Dense QCD matter tackled by experiments, observations, and theory

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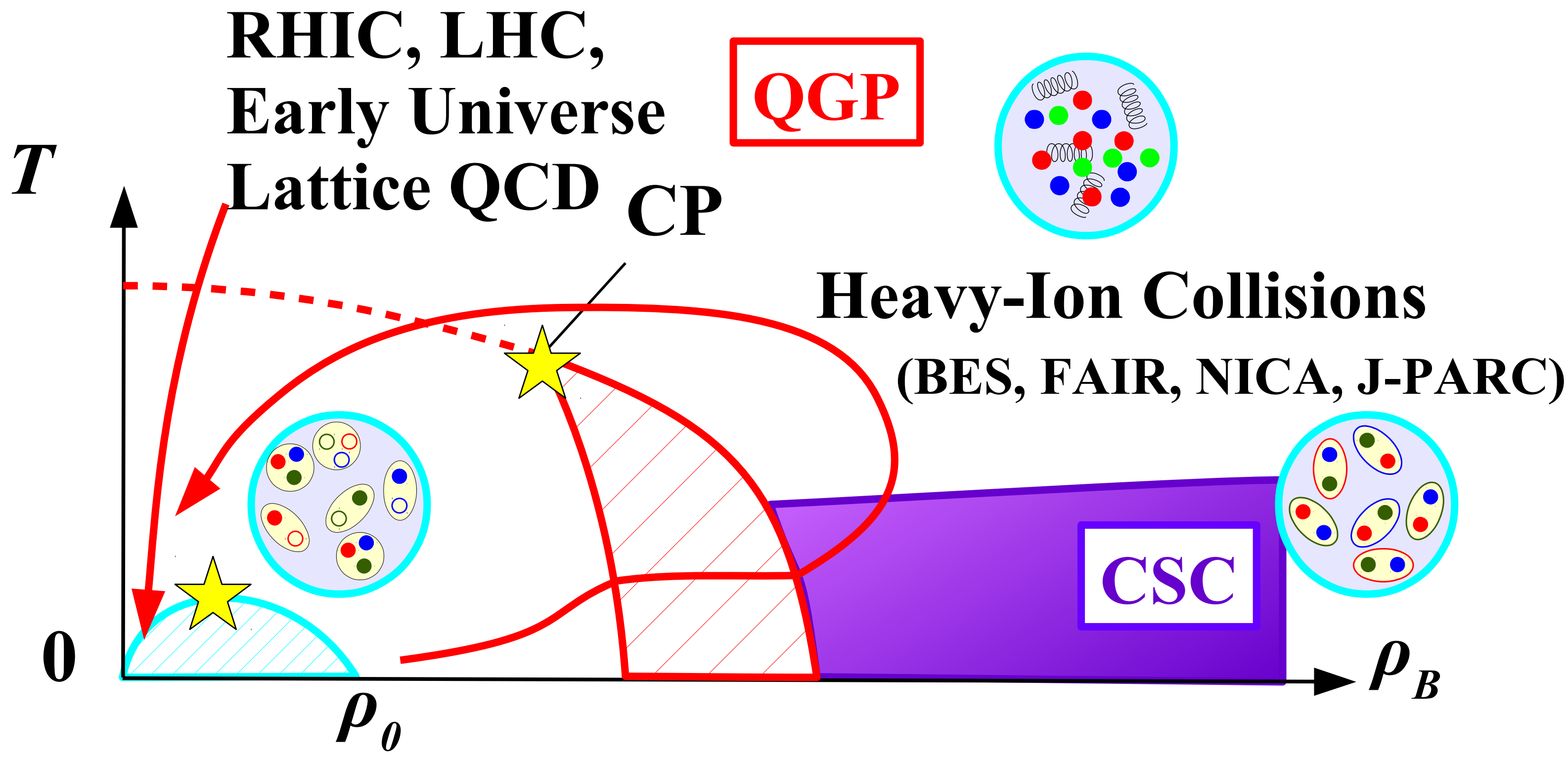
KEK Theory Workshop 2020, Dec. 15-18, 2020, Online/KEK.

- **Introduction**
  - **Celestial dense QCD matter (observations)**
  - **Conjectured dense QCD matter Equation of State**
- **Model-independent and/or first principles approaches**
  - **Approaches based on symmetry and topology**
  - **Finite density lattice QCD**
  - **Heavy-Ion Collisions below RHIC**
- **Summary**

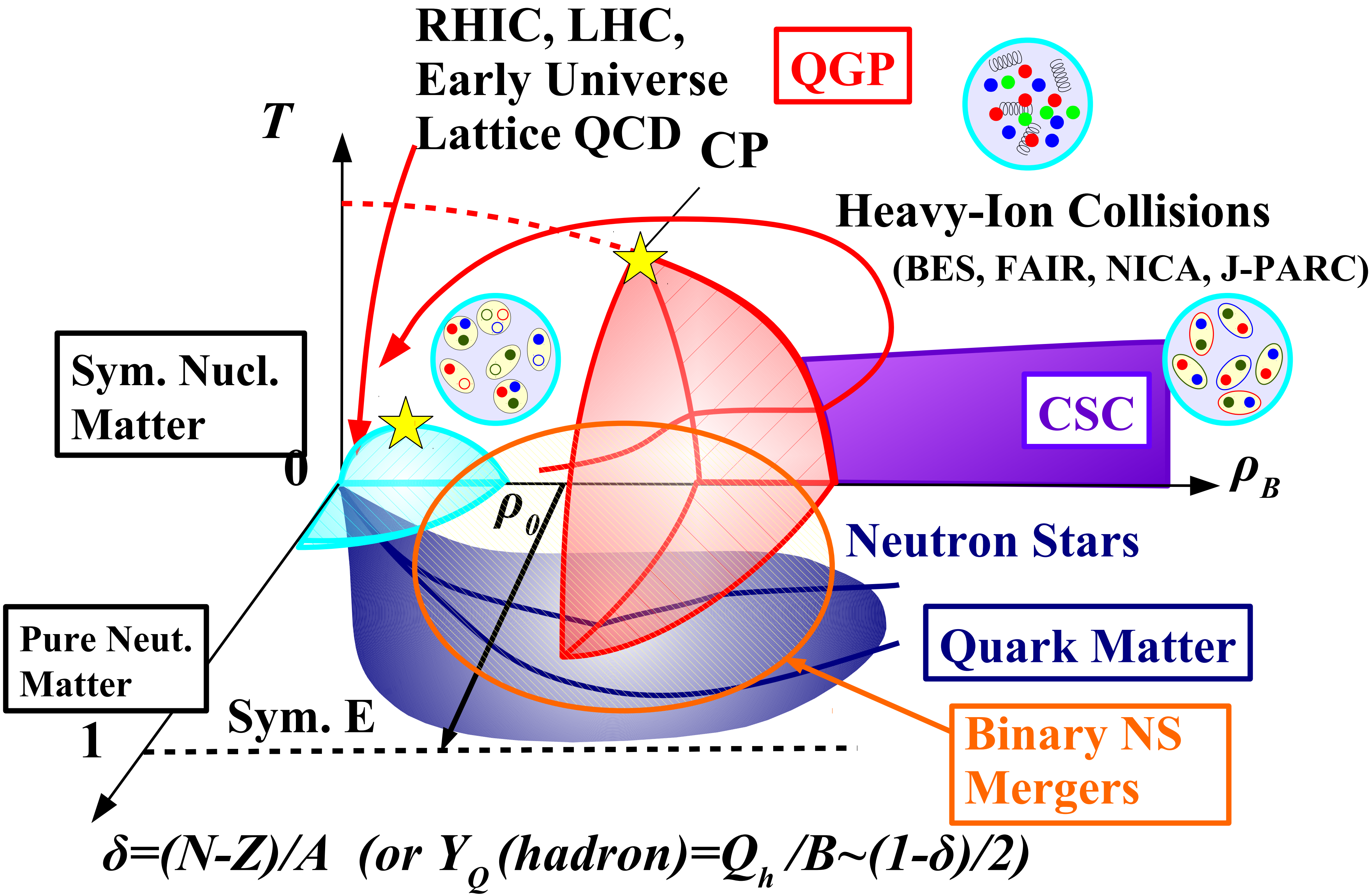
Introduction
Celestial (astronomical) dense QCD matter
Dense QCD matter EOS conjectures

QCD Phase Diagram



AO, 1712.01088

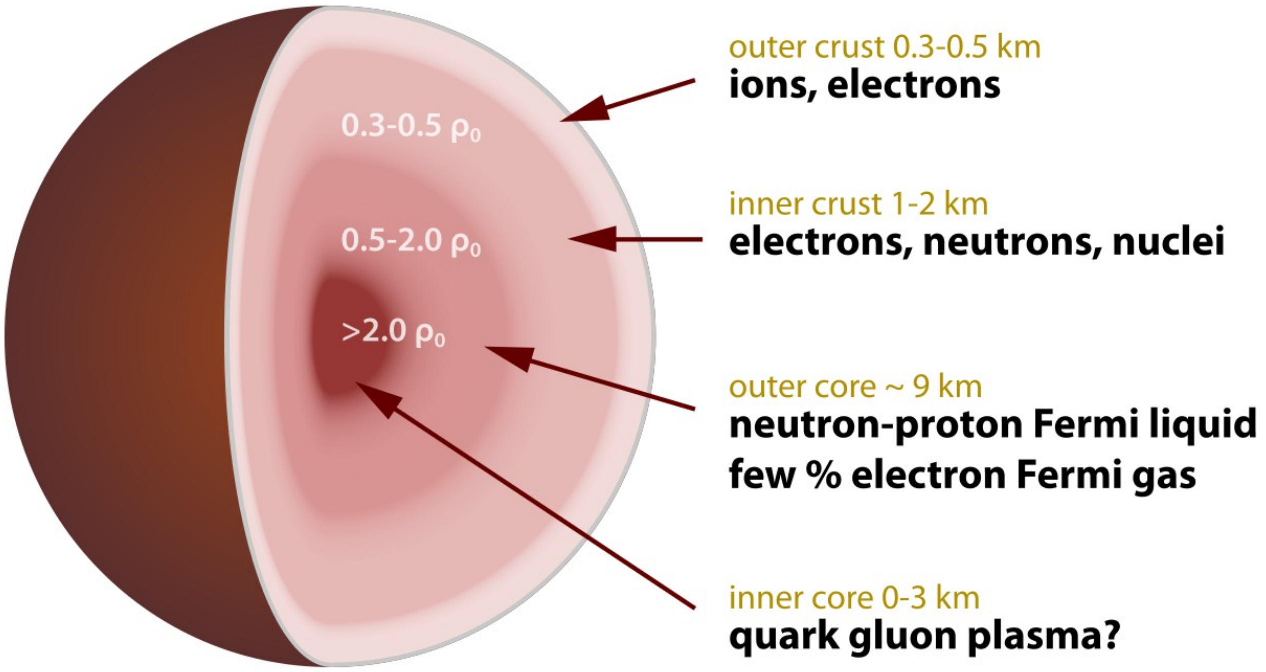
QCD Phase Diagram



AO, 1712.01088

Neutron Stars

- **Central density is expected to reach (2-10)ρ_0**
  - **NS Core is the most-dense stable matter in the universe.**
  - **NS is a clear doorway to dense QCD matter (clearer than HIC).**
  - **By using the Mass-Radius (M-R) curve,
    one can access dense matter information inside neutron stars.**



