# Exclusive $b \to s\mu\mu$ Processes

## Toward Precision Probes of the Standard Model

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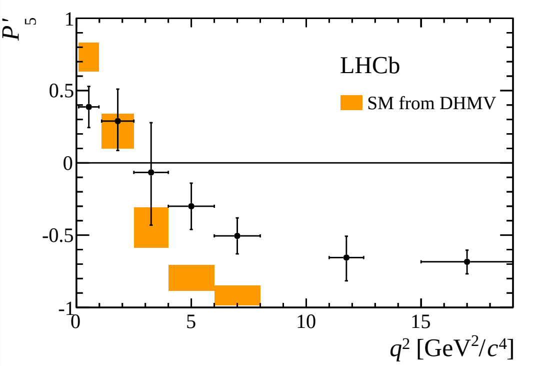
# Prelude

My intention is to enable those members of the audience that are so far unfamiliar with the theoretical aspects of $b \to s\ell\ell$ to develop an understanding of how these types of measurements ...



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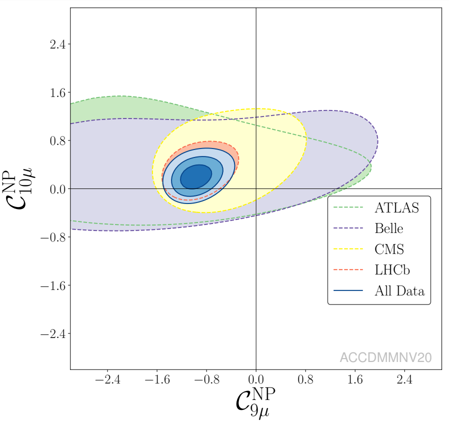
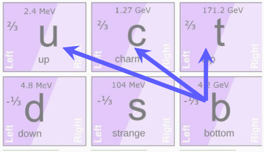
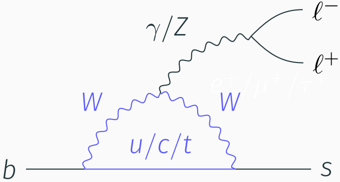



...lead to claims of tensions with SM at and above the $5\,\sigma$ level.



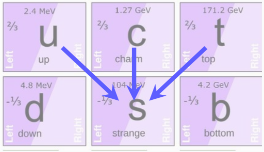

- w/o change of el. charge




only arises at loop level

# Theory for $b \to s\{q\bar{q}, \gamma, \ell^+\ell^-\}$



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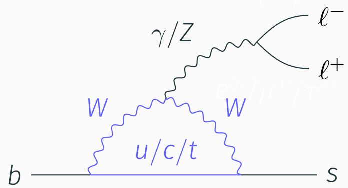


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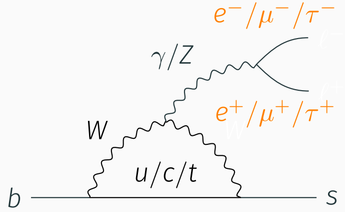
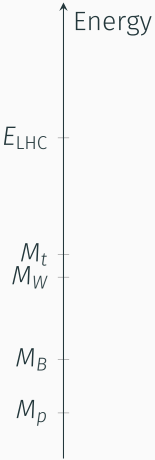
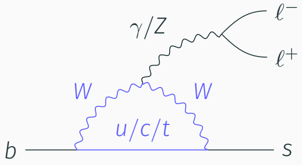



only arises at loop level

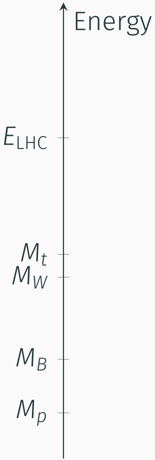
lepton-flavour-universal gauge couplings!

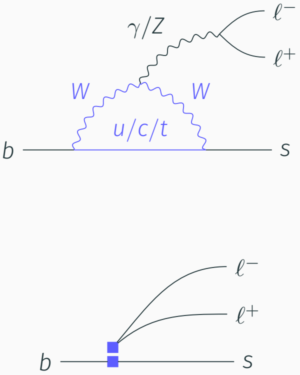
- widely used tool of theoretical physics



- widely used tool of theoretical physics
- replaces dynamical degrees of freedom (here: $t, W, Z$) by coefficients $\mathcal{C}_i$ and static structures in local operators (here: $\Gamma_i$)




$$\sum_i \mathcal{C}_i \times [\bar{s}\,\Gamma_i\, b] \times \ldots$$

# Weak Effective Theory: $b \to s\ell\ell$ SM Operators

in the SM the we find the following $D = 6$ effective operators

$$\mathcal{L}_{\text{SM}}^{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_i \mathcal{C}_i\, \mathcal{O}_i + \lambda_c \sum_i \mathcal{C}_i^{\,c}\, \mathcal{O}_i^{\,c} + \lambda_u \sum_i \mathcal{C}_i^{\,u}\, \mathcal{O}_i^{\,u}\right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R T^A b) G^A_{\mu\nu}$$

$$\mathcal{O}_9 = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_1{}^q = (\bar{q}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L q) \qquad \mathcal{O}_2{}^q = (\bar{q}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a q)$$

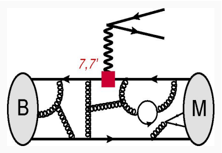
$$\mathcal{O}_i = (\bar{s}\gamma_\mu P_X b) \sum_q (\bar{q}\gamma^\mu q)$$

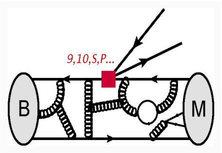
with $\lambda_q \equiv V_{qb} V_{qs}^*$

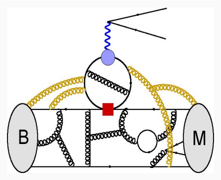
- ▶ very complicated structure compared to the tree-level decays

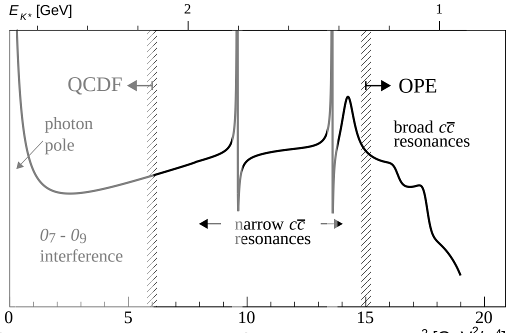
SM contributions to $\mathcal{C}_i(\mu_b)$ known to NNLL [Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]

