Foundation of Synchrotron and Storage Ring

Kaoru Yokoya, KEK

3rd International School on Beam Dynamics and Accelerator Technology (ISBA)

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Evolution of Accelerator

- DC Accelerator
  - Cathode Ray Tube
  - van de Graaf, Cockcroft-Walton
- Cyclotron
- **Synchrotron**
  - **Weak focusing**
  - **Strong focusing**
- **Storage Ring**
- Linear Accelerator
- **Colliders**

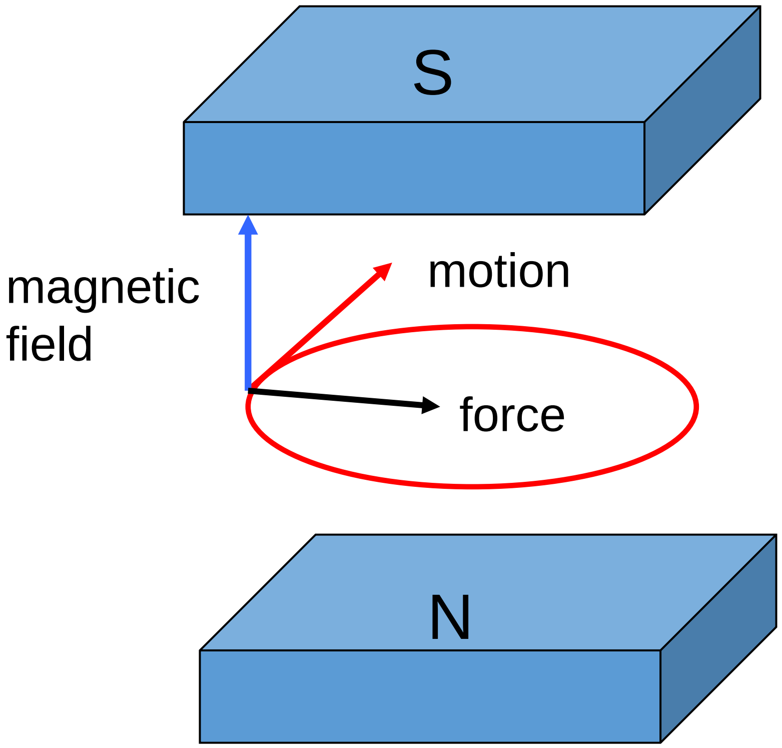
- I am giving 2 lectures, “Synchrotron” and “Collider”
- “Synchrotron” overlaps much with Yujong’s lecture
- So, I will allocate ~1.5hrs + 2.5 hrs, i.e., longer time for “Collider”

Accelerator Development and Physics

- All along the evolution of modern physics
  - Discovery of electron (end of 19th century) by cathode ray tube
  - Some period without accelerator (first 30 years of 20th century)
    - nucleus
    - neutron , neutrino, positron, muon, pion…
    - Do you remember how these particles were found?
  - The, era of synchrotron (weak focus)
    - Anti-proton, ρ, ω, Λ, Ω, … → quark theory
  - Then, strong focusing synchrotron
    - ν_μ, J/ψ
  - And colliders
    - Z, W, …, t-quark, …..
    - Higgs

Charged Particle Motion in Magnetic Field

- A charged particle draws a circle
- $p = e \times B \times \rho$

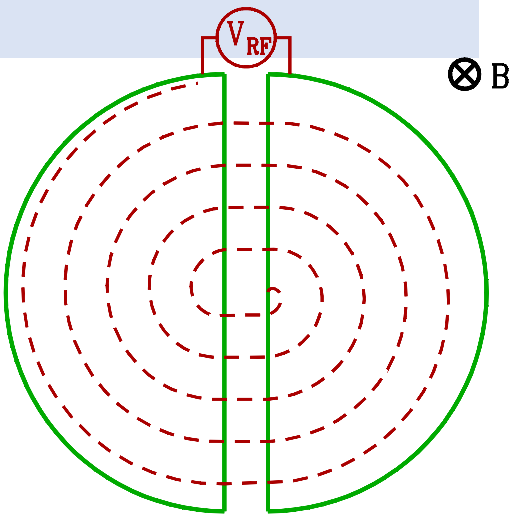


- p: particle momentum
- e: electric charge
- B: magnetic field
- ρ: orbit radius

p [GeV/c] = 0.3 x B [T] x ρ [m]

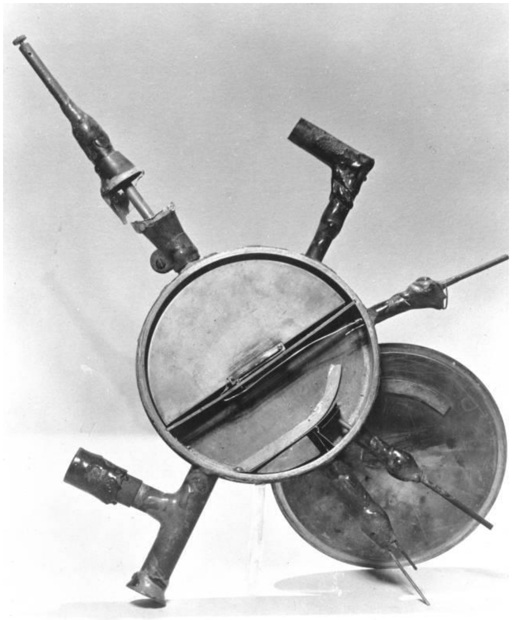
Cyclotron

- Jan. 1931
- Berkeley, California
- Lawrence & Livingston, Phys. Rev. 40, 19, (1932)
- Nobel prize 1939



First cyclotron
diameter 13cm
Proton energy 80keV
Wikipedia says ~25$

The largest cyclotron by now is TRIUMF (17m)



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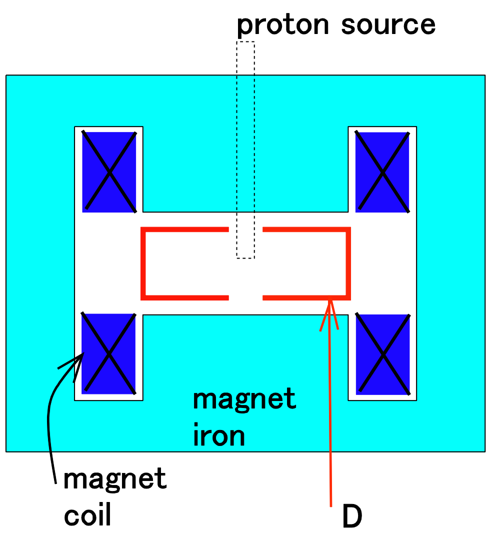


Cyclotron (2)

- Cyclotron makes use of the fact that the time for one turn is independent of the particle energy
  - → Fundamental principle of Cyclotron

$$p = eB\rho = mv$$
$$\Rightarrow T = \frac{2\pi\rho}{v} = \frac{2\pi m}{eB}$$
$$T = \text{time for one revolution}$$

- But, $p = mv$ is an approximation.
  Exact formula with the special relativity is
  $$p = mv/\sqrt{1-(v/c)^2}$$
- "T= independent of p" breaks down when v approaches c.
  $$T = \frac{2\pi m}{eB}\frac{1}{\sqrt{1-(v/c)^2}}$$
- Large difference between electron and proton

Exercise

Assuming that the first cyclotron had maximum orbit diameter 13cm and reached the maximum proton energy 80keV, compute

- The magnetic field
- Frequency of the voltage

(caveats: these may differ a bit from the real one.)

