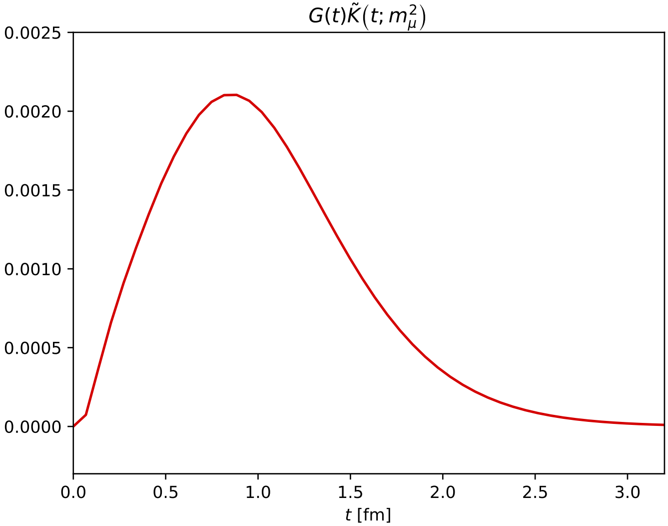

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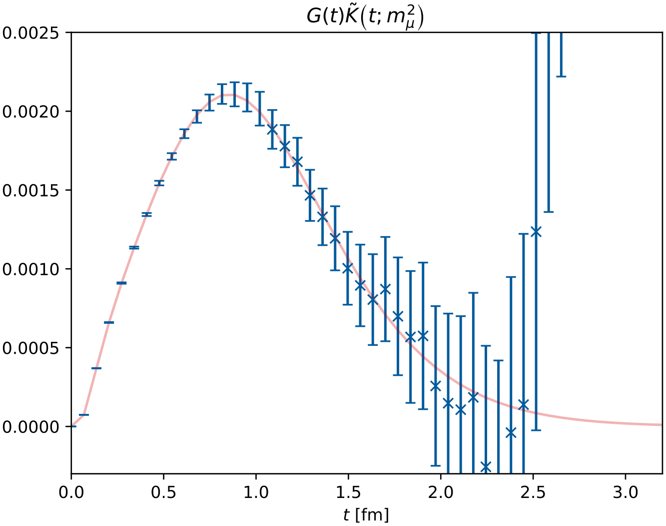

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Lattice data very noisy at large t

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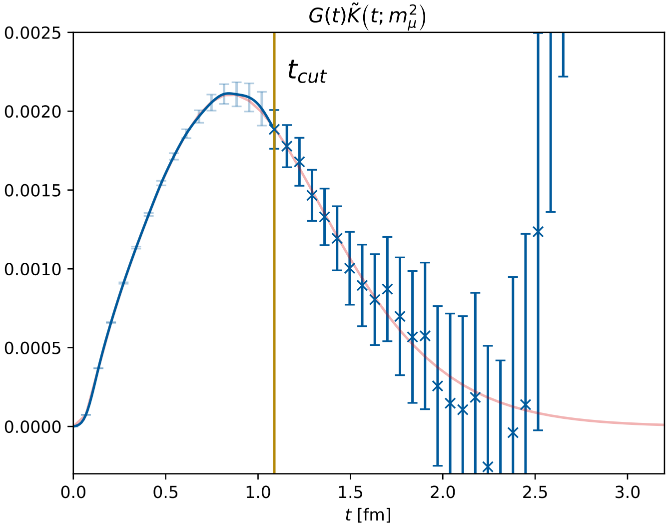

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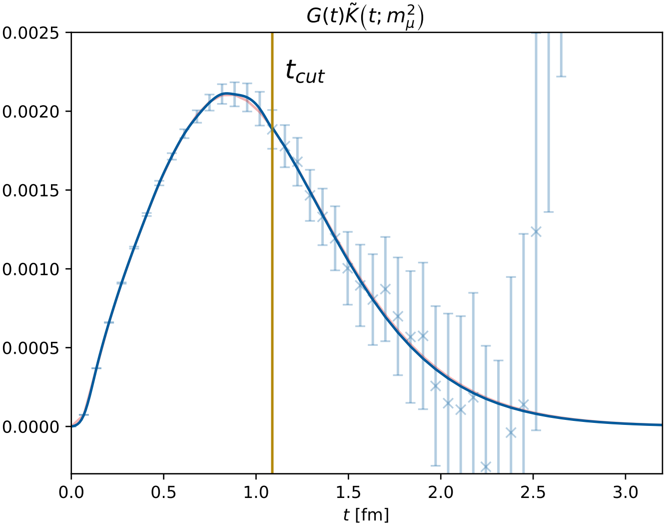