How do these operator contribute? Schematically




[sketch from Blake, Gershon, Hiller 1501.03309]

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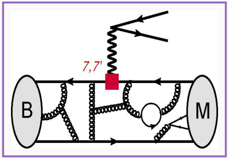


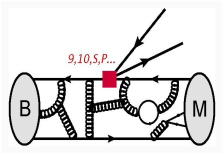


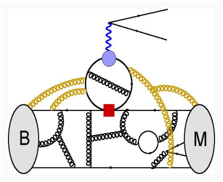


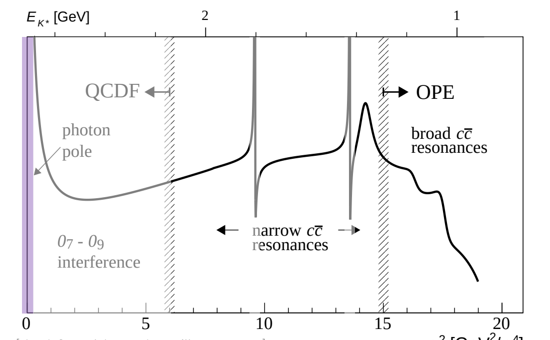


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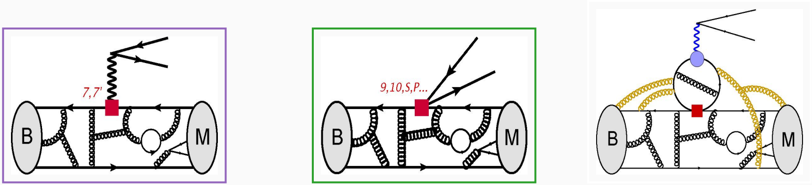


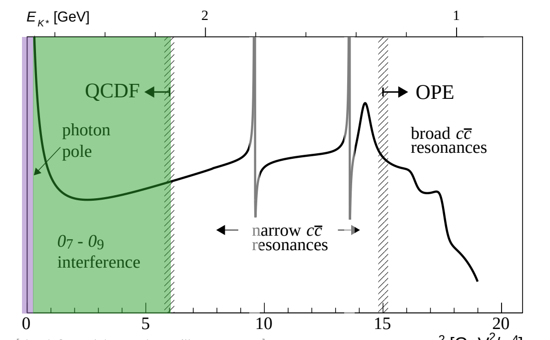


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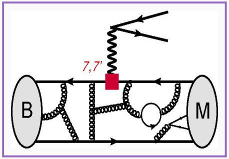


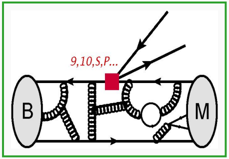


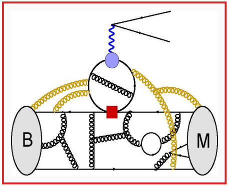


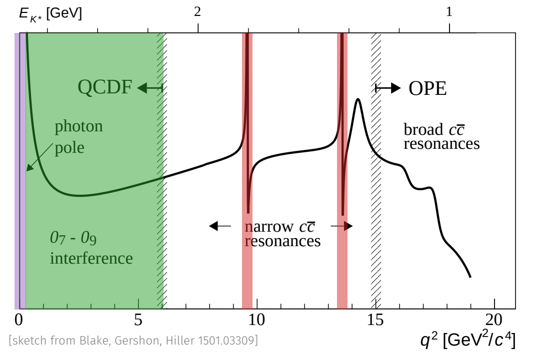


[sketch from Blake, Gershon, Hiller 1501.03309]

in the presence of BSM effects, complete basis of semileptonic operators by adding

$$\mathcal{L}_{\rm BSM}^{\rm eff} = \mathcal{L}_{\rm SM}^{\rm eff} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_i \mathcal{C}_i\,\mathcal{O}_i\right]$$

with $i$ running over $9'$, $10'$, $S$, $S'$, $P$, $P'$, $T$, $T5$:

$$\begin{aligned}
\mathcal{O}_{9'} &= \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell) & \mathcal{O}_{10'} &= \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu\gamma_5 \ell)\\
\mathcal{O}_{S} &= \frac{\alpha}{4\pi}(\bar{s}P_R b)(\bar{\ell}\ell) & \mathcal{O}_{S'} &= \frac{\alpha}{4\pi}(\bar{s}P_L b)(\bar{\ell}\ell)\\
\mathcal{O}_{P} &= \frac{\alpha}{4\pi}(\bar{s}P_R b)(\bar{\ell}\gamma_5\ell) & \mathcal{O}_{P'} &= \frac{\alpha}{4\pi}(\bar{s}P_L b)(\bar{\ell}\gamma_5\ell)\\
\mathcal{O}_{T} &= \frac{\alpha}{4\pi}(\bar{s}\sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu}\ell) & \mathcal{O}_{T5} &= \frac{\alpha}{4\pi}(\bar{s}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu}\gamma_5\ell)
\end{aligned} \tag{1}$$

- $\mathcal{C}_i = 0$ in the SM for all of these operator!

# Weak Effective Theory: Summary



- WET makes calculation in the SM possible in the first place
  - separates long-distance from short-distance physics
  - resums potentially large logarithms
- "divide and conquer"
- transparently allows to account model-independently for the effects of physics beyond the SM
  - interface to model builders ...
  - ...although transitioning to SM Effective Field Theory, which can help to related constraints amongst the various Weak Effective Theories (*i.e.*, relate constraints in $b \to c\tau\nu$ with constraints in $b \to s\ell^+\ell^-$)

# Hadronic Matrix Elements & SM Predictions

- the Lagrangian with its effective operators describes the decay of a free $b$ quark
- however, the quarks are confined in hadrons
- to describe the decay we require further information about the $b$ quark inside the initial state hadron $H_b$ (and similarly about the $s$ inside the final state hadron $H_s$)
- additionally, we need to account for one weak interaction + possibly multiple electromagnetic interactions, all of which are described by $\mathcal{L}_{\text{SM}}^{\text{eff}}$