Answer

- Use nonrelativistic formula

$$p = mv, \qquad E_{kin} = \frac{1}{2}mv^2$$

- momentum

$$\begin{aligned} p &= \sqrt{2mE_{kin}} \\ &= \sqrt{2 \times 938\mathrm{MeV}/c^2 \times 80\mathrm{keV}} = 12.25\mathrm{MeV}/c^2 \end{aligned}$$

- Magnetic field

$$B = \frac{p_{[\mathrm{GeV}/c]}}{0.3\rho_{[m]}} = \frac{12.25 \times 10^{-3}}{0.3 \times 0.065} = 0.63\mathrm{Tesla}$$

- velocity

$$v = \sqrt{2E_{kin}/m} = \sqrt{2 \times 80\mathrm{keV}/938\mathrm{MeV}/c^2} = 0.013\,c$$

- frequency

$$f = \frac{v}{2\pi\rho} = 9.6MHz$$

Synchrotron

- Limitation of Cyclotron
  - Special relativity
  - Huge magnet (must fill 2D area)
- Is it possible to confine the orbit with radius=constant?
  - If so, the magnets must fill only 1D area
  - Possible if *B* is time-dependent

$$p(t) = eB(t)\rho$$

- Acceleration
  - Acceleration only a few points in the ring
  - Time for one turn changes as acceleration

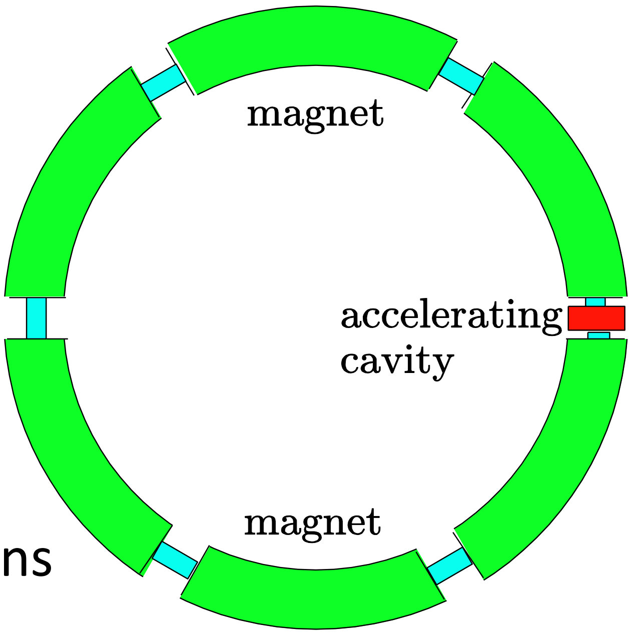
$$T(t) = C/v(t)$$

    C = circumference

  - RF frequency must vary as
    (*h* = integer = harmonic number)

$$f_{RF} = hf_0(t) \quad (f_0 = 1/T)$$
$$C = h\lambda_{RF}$$

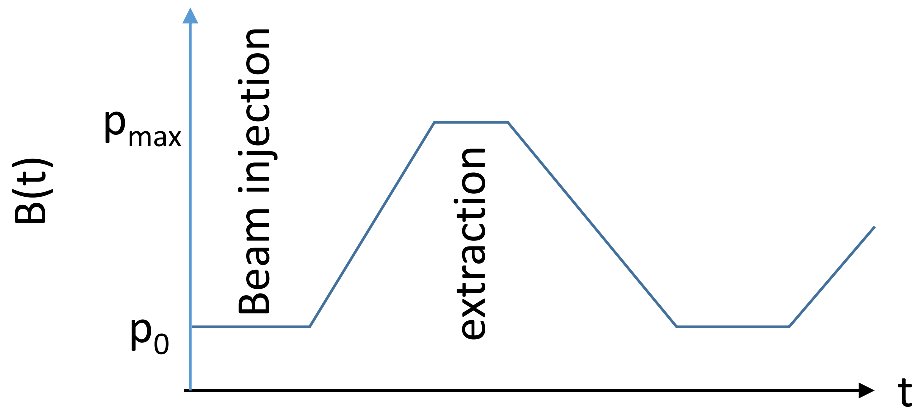
  - Actually, f_{RF} can be constant for electrons ~> 10 MeV



Operation Cycle of a Synchrotron

1. Inject the beam
2. Raise the magnetic field as accelerating the beam
3. Extract the beam
4. Lower the magnetic field to prepare for the next injection

Inevitably pulsed beam. No continuous beam.



Phase Stability

- Group of particles (bunch) has finite size
- The accelerating field is not constant but oscillates sinusoidally
  - Not all the particles are accelerated by the same amount
- What happens if a particle is accelerated more (less) than the average?
  - Are they kept accelerated more (less) ?
- Principle of phase stability
  - V.I.Veksler, Dokl.Akad.Nauk SSSR 43, 346 and 44, 393 (1944)
  - E.M.McMillan, Phys.Rev.68 (1945) 143

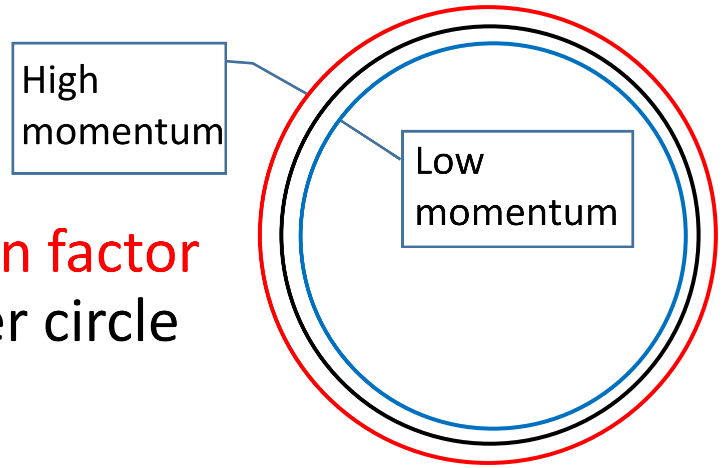
Phase Slippage

- Particles in a bunch have a spread of momentum (energy)
- Revolution time *T* is a function of momentum deviation

$$T = \left(1 + \eta \frac{\Delta p}{p}\right) T_0$$

- η has 2 components:

$$\eta = \alpha_p - \frac{1}{\gamma^2}$$



  - α_p: called momentum compaction factor
    higher energy particle → larger circle

$$C = \left(1 + \alpha_p \frac{\Delta p}{p}\right) C_0$$

  - $1/\gamma^2$: higher energy particle → higher velocity

Synchrotron Oscillation (1)

