Glueball dark matter in SU(N) lattice gauge theory

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In Collaboration with
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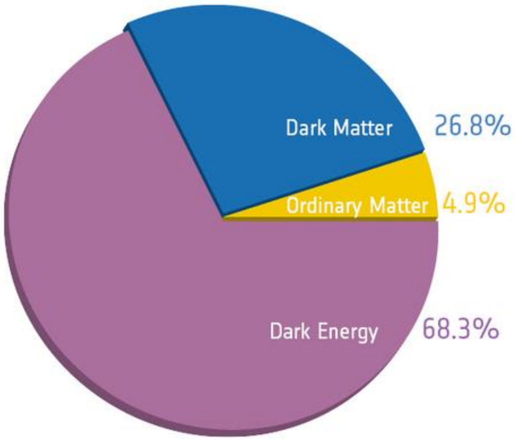
Based on
N. Yamanaka et al., arXiv:1910.01440 [hep-ph]
N. Yamanaka et al., arXiv:1910.07756 [hep-lat]
N. Yamanaka et al., arXiv:1911.03048 [hep-lat]

2020/08/07
APLAT2020

Motivation

Dark matter is representing a significant fraction of the energy content of the Universe

But, we do not know what it is…



Many candidate theories are under discussion

(WIMPs, axions, blackholes, entropic gravity …)

Let us consider the SU(N) Yang-Mills theory : $\mathcal{L}_{\rm YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$

Generation of mass scale, logarithmic dependence, no important fine-tuning

⇒ Theory with very high naturalness

Lightest particles are glueballs ! ⇒ SU(N) glueballs are candidate of DM

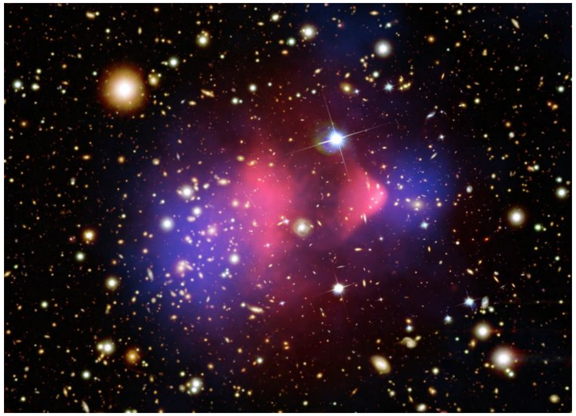
Important feature of DM : Self-interaction

(= DM-DM scattering)

DM self-interaction is constrained by observations (collisions of galaxies, structure formation)

⇒ We need to evaluate

the interglueball force!



Bullet cluster : collision of galaxies

Object of study

Glueballs of SU(N) Yang-Mills theory are good candidates of dark matter

In this work, we study the interglueball scattering on lattice which is the only way to quantify nonperturbative physics of nonabelian gauge theory.

The Yang-Mills theory depends only on the scale parameter Λ (given N_c): can we determine Λ from observation?

Object:

In this work, we study the interglueball scattering of SU(2) Yang-Mills theory on lattice, and set constraint on its scale parameter Λ.

Setup

We consider the SU(2) pure Yang-Mills theory

- Standard SU(2) plaquette action :

  Lattice spacings : β = 2.1, 2.2, 2.3, 2.4, 2.5

  Volume : 10^3x12 ∼ 16^3x24

  Confs. generated with pseudo-heat-bath method (1 M confs.)

- Use SX-ACE (@RCNP, Osaka U.), vector machine

- Improvement of glueball operator : APE smearing

  We use all space-time translational and cubic rotational symmetries to effectively increase the statistics (like the all-mode average for meson and baryon observables)

  Reduction of the statistical error w/ cluster decomposition principle

  chiQCD Collaboration, PRD**97**, 034507 (2018)

Scale determination

We do not know the scale of the YM theory, so we leave it as a free parameter Λ

Nevertheless, all quantities calculated on lattice depend on Λ

$\Rightarrow$ We express all quantities in unit of Λ (and finally constrain Λ from other data).

Relation between Λ and string tension:

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$$

$$= 0.586(41) \qquad \text{(for SU(2))}$$

Fitted from the analysis of the running coupling

C. Allton et al., JHEP **0807** (2008) 021
M. Teper, Acta Phys. Polon. B **40** (2009) 3249

String tension in SU(2) YM :

β	$a\sqrt{\sigma}$
2.1	0.608(16)
2.2	0.467(10)
2.3	0.3687(22)
2.4	0.2660(21)
2.5	0.1881(28)

M. Teper, Phys. Lett. B **397** (1997) 223; hep-th/9812187

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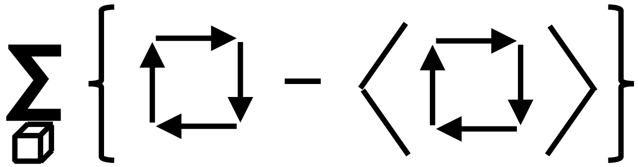
String tension in SU(2) YM :

β	$a\sqrt{\sigma}$	a (in unit of Λ^{-1})
2.1	0.608(16)	0.356(27)
2.2	0.467(10)	0.273(20)
2.3	0.3687(22)	0.216(15)
2.4	0.2660(21)	0.156(11)
2.5	0.1881(28)	0.110(8)

⇒ Lattice spacing is now expressed in unit of Λ

Glueball operator and operator improvement

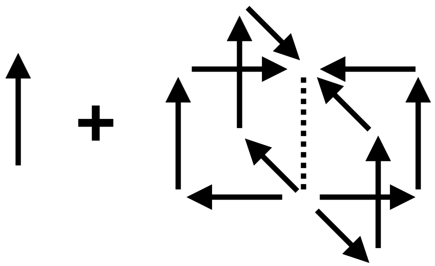
0++ glueball operator:

$\Phi =$ 

Glueball has vacuum expectation value
→ Subtract
Sum over cubic rotational invariance

APE smearing :