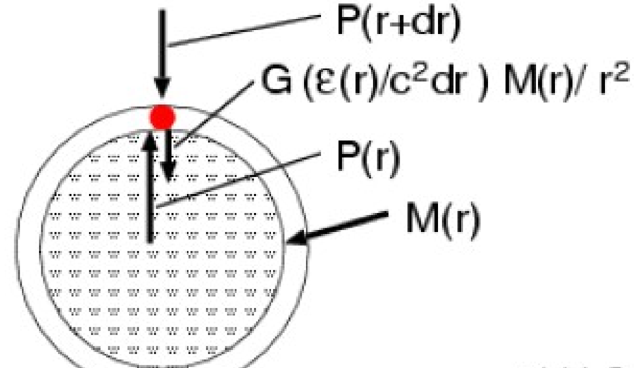
Wikipedia

Mass-Radius (M-R) curve and EOS

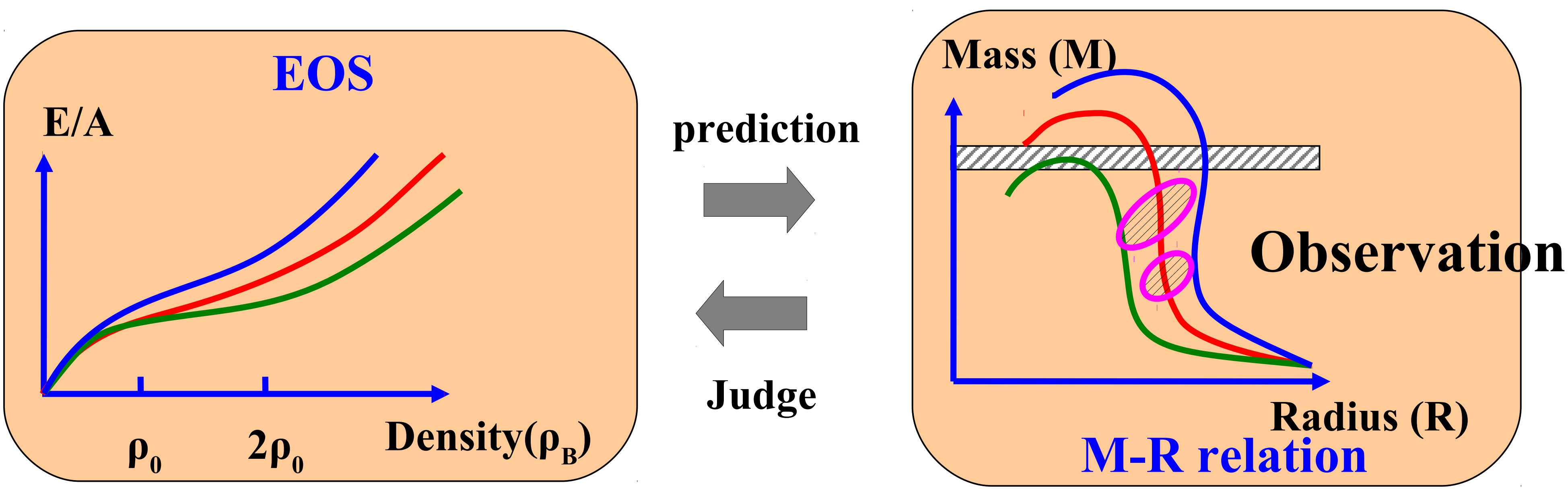
- **M-R curve and NS matter EOS have 1 to 1 correspondence, via the TOV(Tolman-Oppenheimer-Volkoff) equation (=GR Hydrostatic Eq.).**

$$\frac{dP}{dr} = -G\frac{(\varepsilon/c^2+P/c^2)(M+4\pi r^3 P/c^2)}{r^2(1-2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2, \quad P=P(\varepsilon) \quad (\text{EOS})$$



Non-Rela. $\frac{dP}{dr} = -G\frac{\varepsilon(r)/c^2 \; M(r)}{r^2}$



Maximum Mass of Neutron Stars

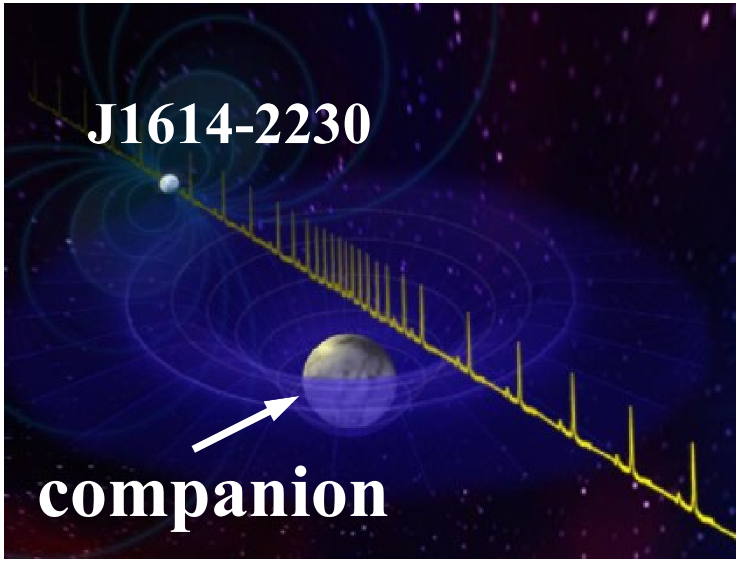
- **Maximum mass > 2 $M_\odot$**
  - **NS mass measured by using Shapiro delay shows the existence of a 2 $M_\odot$ NS kinematically (2010) !**
  - **Many of EOSs before < 2010 were ruled out.**
  - **Hadrons w/ strangeness are difficult to exist in NSs ?**
  - **Further confirmations**

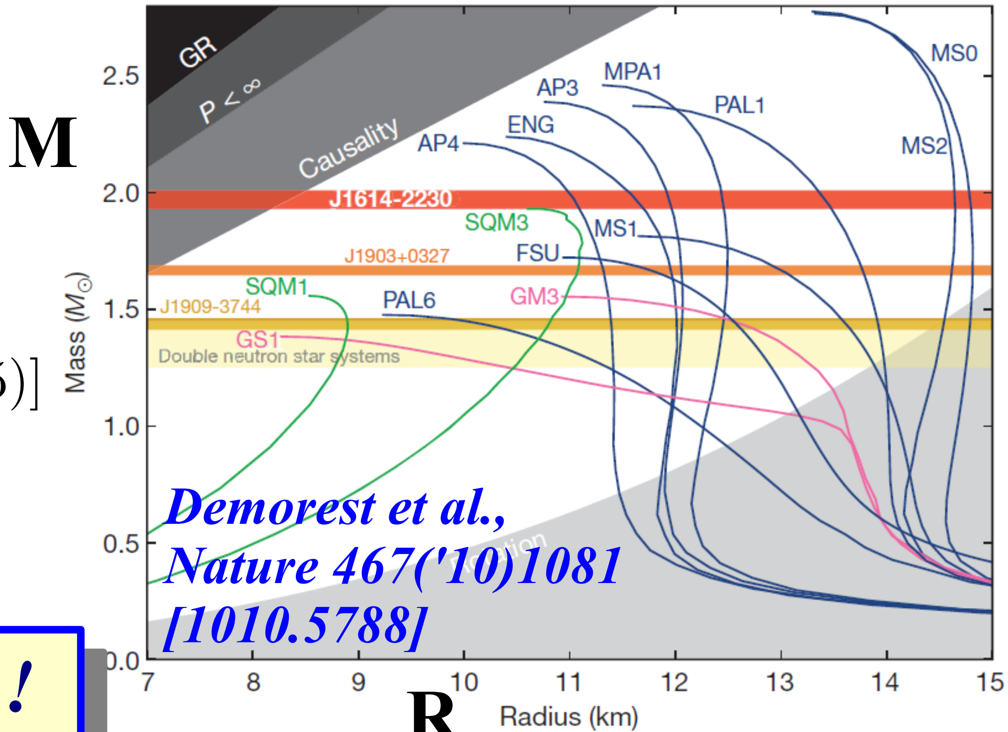
$1.97 \pm 0.04 M_\odot$ (Demorest+,1010.5788)

[$1.928 \pm 0.017 M_\odot$ (Fonceca+,1603.00545)]

$2.01 \pm 0.04 M_\odot$ (Antoniadis+,1304.6875)

$2.27 \pm 0.16 M_\odot$ (Linares+,1805.08799)





Demorest et al., Nature 467('10)1081 [1010.5788]

NS EOS is stiff at high densities !

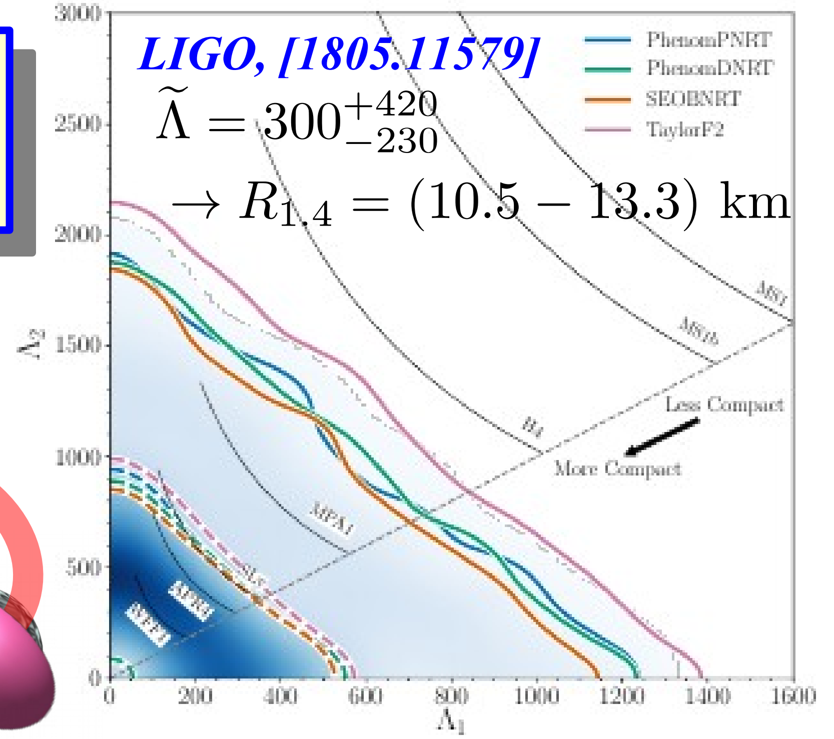
Binary Neutron Star Mergers (BNSM)

- **Gravitational wave from a BNSM event (GW170817)**
  - **NS radius is constrained from the tidal deformability $\bar{\Lambda}$**
    $R_{1.4}$=(10.5-13.3) km *LIGO [1805.11579]*

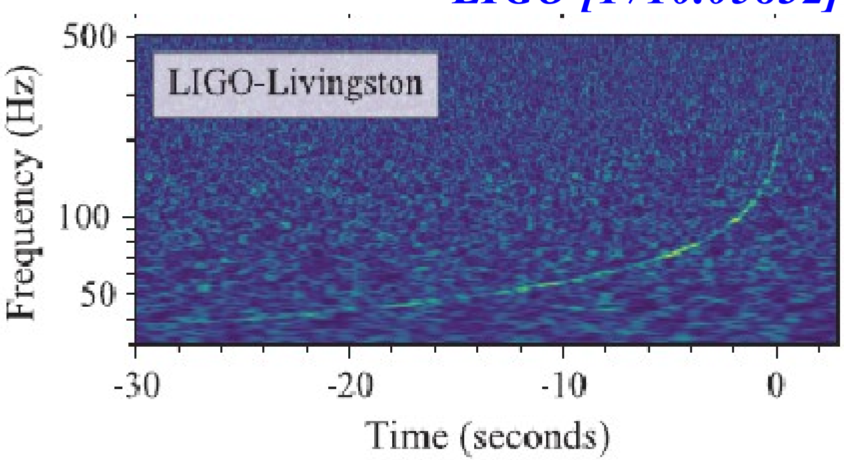
$$R_{1.4} \simeq (11.5 \pm 0.3) \frac{\mathcal{M}}{1.187 M_\odot} \left(\frac{\bar{\Lambda}}{800} \right)^{1/6}$$

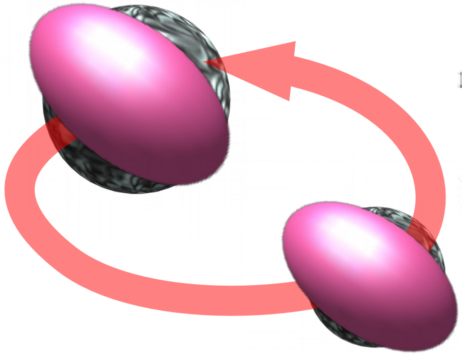
Lattimer ('20)
doi:10.7566/JPSCP.31.011021

NS Radius is relatively small
→ NS EOS needs to be soft at low ρ.



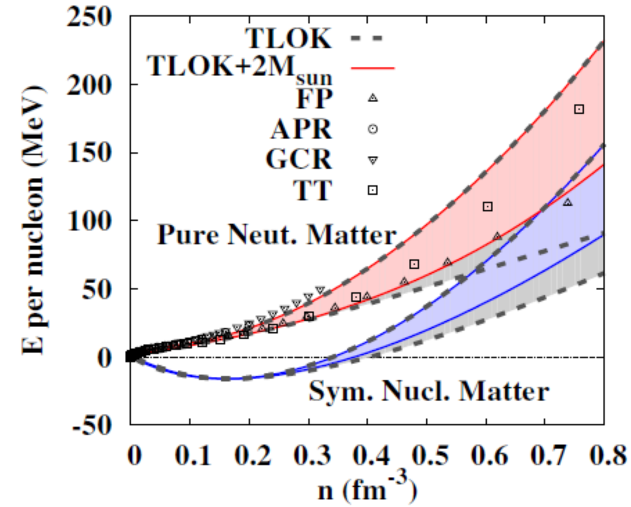
LIGO [1710.05832]





How can we deduce dense matter EOS ?