- Suppose the bunch comes to the accelerating cavity at the phase around $\phi = \phi_s$ (see figure)
- Motion of a particle : $\phi = \phi_s + \Delta\phi$, $\varepsilon = \Delta E / E_0$
- Acceleration: (n = number of turns)

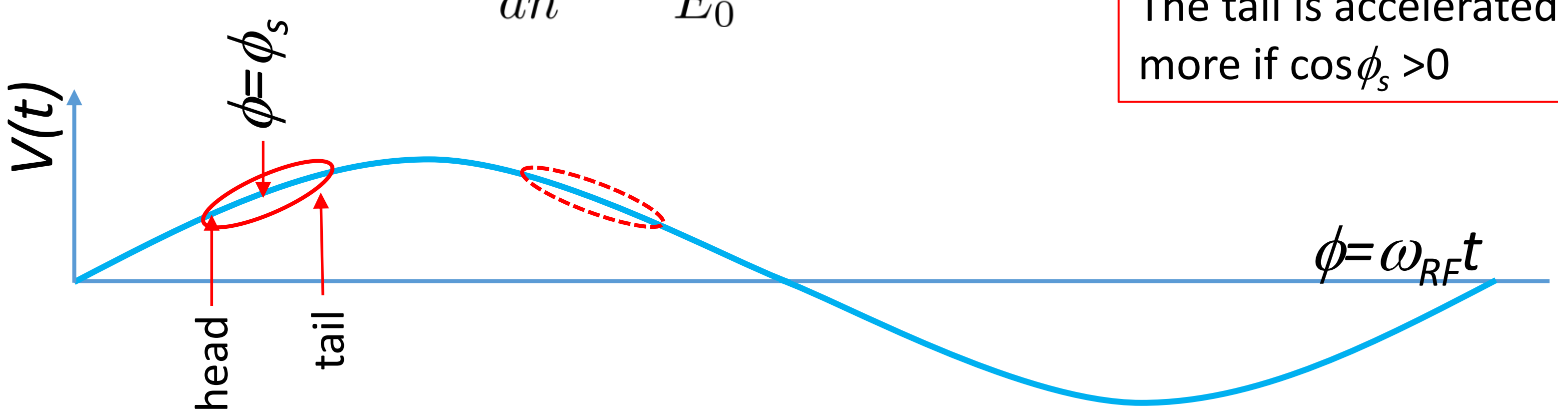
$\Delta\phi > 0$ at the tail

$$\frac{dE}{dn} = eV_{RF} \sin(\phi_s + \Delta\phi) \approx eV_{RF} (\sin\phi_s + \Delta\phi \cos\phi_s)$$

- The first term $eV_{RF} \sin\phi_s$ is compensated for either
  - by the beam acceleration
  - or by the synchrotron radiation loss (for electron)
- Therefore

$$\frac{d\varepsilon}{dn} = \frac{eV_{RF}}{E_0} \cos\phi_s \Delta\phi$$

The tail is accelerated more if $\cos\phi_s > 0$



Synchrotron Oscillation (2)

- Phase slippage:

$$\frac{d\Delta\phi}{dn} = \Delta T \omega_{_{RF}} = \eta \frac{\Delta p}{p} T_0 \omega_{_{RF}} = \frac{2\pi h}{\beta^2} \eta \varepsilon \qquad (\beta = v/c)$$

- Now, we have a simultaneous equation

$$\begin{aligned} \frac{d\varepsilon}{dn} &= \frac{eV_{_{RF}}}{E_0} \cos\phi_s \Delta\phi \\ \frac{d\Delta\phi}{dn} &= \frac{2\pi h}{\beta^2} \eta \varepsilon \end{aligned}$$

- Equation of ε

$$\frac{d^2\varepsilon}{dn^2} = \frac{2\pi h}{\beta^2} \frac{V_{_{RF}}}{E_0} \times \eta \cos\phi_s \times \varepsilon$$

- The oscillation is stable if $\eta \cos \phi_s < 0$

Synchrotron Oscillation (3)

- This oscillation is called “Synchrotron oscillation”
- Synchrotron oscillation tune

$$\nu_s = \sqrt{-\eta \cos\phi_s \frac{h}{2\pi\beta^2} \frac{V_{RF}}{E_0}}$$

  - Usually ν_s << 1
- The discovery of phase stability made synchrotron possible

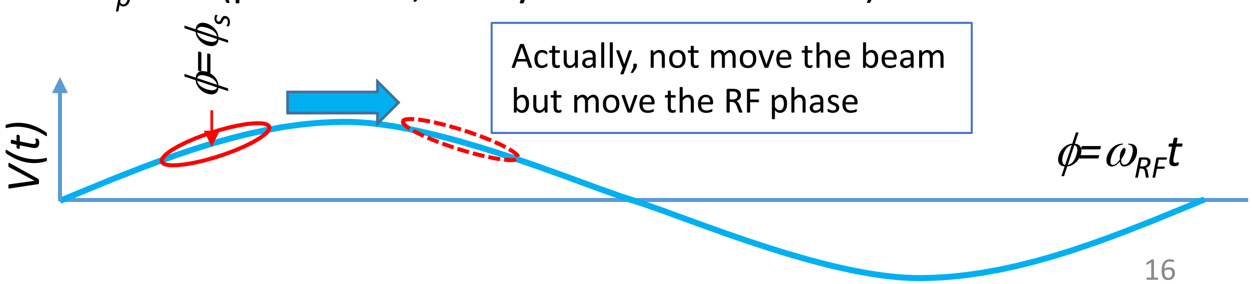
Transition Energy

$$\eta = \alpha_p - \frac{1}{\gamma^2}$$

- Usually, $\alpha_p > 0$
- γ varies as acceleration
- Can become $\eta = 0$ at the special energy (transition energy) during acceleration

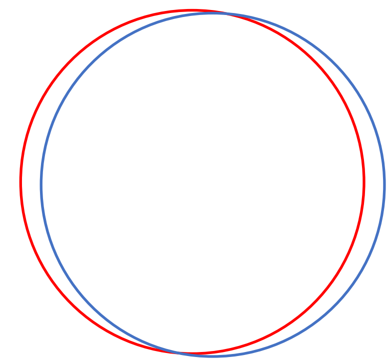
$$\gamma = \gamma_t \equiv \frac{1}{\sqrt{\alpha_p}}$$

- For the phase stability ($\eta \cos \phi_s < 0$), $\cos\phi_s$ must change sign at this moment
- The beam is unstable at the moment $\eta = 0$
- Jump the RF phase at $\eta = 0$, but beam loss and degradation
- To avoid transition energy
  - Design a ring such that γ_t is out of the acceleration range
  - Or, design such that $\alpha_p < 0$ (possible!!, many other side effects)