$U^{(n+1)}$ so as to maximize $\mathrm{Re\,Tr}\,[\,U^{(n+1)}\,V^{(n)\dagger}\,]$

where $V^{(n)} = \alpha \times$ 

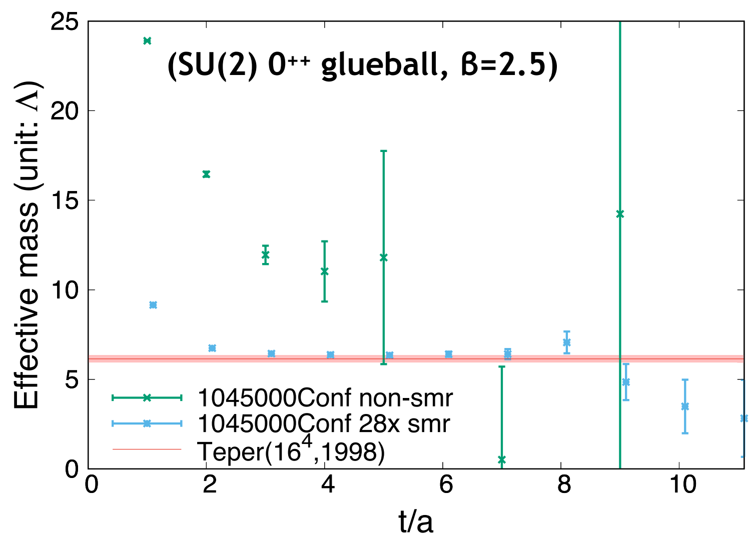
⇒ Gaussian spread: $2\sqrt{\frac{n}{4+\alpha}}$
(in lattice unit)

Ape Collaboration, PLB **192** (1987) 163
N. Ishii et al., PRD **66**, 094506 (2002)

Optimal parameters
for SU(2), β=2.5:

n = 28

α = 2.0

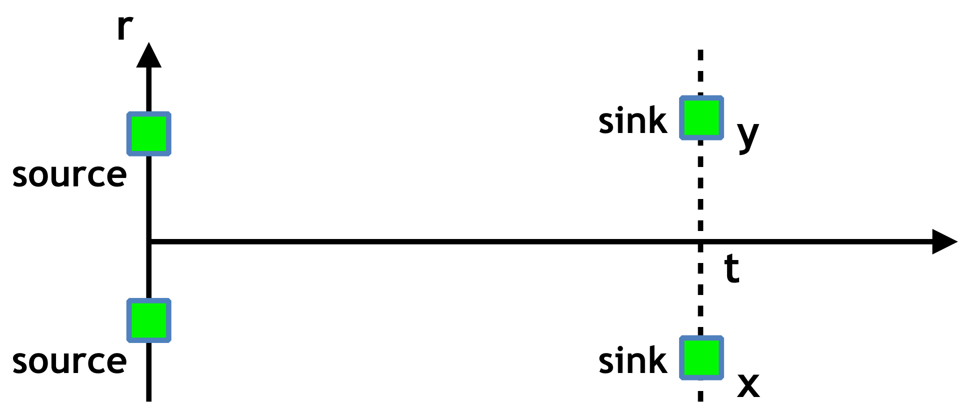


Nambu-Bethe-Salpeter amplitude

The information of the scattering is included in the following n-point correlator (Nambu-Bethe-Salpeter amplitude):

$$C_{\phi\phi}(t, \mathbf{x}-\mathbf{y}) \equiv \frac{1}{V}\sum_{\mathbf{r}} \langle 0 \,| T[\phi(\mathbf{x}+\mathbf{r},t)\phi(\mathbf{y}+\mathbf{r},t)\cdot \mathcal{J}(0)] |\, 0\rangle$$

$\mathcal{J}(0)$ **: source op.**



- **2-glueball (0^{++}) state mixes with all other multi-glueball states:**

  ⇒ The source may be chosen as 1-body, 2-body, etc, on convenience.

  We choose 1-body source, signal noisy is noisy with higher-body source.

- **The NBS amplitude obeys the Schroedinger equation below inelastic threshold**

Nambu-Bethe-Salpeter amplitude

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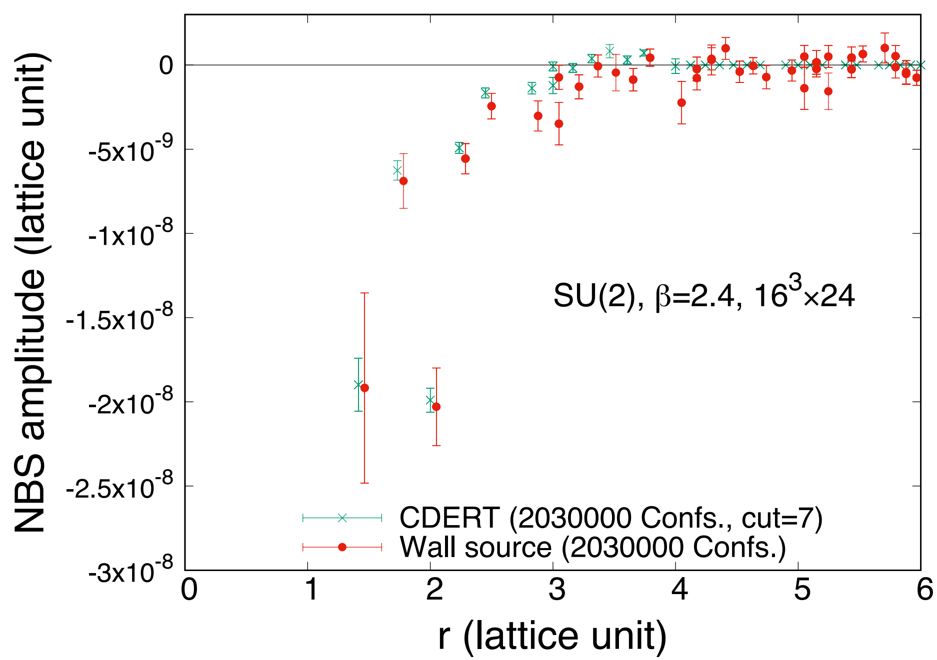
Result of NBS amplitude calculation:



- **2-glueball … ates:**

  ⇒ The sou … nvenience.

  We ch … gher-body source.