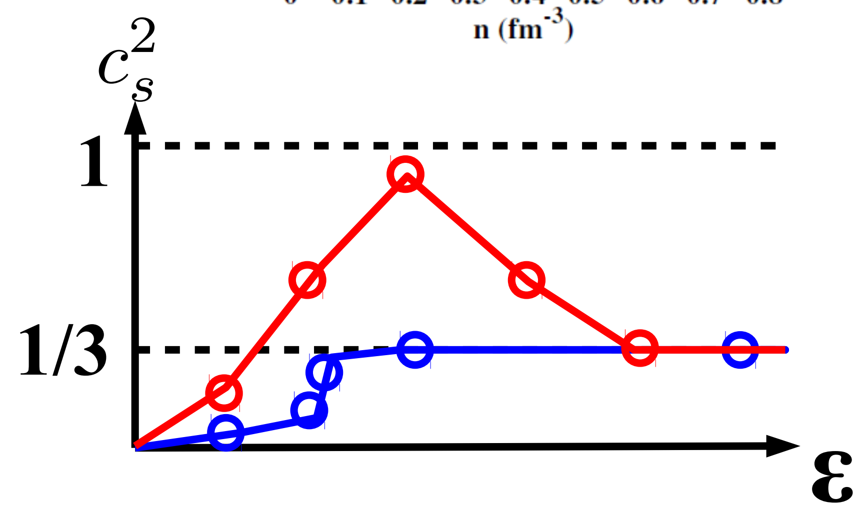
- **Low density EOS ($\rho < 2\rho_0$) is (and will be) constrained by, e.g., Symmetry Energy**
  E.g. Tews, Lattimer, AO, Kolomeitsev, 1611.07133
  - **Naive extrapolation violate causality.**



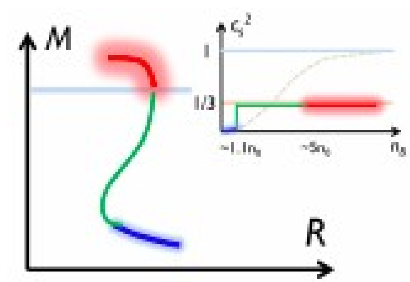
- **Extrapolation / Interpolation**
  - **Speed of sound squared is given by a piecewise linear function of ε.**
    Tews, Margueron, Reddy, 1804.02783; Annala, Gorda, Kurkela, Nättilä, Vuorinen, 1903.09121

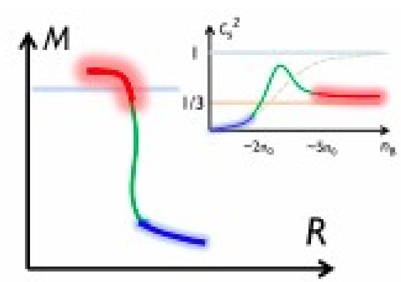
$$c_s^2(\varepsilon) = dp/d\varepsilon \quad (0 \le c_s^2 \le 1)$$

$$p(\varepsilon) = \int_0^\varepsilon d\varepsilon' c_s^2(\varepsilon'),\ n(\varepsilon)\mu(\varepsilon) = \varepsilon + p(\varepsilon)$$



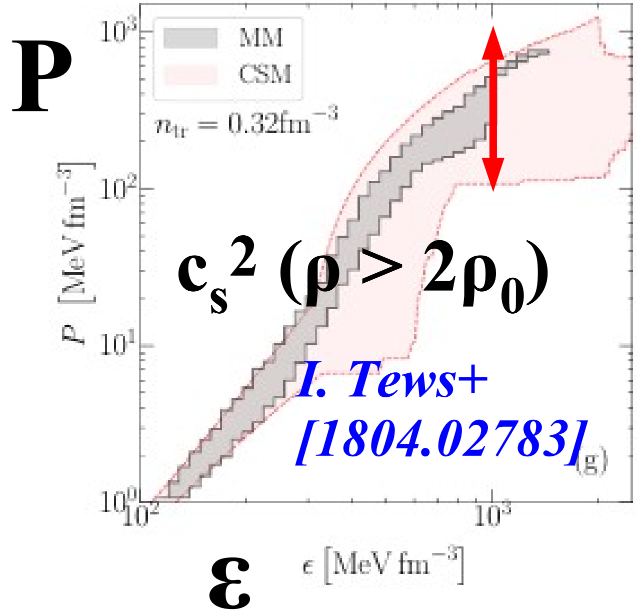
  - **Interpolating the free energies with causality examined.**
    Baym, Furusawa, Hatsuda, Kojo, Togashi, 1903.08963

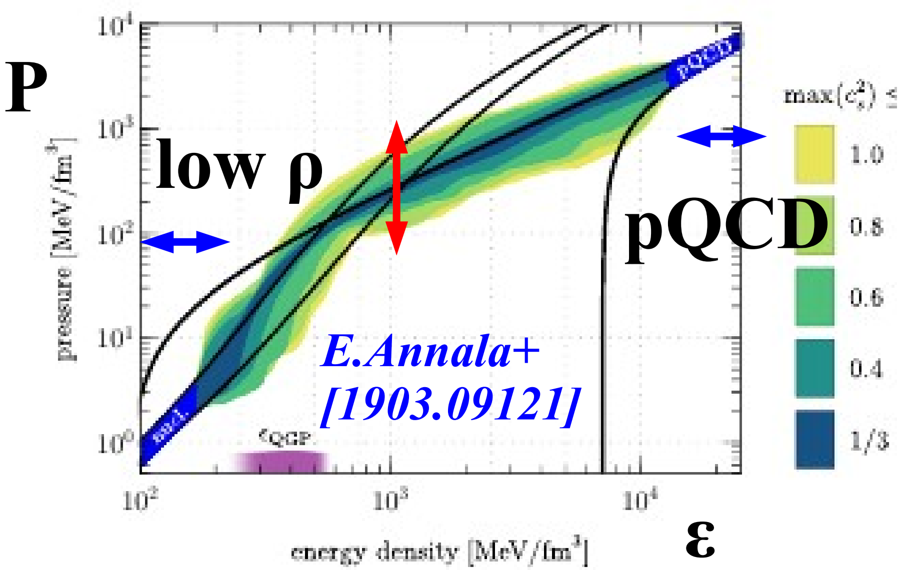


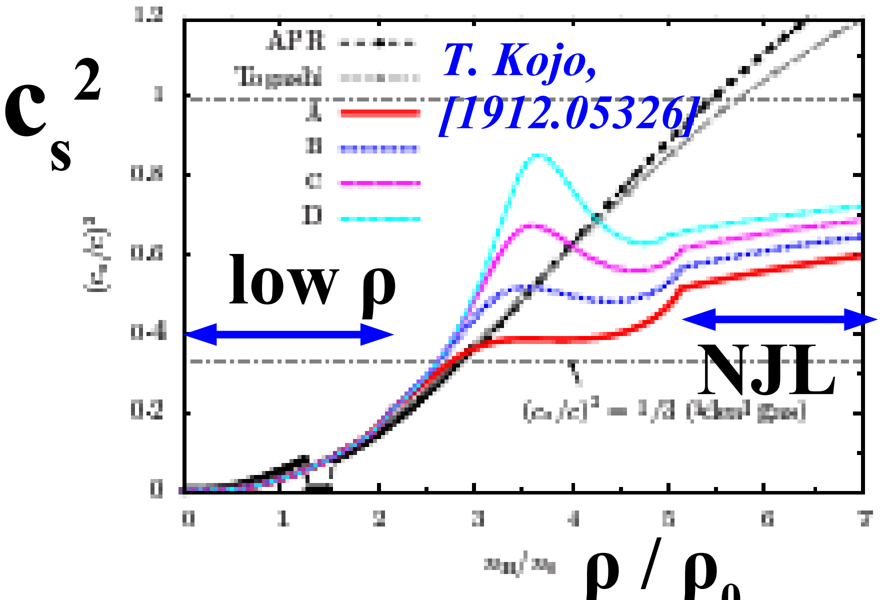


Dense Matter EOS from NS and BNSM

- Low density EOS ($\rho < 2\rho_0$) from nuclear physics inputs.
- High density EOS from extrapolation, Nambu–Jona-Lasino model ($\rho>5\rho_0$), or pQCD ($\rho>100\ \rho_0$)
- Middle density EOS by interpolation







EOS at $2\rho_0<\rho<10\rho_0$ should be given in more reliable methods !

Maximum mass and Radius (at 1.4$M_\odot$) of Neutron Stars implies that NS matter EOS is soft at low densities and stiff at high densities.

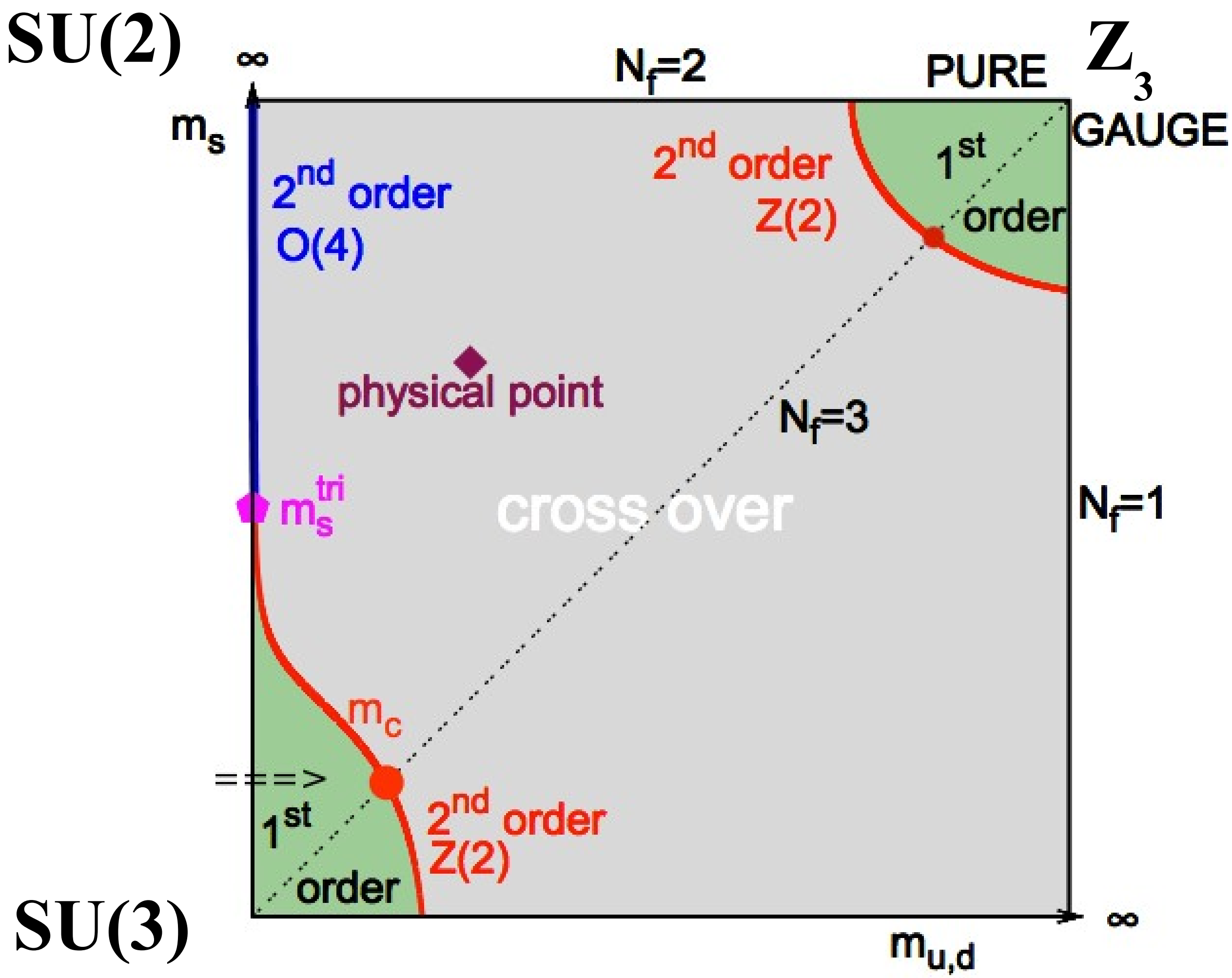
Together with the causality, dense matter EOS seems to be constrained much than before. However, uncertainty is still large. (e.g. factor 5 in pressure at 6 ρ_0)

We need (want) to determine EOS based on model-independent and/or first-principles theories, or terrestrial (laboratory) experimental data obtained from actually formed dense matter.

