What microstate geometries tell us

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KEK Theory Workshop

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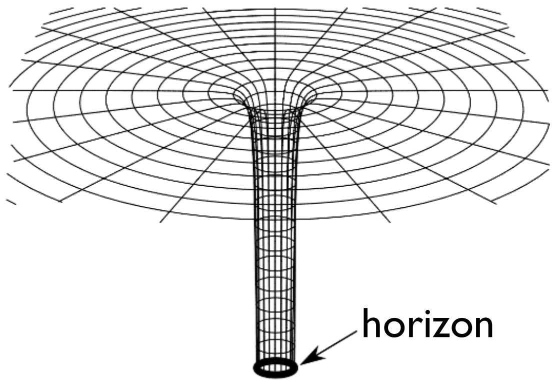
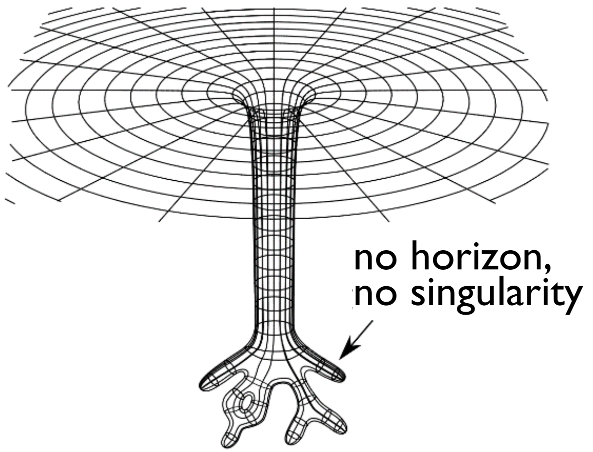
- Black hole microstate geometries



Black hole solution



Microstate geometry

- Superstrata (2015)-
  - Developments
  - Implications
  - Limitations

References

- Warner, Lectures on microstate geometries (2019) and superstrata (2020): https://sites.google.com/view/qbh-structure
- MS, 2002.01592 (superstrata)
- Mayerson, 2010.09736 (physical properties of superstrata, and observations)

Introduction

Black hole puzzles

- Information loss problem [→ P.-M. Ho's talk and T. Ugajin's talk]
  - Page curve, Islands…
- Entropy (microstate) problem

$$S_{\rm BH} = \frac{A}{4G_N}$$

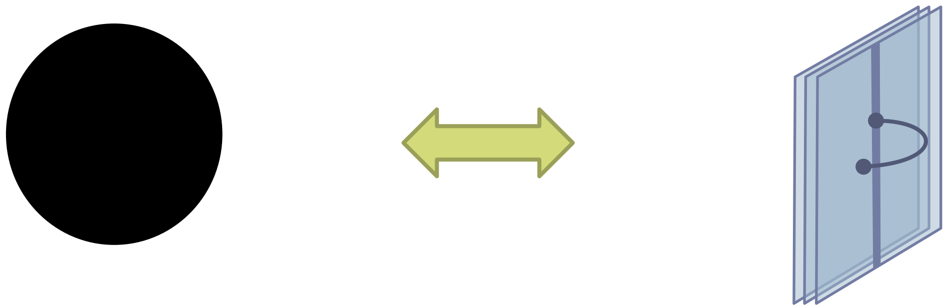
Schwarzschild: $S_{\rm BH} = 10^{77}(M/M_\odot)^2$

Cf. No-hair theorem: $e^S = 1$

— *Where* are the microstates?

Holographic counting

Gravity　　　　　　　　Field theory



E.g. 3-charge susy BH in 5D

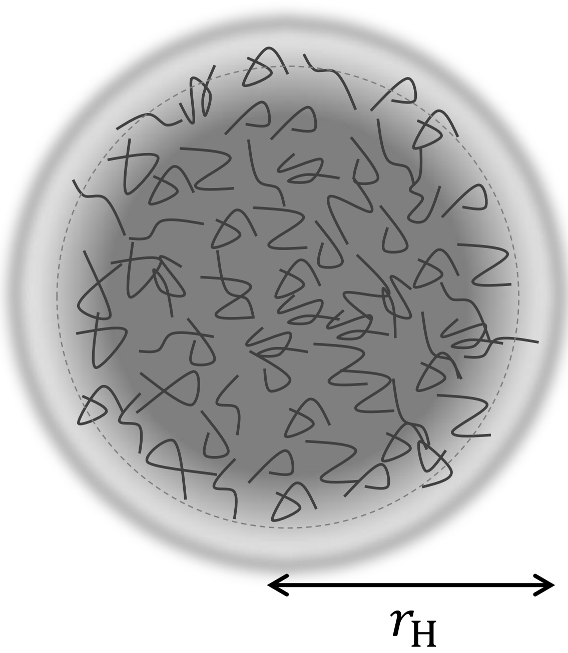
- D1-D5-P system, Type IIB on $S^1 \times \mathcal{M}_4$, $\mathcal{M}_4 = T^4$ or K3
- $S_{\mathrm{BH}} = S_{\mathrm{CFT}} = 2\pi\sqrt{N_1 N_5 N_P - J^2}$

[Strominger-Vafa '96] [BMPV '96]

— What is the *gravity picture* of these microstates?

Gravity picture?

Conjecture: BH microstates are some quantum gravity / stringy state spreading over horizon scale



- Fuzzball conjecture [Mathur, ca. 2000-]
- Firewall [AMPS, …]
- Yuki Yokokura's talk

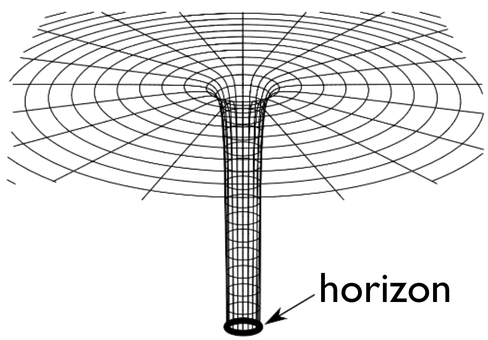
Info. loss problem would be trivially resolved

Gravity microstates

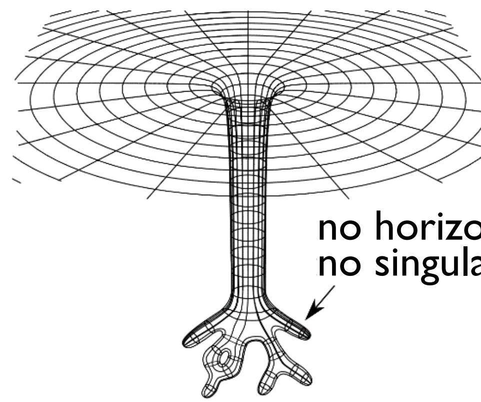
- Smooth and horizonless (unitary scattering amplitude is defined)
- Has the same M, J, Q as the BH
- General microstates of general BHs: not describable within supergravity

Microstate geometries

Fact:
For some BHs, some microstates are described by well-behaved solutions of supergravity (low-E eff. theory of string theory):
"microstate geometries"



Black hole solution



Microstate geometry (schematic)

- A *top-down* approach to understanding BH microphysics

A classification

[Bena-Warner 1311.4538]

- **Microstate geometry**
  Smooth horizonless solution of sugra,
  valid in sugra approximation.
- **Microstate solution**
  Smooth horizonless solution of sugra, or
  a physical limit of it. May have large curvature.
  May have singularities allowed in string theory.
- **General fuzzball**
  Everything else.