- **The NBS a … v inelastic threshold**

HALQCD method

Extract the interglueball potential from the NBS amplitude by inversely solving Schroedinger equation

$$\left[\frac{1}{4m_\phi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{1}{m_\phi}\nabla^2 + \frac{(\mathbf{r}\times\nabla)^2}{2m_\phi r^2}\right]R(t,\mathbf{r}) = \int d^3\mathbf{r}' U(\mathbf{r},\mathbf{r}')R(t,\mathbf{r}')$$

$$R(t,\mathbf{r}) \equiv \frac{C_{\phi\phi}(t,\mathbf{r})}{e^{-2m_\phi t}}$$

N. Ishii et al., PLB **712** (2012) 437.

- Crucial advantage : do not need ground state saturation

➡ Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes very noisy before ground state saturation

- Inelastic threshold for glueball = $3m_\varphi$: high enough to use low t

HALQCD method

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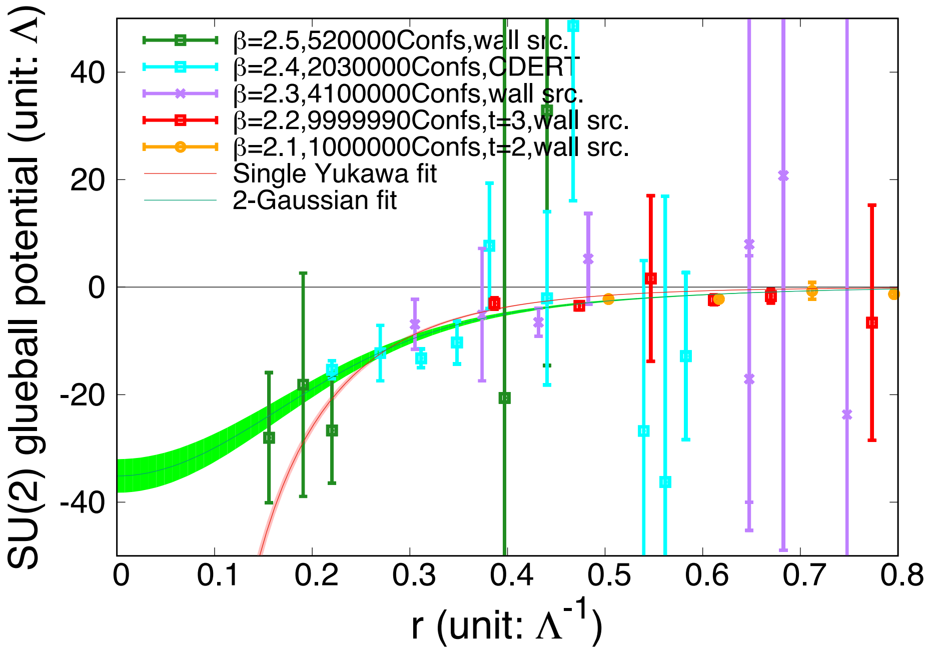
- Crucial advantage : do not need ground state saturation

➡ Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes very noisy before ground state saturation

- Inelastic threshold for glueball = $3m_\varphi$: high enough to use low t

- Subtract centrifugal force for removing higher angular momenta

Result



We test two fitting forms:

- **Yukawa fit:**

$$V_Y(r) = V_1 \frac{e^{-m_\phi r}}{4\pi r}$$

V_1 = -231 ± 8 **x^2 d.o.f. = 1.3**

- **2-Gaussian fit:**

$$V(r) = V_1 e^{-\frac{(m_\phi r)^2}{8}} + V_2 e^{-\frac{(m_\phi r)^2}{2}}$$

V_1 = (-8.5 ± 0.5)Λ
V_2 = (-26.6 ± 2.6)Λ **x^2 d.o.f. = 0.9**

DM cross section is derived from phase shift calculated with the potentials

$$\sigma_{\rm tot} = \frac{4\pi}{k^2}\sin^2[\delta(k \to 0)]$$

Yukawa: **σ_{tot} = (2.5 - 4.7)$Λ^{-2}$ (stat.)**

2-Gaussian: **σ_{tot} = (14 - 51)$Λ^{-2}$ (stat.)**

σ_{tot} = (2 - 51) $Λ^{-2}$ (stat.and sys.)

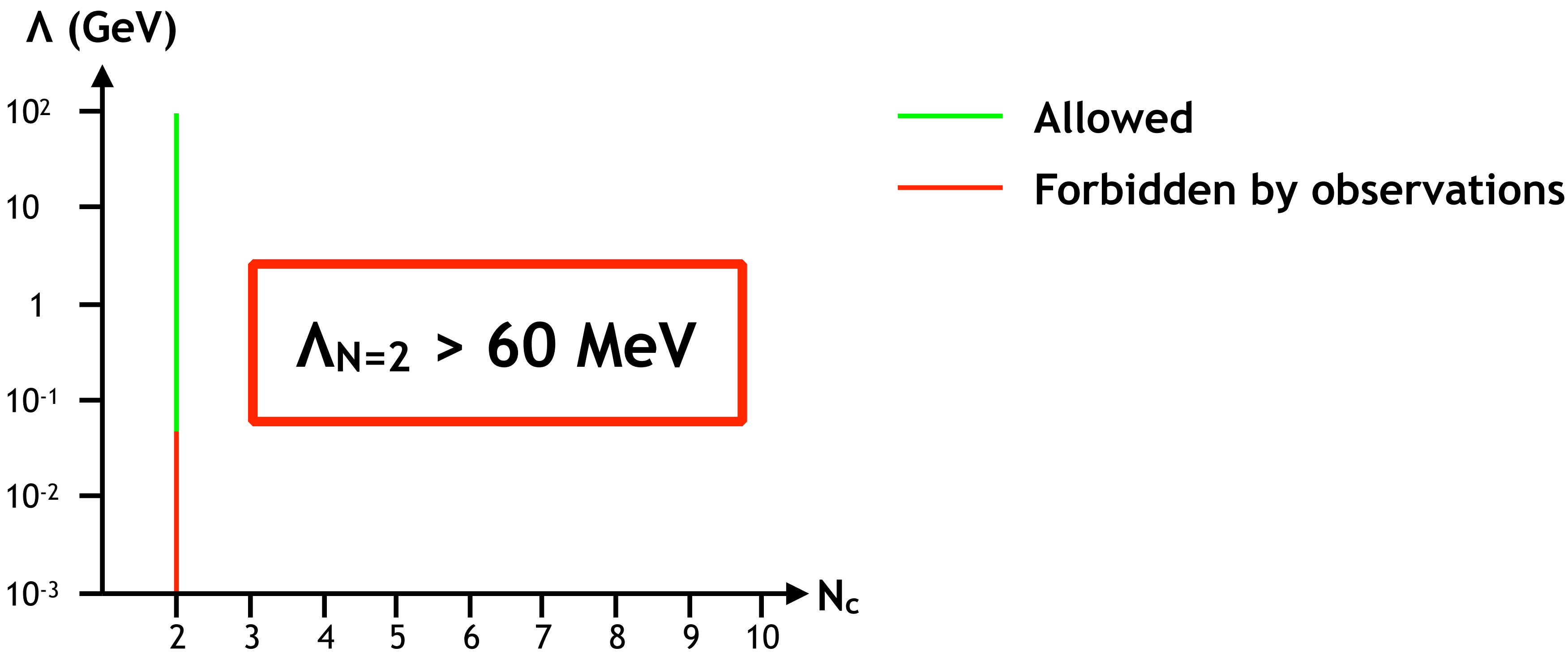
(sys. due to fitting forms)