Model-independent and/or first principles approaches to dense QCD

Another "phase" diagram (Columbia plot)

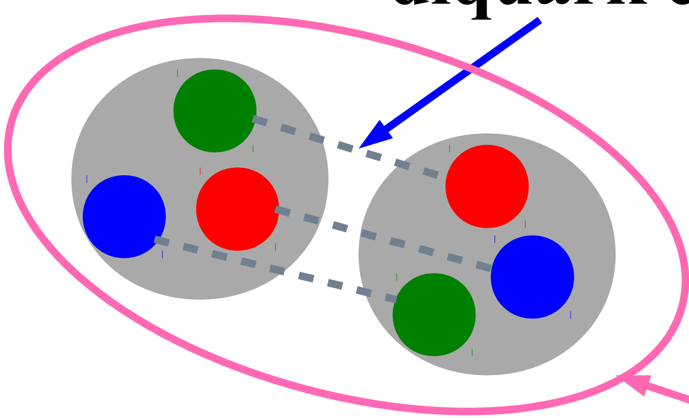
- **Order of the phase transition at zero baryon density (finite T) is strongly constrained by symmetry.**

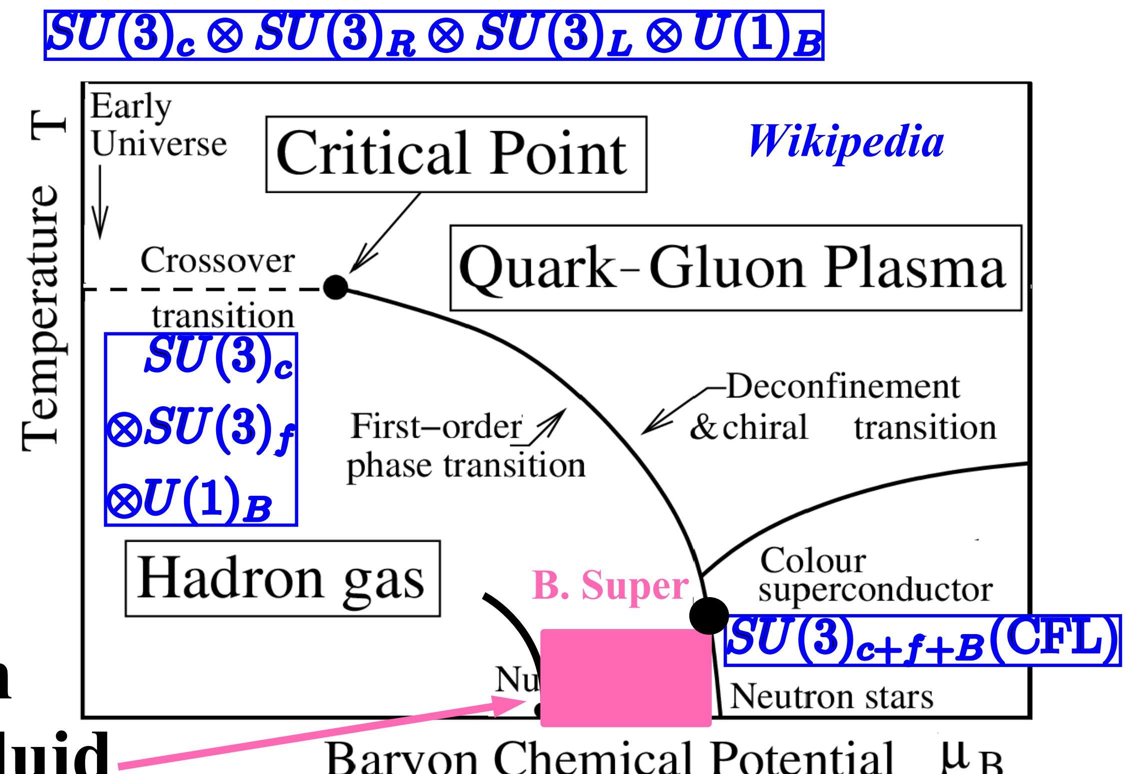


P. de Forcrand, M. D'Elia [1702.00330]

QCD phase diagram from symmetry

- **QCD phase transition at zero baryon density is Crossover.**
  Pisarski-Wilczek ('84), Y.Aoki+(hep-lat/0611014)
- **QCD phase transition at T=0 (finite ρ) is first order or crossover.**
  - **NJL model suggests the first order chiral phase transition.**
    Nambu, Jona-Lasinio ('61), Asakawa, Yazaki ('89), e.g. Hatsuda, Kunihiro ('94)
  - **Transition from baryon superfluid to color superconductor (CSC) may be crossover (Quark-Hadron continuity)**
    Schafer, Wilczek ('99), N.Yamamoto+('07)



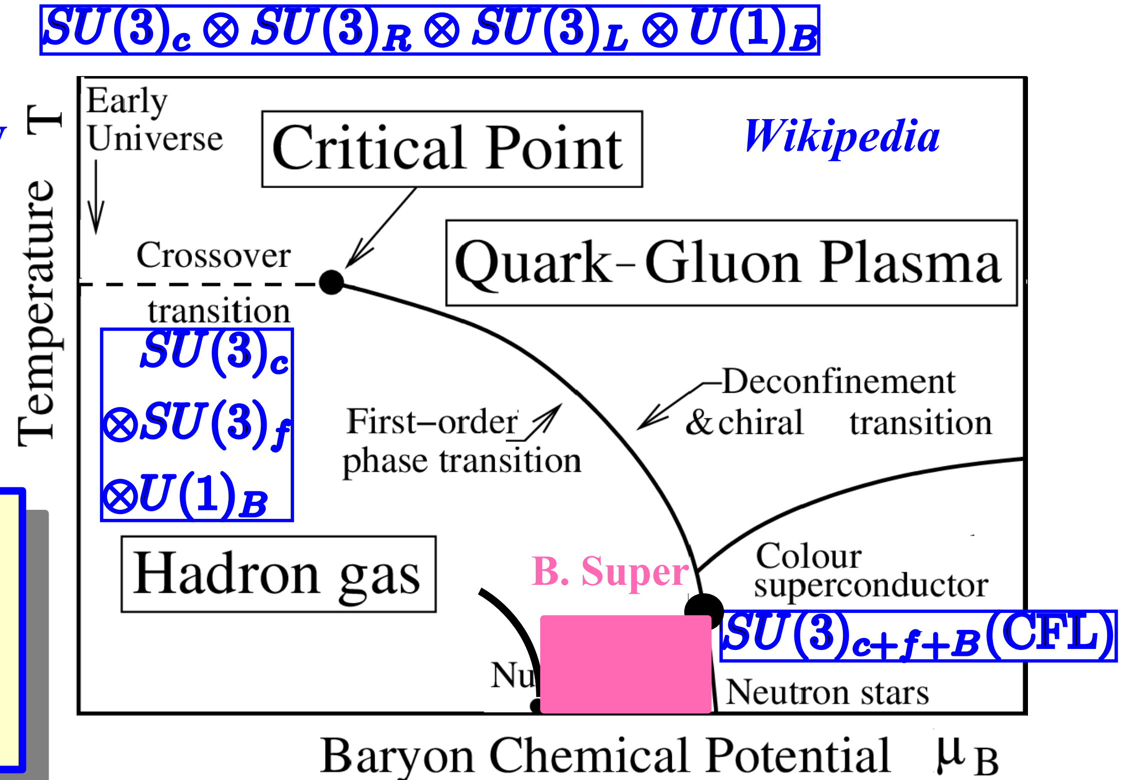


QCD phase diagram from topology (→ Yesterday's talks)

- **Standard symmetry argument is not enough to decide the order of the finite density QCD phase transition.**
  → How about the topology ?

- **Local order parameters cannot capture topological order.**
  X.G.Wen('90); Gaiotto+('15); Yesterday (Hidaka, Shiozaki, Yokokura, Tiwari, ...)

  - **Emergent Z_3 two-form symmetry in the Color-Flavor-Locking (CFL) phase cannot be spontaneously broken.**
    → Quark-hadron continuity is a consistent scenario as a quantum phases of matter.
    Hirono, Tanizaki ('19, '19)



QCD phase transition at finite density can still be crossover.

We cannot distinguish at present baryon superfluidity and color superconductor by local order parameters or topological order.

Thus to determine the order of finite density QCD transition, explicit (hopefully) first-principles calculations or experiments directly producing dense matter is necessary.

Lattice QCD approaches to dense matter

- **Lattice QCD is the first-principles non-perturbative framework.**
  - **But there exists the sign problem at finite density.**
    Fermion det. is complex at finite density
    → Strong cancellation of the Boltzmann weight at large volume
    It is difficult to study dense matter using LQCD

$$\det D(\mu) = (\det D(-\mu^*))^* \to S_{\text{eff}} = S - \log\det D \in \mathbb{C}$$

$$\mathcal{Z} = \int \mathcal{D}x \, \exp(-S(x)), \; \mathcal{Z}_{\text{pq}} = \int \mathcal{D}x \, |\exp(-S(x))|$$

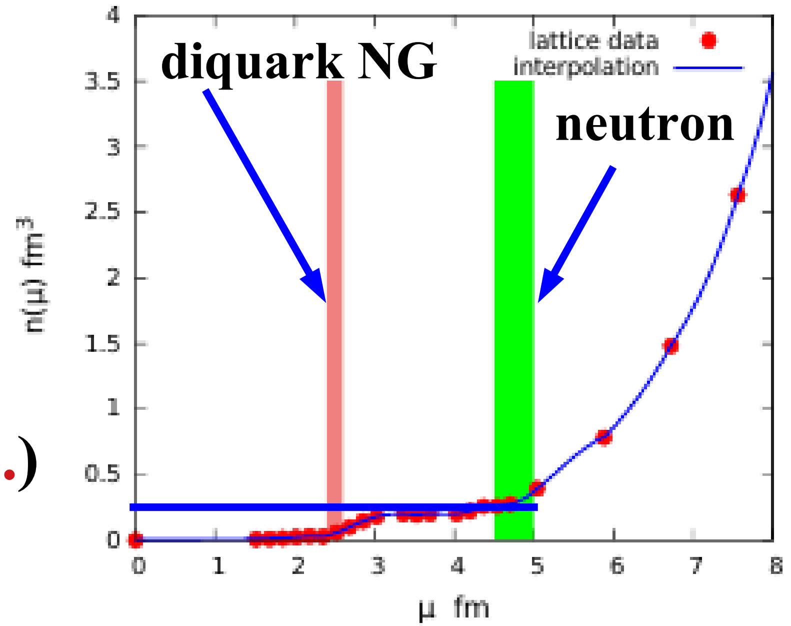
$$\text{APF} = \langle e^{i\theta} \rangle = \mathcal{Z}/\mathcal{Z}_{\text{pq}} \to 0 \quad (V \to \infty)$$

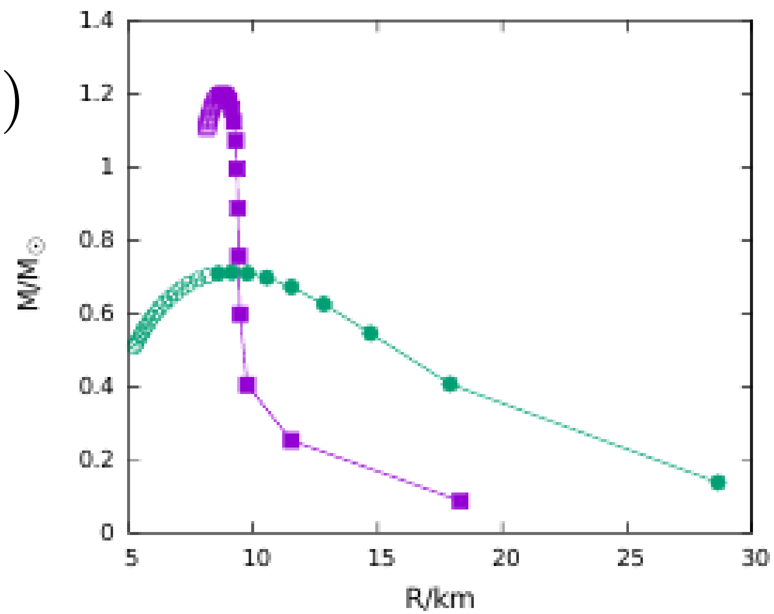
  - **Standard approaches (Reweighting, Taylor exp., Imag. μ, ..)**
    → Useful at μ/T < 1, but not enough to discuss dense QCD matter
  - **A New Hope: Complexified variable methods**
    (Complex Langevin method, Lefschetz thimble method, Path optimization method, ...)