This talk (mainly)

The distinction is not clear-cut.

Microstate geometries

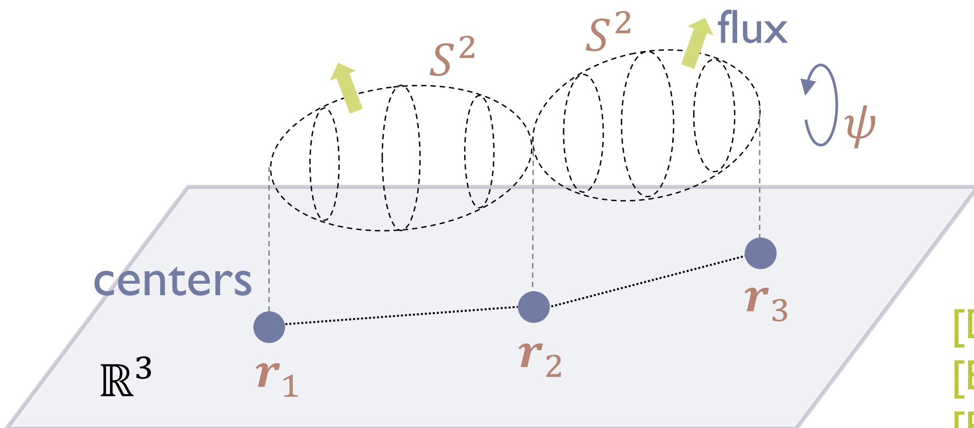
Let's look at some microstate geometries.

1. Multi-center bubbled geometries (2006-)
2. Superstrata (2015-)

These are supersymmetric (BPS). No Hawking rad.

Multi-center bubbled geometry

Multi-center bubbled geometries



[Denef+Bates 2003]
[Bena+Warner 2006]
[Berglund+Gimon+Levi 2006]

- Supersymmetric microstate geometries for 5D / 4D (asymptotically $\mathbb{R}^{1,4}$ or $\mathbb{R}^{1,3} \times S^1_\psi$)

5D BH/BR (SV/BMPV BH)

Reduced to 4D BH

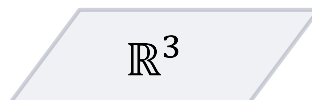
5D: smooth (up to orbifold singularity)

4D: singular source

Construction

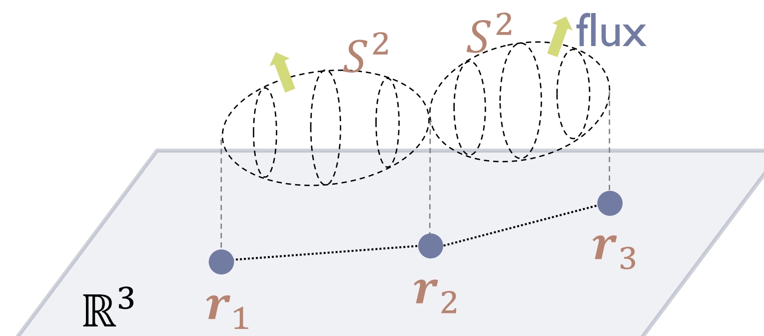
$$ds_5^2 = -Z^{-2/3}(dt + \mu(d\psi + A) + \omega)^2 + Z^{1/3}\left(V dr^2 + V^{-1}(d\psi + A)^2\right)$$

- **0th layer**: base



- **1st layer**: harmonic funcs on the base

$$H^I = \sum_i \frac{q_i^I}{|\boldsymbol{r} - \boldsymbol{r}_i|}$$



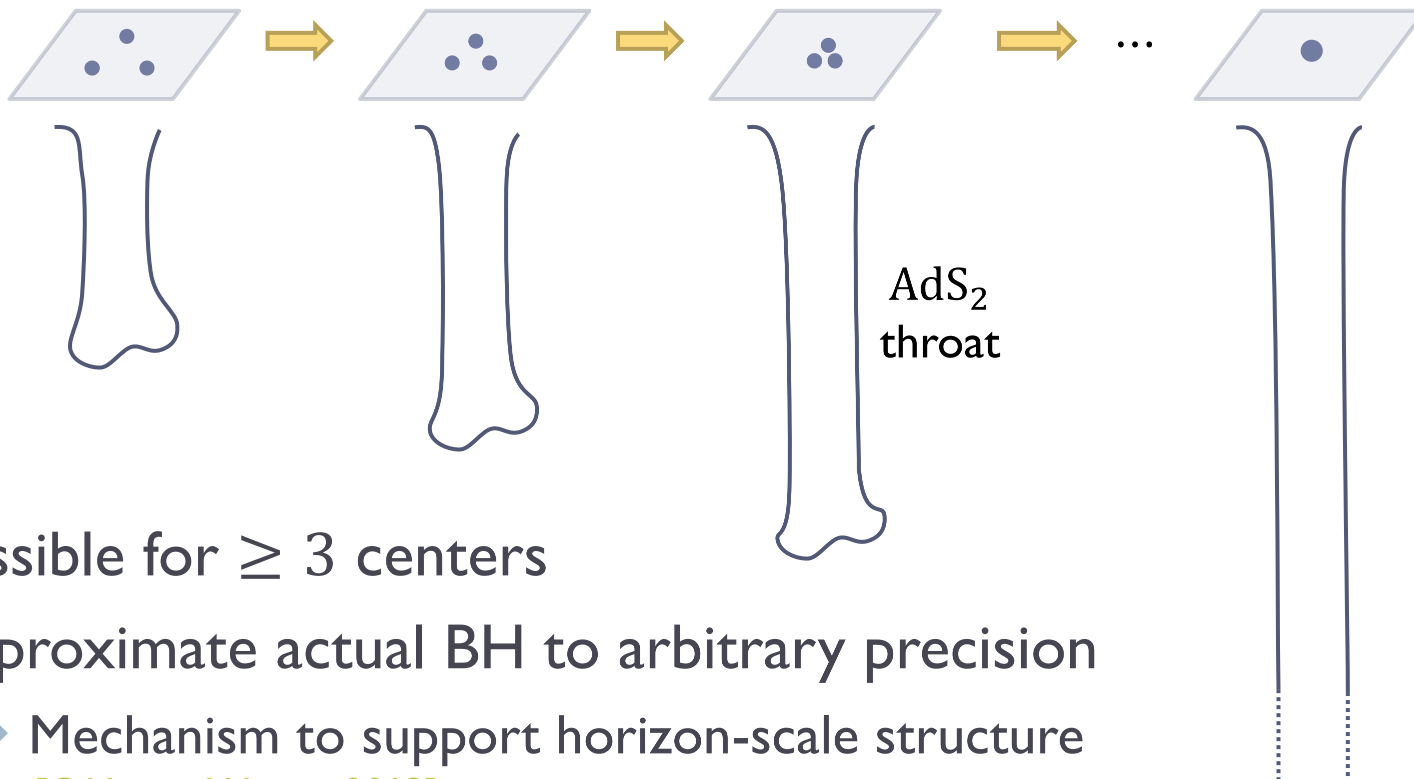
- **2nd layer**: 1-form on the base, with 1st-layer fields as source

$$*_3\, d\omega = \langle H, dH \rangle$$

  - The position of centers are not arbitrary but constrained by the integrability of the 2nd layer eq