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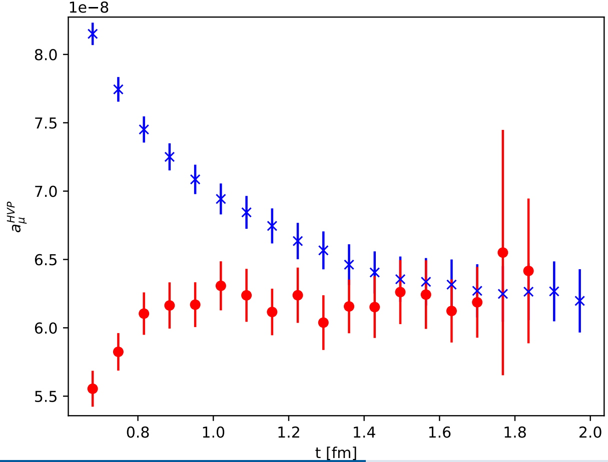
Bounding Method



$$G(t) = \begin{cases} G(t) & t \leq t_{cut} \\ G(t_{cut})e^{-E(t-t_{cut})} & t > t_{cut} \end{cases}$$

Upper: $E = E_0$

Lower: $E = E_{eff} = \log\left(\frac{G(t_{cut})}{G(t_{cut}+1)}\right)$

Flavour Breaking Expansion

Flavour breaking expansion [1509.00799]

$$
\begin{aligned}
a_{\mu,a} =& a_{\mu,0} + 2\alpha\delta\mu_a + \beta_0\frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + 2\beta_1\delta\mu_a^2 + \beta_0^{EM}(e_u^2 + e_d^2 + e_s^2) \\
&+ 2\beta_1^{EM}e_a^2 + \gamma_0^{EM}(e_u^2\delta m_u + e_d^2\delta m_d + e_s^2\delta m_s) + 2\gamma_1^{EM}e_a^2\delta\mu_a \\
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\end{aligned}
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δm is sea quark mass
$\delta\mu$ is valance quark mass
$e_f, f = u, d, s$ is sea quark charge
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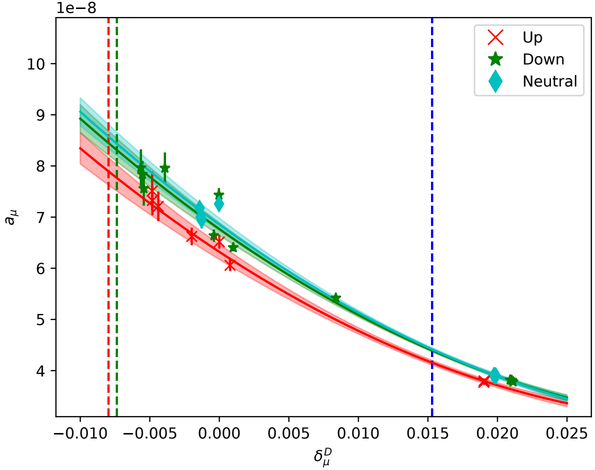
Define Dashen mass $\delta\mu_a^D$ to absorbs EM effect in neutral pseudoscalar meson masses

Results - a_μ^{HVP}

$32^3 \times 64$ Lattice results



Remember flavour breaking expansion.

Fit for sea and valance quark mass and charge.

Sea quark mass δm fixed to physical.

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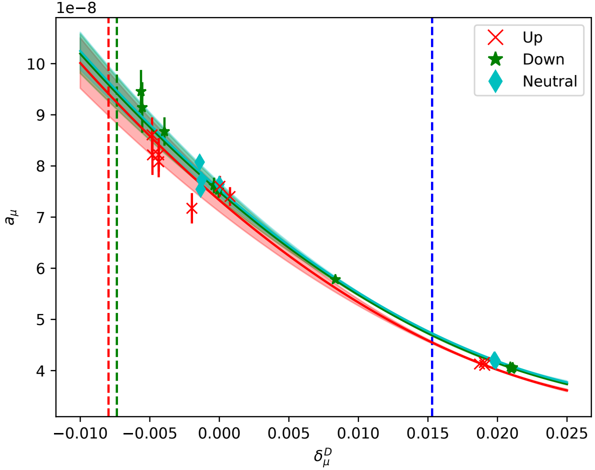
$48^3 \times 96$ Lattice results