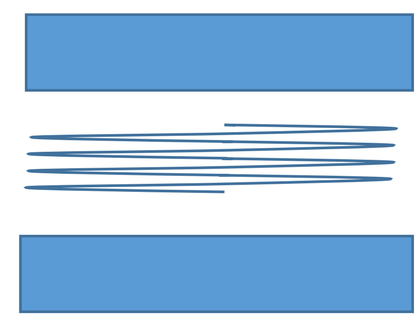


Betatron Oscillation

- If a particle velocity has an off-circle motion, the particle draws a circle with a shifted center but does not go far away (horizontal focusing)
- But once a particle gets vertical velocity, then the particles eventually hit the magnet and is lost.
- There must be a force which gives a vertical force to the particle to bend the orbit back to the medium plane.
- Transverse motion is called “Betratron Oscillation” due to a historical reason
- Next-page figure: The magnet gap is larger outside
  - Stability of betratron oscillation



top view



side view

Betatron

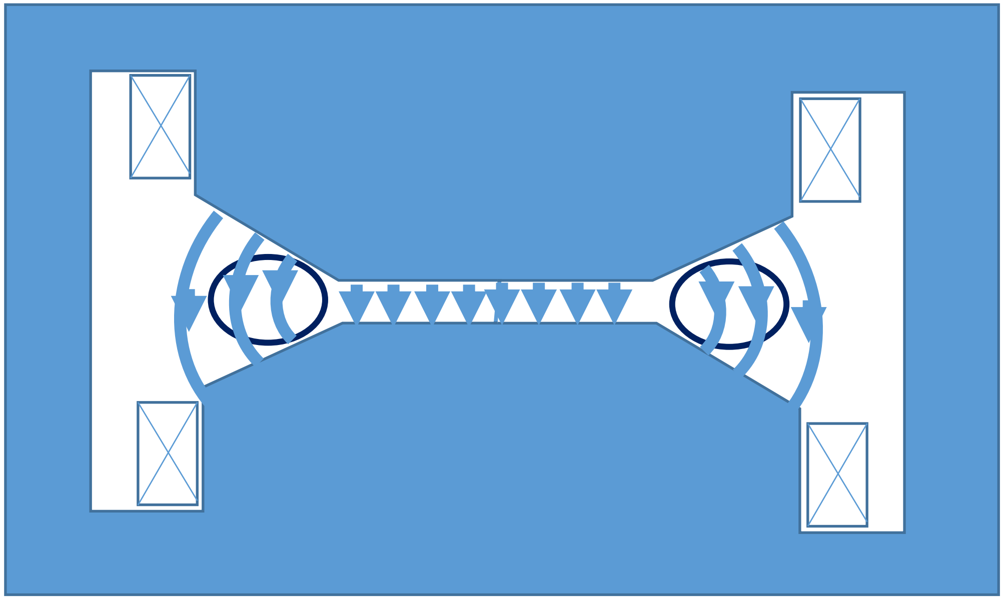
- Donald William Kerst, built ~1940 (Phys. Rev. 60, 47 July 1941)
- Induction accelerator (time-dependent magnetic field)

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow E_\phi = \frac{1}{2\pi\rho}\frac{\partial \Phi}{\partial t}, \qquad \left(\Phi \equiv \int B \cdot dS\right)$$

- Actually, only for electron (i.e., beta particle)
  - Proton requires a too big magnet
- Maximum ~300MeV
  - synchrotron radiation
  - Heavy magnet
  - Eddy current
- Betatron condition (to keep the particle on *r*=const.)

$$B(r) = B_{\rm avr}(r)/2$$

$$B_{\rm avr}(r) = \frac{1}{\pi r^2}\int_{<r} B \cdot dS$$



Equation of Betatron Motion

- Transverse motion is called "betatron motion" due to a historical reason (like "synchrotron motion")
- Coordinate
  - *s*: direction of (circular) motion
  - *x*: outwards on the ring plane
  - *y*: vertical so that (x,y,s) is right-handed
- Equation of motion:

$$\frac{d^2x}{ds^2} + K_x x = 0, \qquad K_x = \frac{e}{p_0}\frac{\partial B_y}{\partial x} + \frac{1}{\rho_0^2}$$

$$\frac{d^2y}{ds^2} + K_y y = 0, \qquad K_y = -\frac{e}{p_0}\frac{\partial B_y}{\partial x}$$

Focusing Magnets

- Use a magnet like →

$$B_y = B_0 \left(1 - \frac{nx}{\rho_0}\right)$$

$$B_x = -B_0 \left(\frac{ny}{\rho_0}\right)$$

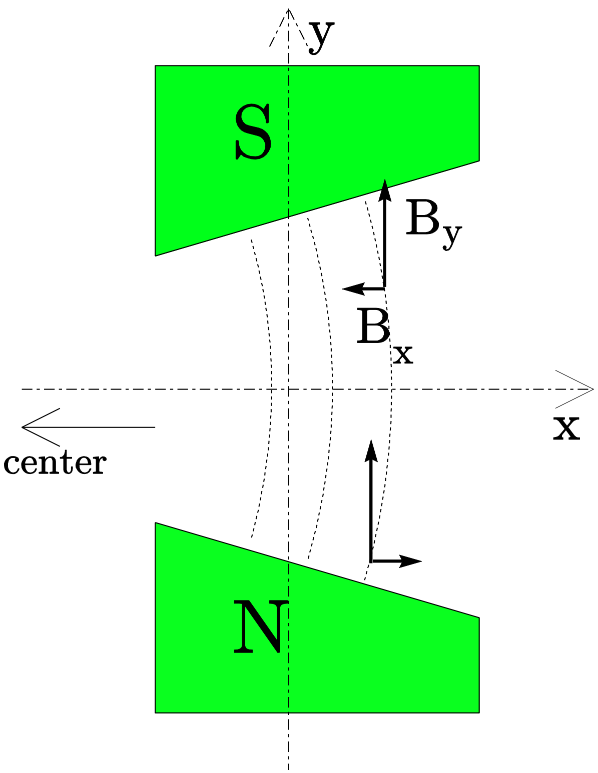
$$\text{Note}: \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

n = field index

- Then the equation becomes

$$\frac{d^2x}{ds^2} + \frac{1-n}{\rho_0^2}x = 0,$$

$$\frac{d^2y}{ds^2} + \frac{n}{\rho_0^2}y = 0$$



Weak Focusing

- Then, ($\theta = s/\rho = 2\pi s/C$, C: circumference)

$$x \propto \sin\sqrt{1-n}\theta, \qquad y \propto \sin\sqrt{n}\theta$$

- Stable both in x and y, iff $0 < n < 1$.
- Number of oscillations in one turn is called "tune", usually denoted by ν or Q
  - $\nu_x = \sqrt{1-n}$, $\nu_y = \sqrt{n}$ in the above case
  - $0 < \nu_{x,y} < 1$
  - In the case of flat magnet, $\nu_x = 1$, $\nu_y = 0$
- This focusing is called "weak focusing"