Scaling solutions

[Denef]
[Bena, Warner et al. 2006, 07]



- Possible for ≥ 3 centers
- Approximate actual BH to arbitrary precision
  - → Mechanism to support horizon-scale structure [Gibbons+Warner 2013]
- Gap expected from CFT: $\Delta E \sim 1/N, \quad N \equiv N_1 N_5$

Counting: not enough

- Their entropy is parametrically smaller than the BH entropy

$$S_{\text{geom}} \ll S_{BH}$$

[Bena et al. 1006.3497]
[de Boer et al., 2008-09]

→ Multi-center bubbled geometry are not typical microstates.

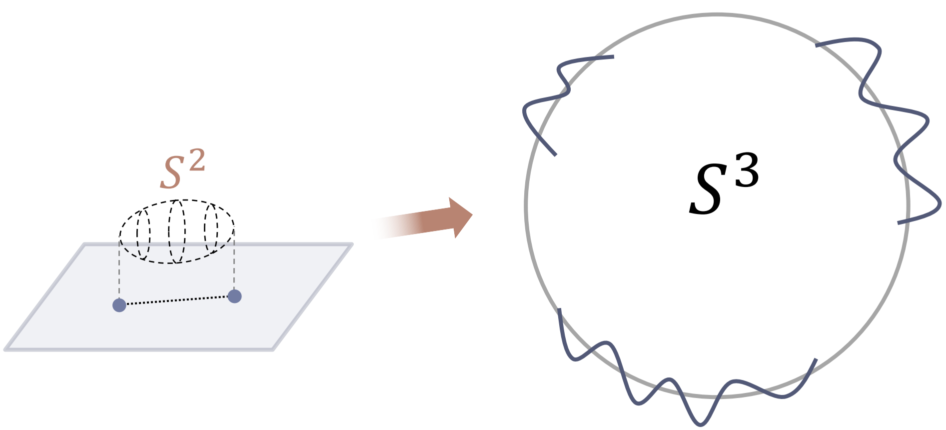
- Superstrata have larger entropy

Superstrata

Superstrata

[de Boer, MS, 2010, 2012] [Bena, de Boer, MS, Warner 2011]
[Bena, Giusto, Russo, MS, Warner 2015]
[Bena, Giusto, Martinec, Russo, MS, Turton, Warner 2016-17]

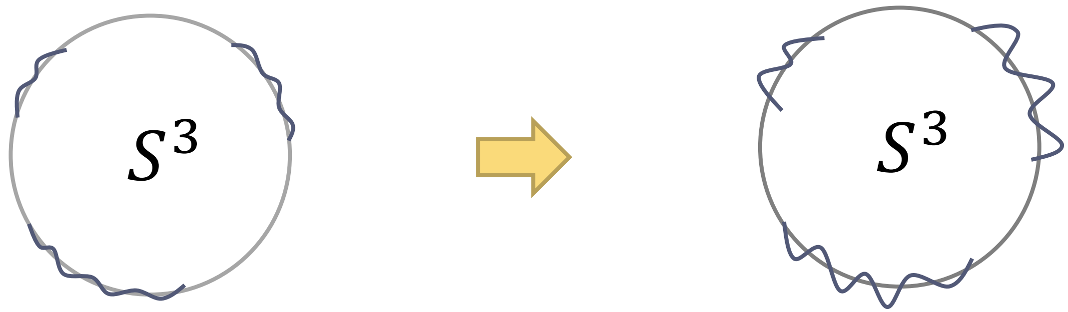
- More general than multi-center bubbled geometry
- Susy solutions of 6D sugra ($\mathrm{AdS}_3 \times S^3$, $\mathbb{R}^{1,4} \times S^1$, etc.)
  - Microstates for D1-D5-P BH with $N_1, N_5, N_P,$ and J (SV/BMPV)
  - Can use AdS/CFT (D1-D5 CFT)
- Fluctuate the non-trivial S^3 (and AdS_3)



S^2 in the multi-center bubbled solution in 5D becomes S^3 in 6D.

Superstrata

- Many possible Fourier modes (k, m, n)
- They represent "supergravitons", non-linearly completed



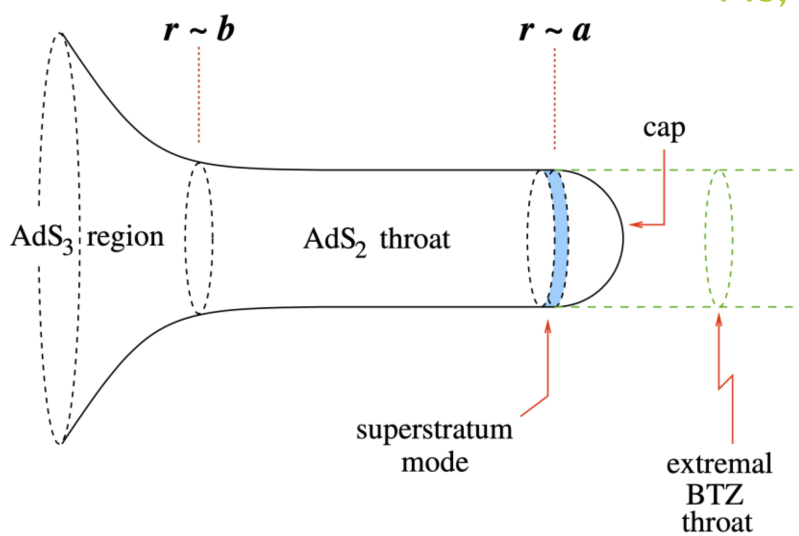
Linear (infinitesimal) fluctuation.
No backreaction.

Non-linear (finite) fluctuation.
There is backreaction.