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formally, we require matrix elements of all possible contributions of the Lagrangian

$\mathcal{T}$: time ordering

$$\mathcal{A} \propto \langle H_s| \mathcal{T} \exp\left[i \int d\tau \mathcal{L}_{\mathrm{SM}}^{\mathrm{eff}}(\tau)\right] |H_b\rangle = 0 + \langle H_s| \mathcal{L}_{\mathrm{SM}}^{\mathrm{eff}}(0) |H_b\rangle + \langle H_s| \mathcal{T} \int d\tau \mathcal{L}_{\mathrm{SM}}^{\mathrm{eff}}(\tau)\mathcal{L}_{\mathrm{SM}}^{\mathrm{eff}}(0) |B\rangle + \dots$$

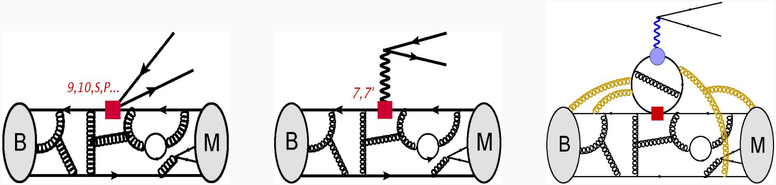
- here, we are discussing $b \to s\ell\ell$ transitions only!

- examples for exclusive decays mediated by $b \to s\ell\ell$ include
  - $\overline{B} \to \overline{K}^{(*)}\ell^+\ell^-$ pseudoscalar and vector final states
  - $\overline{B}_s \to \phi\ell^+\ell^-$ vector final state w/ $s$ spectator
  - $\Lambda_b \to \Lambda\ell^+\ell^-$ baryonic cousin to $\overline{B} \to \overline{K}\ell^+\ell^-$
  - $\Lambda_b \to pK^-\ell^+\ell^-$ baryonic cousin to $\overline{B} \to \overline{K}^*\ell^+\ell^-$

Virtually identical amplitude anatomy for all these decays!

# Anatomy of Exclusive $b \to s\ell^+\ell^-$ Decay Amplitudes






$$\mathcal{A}_\lambda^\chi = \mathcal{N}_\lambda \left\{ (\mathcal{C}_9 \mp \mathcal{C}_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[ \mathcal{C}_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

nomenclature $\lambda$: dilepton ang. mom., $\chi$: lep. chirality

- $\mathcal{F}_\lambda$ local form factors of dimension-three currents: $\bar{s}\gamma^\mu b$ & $\bar{s}\gamma^\mu\gamma_5 b$
- $\mathcal{F}_\lambda^T$ local dipole form factors of dimension-three current: $\bar{s}\sigma^{\mu\nu} b$
- $\mathcal{H}_\lambda$ nonlocal form factors of dimension-five nonlocal operators

$$\int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\rm em}(x), \sum \mathcal{C}_i O_i(0) \}$$

all three needed for consistent description to leading-order in $\alpha_e$

# Observables: Branching Fraction

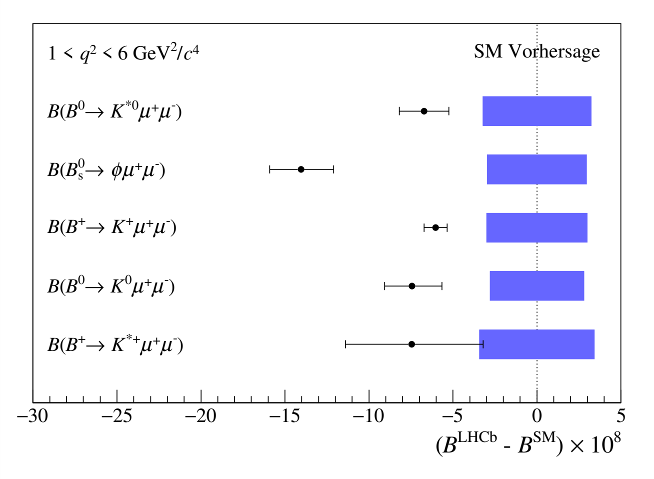


- simplest observable: how frequently does a $\overline{B} \to \overline{K}^* \ell^+ \ell^-$ decay happen?
- needs to acount for each amplitude, with their various angular momentum states $\lambda$ and lepton chiralities $\chi$

$$\frac{d\mathcal{B}}{dq^2} \propto \tau_B \left[ \sum_{\chi = L,R} \sum_{\lambda} |\mathcal{A}_\lambda^\chi|^2 \right]$$

- very sensitive to the local form factors!

$\Rightarrow$ largest theory uncertainty of all observables however ... measurements are systematically below predictions




[Albrecht, Langenbruch 2018]

Idea: test lepton-flavour universality through ratios of $\mathcal{B}$

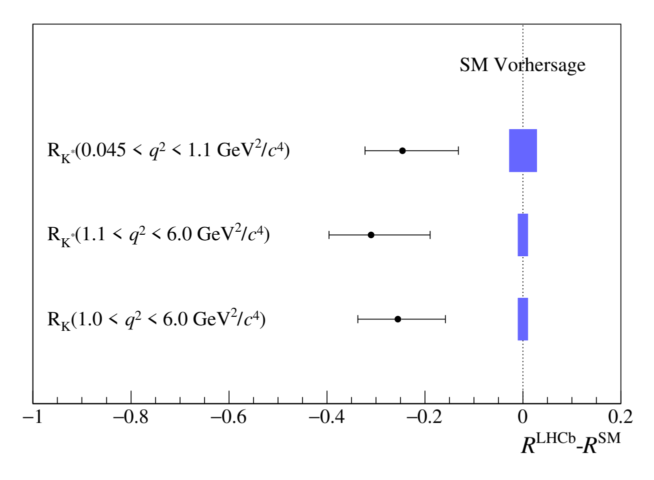
$$\left.\frac{d\mathcal{B}(H_b \to H_s \ell^+\ell^-)}{dq^2}\right|_{\text{SM}} \propto \quad \#1 + \frac{m_\ell^2}{q^2} \quad \#2$$

- for $q^2 \geq 1\,\text{GeV}^2$, the lepton-mass specific factor $m_\ell^2/q^2$ is negligible and hence term #2 is irrelevant
- term #1 then cancels in every $q^2$ point

$\Rightarrow$ $R_{H_s} \equiv \mathcal{B}^{(\mu)}/\mathcal{B}^{(e)} \simeq 1$ for every $H_s$ and in that $q^2$ interval

- deviation from 1 is a brilliant SM null test, th. uncertainties $\sim 1\%$
  - reasonable SM uncertainty estimates must include electromagnetic effects!
- works even for decays such as $\overline{B} \to \overline{K}\pi\pi\ell^+\ell^-$ or $\Lambda_b \to pK^-\ell^+\ell^-$, for which we have no reliable theory predictions at all!