Particle Discovery before the Era of Accelerator

- Neutron 1932 (α on beryllium)
- Neutrino ~1932 (to explain beta decay)
- positron 1932 (from cosmic ray)
- muon 1937 (cosmic ray)
- π meson 1947 (cosmic ray)

- Accelerators, improved to high energies, started to discover new particles in 1950's

GeV-class Synchrotrons

- 1950's
- A few GeV proton synchrotrons
  - Cosmotron (BNL) 3.3GeV
  - Bevatron (LBL) 6.2GeV
- Many new particles found
  - anti-proton, anti-neutron
  - ρ, ω,...., Λ, Σ, Ξ, Ω,....
  - Systematic description introducing "Quarks" by Gell-Mann in 1964

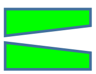
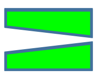
Cosmotron

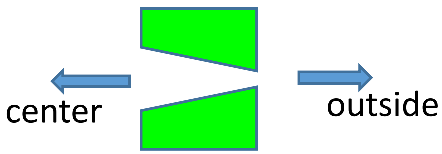
Bevatron

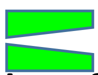
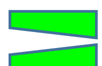
- Weak-focusing synchrotron
  - 10,000 tons of magnets
- Lawrence Berkeley Laboratory
- Start operation in 1954
- Bev.. = Billion Electron Volt
  = Giga Electron Volt (GeV)
- Discovered antiproton in 1955

http://www.lbl.gov/image-gallery/image-library.html

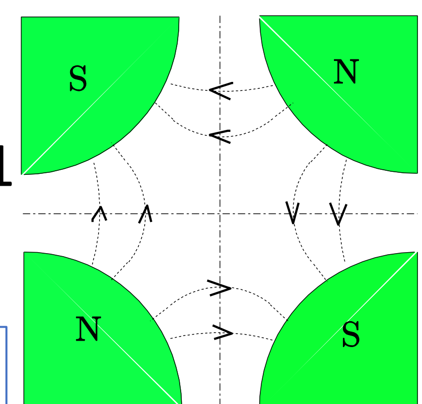
Strong Focusing (1)

- Very big magnets would be needed to go beyond a few GeV
- A new principle was found by Earnest Courant, et.al.
  - E. D. Courant, M.S. Livingston, H. S. Snyder, Phys.Rev. 88 (1952)
- Note that no single magnet can focus simultaneously in x and y.
- : defocus in x, focus in y
- : focus in x, defocus in y



- **The field index *n* need not be a constant over the ring**
- Arranging  and  alternately, the beam can be focused both in x and y
- Large positive n>>1 and large negative n<<-1

Can also be done by quadrupole magnets →



Strong Focusing (2)

- Why focus + defocus = focus ?
- Two lenses with the focal length f_1, f_2 placed with the distance *d* make a lens of focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

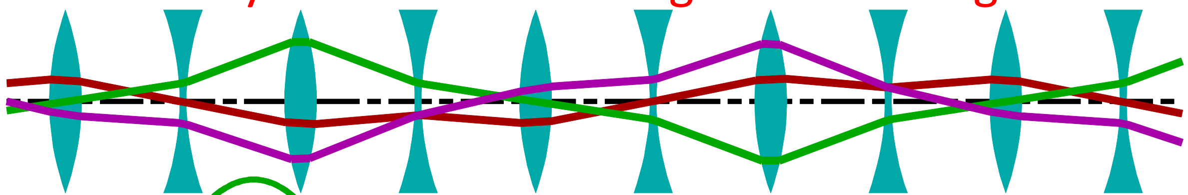
- If $f_1 = -f_2 = f$, then $F = \frac{f^2}{d}$
  i.e., a focusing lens

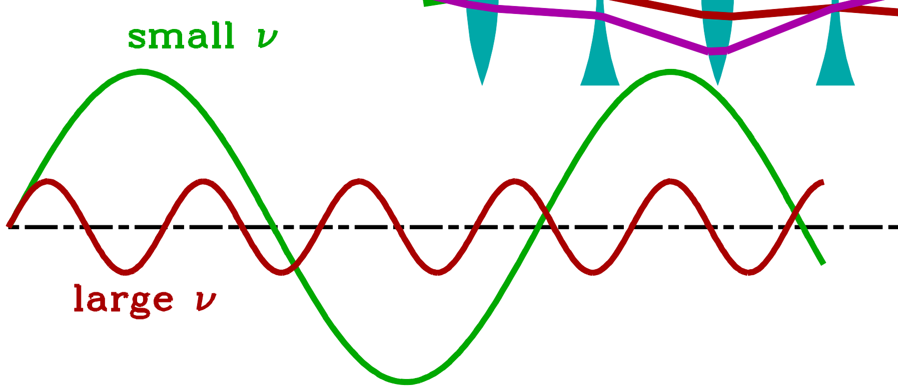
If you are familiar with matrix formalism

$$\begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 - d/f_1 & d \\ -1/f_1 - 1/f_2 + d/f_1 f_2 & 1 - d/f_2 \end{pmatrix}$$

Strong Focusing (3)

- With many focusing and defocusing lenses, the orbit oscillates many times during one turn
  - ν_x, ν_y can be > 1, even >> 1, can be even ~100 in modern synchrotrons
- The beam size becomes much smaller than weak-focusing → magnet becomes much smaller
- The price to pay was the accuracy of the field and alignment of magnets





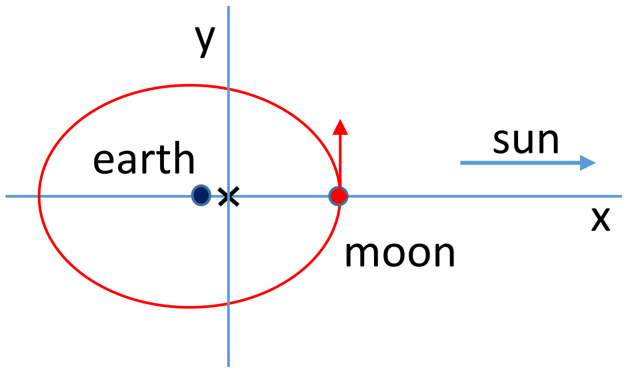
- Strong focusing synchrotrons in early times
  - CERN-PS 28GeV, 1959
  - BNL AGS 33GeV, 1960
  - Serpukov 76GeV, 1967

Hill's Equation

- Both x and y motions are written in the form

$$\frac{d^2x}{ds^2} + K(s)x = 0, \qquad K(s+C) = K(s) \quad (C = \text{ring circumference})$$