"Successful" LQCD calc. of dense QCD

- G_2 QCD *O. Hajizadeh, A. Maas, 1702.08724*
  - Exceptional Lie group G_2
  - Subgroup of SO(7) (baryons=fermion) 7 (14) quark (gluon) colors
  - Representations are real. (No Sign prob.)
  - EOS from G_2 QCD (T ~ 0) → Neutron Star MR curve

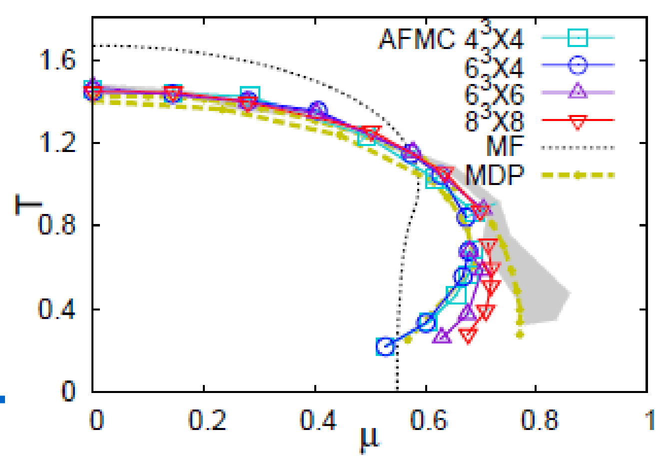
$$p(\mu) = \int_{\mu_0}^{\mu} d\mu' n(\mu'), \varepsilon(\mu) = n(\mu)\mu - p(\mu)$$





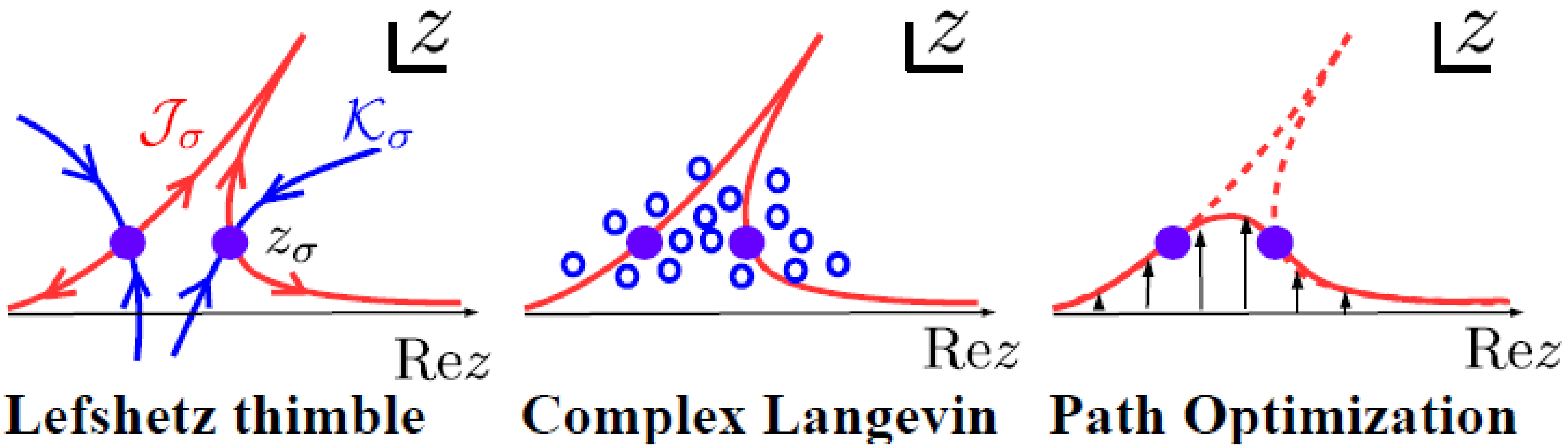
- Strong coupling limit

de Forcrand, Fromm ('10), T.Ichihara, AO, T.Z.Nakano, PTEP2014,123D02.



Complexified Variable Methods

- **Complex Langevin method**
  Parisi-Wu('81), Klauder('83), Aarts+('11), Nagata+('16), Seiler+('13), Ito+('16).
- **Lefschetz thimble method**
  E. Witten ('10), Cristoforetti+('12),Fujii+('13), Alexandru+('16).
- **Path Optimization method**
  Mori+(1705.05605, 1709.03208, 1904.11140, 2007.04167);
  Kashiwa+ (1903.03679, 1805.08940); Alexandru+(1804.00697, 1807.02027),
  Bursa, Kroyter (1805.04941)



Complex Langevin method (→ Nishimura's talk)

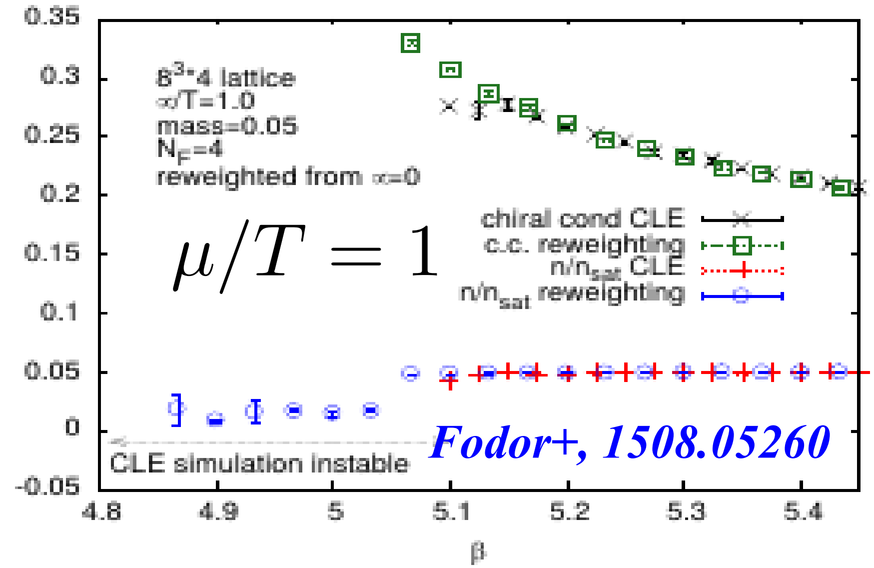
Parisi ('83), Klauder ('83), Aarts et al. ('11), Nagata et al. ('16); Seiler et al. ('13), Ito et al. ('16, …, '20)

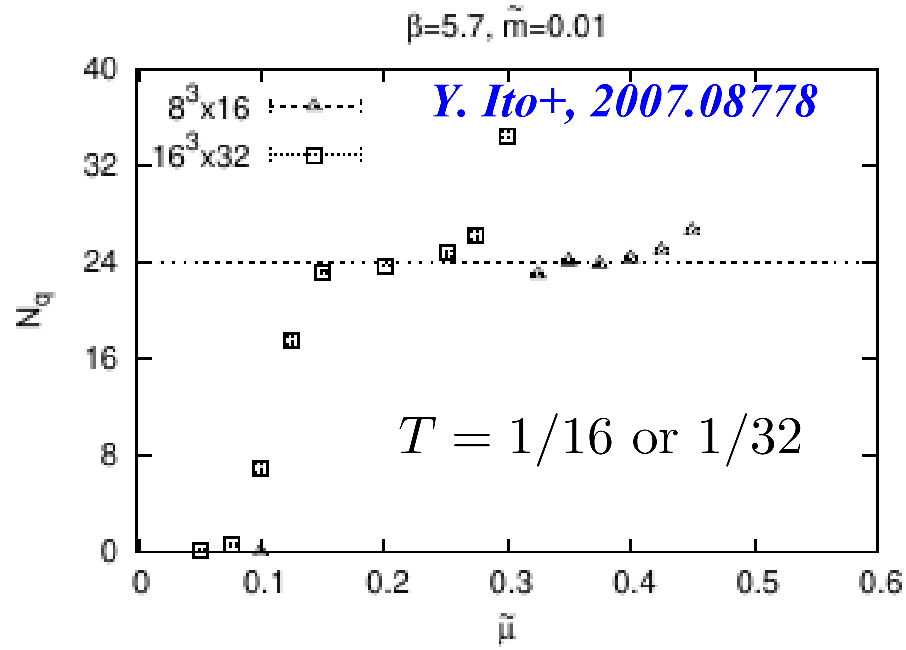
- **Configurations are obtained by solving the complex Langevin eq. (no sign prob.).**

$$\frac{dz_i}{dt} = -\frac{\partial S}{\partial z_i} + \eta_i(t) (\eta_i : \text{White noize})$$

$$\langle \mathcal{O}(x) \rangle = \langle \mathcal{O}(z) \rangle$$

- **Most successful LQCD approach to dense QCD matter (at present)**
  - **At high T ($\beta=2N_c/g^2>5.1$), CLM and reweighting results agree.**
  - **"Fermi sphere" is observed at low T**
- **Problem: CLM can give converged but wrong results under some thermodynamic conditions (μ, T).**





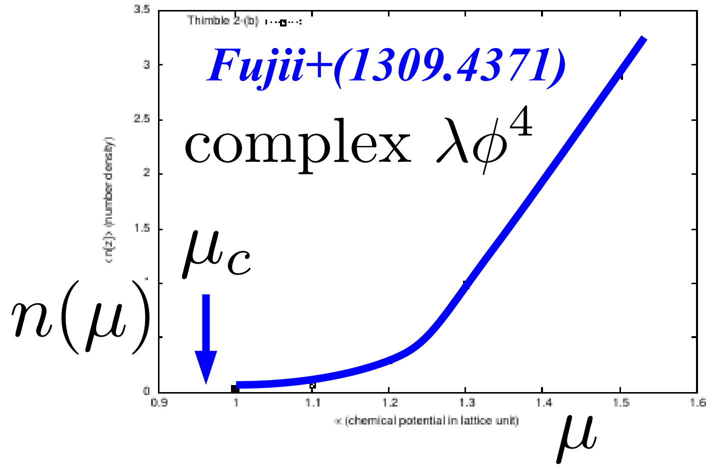
Lefschetz thimble method

E. Witten (1001.2933), Cristoforetti+ (1205.3996), Fujii+ (1309.4371), Alexandru+ (1512.08764).

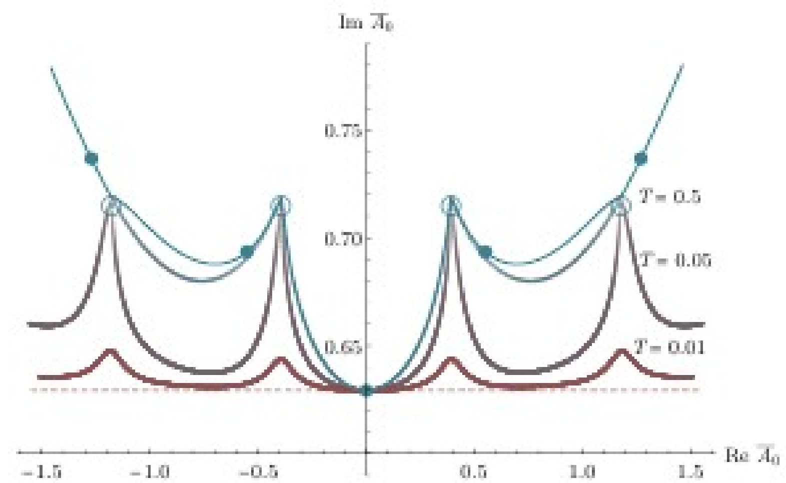
- **Solving the flow eq. from a fixed point σ provides integration path w/ const. Im(S).**

$$\mathcal{J}_\sigma : \frac{dz_i(t)}{dt} = \overline{\left(\frac{\partial S}{\partial z_i}\right)} \to \frac{dS}{dt} = \sum_i \left|\frac{\partial S}{\partial z_i}\right|^2 \in \mathbb{R}$$

$$\mathcal{C} = \sum_\sigma n_\sigma \mathcal{J}_\sigma$$



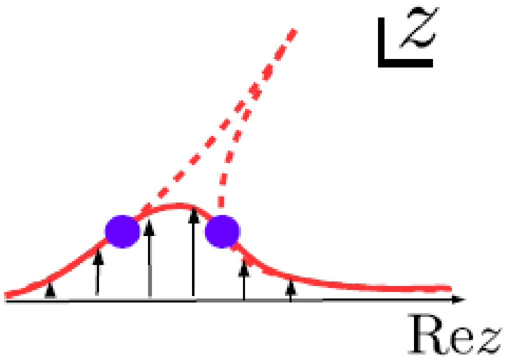
- **Thimbles are approximately obtained by solving the flow eq. to some fictitious time (Generalized LTM).**
- **Problem:**
  - **Phase from the Jacobian, Different Phases of Multi-thimbles (residual and global sign pr.)**
  - **Degeneracy from gauge dof needs to be removed.**