Example of superstrata

- One example: the $(1,0,n)$ solution

[Bena, Giusto, Martinec, Russo, MS, Turton, Warner 2016-17]



- KK momentum excitation on top of a 2-center bubbling sol.
- Approaches a BH as $a \to 0$, although based on 2 centers
- Is simple and allows analytical study.
  Extensively used for concrete computations

Construction (1)

- **0th layer**: choose almost HK base B_4 (normally $\mathbb{R}^4$)
- **1st layer**: funcs and forms on B_4

D1	D5	fluctuation
$Z_1 = \ \dots\ ,$	$Z_2 = \ \dots\ ,$	$Z_4 = b\, z_{k,m,n}(r,\theta,v,\phi,\psi)$
$\Theta_2 = \ \dots\ ,$	$\Theta_1 = \ \dots\ ,$	$\Theta_4 = b\, \vartheta_{k,m,n}(r,\theta,v,\phi,\psi)$

P

- **2nd layer**: $\omega, \mathcal{F}$ on B_4, with 1st-layer fields as source

→ Regularity of 6D geometry fixes integration constants and requires us to modify Z_1 at $\mathcal{O}(b^2)$ in a specific way ("***coiffuring***")

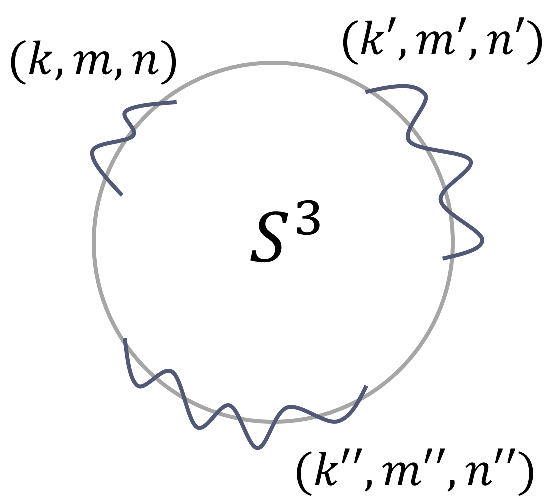
Construction (2)

- Multi-mode solution: sum over modes

$$Z_4 = \sum_{k,m,n} b_{k,m,n} z_{k,m,n}$$

$$\Theta_4 = \sum_{k,m,n} \left(b_{k,m,n} \vartheta_{k,m,n} + c_{k,m,n} \hat{\vartheta}_{k,m,n}\right)$$

"Supercharged" modes [Ceplak, Russo, MS '18]



- Supercharged modes crucial for regularity [Heidmann, Warner '19]
- Hard to explicitly write down the general multi-mode solution
- General solution: described by holomorphic func of 3 variables.
  - So far, solution with holon. func of 1 variable constructed, e.g. for $(1,0,n)$. [Heidmann, Mayerson, Walker, Warner '19].
  - Cf. consistent truncation to 3D

Construction (3)

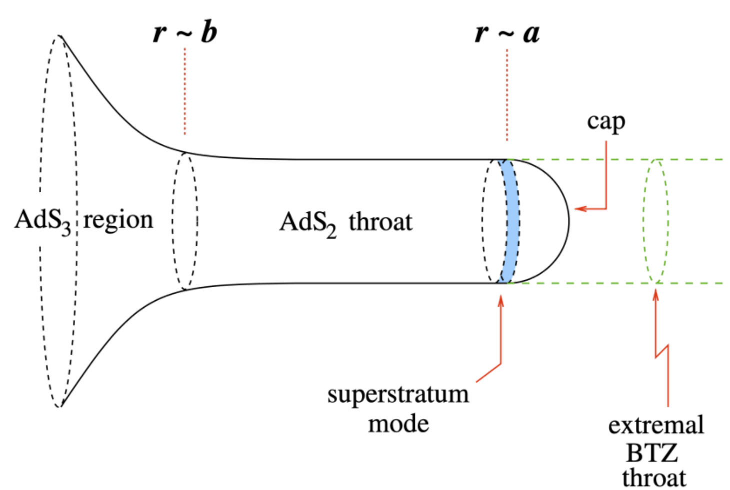
Regularity requirement gives

$$N_1 N_5 \sim a^2 + b^2 \qquad\qquad b^2 \sim \sum_{k,m,n} \left(\left| b_{k,m,n} \right|^2 + \left| c_{k,m,n} \right|^2 \right)$$

For deep, scaling geometries,

$$b^2 \sim N_1 N_5,$$

$$\frac{a^2}{b^2} \sim \frac{J}{N_1 N_5} \ll 1$$



The explicit form

10D (type IIB) fields

$$ds^2_{10} = \sqrt{\frac{Z_1 Z_2}{\mathcal{P}}}\, ds^2_6 + \sqrt{\frac{Z_1}{Z_2}}\, ds^2(\mathcal{M}),$$

$$ds^2_6 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\left[du + \omega + \frac{\mathcal{F}}{2}(dv+\beta)\right] + \sqrt{\mathcal{P}}\, ds^2(\mathcal{B}),$$

$$e^{2\Phi} = \frac{Z_1^2}{\mathcal{P}}, \qquad B_2 = -\frac{Z_4}{\mathcal{P}}(du+\omega)\wedge(dv+\beta) + a_4\wedge(dv+\beta) + \delta_2,$$

$$C_0 = \frac{Z_4}{Z_1}, \qquad C_2 = -\frac{Z_2}{\mathcal{P}}(du+\omega)\wedge(dv+\beta) + a_1\wedge(dv+\beta) + \gamma_2,$$

$$C_4 = \frac{Z_4}{Z_2}\mathrm{vol}(\mathcal{M}) - \frac{Z_4}{\mathcal{P}}\gamma_2\wedge(du+\omega)\wedge(dv+\beta) + x_3\wedge(dv+\beta),$$

$$C_6 = \mathrm{vol}(\mathcal{M})\wedge\left[-\frac{Z_1}{\mathcal{P}}(du+\omega)\wedge(dv+\beta) + a_2\wedge(dv+\beta) + \gamma_1\right]$$

$$\mathcal{P} \equiv Z_1\, Z_2 - Z_4^2.$$

$$ds^2(\mathcal{B}) = h_{mn}(x,v)dx^m dx^n.$$

$$\Theta_1 \equiv \mathcal{D}a_1 + \dot\gamma_2 - \dot\beta\wedge a_1, \quad \Theta_2 \equiv \mathcal{D}a_2 + \dot\gamma_1 - \dot\beta\wedge a_2, \quad \Theta_4 \equiv \mathcal{D}a_4 + \dot\delta_2 - \dot\beta\wedge a_4,$$

$$\Sigma_1 \equiv \mathcal{D}\gamma_2 - a_1\wedge\mathcal{D}\beta, \quad \Sigma_2 \equiv \mathcal{D}\gamma_1 - a_2\wedge\mathcal{D}\beta, \quad \Sigma_4 \equiv \mathcal{D}\delta_2 - a_4\wedge\mathcal{D}\beta,$$

$$\Xi_4 \equiv \mathcal{D}x_3 - \dot\beta\wedge x_3 - \Theta_4\wedge\gamma_2 + a_1\wedge\Sigma_4,$$