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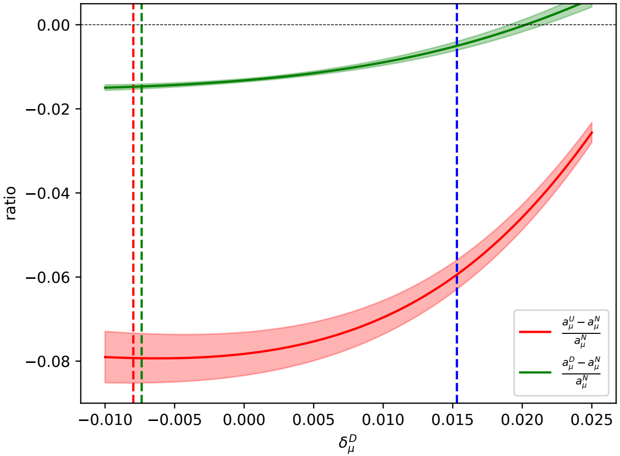
Compare charged fits to neutral fit

$$\frac{a_\mu^{charged} - a_\mu^{neutral}}{a_\mu^{neutral}}$$

Results - Charge Effect

$32^3 \times 64$ Lattice results



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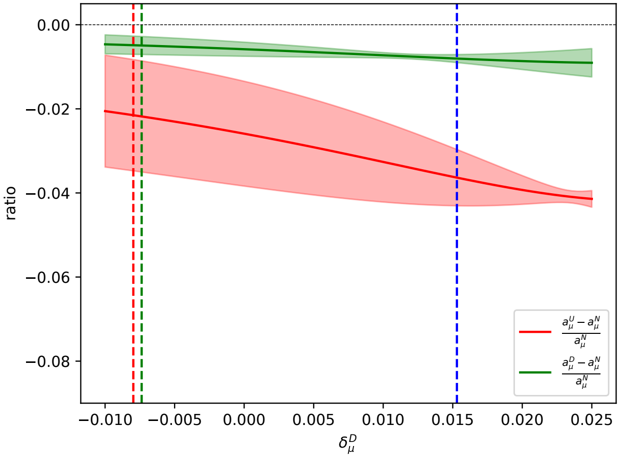

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QED contributions $\sim$ 2% effect

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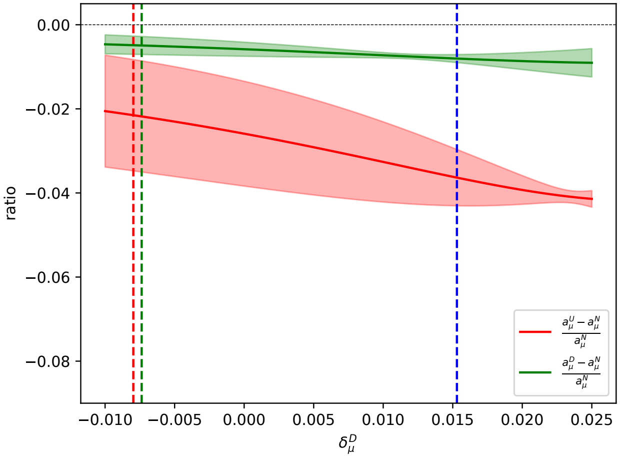

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QED contributions $\sim$ 0.2% effect

Remember $\alpha_{QED} \sim$ 10 times physical.

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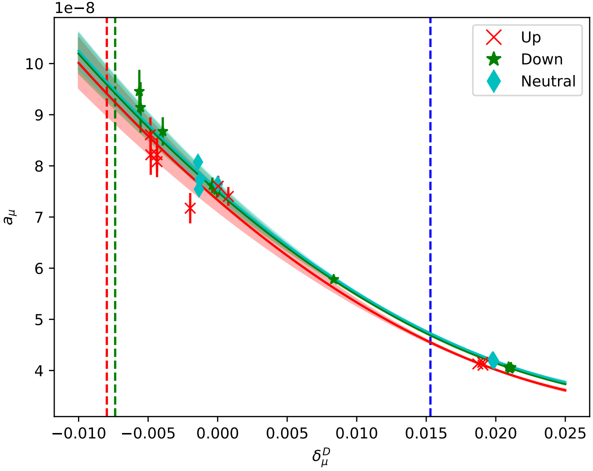