- This sort of 2nd order linear differential equation with a periodic coefficient is called "Hill's equation"
  - First introduced by G.W.Hill as the first order motion of the moon with the earth and the sun taken into account
    - Rotating coordinate (1 year) with centrifugal force with infinitely distant sun $\left(D \to \infty \quad \text{with} \quad M_{\text{sun}}/D^3 \quad \text{fixed}\right)$
    - Origin: center-of-mass of earth+moon
  - First, find a periodic solution (corresponding to the closed orbit in accelerator physics)
  - Then, include the deviation (horizontal and vertical betatron oscillation) and find the "tunes"
    - As the eigenvalue of infinite dimension matrix



Math of Betatron Motion

- General solution of a Hill's equation can be written as

$$x(s) = \Re A f(s)$$

- where *A* is an arbitrary complex constant and *f*(s) is a complex function and has the property (Floque's solution)

$$f(s+C) = e^{2\pi i\nu} f(s)$$

- Usually, we parametrize *f(s)* as

$$f(s) = \sqrt{\beta(s)} e^{i\phi(s)}, \qquad \phi(s+C) = 2\pi\nu + \phi(s)$$

- *β(s)* and *ϕ(s)* have the periodicity

$$\beta(s+C) = \beta(s), \qquad \phi(s+C) = 2\pi\nu + \phi(s)$$

- and the relation

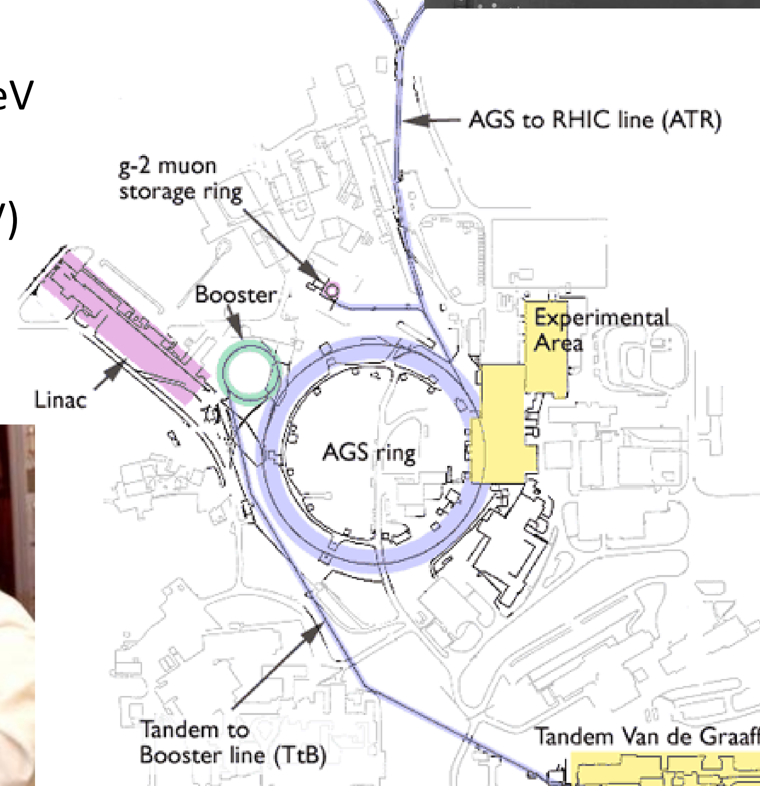
$$\phi(s) = \int_0^s \frac{1}{\beta(s)} ds$$

- So, *β(s)* is related both to the betatron amplitude and to the phase advance :
  - A fundamental quantity of synchrotron beam dynamics
- But I do not go into detail. See Yujong's lecture.

AGS: Alternating Gradient Synchrotron

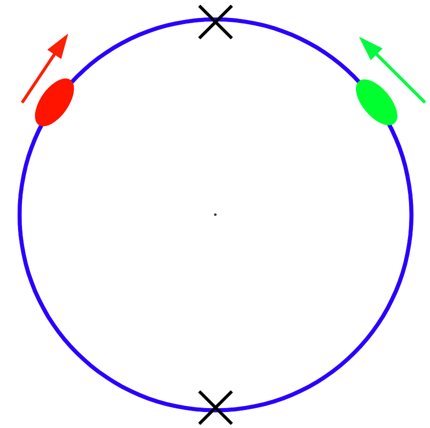
- First strong-focusing synchrotron
  - I do not know the exact magnet aperture of Bevatron except the photo on page 24 (with people sitting in the magnet)
  - The diameter of the beam pipe of AGS is 173x78 mm, nonetheless this is even large in the present standard
- Brookhaven Laboratory (BNL)
- Proton synchrotron,
  - diameter 257m, tunes ~8.7, max energy 33GeV
- Started operation in 1960
  - Almost at the same time as CERN CPS (28GeV)
- Discovery of new particles
  - ν_μ
  - J/ψ , charm quark

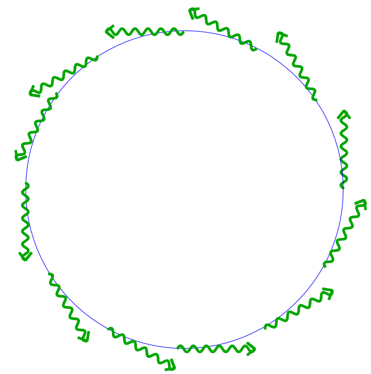
Sam Ting



Storage Ring

- A synchrotron can store a beam for milli-seconds to days
- Usage of storage ring
  - Collider
  - Synchrotron light source
  - Beam manipulation
    - Low emittance
    - Buncher/debuncher
    - Stacking
- Principle is the same as synchrotron but
  - no need of rapid acceleration (in somecase even no acceleration, i.e., full energy injection)
  - longer beam life required (e.g., better vacuum)
  - insertion structure (colliding region, undulator, etc) required, depending on the purpose





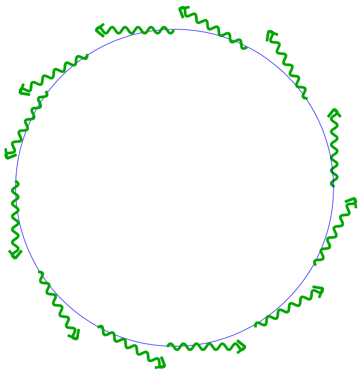
Electron Storage Ring

- Electron Storage Ring is somewhat different from a normal synchrotron because of “synchrotron radiation”

Synchrotron Radiation (1)

- Charged particles lose energy by synchrotron radiation
- proportional to $1/m^4$
  - Almost negligible from protons but visible in LHC
- Loss per turn (electron)

$$U = 0.088_{[\mathrm{MeV}]} \frac{E^4{}_{[\mathrm{GeV}]}}{\rho_{[\mathrm{m}]}}$$



- Photons are emitted almost in the forward cone of angle $1/\gamma$
- Average photon energy

$$E_\gamma = 0.683_{[\mathrm{keV}]} \frac{E^3{}_{[\mathrm{GeV}]}}{\rho_{[\mathrm{m}]}}$$

- Wavelength

$$\lambda_{critical} \equiv \frac{\lambda_{critical}}{2\pi} = \frac{2}{3} \frac{\rho}{\gamma^3}$$