0^{th} layer

$$\tilde d J^{(A)} = \partial_v(\beta\wedge J^{(A)}).$$

$$\mathcal{D}\beta = *_4\mathcal{D}\beta.$$

1^{st} layer

$$\partial_v[*_4(\mathcal{D}Z_1 + \dot\beta Z_1) + \beta\wedge\Theta_2] = \tilde d\Theta_2,$$

$$\partial_v[*_4(\mathcal{D}Z_2 + \dot\beta Z_2) + \beta\wedge\Theta_1] = \tilde d\Theta_1,$$

$$\partial_v[*_4(\mathcal{D}Z_4 + \dot\beta Z_4) + \beta\wedge\Theta_4] = \tilde d\Theta_4$$

$$\mathcal{D} *_4 (\mathcal{D}Z_1 + \dot\beta Z_1) = -\Theta_2\wedge\mathcal{D}\beta,$$

$$\mathcal{D} *_4 (\mathcal{D}Z_2 + \dot\beta Z_2) = -\Theta_1\wedge\mathcal{D}\beta,$$

$$\mathcal{D} *_4 (\mathcal{D}Z_4 + \dot\beta Z_4) = -\Theta_4\wedge\mathcal{D}\beta.$$

2^{nd} layer

$$(1 + *_4)\mathcal{D}\omega + \mathcal{F}\,\mathcal{D}\beta = Z_1\Theta_1 + Z_2\Theta_2 - 2Z_4\Theta_4 - 2(Z_1 Z_2 - Z_4^2)\psi,$$

$$\begin{aligned} *_4\mathcal{D} *_4 L &+ 2\dot\beta_m L^m - *_4(\psi\wedge\mathcal{D}\omega) \\ &= -\frac{1}{4}(Z_1 Z_2 - Z_4^2)\dot h^{mn}\dot h_{mn} + \frac{1}{2}\partial_v[(Z_1 Z_2 - Z_4^2)h^{mn}\dot h_{mn}] \\ &\quad + (\dot Z_1 \dot Z_2 - \dot Z_4^2) + (Z_1\ddot Z_2 + Z_2\ddot Z_1 - 2Z_4\ddot Z_4) \\ &\quad - \frac{1}{2} *_4 \Big[(\Theta_1 - Z_2\psi)\wedge(\Theta_2 - Z_1\psi) - (\Theta_4 - Z_4\psi)\wedge(\Theta_4 - Z_4\psi) \end{aligned}$$

$$L \equiv \dot\omega + \frac{\mathcal{F}}{2}\dot\beta - \frac{1}{2}\mathcal{D}\mathcal{F}.$$

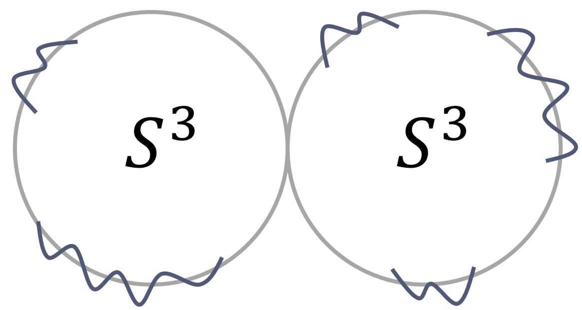
Holographic dictionary

Gravity		CFT
mode $(k, 0{,}0)$	⟷	chiral primary χ_k
mode (k, m, n)	⟷	descendant $(J_0^+)^m (L_{-1})^n \chi_k$
"supercharged" mode	⟷	superdescendant $QQ(J_0^+)^m (L_{-1})^n \chi_k$
Superstrata with mode (k, m, n), amplitude $b_{k,m,n}$	⟷	$[(J_0^+)^m (L_{-1})^n \chi_k]^{N_{k,m,n}}$, $N_{k,m,n} \sim \lvert b_{k,m,n} \rvert^2$

- Implied by the construction

Multi-cycle superstrata

- Superstrata based on multiple 3-cycles are also possible in principle



- Holographic dictionary for multi-cycle superstrata (or > 2 center bubbling solutions) not known

Aspects of superstrata

Precision holography (1)

— Matching of correlators in microstates between bulk geometry and CFT states [Kanitscheider, Skenderis, Taylor '06-08]

- Protected BPS 3-point function:

$$\langle HLH \rangle = \langle H|L|H \rangle$$

  - H: heavy ($\Delta \sim N$) 1/8-BPS op. dual to a superstratum
  - L: light ($\Delta \sim 1$) chiral-primary op. dual to a sugra field

In gravity, $\langle H|L|H \rangle$ = (1-pt func in superstratum backgnd)

→ Compare this with CFT computations.

Note: for typical thermal states, $\langle H|L|H \rangle$ would be almost state independent.

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Precision holography (2)

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[Giusto, Moscato, Russo ’15]
[Giusto, Rawash, Turton ’19, ’20]

- Confirm holographic dictionary for superstrata
- Mixing between single- and multi-trace operators (important even in sugra e.g for extremal correlators)
- Confirm coiffuring (non-trivial term in harmonic func at $\mathcal{O}(b^2)$)
  - → Power of CFT in predicting non-trivial features of bulk geom
  - → Useful for finding more general classes of superstrata?
- Entanglement entropy [Giusto, Moscato, Russo ’15]

Counting superstrata

- CFT counting: use holographic dictionary [MS, 2020]
- Bulk counting: compute symplectic form for superstrata [Mayerson, MS, 2020]

$$S_{\text{strata}} \sim \left[2\left(N^2 - NN_P + N_P^2\right)^{3/2} - (N - 2N_P)(2N - N_P)(N + N_P)\right]^{1/4}$$

$$\sim N^{1/2} N_P^{1/4} \qquad (N \equiv N_1 N_5 \ll N_P)$$

$$S_{\text{BH}} \sim N^{1/2} N_P^{1/2}$$

$$S_{\text{strata}} \ll S_{\text{BH}}$$

Superstrata are parametrically too few to account for BH entropy

Counting superstrata: detail (1)

- Superstrata = 3-charge states with $N, N_P = L_0, J = J_0^3$
- Partition function:

$$Z(p,q,y) = \sum_{N,N_P,J} D(N,N_P,J)\, p^N q^{N_P} y^J$$

$$p \equiv \mathrm{e}^{-\alpha}, \quad q \equiv e^{-\beta}, \quad y \equiv e^{-\gamma}$$

Counting superstrata: detail (2)

- Contribution from states based on a chiral primary $|\psi\rangle$:

$$\log z^{\text{bos}}_{|a\rangle} = \sum_{r=1}^{\infty} \frac{p^r y^{\frac{ar}{2}}}{r(1-q^r)(1-q^r y^r)} \left[\frac{1-2(-1)^r q^r y^{\frac{r}{2}} + q^{2r} y^r}{1-p^r} - \frac{q^{(2-a)r} y^{(1-a)r}(1-2(-1)^r y^{\frac{r}{2}} + y^r)}{1-(pqy)^r} \right]$$

$$\log z^{\text{fer}}_{|a\rangle} = -\sum_{r=1}^{\infty} \frac{p^r y^{\frac{ar}{2}}}{r(1-q^r)(1-q^r y^r)} \left[\frac{(-1)^r - 2q^r y^{\frac{r}{2}} + (-1)^r q^{2r} y^r}{1-p^r} - \frac{q^{(2-a)r} y^{(1-a)r}((-1)^r - 2y^{\frac{r}{2}} + (-1)^r y^r)}{1-(pqy)^r} \right]$$