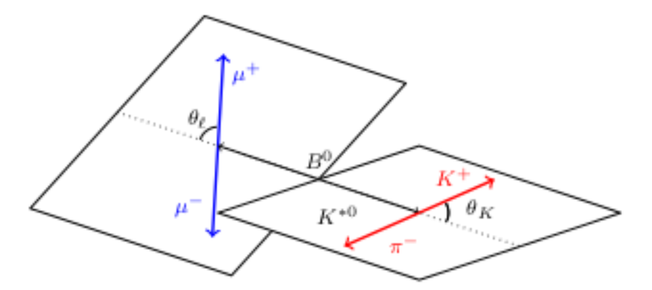
again, measurements are systematically below predictions




[Albrecht, Langenbruch 2018]

Three independent decay angles in $\overline{B} \to \overline{K}^{*}\ell^{+}\ell^{-}$ (similar for other decays!)

- $\theta_\ell$ helicity/polar angle of the lepton pair
- $\theta_K$ helicity/polar angle of the $\overline{K}\pi$ pair
- $\phi$ azimuthal angle between the two decay planes




[LHCb-PAPER-2013-019]

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angular distribution

$$\frac{1}{\mathcal{B}} \frac{d^4\mathcal{B}}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \sum_i S_i(q^2)\, f_i(\cos\theta_\ell, \cos\theta_K, \phi)$$

gives rise to 12 angular observables $S_i(q^2)$!

- numerator of each $S_i$ comprised of the same amplitudes as $\mathcal{B}$
- but: non-diagonal terms like $S_{6s} \propto \mathrm{Re}\,\mathcal{A}_\perp \mathcal{A}_\parallel^*$ provide complementary access to Wilson coefficients compared to $\mathcal{B}$
- normalization to $\mathcal{B}$ ensures (partial) cancellation of theory uncertainties

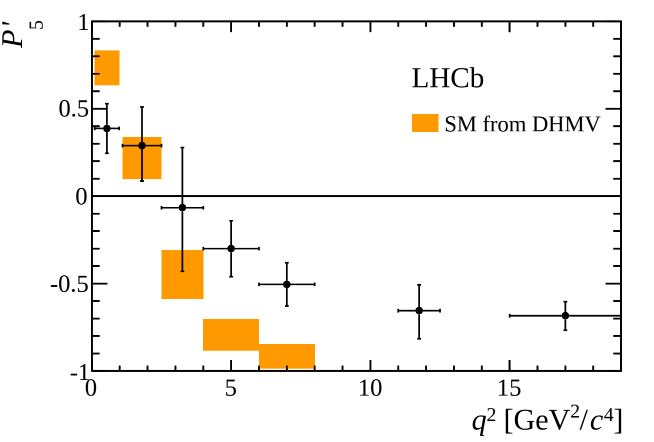
Some of the angular observables (or linear combinations thereof) are better known under other names

- forward-backward asymmetry: how often does the negative charged lepton fly into the opposite direction of the kaon vs in direction of the kaon?

$$A_{\mathrm{FB}} \propto S_{6s} + \frac{1}{2} S_{6c}$$

  Parity violating observable, sensitive to interference of vector and axialvector currents!
- longitudinal polarisation: how often is the kaon longitudinally polarized out of all decays
  more complicated expression, dominantly sensitive to local form factors

But what about $P_5'$?




[LHCb]

But what about $P'_5$?

idea: construct basis of angular observables in which the impact of local form factors ($\mathcal{F}_\lambda$) is reduced. [Descotes-Genon,Matias,Ramon,Virto '12]

- clever use of symmetries among the decay amplitudes
- affected fits when theory and experimental correlations were unknown or only poorly known
- still useful to illustrate tensions between SM predctions and measurements

If experimental and theoretical correlations are accounted for, the choice of basis makes no difference!

# Theory Inputs: Local Form Factors



| | $B \to K$ | $B \to K^*$ | $B_s \to \phi$ | $\Lambda_b \to \Lambda$ |
|---|---|---|---|---|
| # of FFs | 3 | 7 | 7 | 10 |
| $q^2 \lesssim 10\,\text{GeV}^2$ | LCSR ($\times 1$) | LCSR ($\times 2$, *) | LCSR ($\times 2$) | LQCD (†) |
| $q^2 \gtrsim 15\,\text{GeV}^2$ | LQCD ($\times 2$) | LQCD ($\times 1$, *) | LQCD ($\times 1$) | LQCD ($\times 1$) |

LQCD Lattice QCD simulations, systematically improvable

LCSR Light-Cone Sum Rules calculations, with hard-to-quantify systematic uncertainties, with either

- rule of thumb: $\sim 10\%$ uncertainty, but correlations are usually known

$\Rightarrow$ largest impact in branching fraction, but reduced uncertainties in ratios

(*) assuming that the $K^*(892)$, which is a $K\pi$ resonance, can be replaced with a stable bound state

(†) large uncertainties due to extrapolation

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| $q^2 \gtrsim 15\,\text{GeV}^2$ | LQCD $(\times 2)$ | LQCD $(\times 1, *)$ | LQCD $(\times 1)$ | LQCD $(\times 1)$ |

- different excl. decay modes provide complementary systematic effects
- experimental data also provides information on the local form factors

$\Rightarrow$ global analyses: nontrivial crosschecks of the computation methods

! small $q^2$, which drives anomalies, dominated by LCSRs, which are least reliable method

✓ no conceptual problem for LQCD to reach small $q^2$

$\Rightarrow$ good prospects for improvement

$$\mathcal{H} \sim \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\rm em}(x), \sum \mathcal{C}_i O_i(0) \}\, |H_b\rangle$$