- This energy loss must be compensated for by RF acceleration
  - Therefore, an electron storage ring must have RF acceleration system even if net acceleration is not necessary

Synchrotron Radiation (2)

- Synchrotron radiation defines the fundamental limitation of electron storage ring to go to very-high energy physics
- Nonetheless, the synchrotron radiation causes not only unwelcomed effects but
  - can be used as light source
  - radiation damping
    - Stabilize beam oscillation
    - Lower the emittance
    - Used for damping rings for linear and circular colliders

Radiation Damping (Longitudinal)

- Synchrotron radiation loss is larger for higher-energy particle

$$\frac{dE}{dt} = -\mathrm{const.} \times B^2 E^2$$

- → damping of synchrotron oscillation (damping of energy spread)
- Number of revolution needed for the radiation damping of energy spread is approximately ($R=C/2\pi$)

$$n = \frac{E}{U} \quad \Rightarrow \quad \tau = \frac{E}{U} T_0 \approx 0.237_{[\mathrm{ms}]} \frac{\rho_{[\mathrm{m}]} R_{[\mathrm{m}]}}{E^3_{[\mathrm{GeV}]}}$$

Equilibrium Energy Spread

- Longitudinal effect of radiation is not damping only
- Energy loss occurs randomly, which causes the spread of the beam energy
- The equilibrium energy spread in an electron storage ring is approximately

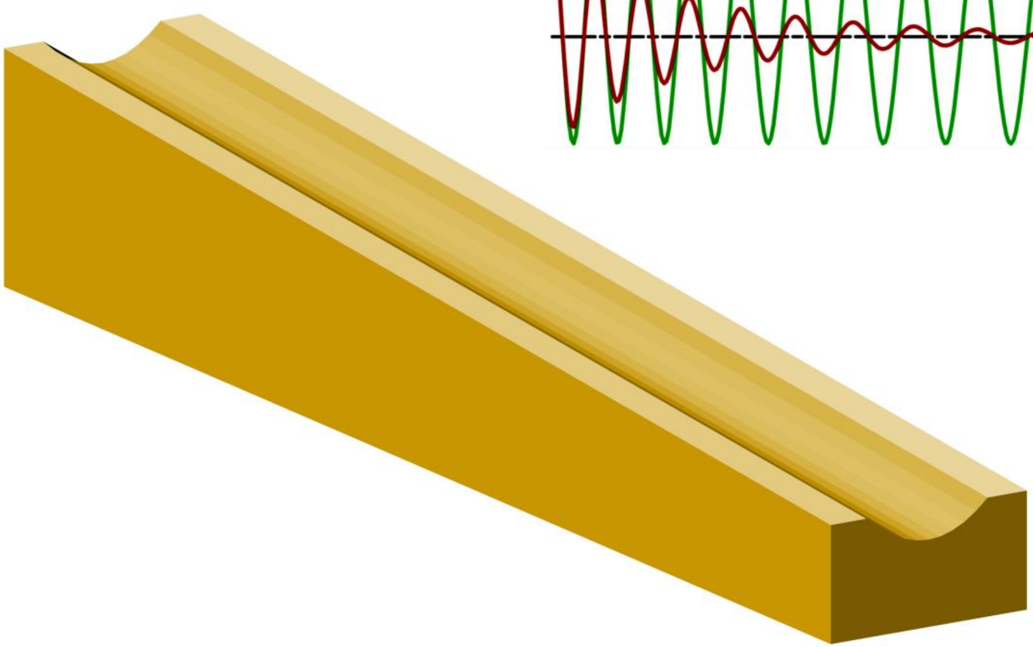
$$\frac{\sigma_\varepsilon}{E_0} = 0.857 \times 10^{-3} \frac{E_{[\mathrm{GeV}]}}{\sqrt{\rho_{[\mathrm{m}]}}}$$

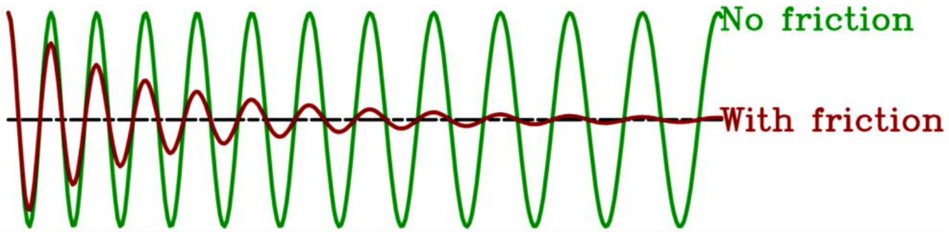
- The equilibrium bunch length (ω_0 = revolution angular frequency $2\pi/T$)

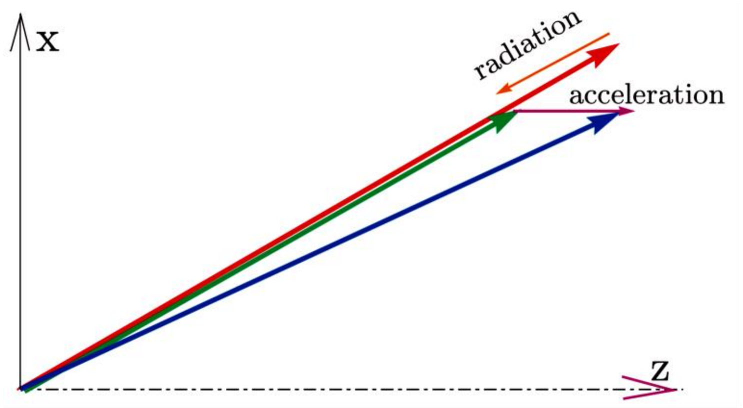
$$\sigma_z = \frac{c\alpha_p}{\nu_s \omega_0} \frac{\sigma_\varepsilon}{E_0}$$

Radiation Damping (Transverse)

- Momentum loss in the direction of motion (like frictional force) → damping of transverse oscillation
- The transverse damping time is about 2x the longitudinal damping time.