a: value of J for $|\psi\rangle$

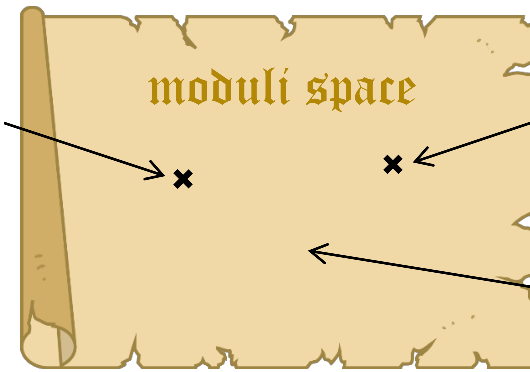
- The total partition function:

$$\log Z_{T^4}(p,q,y) = 2\log z^{\text{bos}}_{|+\rangle} + 4\log z^{\text{bos}}_{|0\rangle} + 2\log z^{\text{bos}}_{|-\rangle} + 2\log z^{\text{fer}}_{|+\rangle} + 4\log z^{\text{fer}}_{|0\rangle} + 2\log z^{\text{fer}}_{|-\rangle}$$

$$\log Z_{\text{K3}}(p,q,y) = 2\log z^{\text{bos}}_{|+\rangle} + 20\log z^{\text{bos}}_{|0\rangle} + 2\log z^{\text{bos}}_{|-\rangle}.$$

→ Can be estimated using thermodynamics

Lifting



- Single-ctr BH exists everywhere and contributes to susy index.
- BH microstates must also exist everywhere and contribute to index.
- ≥3 center bubbled solns and multi-cycle superstrata lift at generic points in moduli space [Dabholkar, Giuca, Murthy, Nampuri '09] [Bossard, Lust '19]
  - → No contribution to susy index – irrelevant for microstates?
  - → Multi-cycle superstrata don't change counting
  - → Some superstrata with one S^3 also lift?? [Guo, Mathur, 2021?]
- However, lifting may not actually be physically relevant [Chowdhury, Mayerson '13]

Typical or atypical?

- Superstrata of any use in understanding BH microphysics?
- Are they typical or atypical states of the BH ensemble?
  - $S_{\mathrm{strata}} \ll S_{\mathrm{BH}}$
  - Gap expected of a typical state is reproduced: $\Delta E \sim 1/N$

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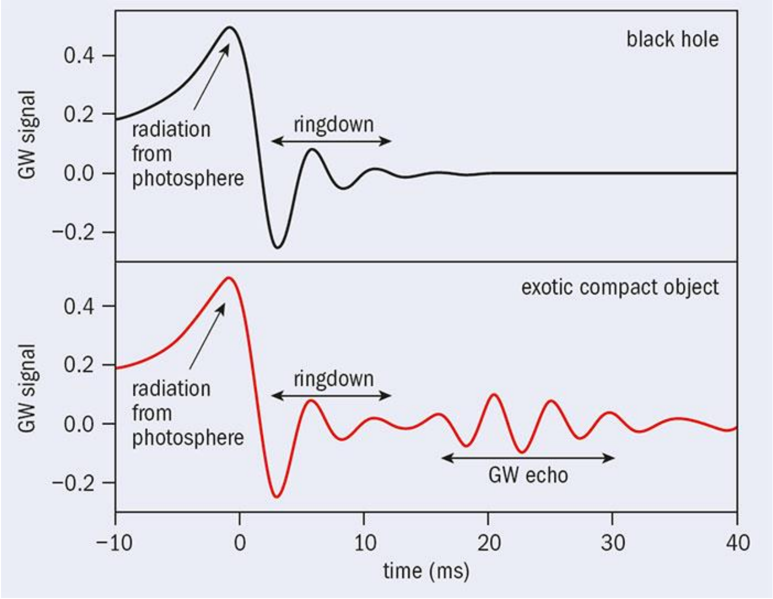
- Whether they are typical or not depends on the question you ask.
- Even if they are atypical, we can use them to study evolution toward more typical states by perturbing them.

Echoes

Gravitational wave echoes [Cardoso, Franzin, Pani '16]

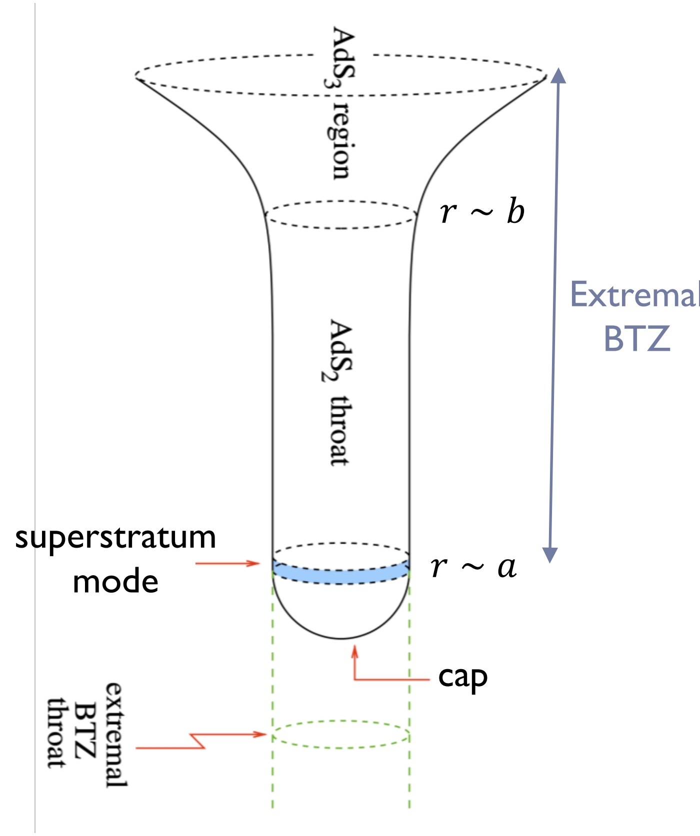


http://cerncourier.com/cws/article/cern/67457

- Can probe near-horizon region
- Various models of ECOs
- Microstate geometries?
  - No microstates for realistic BHs (non-extremal Kerr)
  - Microstate geometries: only top-down model based on string theory
  - Tools to identify universal properties of BH microstates, pointing toward interesting possible observable quantities