- numerically dominant effect from $\bar{s}b\bar{c}c$ operators $O_1^c$ and $O_2^c$, the so-called "charm loop effect"

| | $B \to K$ | $B \to K^*$ | $B_s \to \phi$ | $\Lambda_b \to \Lambda$ |
|---|---|---|---|---|
| # of FFs | 1 | 3 | 3 | 4 |
| $q^2 \lesssim 1\,\mathrm{GeV}^2$ | LCOPE | LCOPE | LCOPE | LCOPE (*) |
| $q^2 \gtrsim 15\,\mathrm{GeV}^2$ | OPE | OPE | OPE | OPE |

OPE reduction to local operators $x^\mu = 0$

LCOPE reduction to operators on the light-cone $x^2 \simeq 0$

(*) next-to-leading power matrix elements cannot presently be computed

both cases: matrix elements of the leading operators are the local form factors

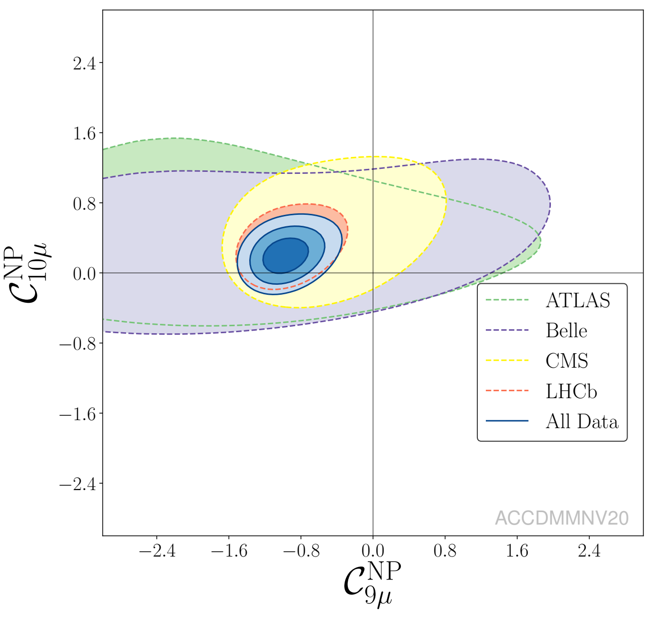
# Phenomenology

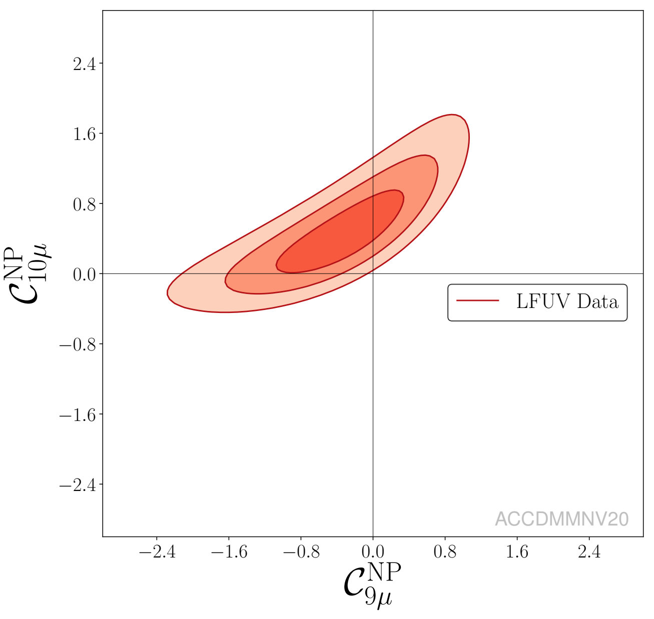
- use universality of $\mathcal{C}_i$ to overconstrain their values from data
- use data on $B \to K^{(*)}\ell^+\ell^-$, $B \to K^*\gamma$, $B_s \to \phi\ell^+\ell^-$, ...
  - available from CLEO
  - available from $B$-factory experiments: BaBar, Belle
  - available from Tevatron experiments: CDF, D0
  - available from LHC experiments: ATLAS, CMS, LHCb
  - LHCb has largest impact in fits due to number of observations and their precision
- make assumptions on relevant $\mathcal{C}_i$
  - 10 per lepton flavour up to mass dimension 6
  - 6 of these can be removed due to smallness observed in data

[Beaujean, Bobeth, Jahn 2015] [Altmannshofer, Niehoff, Straub 2017]

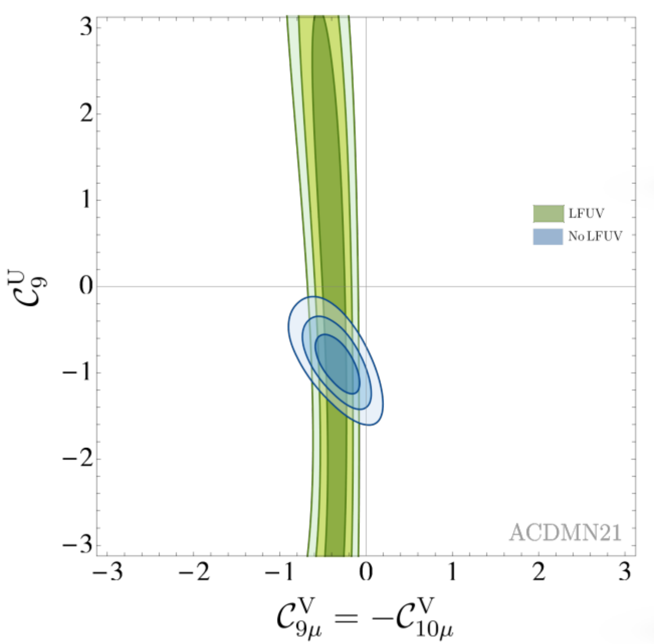
- fit 8 $\mathcal{C}_i$ and $\mathcal{O}(50)$ nuisance parameters (form factors) to theory constraints and $\sim 250$ experimental measurements

- hoping to see Belle 2 and CMS highlighted in the near future!