Alexandru+,1512.08764

Path optimization method

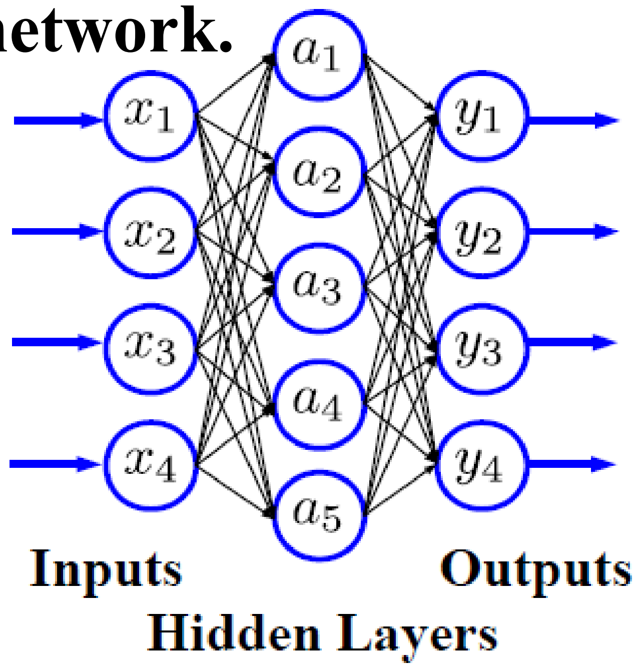
Mori+(1705.05605, 1709.03208, 1904.11140, 2007.04167); Kashiwa+ (1903.03679, 1805.08940); Alexandru+(1804.00697, 1807.02027), Bursa, Kroyter (1805.04941)



- **Partition function is independent of the path, if**
  - **the Boltzmann weight W=exp(-S) is holomorphic,**
  - **and the path does not go across the poles and cuts of W.**
    (det D=0 → Singular point of S_{eff}, Zero point of W)

- **Integration path is optimized to evade the sign problem, using the parameterized function or neural network.**
  Cost function:

$$\mathcal{F}[z(x)] = \mathcal{Z}_{\rm pq} - |\mathcal{Z}| = |\mathcal{Z}| \left(\mathrm{APF}^{-1} - 1\right)$$



Sign Problem → Optimization Problem

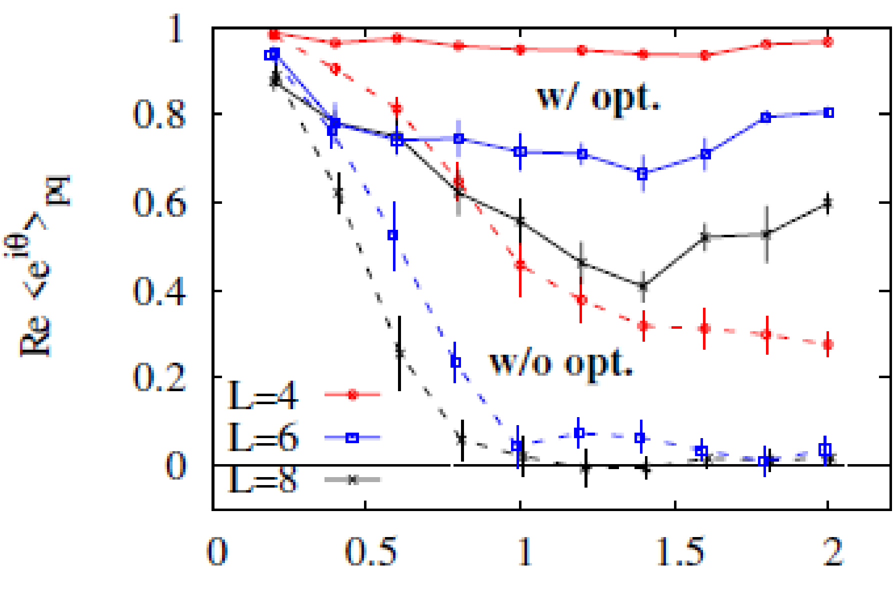
1+1 dim. Complex φ⁴ theory at finite μ

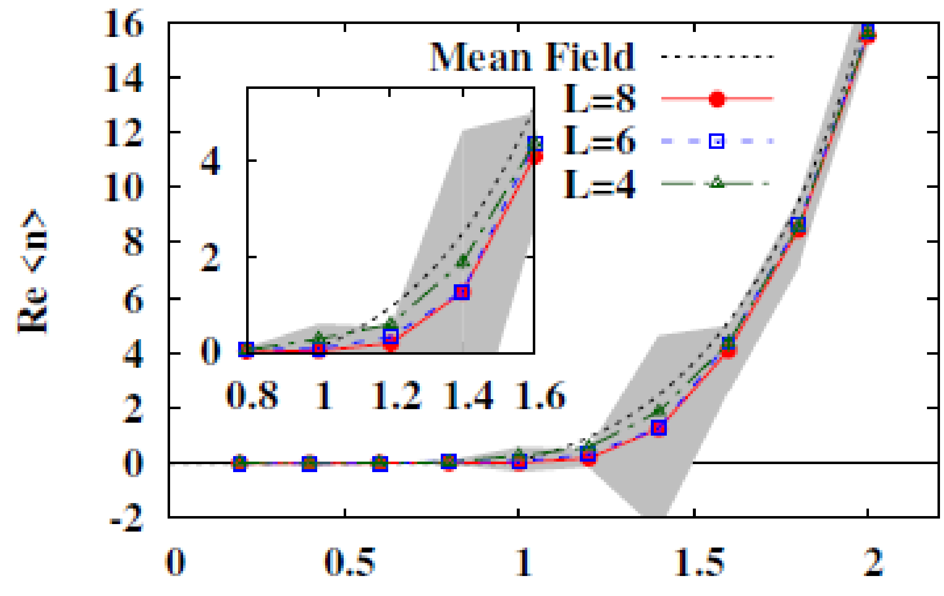
- **Complex φ⁴ theory** $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda(\phi^* \phi)^2$
- **Action on Eucledean lattice at finite μ.**
  G. Aarts, PRL102('09)131601; H. Fujii, et al., JHEP 1310 (2013) 147.

$$S = \sum_x \left[\frac{(4+m^2)}{2} \phi_{a,x}\phi_{a,x} + \frac{\lambda}{4}(\phi_{a,x}\phi_{a,x})^2 - \phi_{a,x}\phi_{a,x+\hat{1}} - \cosh\mu\, \phi_{a,x}\phi_{a,x+\hat{0}} \right.$$

$$\left. + i\epsilon_{ab} \sinh\mu\, \phi_{a,x}\phi_{b,x+\hat{0}} \right] \quad \left(\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \right)$$

complex **Complexify**





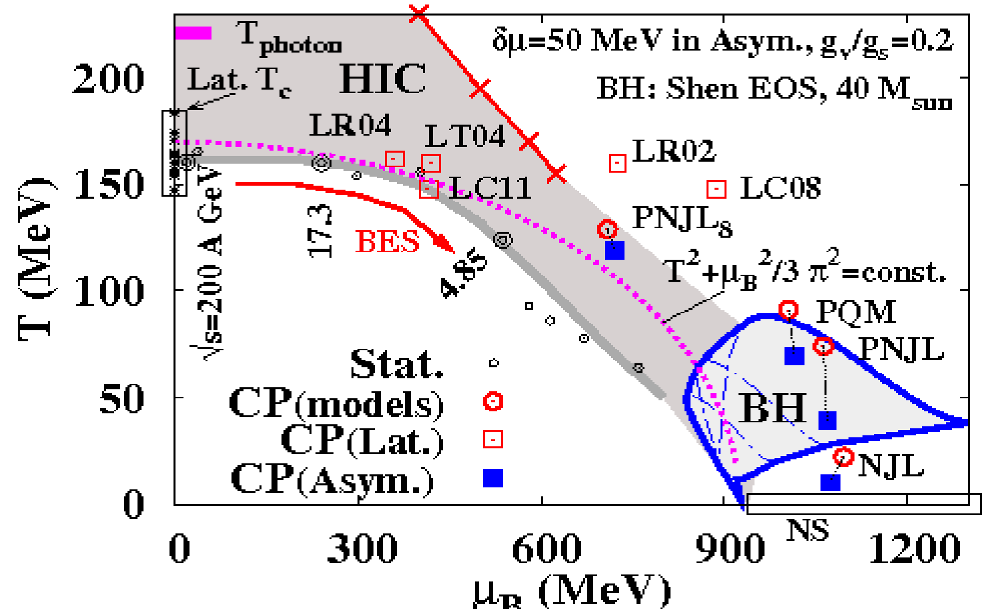
Y. Mori, K. Kashiwa, AO, PTEP 2018 ('18), 023B04 [arXiv:1709.03208]

We (theoretical physicists) have hopes and do not give up, but we may need more time to conclude.

Experimental data should show the exact results of QCD, while interpretation needs theoretical analysis. Is there any possibility to produce dense (~10 ρ_0) QCD matter in laboratory ?

Hints from Current Experimental Data (1)

- **At lower incident energies than RHIC, chemical freeze-out point moves to higher μ_B.**
  → Dense QCD matter may be formed at colliding energies of $\sqrt{s_{NN}}$=(5-20) GeV



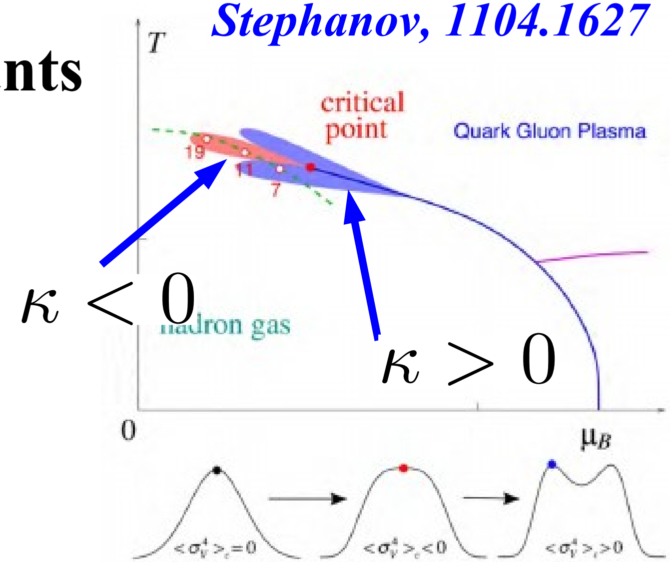
- **In the above colliding energy region, some of the observables show non-monotonic behavior.**
  - **Proton number cumulants (critical point signal?)**