Constraint on SU(N) YM scale parameter from DM X section

Observational constraints: $\frac{\sigma_{tot}}{m_\Phi} < 1.0\ cm^2/g$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).



$\Lambda_{N=2} > 60$ MeV

N_c vs. scale parameter (Λ) diagram

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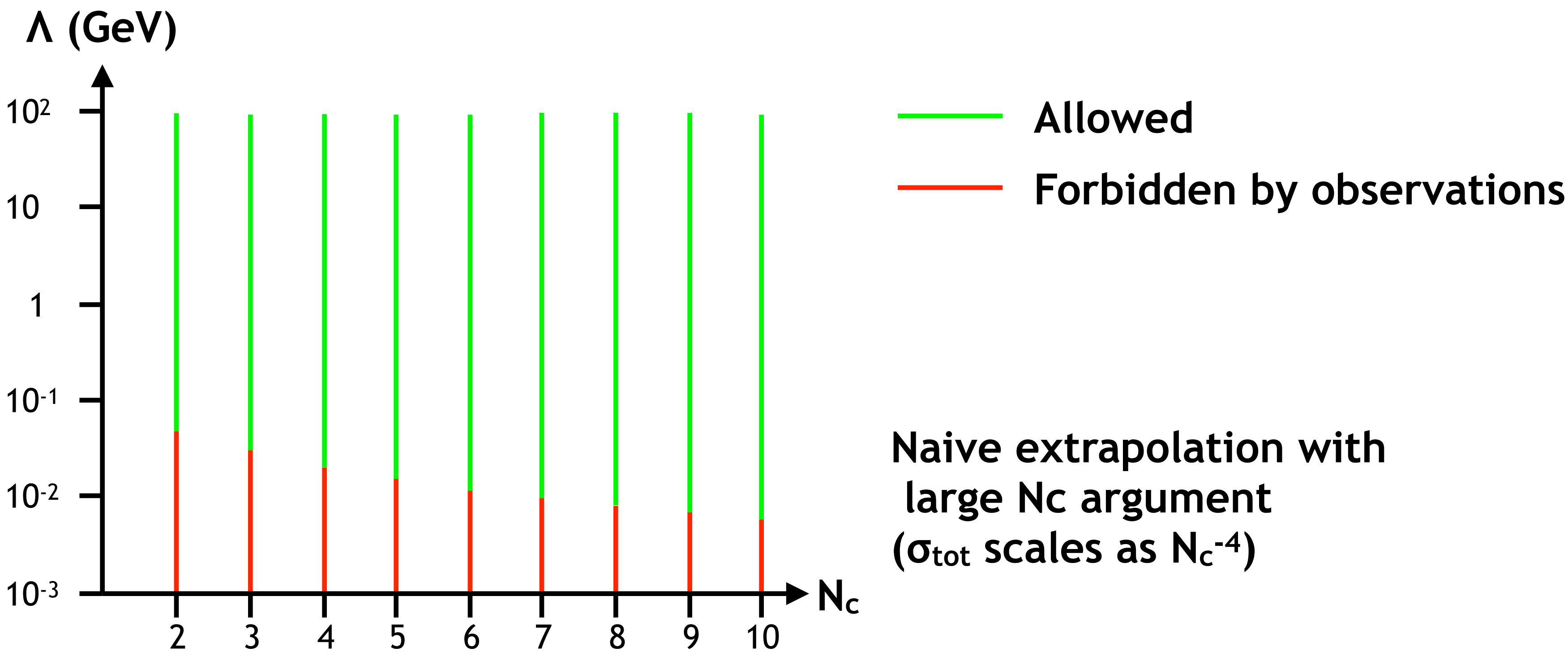


Naive extrapolation with large Nc argument (σ_{tot} scales as N_c^{-4})