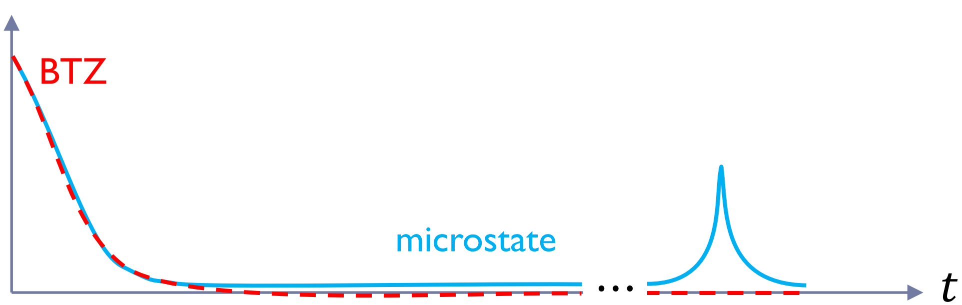
Structure of deep scaling superstrata

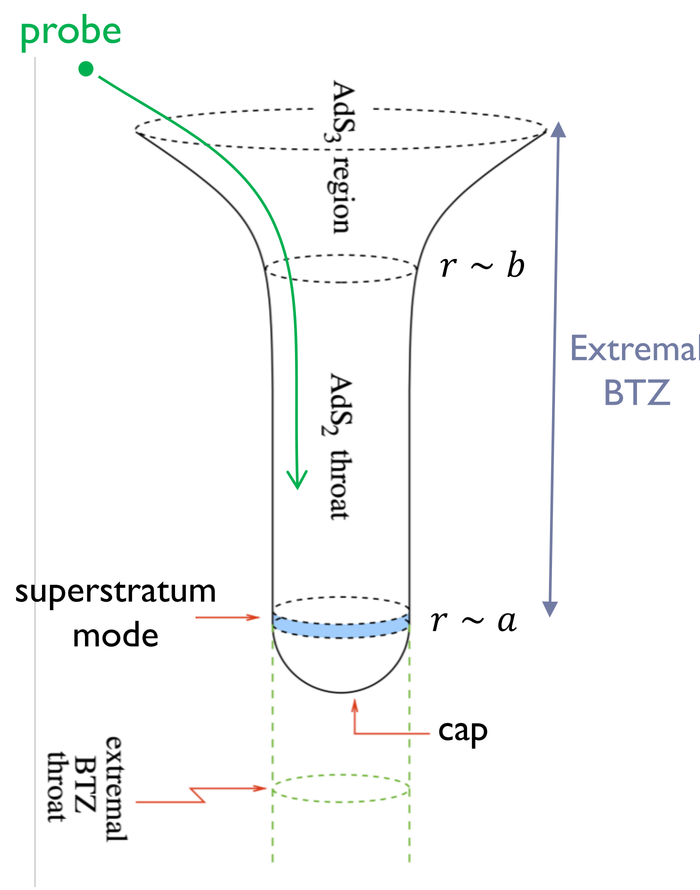
- Let us study "echoes" from in superstrata that is a microstate of the extremal BTZ black hole.
- This also tells us about typical states that they want to evolve into.
- Structure of deep scaling superstrata:
  - If a is small and $a \ll b$, the superstratum approximates extremal BTZ down to $r \sim a$
  - The redshift can be made parametrically of order $N = N_1 N_5$



Probing deep scaling superstrata

- What happens if we throw in a probe? 2-point function? (This is $\langle HLLH \rangle$)
- If it were the extremal BTZ BH:
  - Initial exponential decay
  - Goes to zero at later time
- In a microstate, it must not go strictly to zero and come back after the Poincare time $t \sim e^S$. How well do superstrata do?

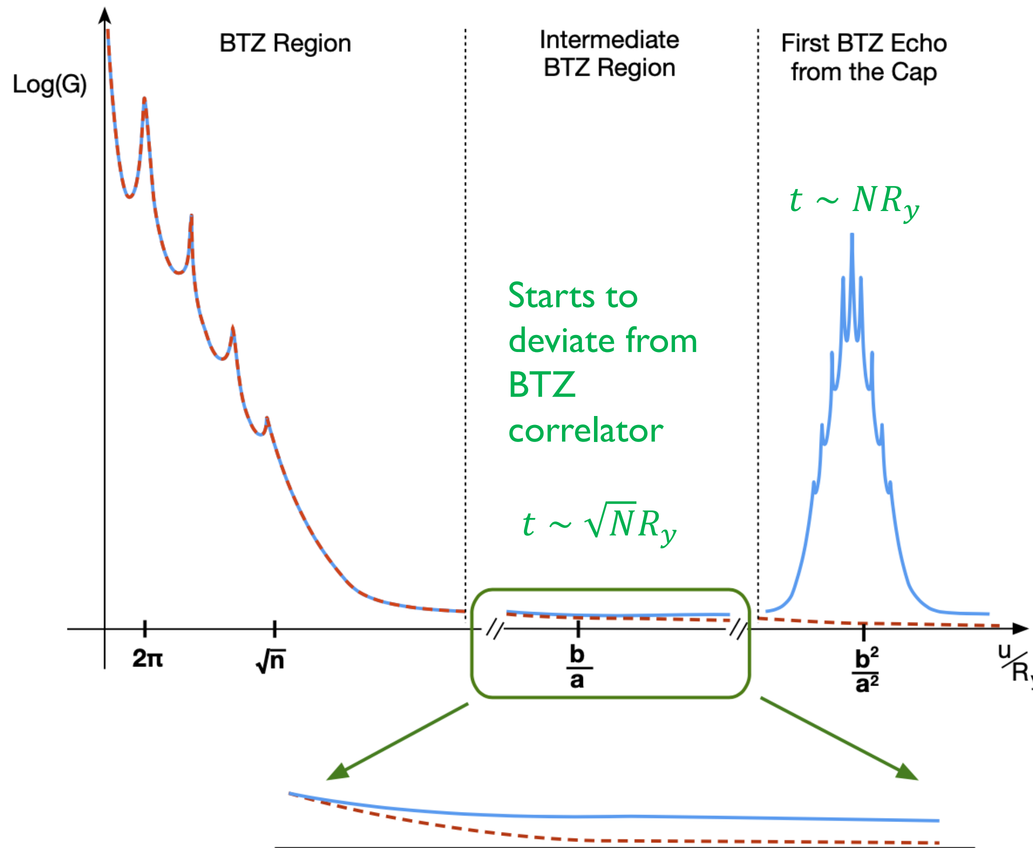




Echoes from superstrata

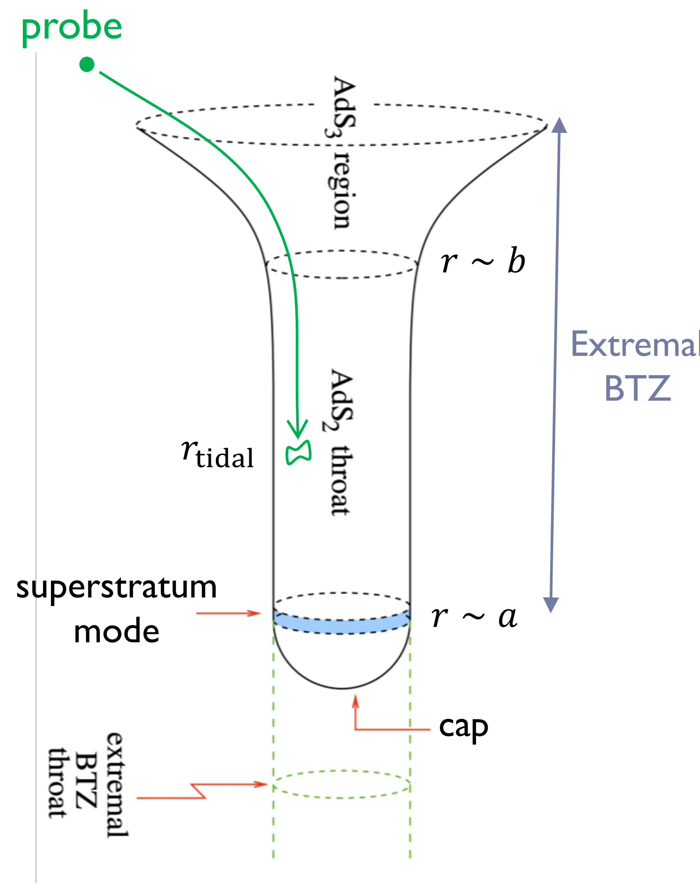
[Bena, Heidmann, Monten, Walker, Warner '19]