2.4
1.6
0.8
0.0
−0.8
−1.6
−2.4
$\mathcal{C}_{10\mu}^{\mathrm{NP}}$
−2.4
−1.6
−0.8
0.0
0.8
1.6
2.4
$\mathcal{C}_{9\mu}^{\mathrm{NP}}$
ATLAS
Belle
CMS
LHCb
All Data
ACCDMMNV20



2.4
1.6
0.8
0.0
−0.8
−1.6
−2.4
$\mathcal{C}_{10\mu}^{\rm NP}$
LFUV Data
ACCDMMNV20
−2.4
−1.6
−0.8
0.0
0.8
1.6
2.4
$\mathcal{C}_{9\mu}^{\rm NP}$


LFUV
No LFUV
ACDMN21
$\mathcal{C}_9^{\mathrm{U}}$
$\mathcal{C}_{9\mu}^{\mathrm{V}} = -\mathcal{C}_{10\mu}^{\mathrm{V}}$
3
2
1
0
−1
−2
−3

# Global Fits 2021: Summary



- based on M. Alguero's update talk at Moriond '21 QCD
- measurements do not agree well with SM predictions
  $p$ values $\sim 1\%$ for all obs. and for the LFU subset!
- BSM contributions in $\mathcal{C}_9$-only scenario increase $p$ value to $\sim 40\%$
- Pulls in $\mathcal{C}_9$-only scenario have reached $7\,\sigma$
- Prefered scenario ($7.3\,\sigma$):
  - lepton-flavour universal shift to $e$ and $\mu$: $\mathcal{C}_9^U = -0.92$
  - $\mu$-specific additional shift: $\mathcal{C}_9^V = -\mathcal{C}_{10}^V = -0.30$

- Are all angular momentum states under control? Does $\mathcal{C}_9$ extracted from $\lambda = \perp$ coincide with $\mathcal{C}_9$ extracted from $\lambda = \parallel$?
  (in the past) yes! new analyses need to update checks!

- The Wilson coefficients are $q^2$ agnostic. Do we see a $q^2$ dependence in the shift to $\mathcal{C}_9$?
  no!

- The Wilson coefficient are process agnostic. Do we see deviations in the best-fit point across different processes?
  yes! 2016 – 2019: $\Lambda_b \to \Lambda\mu^+\mu^-$ showed $\mathcal{C}_9^{\text{BSM}} > 0$
  no! since 2019 LHCb erratum and new data

Excellent agreement in all cross checks!

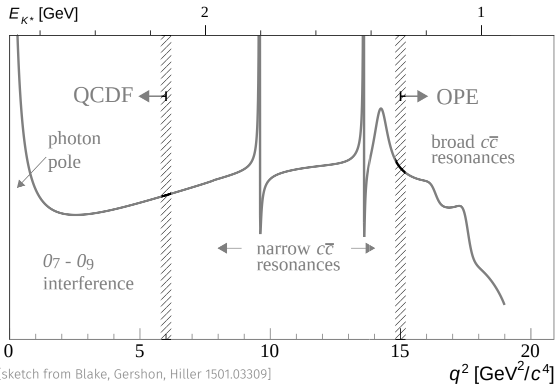
- all present fitting groups rely on following approach:

$$\mathcal{H}_\lambda(q^2) \equiv c_3(q^2) \times \mathcal{F}_\lambda(q^2) + \tilde{\mathcal{V}}_\lambda(q^2)$$

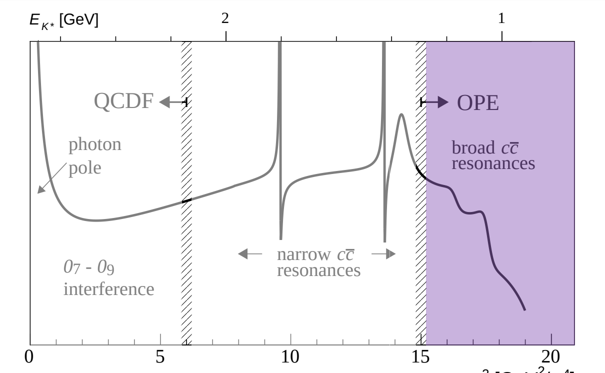
  - use results for local OPE of the nonlocal form factors to leading power ($c_3$)
  - parametrize effect of next-to-leading power operator using $\tilde{\mathcal{V}}_\lambda$
- at $q^2 > 1\,\mathrm{GeV}^2$, no guarantee that this approach works
- need better strategy for precision analyses of present and upcoming data

# Developments

# New Strategy




[sketch from Blake, Gershon, Hiller 1501.03309]

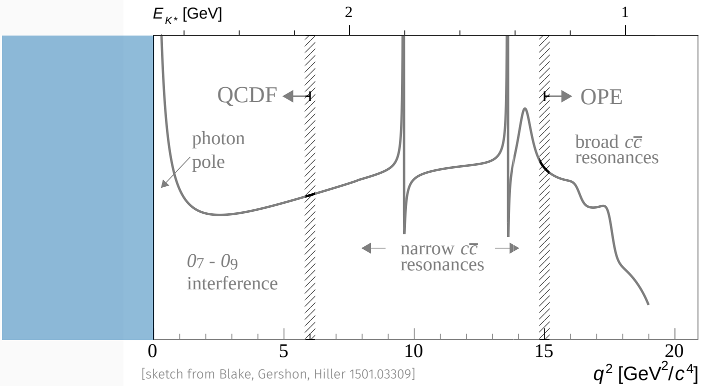


[sketch from Blake, Gershon, Hiller 1501.03309]

- if $|q^2| = \mathcal{O}\left(m_b^2\right)$, expand T-product in local operators
- leading operators have mass dimension three, with universal matching coefficient $c_3(q^2)$

$\Rightarrow \mathcal{H}_\lambda = c_3(q^2)\mathcal{F}_\lambda(q^2) + \dots$

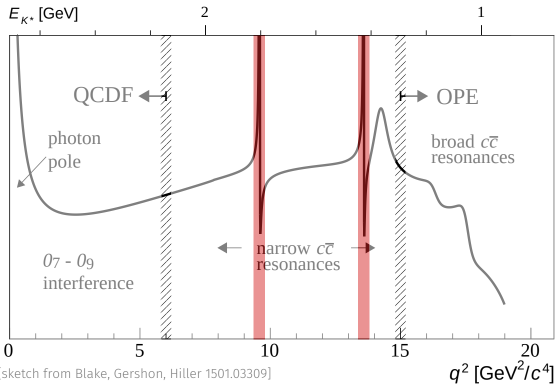
**!** usually applied in integrated form to $q^2 \leq 4M_D^2$