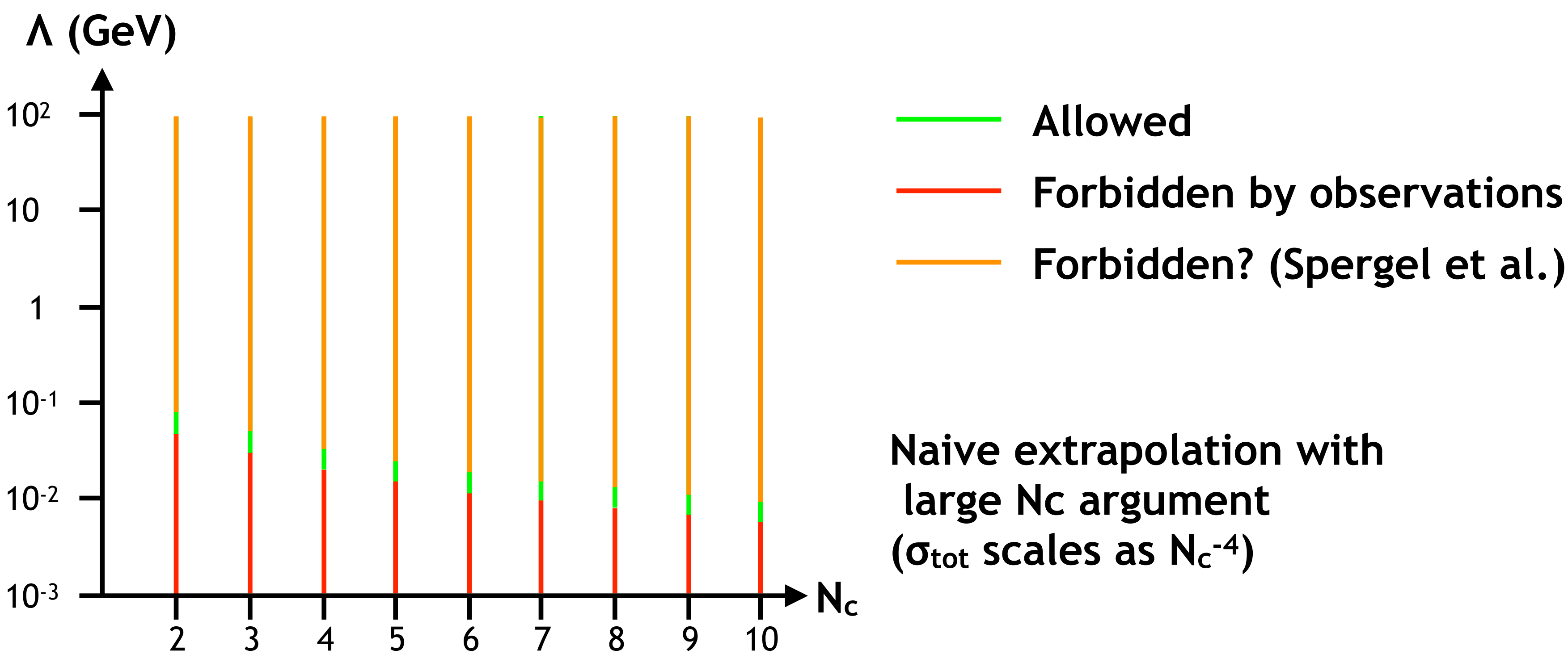
N_c vs. scale parameter (Λ) diagram

Constraint on SU(N) YM scale parameter from DM X section

Observational constraints: $0.45\ \mathrm{cm^2/g} < \dfrac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm^2/g}$

Robust constraint from galactic cluster shape, collisions (upper limit)
A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).

Constraint from Spergel et al. (lower limit), under discussion?
D. N. Spergel et al., PRL **84**, 3760 (2000).

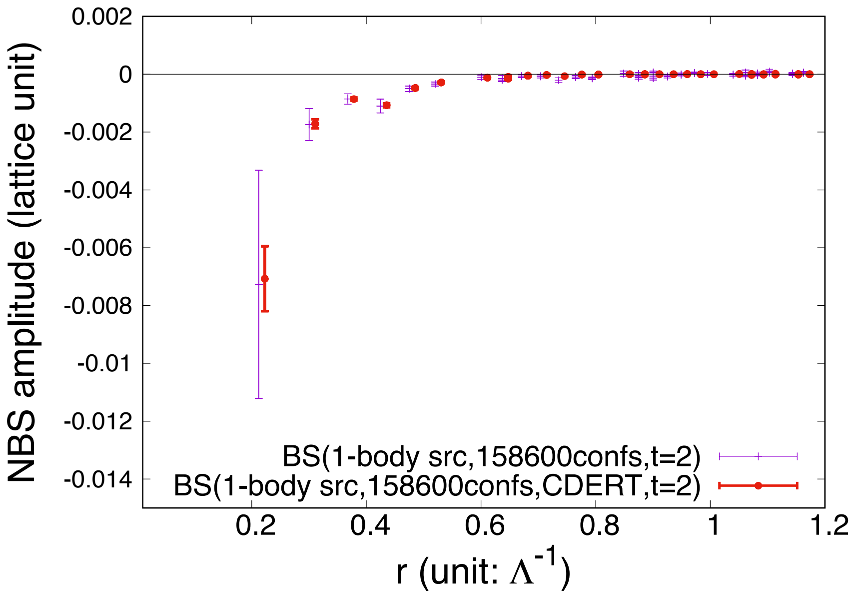


N_c vs. scale parameter (Λ) diagram

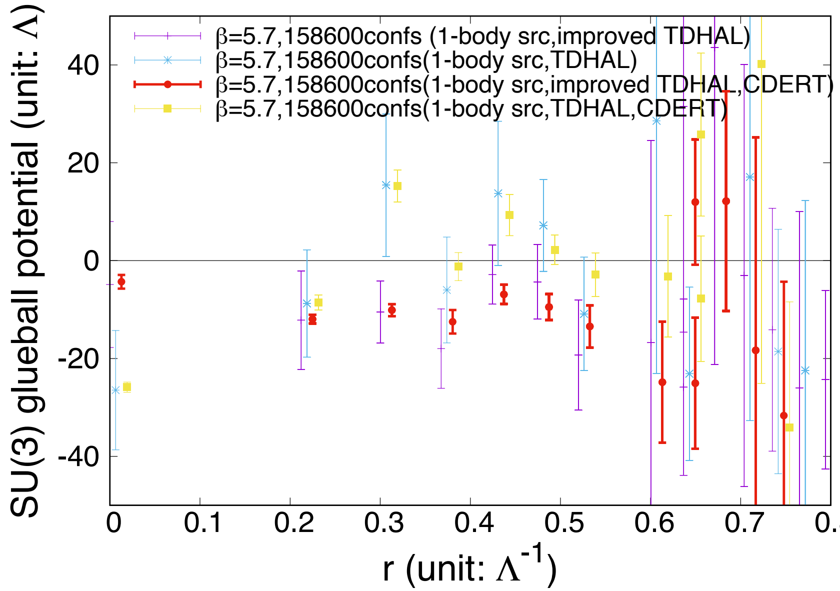
Naive extrapolation with large Nc argument (σ_{tot} scales as N_c^{-4})