Combine quark contributions

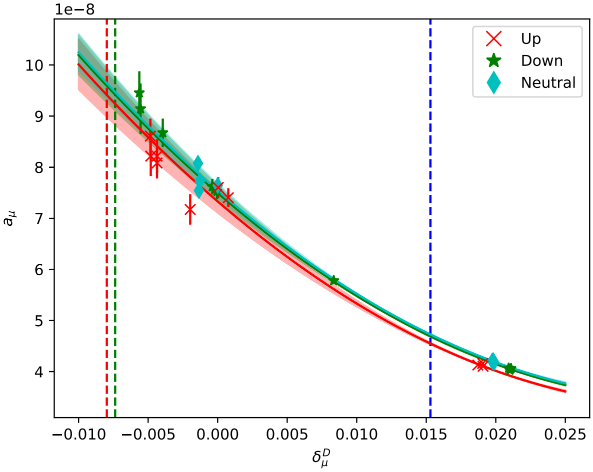
$$a^{HVP}_{\mu,\text{charged}} = \frac{4}{9}(u_c) + \frac{1}{9}(d_c) + \frac{1}{9}(s_c)$$

$$a^{HVP}_{\mu,\text{neutral}} = \frac{4}{9}(u_0) + \frac{1}{9}(d_0) + \frac{1}{9}(s_0)$$

Take the ratio → charge effects

$2\% \pm \quad 1\%$

Results - Charge Effects



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Take the ratio $\rightarrow$ charge effects $0.2\% \pm 0.1\%$

Remember $\alpha_{QED} \sim 10$ times physical.

Summary

Remember

- current lattice results sits at around 0.6 - 2%
- current e^+e^- scattering results at around 0.3%
- QED effects around 0.2%

If lattice results are to reach desired precision, need to consider QED corrections.

Summary

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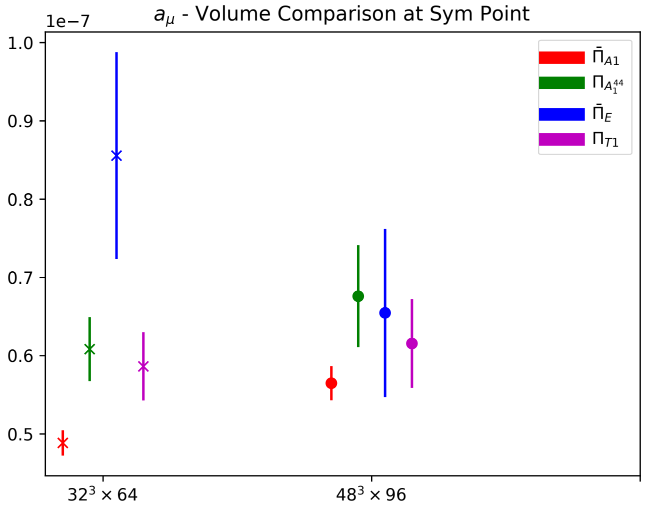
Thank you for listening

BONUS: Finite volume effects



Should all be equal in infinite volume limit [1512.07555]

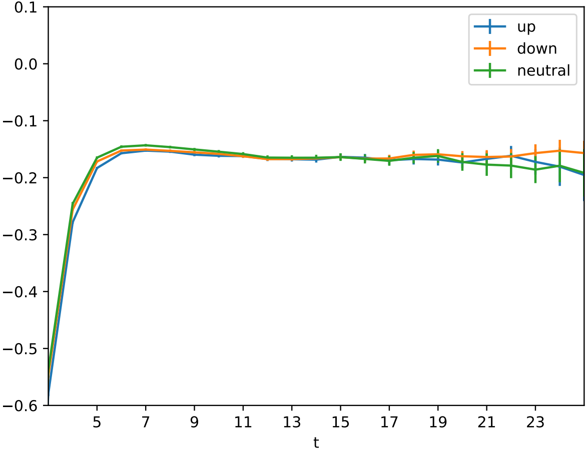
$$A_1 : \sum_i \bar{\Pi}_{ii} = \left(3q^2 - \vec{q}^{\,2}\right) \bar{\Pi}_{A_1},$$

$$T_1 : \bar{\Pi}_{4i} = -\left(q_4 q_i\right) \bar{\Pi}_{T_1}$$

$$A_1^{44} : \bar{\Pi}_{44} = \left(\vec{q}^{\,2}\right) \bar{\Pi}_{A_1^{44}},$$

$$E : \bar{\Pi}_{ii} - \sum_i \bar{\Pi}_{ii}/3 = \left(-q_i^2 + \vec{q}^{\,2}/3\right) \bar{\Pi}_E.$$

BONUS: Charge independant renormalisation



$$J^R_{\mu,f} = Z_V^{m_f} J_{\mu,f}\left(1 + c_V \frac{\partial_\nu T_{\mu\nu,f}}{J_{\mu,f}}\right).$$

$\mathcal{O}(a)$ improvements not fully included, but appear to be charge independent.