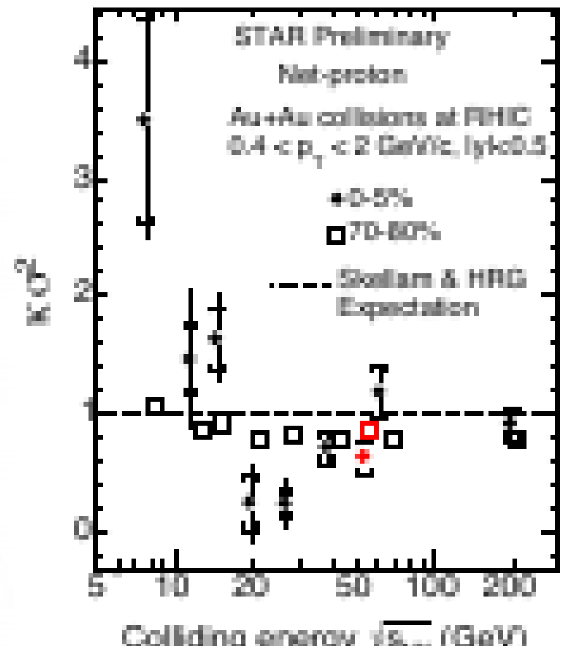
$$\kappa\sigma^2 = C_4/C_2$$

$$C_k = \langle (N_p - \langle N_p \rangle)^k \rangle$$

  - **Directed flow**

Stephanov, 1104.1627

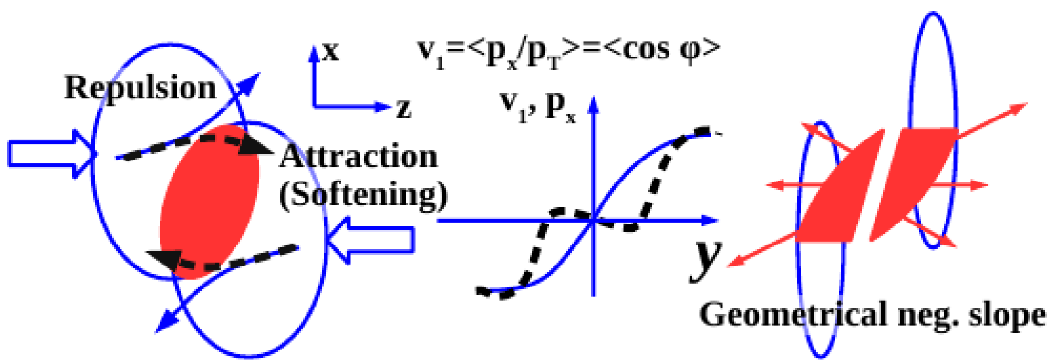


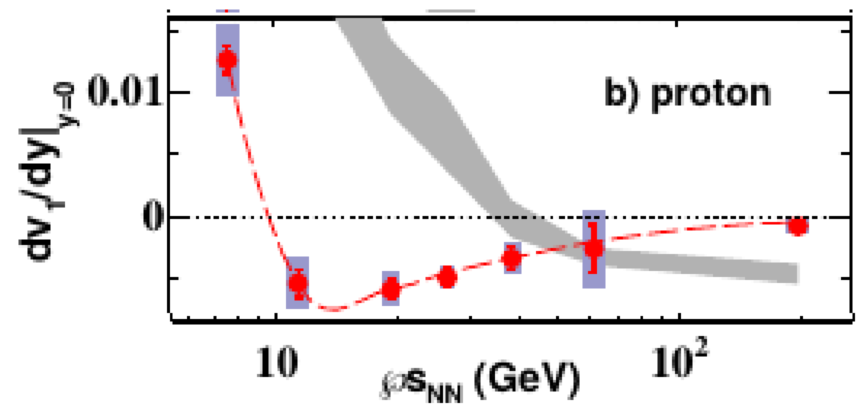


STAR, 2003.12503 (1309.5681)

Hints from Current Experimental Data (2)

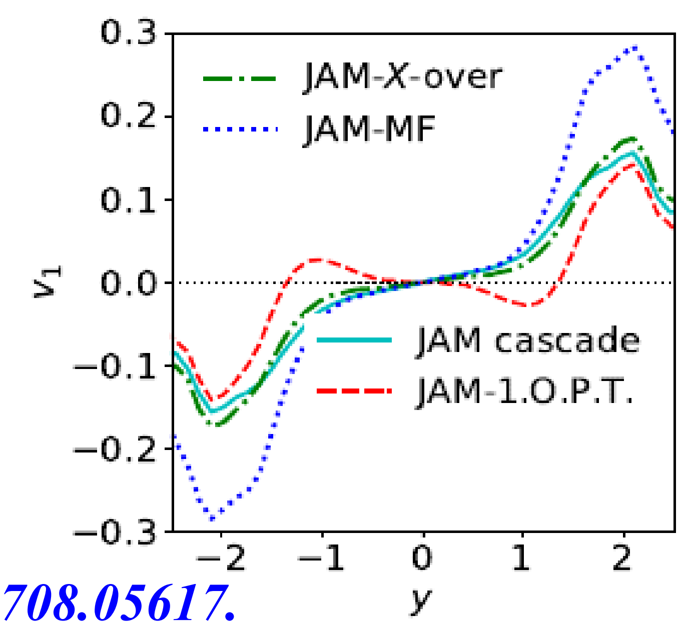
- **Directed flow (v_1, $<p_x>$) is sensitive to dense matter EOS. RHIC data at $\sqrt{s_{NN}}$=10 GeV show negative v_1 of protons.**





STAR, 1401.3043

- **The negative flow implies softening of EOS (Transport model favors 1st order p.t. but three fluid model favors crossover.)**



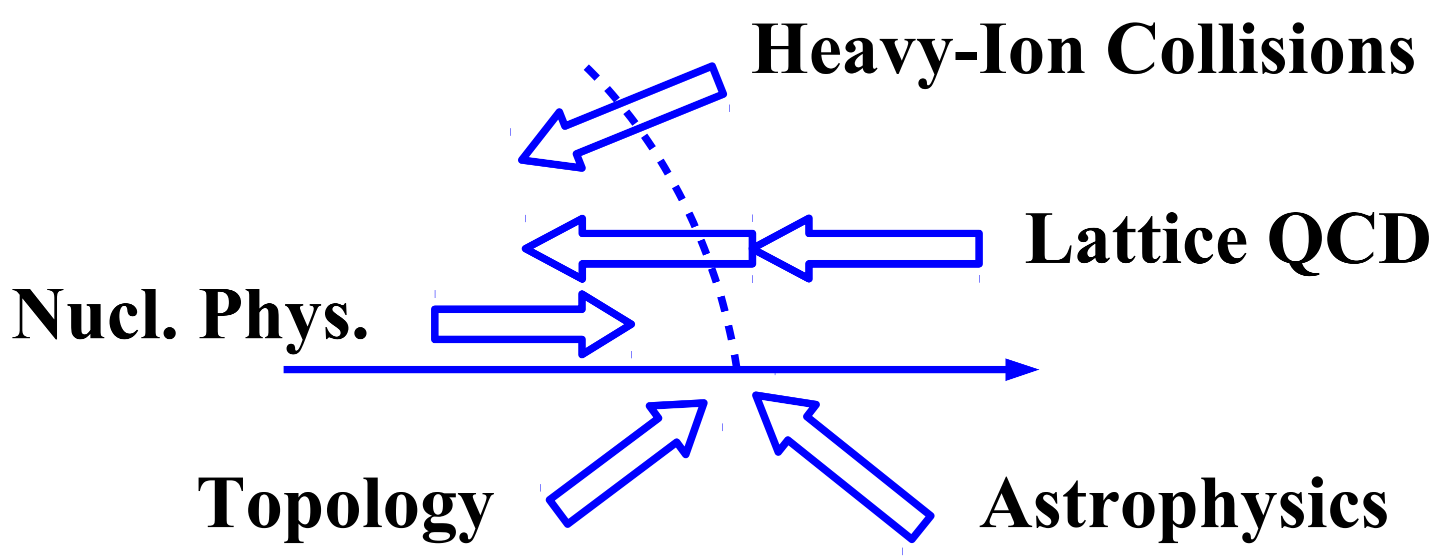
Y.Nara+, 1601.07692, 1708.05617.

Summary

- Dense (QCD) matter is formed in the core region of neutron stars (NSs) and during binary neutron star mergers (BNSMs).
  - Dense matter equation of state (EOS) is NECESSARY to describe NS profile and to predict dynamics of BNSM. (Namely, to understand the sites of r-process nucleosynthesis.)
  - However, there is a large uncertainty in the EOS at "middle" densities ($\rho=(2\text{-}10)\rho_0$).
- Some of recent developments in model-independent and/or first-principles approaches to the order of finite density QCD phase transition are reviewed.
  - Symmetry and topology arguments provide global picture of the phase diagram. Quark-hadron continuity picture survives.
  - Several methods in finite density lattice QCD are being developed. Complex Langevin method gives observables in cold-dense matter in some parameter region.
  - Heavy-ion collisions data at colliding energies of $\sqrt{s_{NN}}$=(5-20) GeV may be suggesting the nature of phase transition of dense QCD matter.

Summary (cont.)

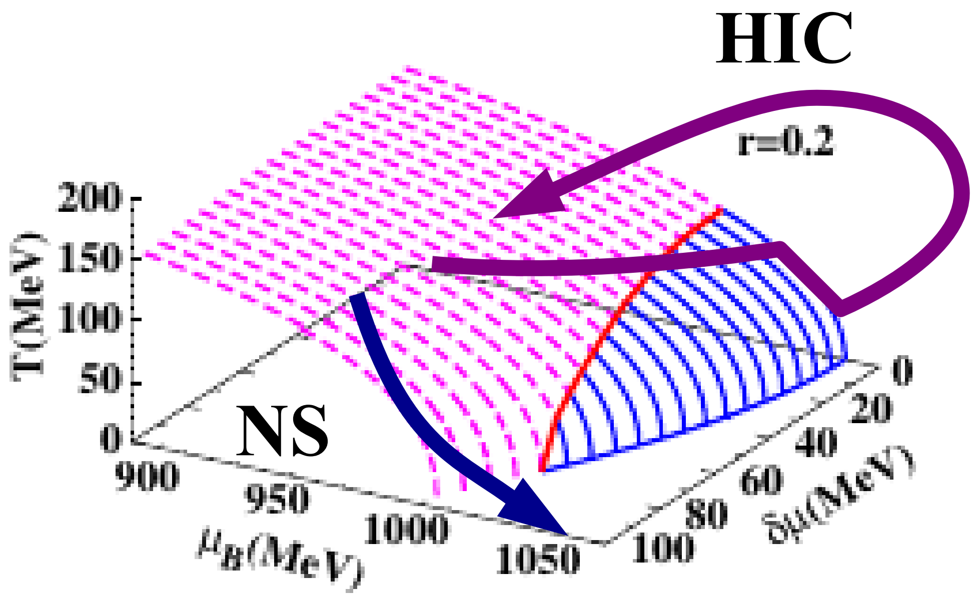
- **Why is it so difficult to determine dense matter EOS ?**
  - **30 years ago, the order of finite T QCD phase transition changed every year ! (First order or crossover from symmetry arguments) Pisarski-Wilczek ('84) → Crossover from LQCD ('06) → EOS ('14)**
  - **It may take 30 years from the conjecture based on symmetry. Schafer-Wilczek ('99) → Order of transition (??) → EOS (??)**
  - **But this time, we have more approaches !**



  - **Even if with large uncertainties, LQCD results are definitely useful.**

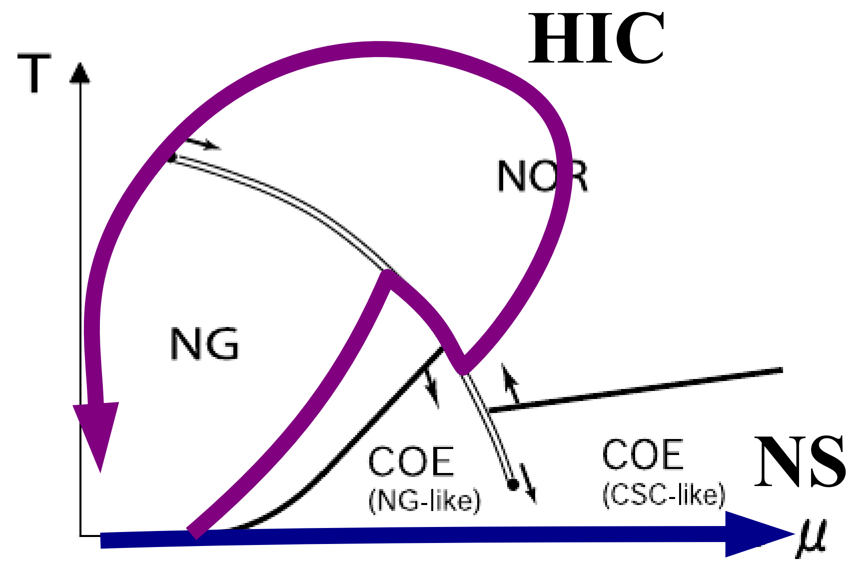
A Premature Conjecture

- Nishimura-san told me "Please say something interesting".
- If phase transition is crossover in NS and 1st order in HIC, the phase diagram may be conjectured to be one of the followings.



Isospin chemical potential effects change the order.