SU(3) result (preliminary)

NBS amplitude:



Potential:



SU(3), β = 5.7, 158600 confs

- **The CDERT is very efficient in reducing statistical error**
- **The removal of centrifugal force makes the potential attractive, like SU(2)**
- **The value of the SU(3) interglueball potential is close to the SU(2) one**

Summary

- Glueballs of the SU(N) Yang-Mills theory are good candidates of dark matter : study of self-interaction is important.
- We calculated the interglueball potential in the SU(2) Yang-Mills theory: Time-dependent HALQCD method is important for the interglueball potential because the signal becomes noisy before the ground state saturation.
- We used the cluster decomposition principle to reduce the statistical noise.
- We removed the centrifugal force : the interglueball potential is attractive.
- We calculated the scattering phase shift and derived the interglueball cross section, and we could constrain Λ of SU(2) YMT for the 1st time from observational data : $\Lambda > 60$ MeV.
- Preliminary result of SU(3) YMT looks consistent with SU(2).

Homeworks:

- Calculations for $N_c > 2$: extrapolate to large N_c.
- Lattice artifacts to be discussed.

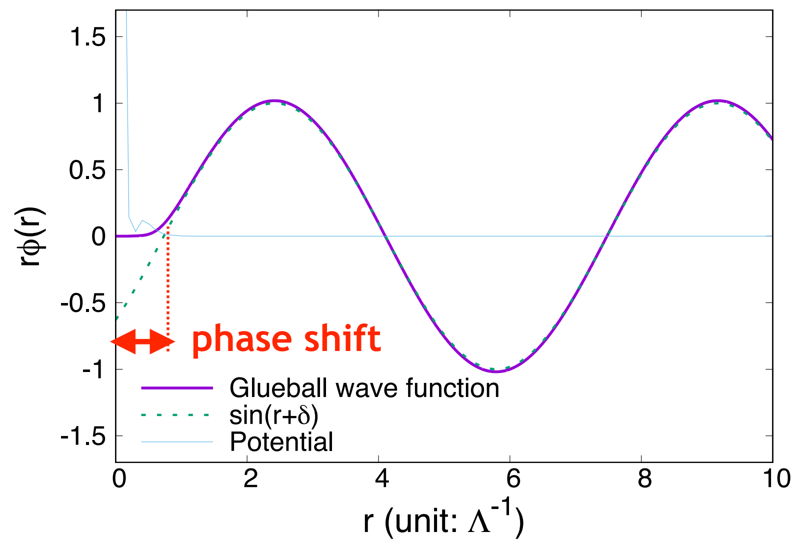
Backup slides

From potential to scattering cross section

Potential ⇒ Scattering phase shift:

Solve $$\left(\frac{\partial^2}{\partial r^2} + k^2 + U(r)\right)\phi(r) = 0$$

$$\Rightarrow \quad \phi(r) \propto \sin[r + \delta(k)] \qquad (r \to \infty)$$



Scattering phase shift ⇒ Cross section:

We are interested in low energy DM cross section, s-wave dominant :

$$\Rightarrow \quad \sigma_{\rm tot} = \frac{4\pi}{k^2}\sin^2[\delta(k \to 0)]$$

Yukawa: σ_{tot} = (2.5 - 4.7)Λ^{-2} (stat.)

2-Gaussian: σ_{tot} = (14 - 51)Λ^{-2} (stat.)

$\Rightarrow$ **σ_{tot} = (2 - 51) Λ^{-2} (stat.and sys.)**

(sys. due to fitting forms)

Reduction of statistical error with cluster decomposition pr.

Cluster decomposition principle:

"If you are far you are uncorrelated"

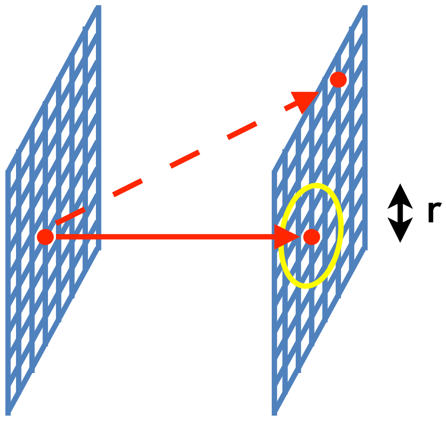
⇒ Almost zero contribution for r > cutoff

For disconnected diagram, noise remains constant

⇒ Integration over r > cutoff accumulates noise

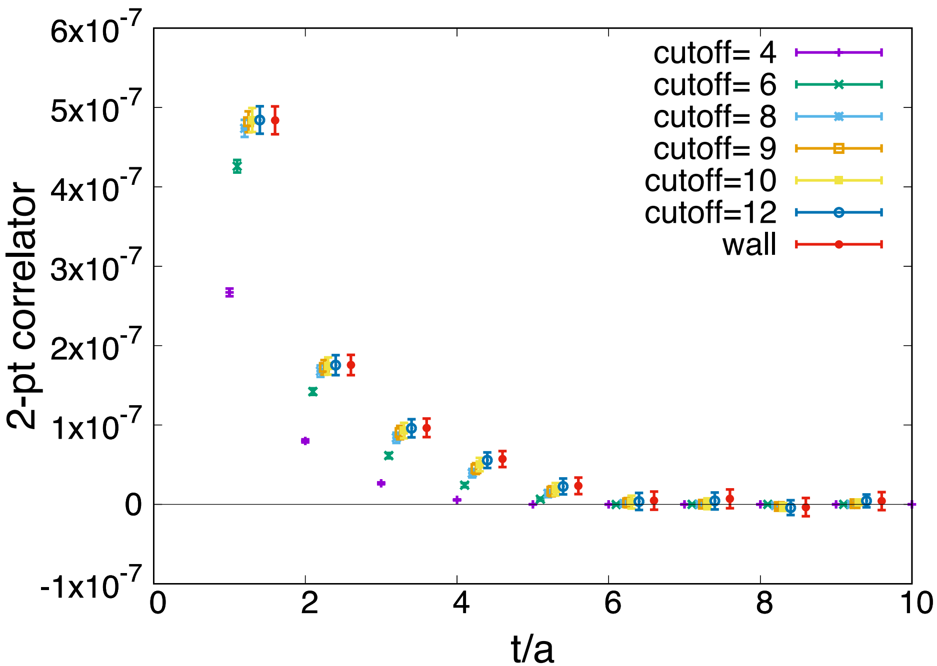
⇒ Remove r > cutoff will reduce the noise?