[sketch from Blake, Gershon, Hiller 1501.03309]

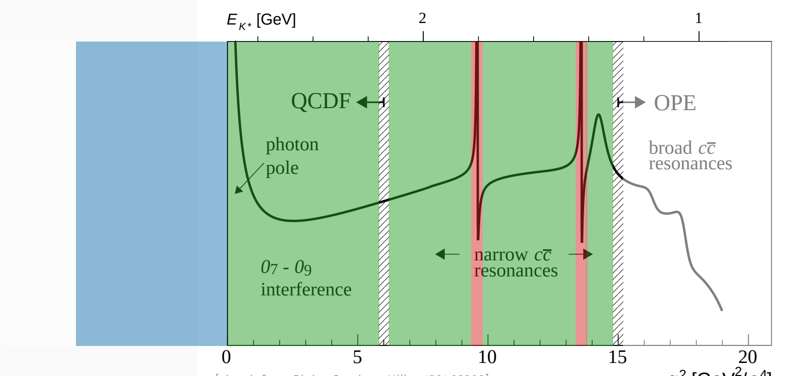
- if $q^2 - 4m_c^2 \ll \Lambda_{\text{had}} m_b$, expand T-product in light-cone operators
- leading operators have mass dimension three, with universal matching coefficient $c_3(q^2)$

$\Rightarrow \mathcal{H}_\lambda = c_3(q^2)\mathcal{F}_\lambda(q^2) + \ldots$



[sketch from Blake, Gershon, Hiller 1501.03309]

- for $q^2 = M^2_{J/\psi}$ and $q^2 = M^2_{\psi(2S)}$, spectrum dominated by hadronic decays
- residues of nonlocal form factors model-independently relate to hadronic decay amplitudes

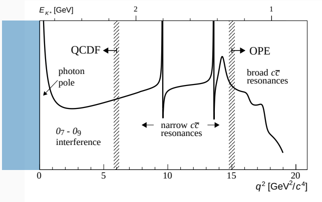



[sketch from Blake, Gershon, Hiller 1501.03309]

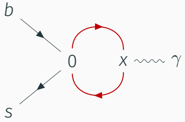
strategy

- compute $\mathcal{H}$ at spacelike $q^2$
- extrapolate $\mathcal{H}$ to timelike $q^2 \leq 4M_D^2$
- include information from hadronic decays $\overline{B} \to \overline{K}^{(*)}\psi_n$
- data-driven approach, ideally carried out with the experimental colleagues

$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$

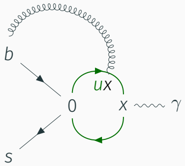



- ▶ expansion in operators at light-like distances $x^2 \simeq 0$ [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- ▶ employing light-cone expansion of charm propagator [Balitsky, Braun 1989]



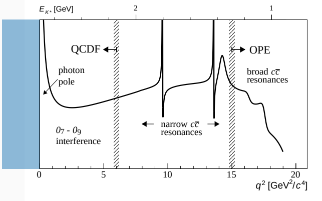

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$+ (\text{coeff \#2}) \times [\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L]$$




$$0 \leq u \leq 1$$

$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$

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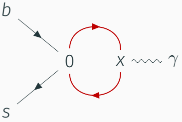


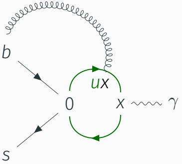


$$0 \leq u \leq 1$$

$$\Rightarrow \mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}} + \text{coeff \#2} \times \tilde{\mathcal{V}}$$

- ▶ leading part identical to QCD Fact. results [Beneke, Feldmann, Seidel 2001&2004]
- ▶ subleading matrix element $\tilde{\mathcal{V}}$ can be inferred from $B$-LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- ▶ recalculating this step we obtain full agreement; cast result in more convenient form

[Gubernari,DvD,Virto '20]

at subleading power in the OPE, need matrix elements of a non-local operator

$$\tilde{\mathcal{V}} \sim \langle M| \bar{s}(0)\gamma^\rho P_L G^{\alpha\beta}(-un^\mu)b(0)\,|B\rangle$$

for $B \to K^{(*)}$ and $B_s \to \phi$ transitions

- matrix element has been calculated in light-cone sum rules

[Khodjamirian et al, 1006.4945]

- physical picture provides that the soft gluon field originates from the $B$ meson
  - analytical results independent of two-particle $b\bar{q}$ Fock state inside the $B$
  - expressions start with three-particle $b\bar{q}G$ Fock state, and their light-cone distribution amplitudes (LCDAs)

$$\langle 0|\,\bar{q}(x)G^{\mu\nu}(ux)\Gamma h_v^b(0)\,|\bar{B}(vM_B)\rangle$$

  - original results missing out on four out of eight three-particle LCDAs

- we recalculate the soft-gluon contributions to the full set of $B \to V$ and $B \to P$ non-local form factors using light-cone sum rules
  - analytic results for restricted set of LCDAs in full agreement with KMPW2010 [Khodjamirian, Mannel, Pivovarov, Wang 2010]
  - result of restricted set fails to reproduce duality thresholds obtained from local form actor sum rules [Gubernari, Kokulu, DvD '18]
  - using the full set of LCDAs, our results reproduce the duality thresholds!
  - our numerical results differ significantly from KMPW2010, but are well understood!