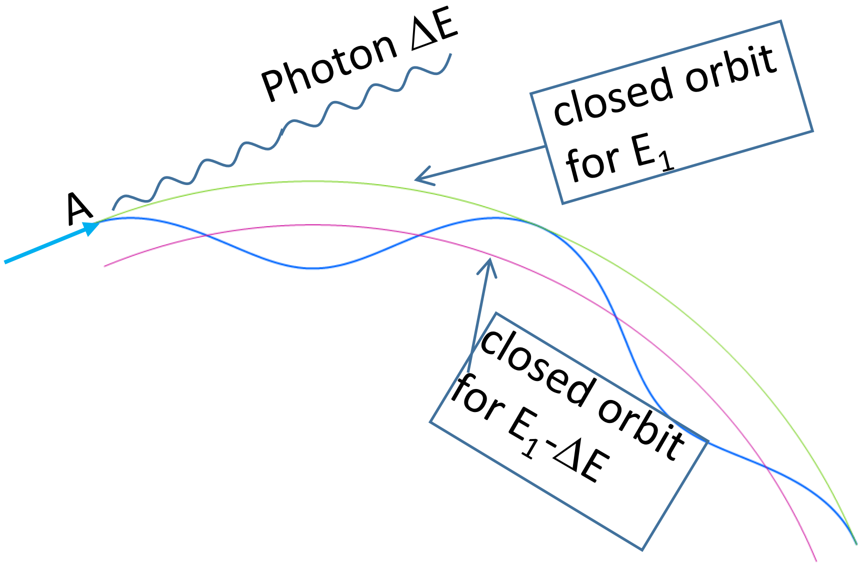






Excitation of Betatron Oscillation

- Transverse effect of synchrotron radiation is not damping only
- Random nature of radiation excites (horizontal) betatron oscillation



- Suppose an electron with energy E_1 is on the closed orbit (green curve)
- Suppose it emitted a photon ΔE at A
- Then, the electron starts betatron oscillation (blue curve) around the closed orbit (red) for the energy $E_1 + \Delta E$
- There is also a case where betatron oscillation stops at the radiation
- Statistical average shows emittance increase

Equilibrium Emittance

- Equilibrium emittance is determined by the balance between the excitation and damping
- General expression of the equilibrium emittance is complex (perhaps, see Yujong's lecture)
- In the simplest case (repeated "FODO" cells)

$$\epsilon_x \approx 1.47 \times 10^{-6}\ _{[\mathrm{rad}\cdot \mathrm{m}]} \frac{R_{c[\mathrm{m}]} E^2_{[\mathrm{GeV}]}}{\rho_{[\mathrm{m}]} \nu_x^3}$$

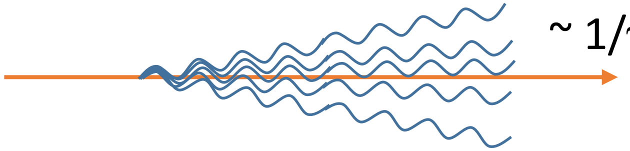
- here, R_c=average orbit radius in FODO cell, ν_x = horizontal tune in FODO arc
- Strong dependence on the focusing ν_{xc}
  - Tighter focusing → smaller (horizontal) emittance
- Many advanced optics (magnet layout) have been invented for lower emittance in particular for light sources

Vertical Emittance (1)

- The equilibrium emittance in the previous page is for horizontal plane only.
- Synchrotron radiation is emitted almost forward
- Excitation occurs only via the energy dependence of the closed orbit (dispersion)
- Therefore, vertical equilibrium emittance comes from
  - Vertical bending magnets, if they are there
  - Vertical-horizontal coupling due to solenoid, machine errors, etc.
    - Usually, coupling by solenoid is compensated by skew-quadrupole magnets
    - So, basically V-H coupling comes from errors
  - Normally, ε_y from this reason is ~1/100 to 1/1000 of ε_x
    - Make ε_x smaller when small ε_y is necessary
    - Normally, light sources do not require $\varepsilon_y \ll \varepsilon_x$

Vertical Emittance (2)

- Strictly speaking, synchrotron radiation is not emitted exactly forward, but has a finite angle *O(1/γ)* . Hence the recoil causes finite vertical emittance



- The equilibrium vertical emittance due to this reason is

$$\epsilon_y = 0.906 \times 10^{-13} [\mathrm{m}] \frac{1}{J_y} \frac{\oint \beta_y / |\rho|^3 \, ds}{\oint 1 / |\rho|^2 \, ds}$$

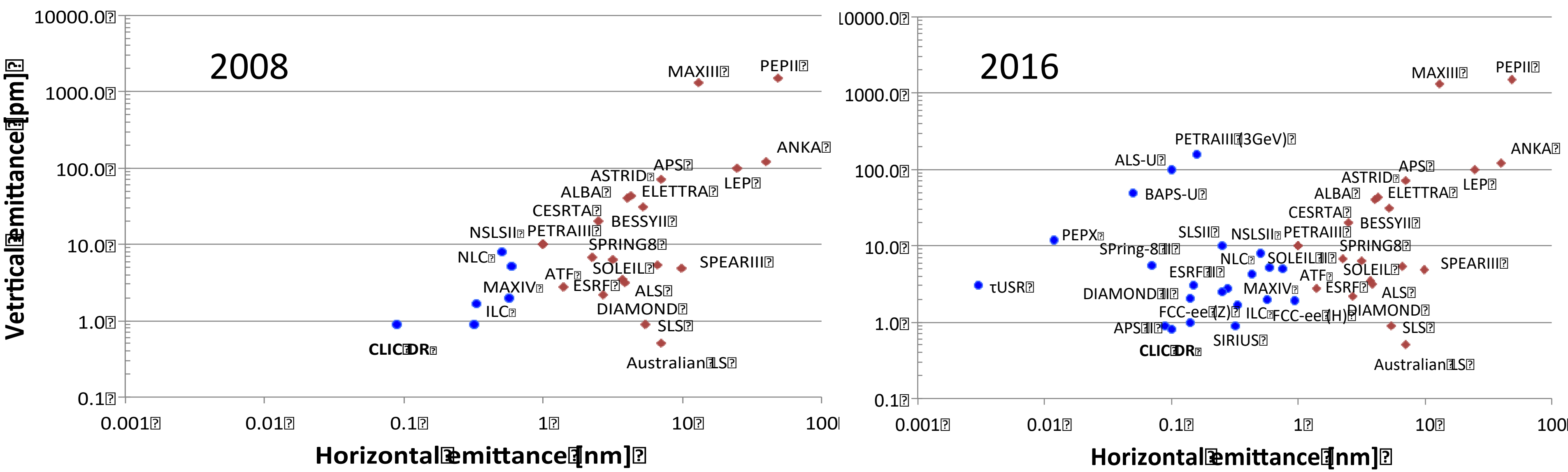
T.O.Raubenheimer, Particle Accelerators 36 (1991) 75

(Sorry, I don't have time to explain Jy. It is about 1. See the lecture by Yujong.)

- This minimum emittance has already been observed

Emittances of Electron Storage Rings

- Horizontal/vertical emittance of existing and planned rings
- red: existing, blue: planned
- Geometric emittance



V. Shiltzev

Use of Synchrotron & Storage Rings

- High Energy Physics
  - Colliders
    - Collider rings (e-, e+, p, ions)
    - E+e- Damping rings (including those for linear colliders)
- Light Sources (electron)
  - Many accelerators have been built to make use of the synchrotron radiation
  - First generation: parasitic use of radiation from bending magnets of colliders
  - Second generation: parasitic use of radiation from insertion magnets of colliders
    - Undulators
  - Third generation: Radiation in a ring dedicated for light source
  - Linear accelerator now also being used as radiation source
  - Fourth generation?: FEL, ERL,
- Industrial, medical, …
- These are not my field → Yamada san yesterday, etc.