chiQCD Collaboration, PRD**97**, 034507 (2018)



wall x wall correlation

Example of glueball 2pt-correlator:



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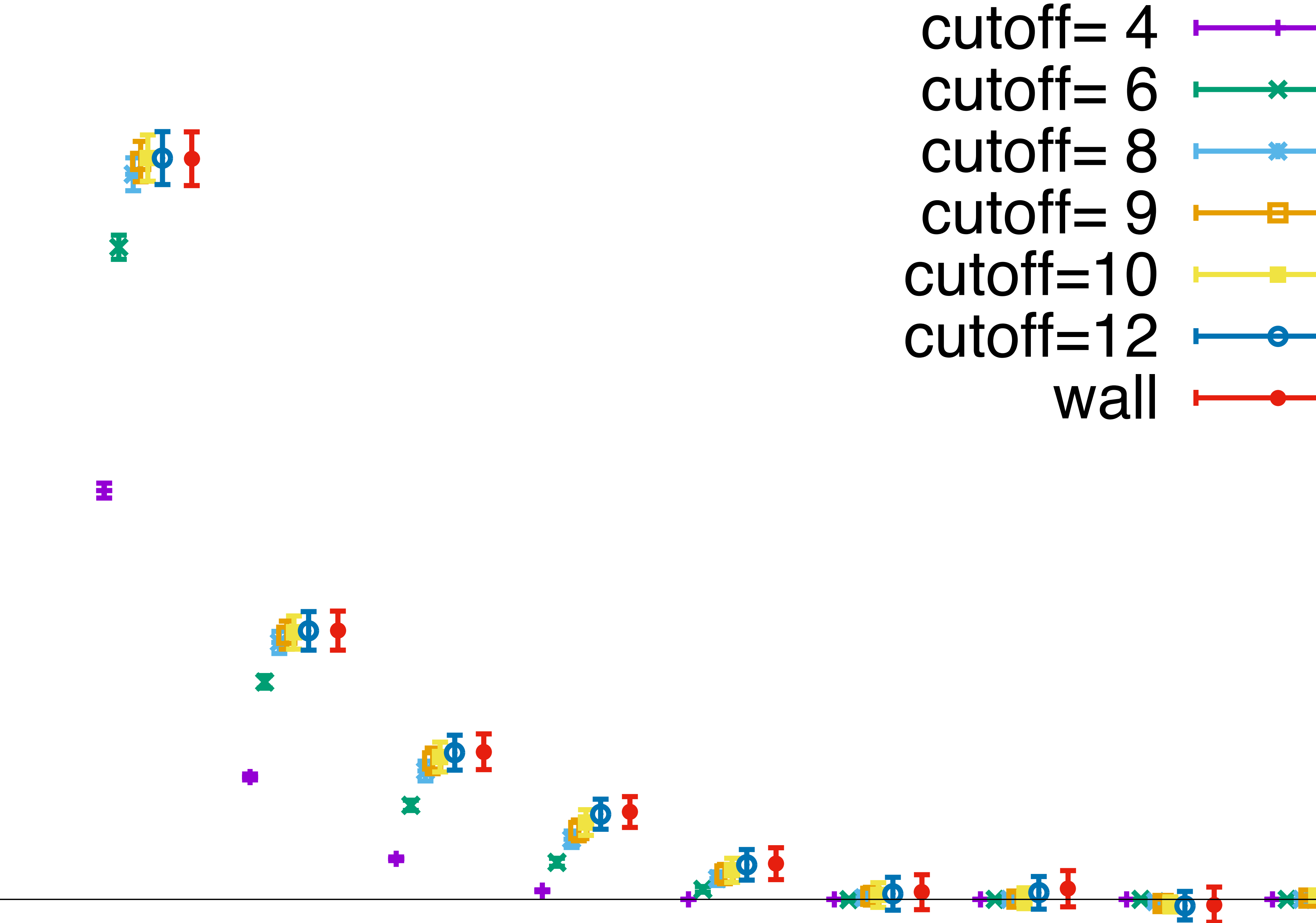