H.Ueda, T.Z.Nakano, AO, M.Ruggieri, K.Sumiyoshi, PRD88('13), 074006 [1304.4331]; AO, H.Ueda, T.Z. Nakano, M.Ruggieri, K.Sumiyoshi, PLB704('11),284 [1102.3753]



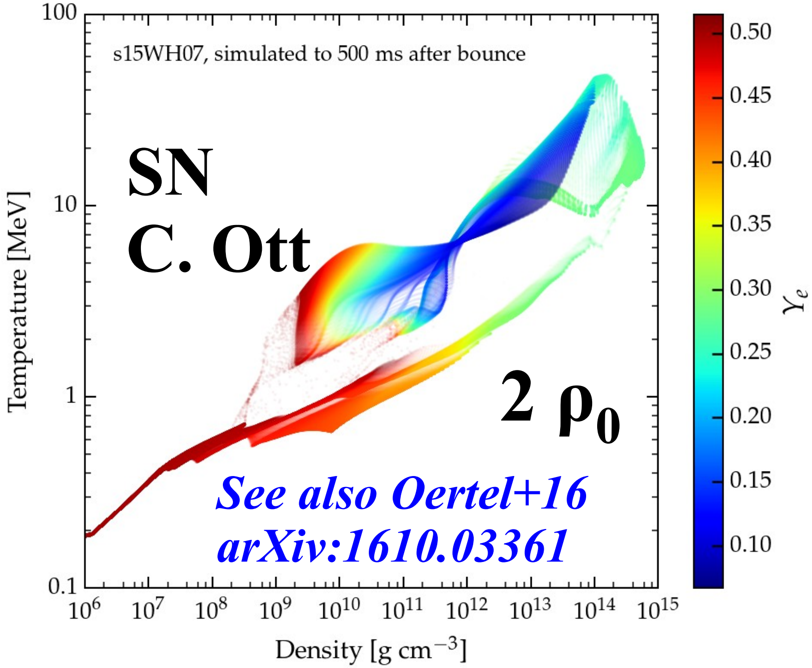
There are two CPs, and T effects change the order.

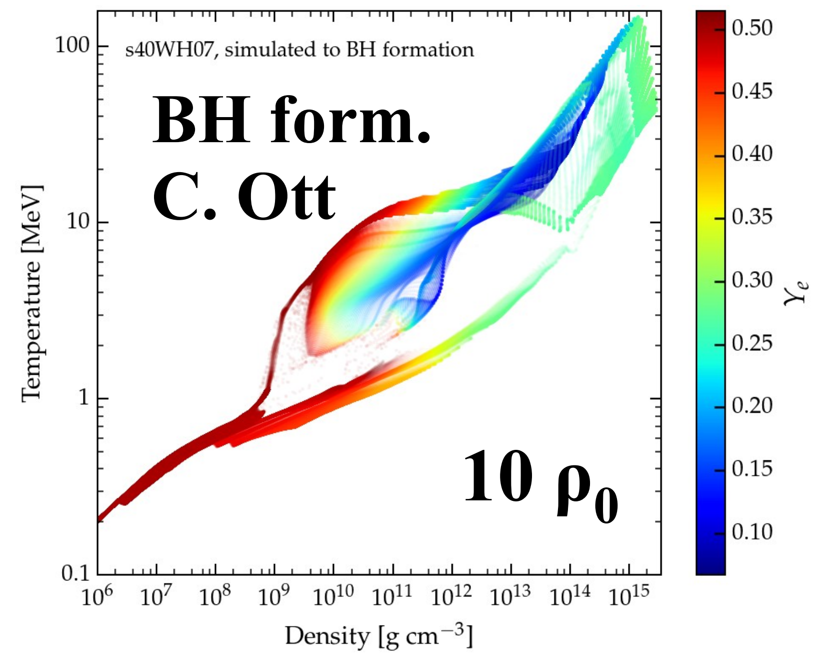
M.Kitazawa, T.Koide, T.Kunihiro, Y.Nemoto, PTP108('02)929 [hep-ph/0207255]; T.Hatsuda, M.Tachibana, N.Yamamoto, G. Baym, PRL97('06) 122001 [hep-ph/0605018]; N.Yamamoto, M.Tachibana,T.Hatsuda,G.Baym, PRD76('07) 074001 [0704.2654]

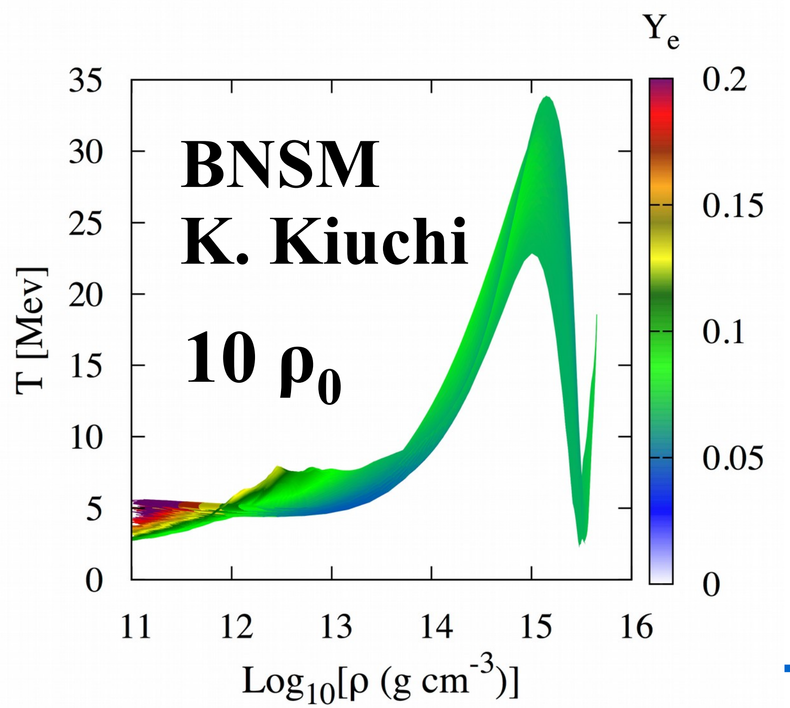
Thank you for your attention !

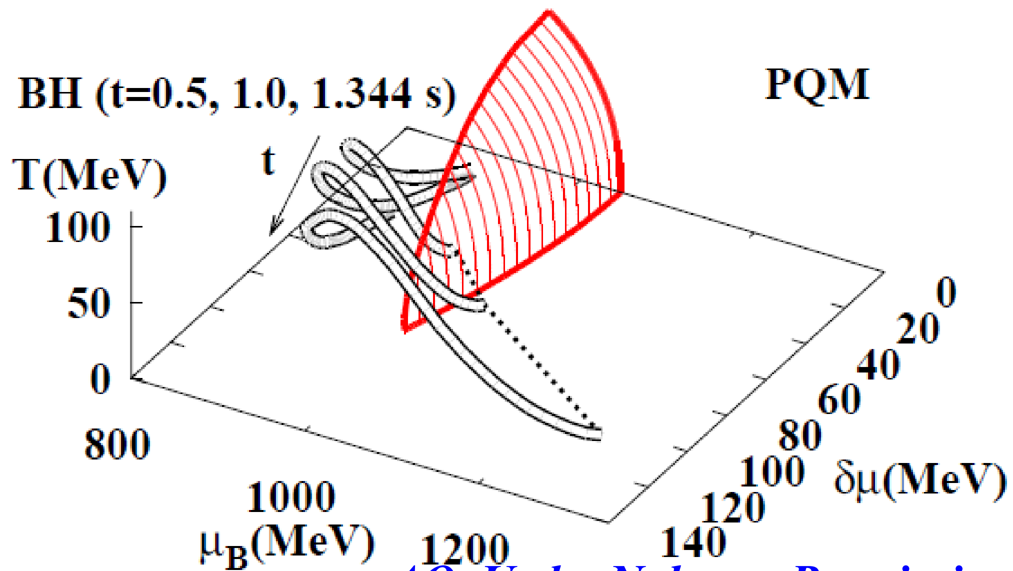
I could not mention many of the works related to dense QCD matter, and feel sorry for the authors of those works.

(ρ, T, Y_e) during SN, BH formation, BNSM





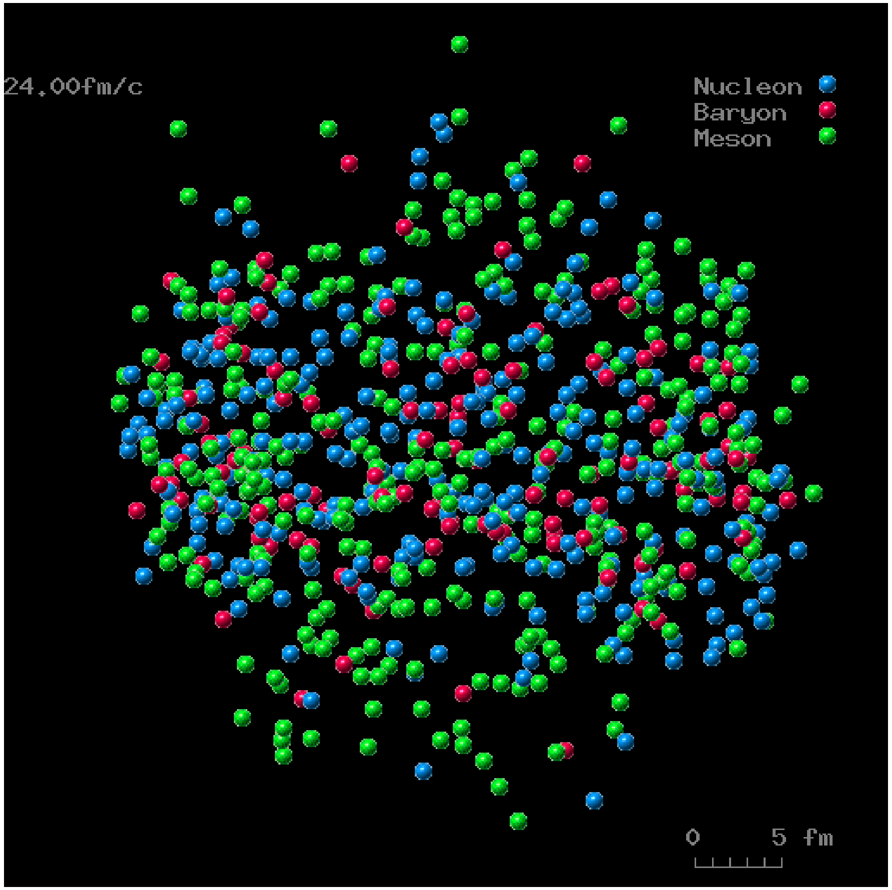


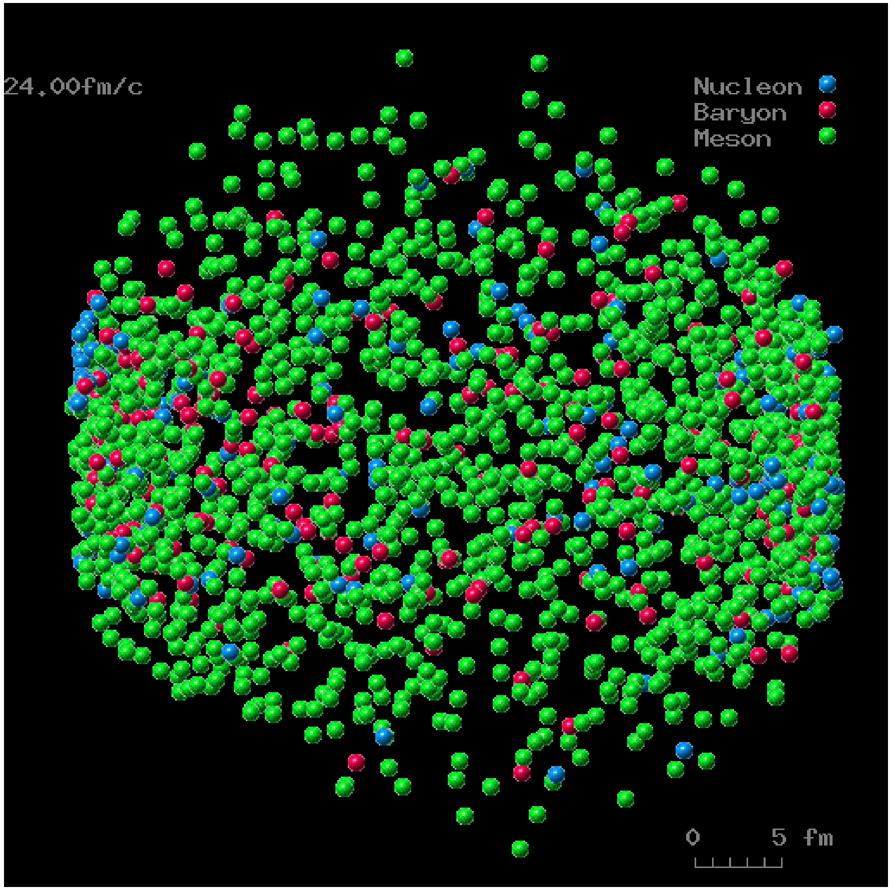


AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284

How do heavy-ion collisions look like ?

Au+Au, 10.6 A GeV | **Pb+Pb, 158 A GeV**





$\sqrt{s_{NN}} \sim$ 5 GeV | **$\sqrt{s_{NN}} \sim$ 20 GeV**

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