- $(1,0,n)$ strata: wave eq. separable
  [Bena, Turton, Walker, Warner '17]
- Matching WKB approx.
- Result:
  - Initial exponential decay mimics BTZ QNMs
  - Info returns (thus no info puzzle), but it is too soon and too coherent
- Multi-mode strata should give quite different results.



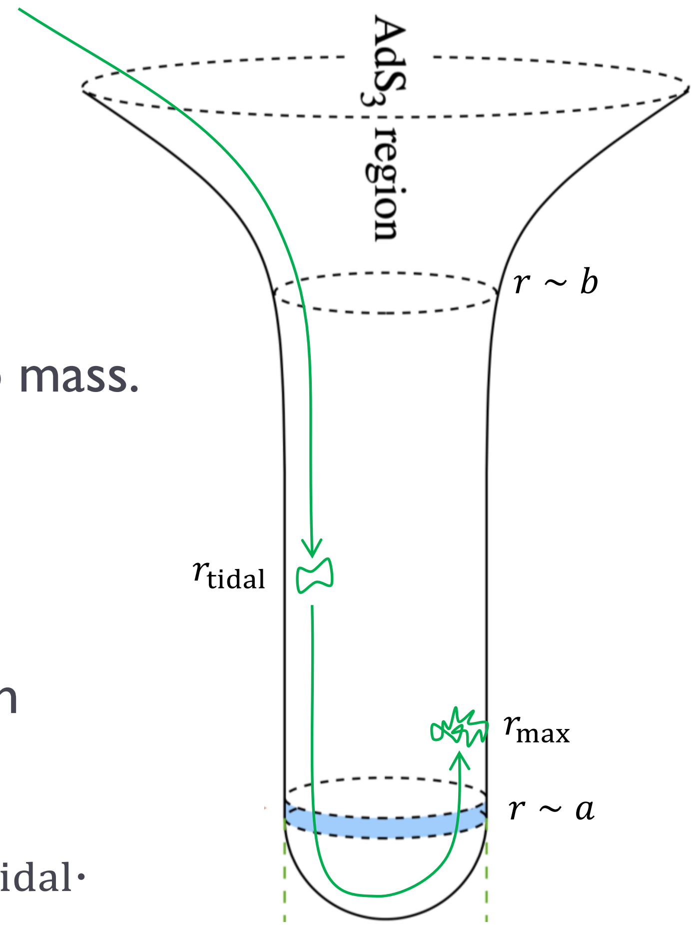
Tidal force [Tyukov, Walker, Warner '17] [Bena, Martinec, Walker, Warner '18]

- Tidal force becomes stringy midway at $r_{\mathrm{tidal}} \sim \sqrt{ab}$
  (cf. It is small everywhere for BTZ)
  - Small bump amplified by the blueshift of the probe as it falls down
  - This happens for any capped geometry with a long BTZ throat.
- Point-particle approximation becomes invalid, because it's really a string!



String probes (1) [Martinec, Warner '20]

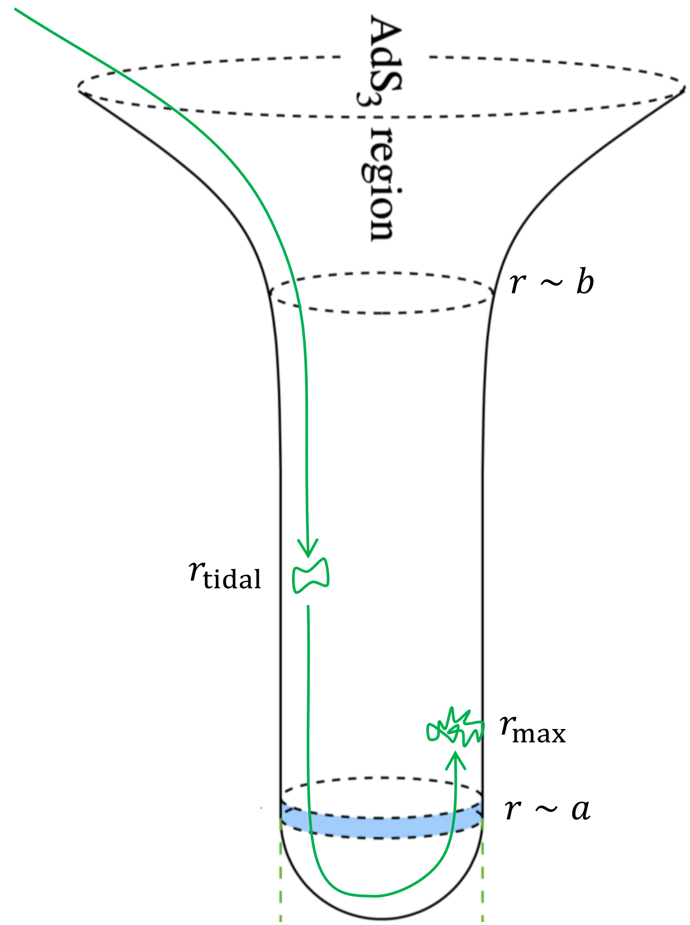
- String worldsheet NL-σ model
  - Radially falling in, with energy E
  - Large E → Penrose limit
  - Same as harmonic oscillator with r-dep mass. Negative mass modes get excited.



- Result
  - After excitation, string gets massive with $m^2 \propto E$
  - String will go back up only to $r_{\text{max}} \ll r_{\text{tidal}}$. r_{max} is indep of E → trapped

String probes (2) [Martinec, Warner '20]

- The string goes up and down, eventually settling down at the bottom of the cap, and thermalizes.
- No sharp echo as predicted in point-particle approximation
- There must be weak echoes by bremsstrahlung of the string (probably much more like real BH!)
- Nice, but we have to be careful in interpreting this result; the capped superstratum is atypical anyway.



Conclusions

Conclusions

- Microstate geometries provide a useful paradigm to explore BH microphysics
- Superstrata
  - The largest known class of microstate geometries
  - Deep scaling geometries: approximates BH to arbitrary precision
  - Various technical developments
  - Not enough to account for BH entropy
  - Probe analysis shows similarity to actual BH, through stringy physics
  - Possible connection to observation
    (GW echoes, QNMs, multipoles, tidal Love numbers…)