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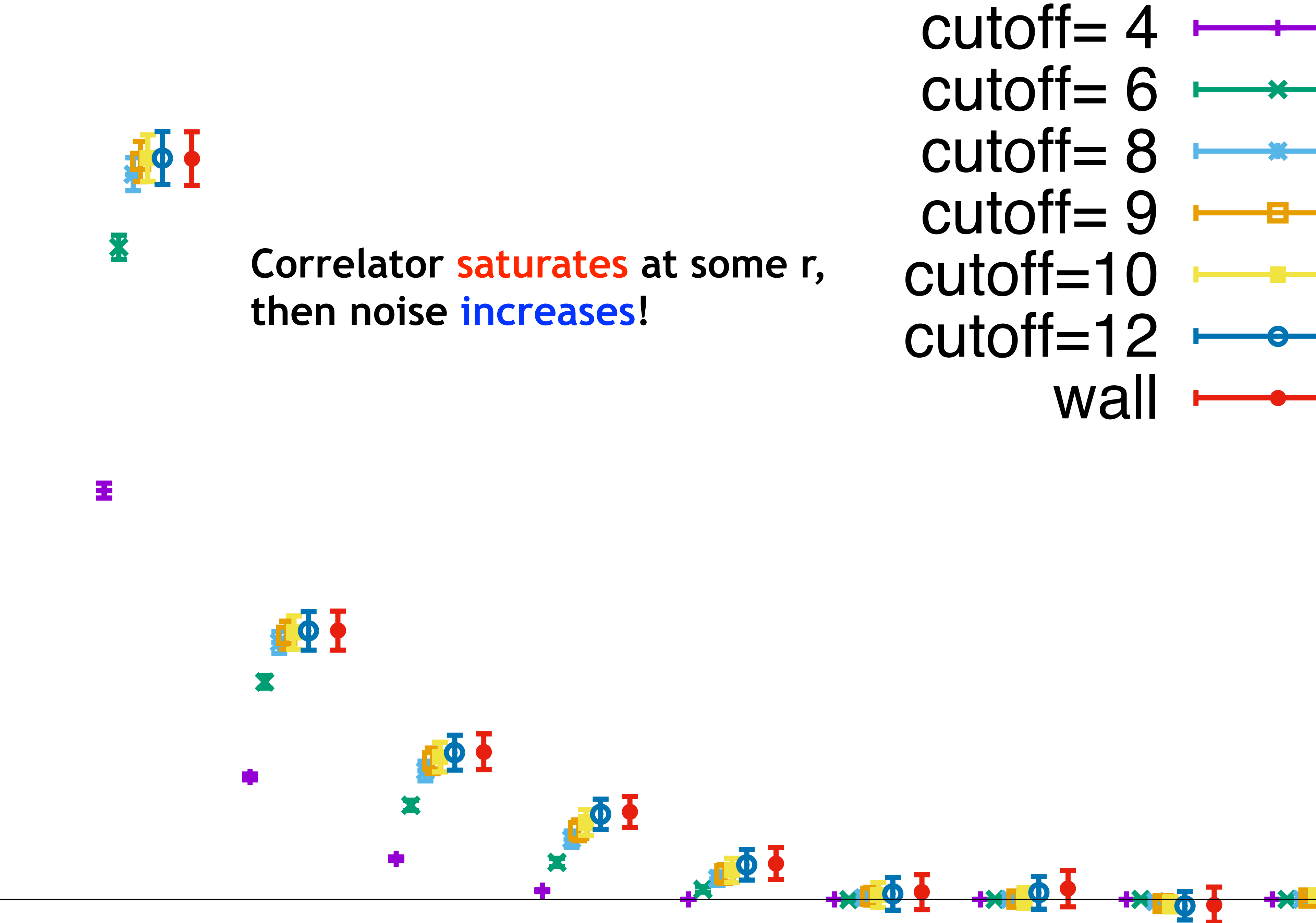

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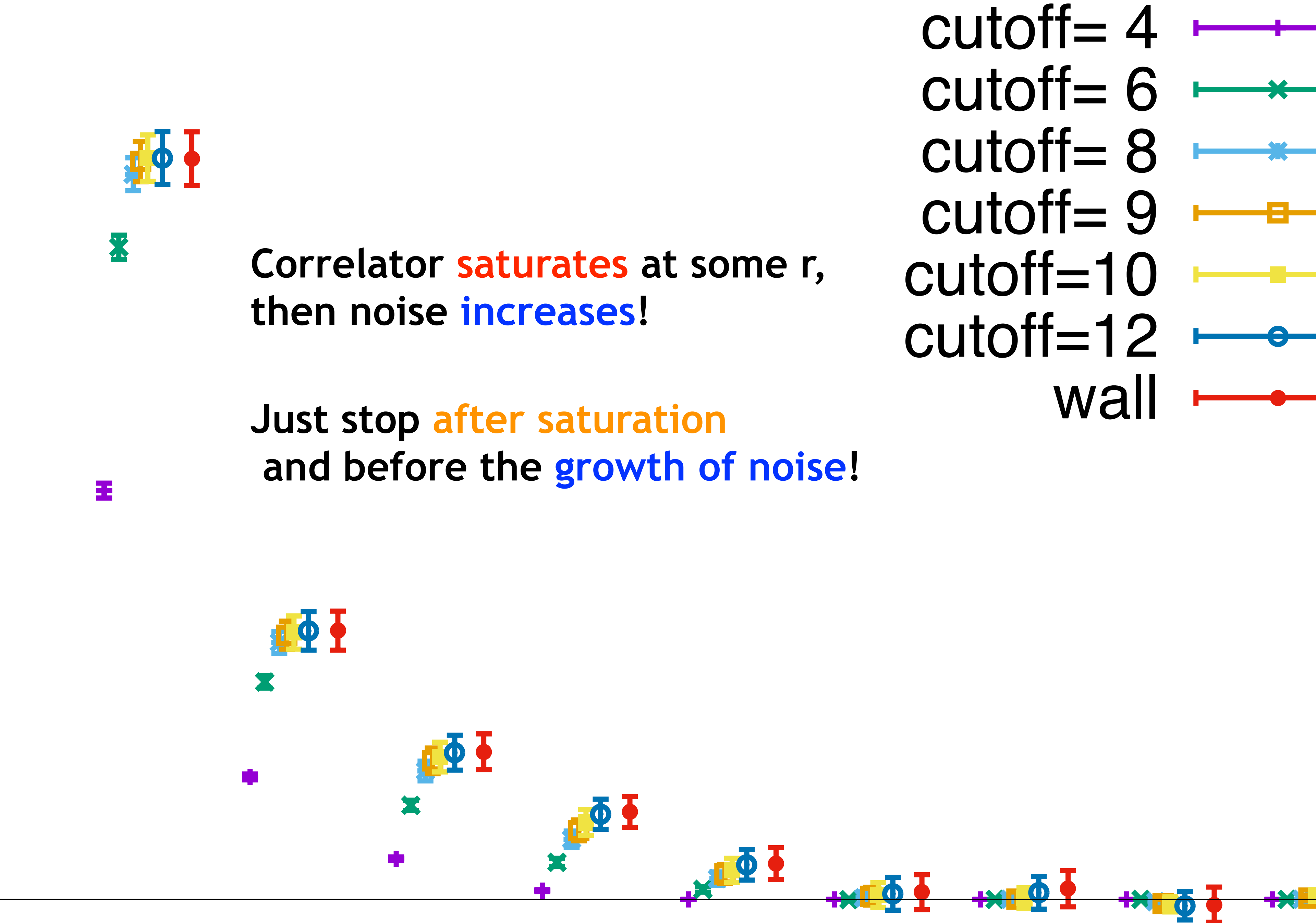

Correlator saturates at some r,
then noise increases!

Just stop after saturation
and before the growth of noise!