# Soft gluon matrix elements



| Transition | $\tilde{\mathcal{V}}(q^2 = 1\,\text{GeV}^2)$ | GvDV2020 | KMPW2010 |
|---|---|---|---|
| $B \to K$ | $\tilde{\mathcal{A}}$ | $(+4.9 \pm 2.8) \cdot 10^{-7}$ | $(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$ |
| $B \to K^*$ | $\tilde{\mathcal{V}}_1$ | $(-4.4 \pm 3.6) \cdot 10^{-7}\,\text{GeV}$ | $(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\text{GeV}$ |
| | $\tilde{\mathcal{V}}_2$ | $(+3.3 \pm 2.0) \cdot 10^{-7}\,\text{GeV}$ | $(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\text{GeV}$ |
| | $\tilde{\mathcal{V}}_3$ | $(+1.1 \pm 1.0) \cdot 10^{-6}\,\text{GeV}$ | $(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\text{GeV}$ |
| $B_s \to \phi$ | $\tilde{\mathcal{V}}_1$ | $(-4.4 \pm 5.6) \cdot 10^{-7}\,\text{GeV}$ | — |
| | $\tilde{\mathcal{V}}_2$ | $(+4.3 \pm 3.1) \cdot 10^{-7}\,\text{GeV}$ | — |
| | $\tilde{\mathcal{V}}_3$ | $(+1.7 \pm 2.0) \cdot 10^{-6}\,\text{GeV}$ | — |

reduction by a factor of $\sim 200$

- new structures in three-particle LCDAs account for factor 10
- updated inputs that enter the sum rules (mostly) linearly account for further factor 10
- similar relative uncertainties, but absolute uncertainties reduced by $\mathcal{O}(100)$

- Taylor expand $\mathcal{H}_\lambda$ in $q^2/M_B^2$ around 0 [Ciuichini et al. '15]
  - + simple to use in a fit
  - - incomaptible with analyticity properties, does not reproduce resonances
  - - expansion coefficients unbounded!

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- use information from hadronic intermediate states in a dispersion relation [Khodjamirian et al. '10]

  $$\mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(q^2) = \int ds \frac{\text{Im}\,\mathcal{H}_\lambda(s)}{(s-s_0)(s-q^2)} + \dots$$

  - \+ reproduces resonances
  - \- hadronic information above the threshold must be modelled
  - \- complicated to use in a fit, relies on theory input in single point $s_0$

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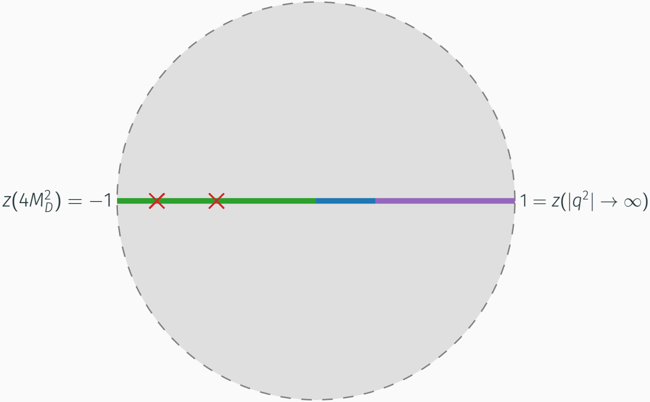
  - \+ reproduces resonances
  - \- hadronic information above the threshold must be modelled
  - \- complicated to use in a fit, relies on theory input in single point $s_0$
- expand the matrix elements in variable $z(q^2)$ that develops branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]
  - \+ resonances can be included through explicit poles (Blaschke fact.)
  - \+ easy to use in a fit
  - \+ compatible with analyticitiy properties
  - \- expansion coefficients unbounded!




light-cone OPE

SL phase space

$J/\psi$,$\psi(2S)$

local OPE

matrix elements $\mathcal{H}$ arise from non-local operator

$$O^\mu(Q;x) \sim \int e^{iQ\cdot y}\, T\{J^\mu_{\text{em}}(x+y), [C_1 O_1 + C_2 O_2](x)\}$$

construct four-point operator to derive a dispersive bound

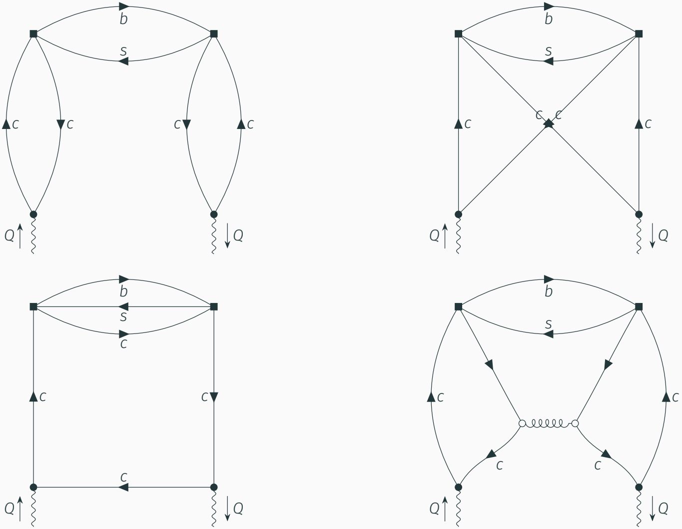
- define matrix element of “square“ operator

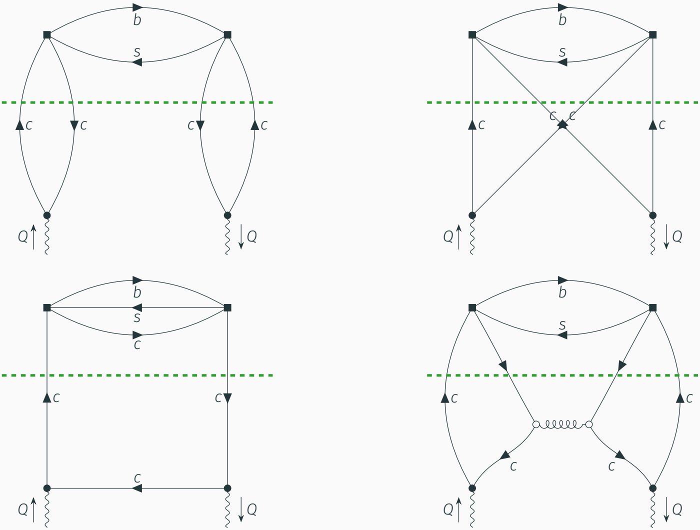
$$\left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right] \Pi(Q^2) \equiv \int e^{iQ\cdot x} \langle 0|\, T\{O^\mu(Q;x) O^{\dagger,\nu}(Q;0)\}\, |0\rangle$$

- as hermiatian operator, vacuum eigenvalues are positive definite!
- for $Q^2 < 0$ we find that $\Pi(Q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

# Cuts of Π



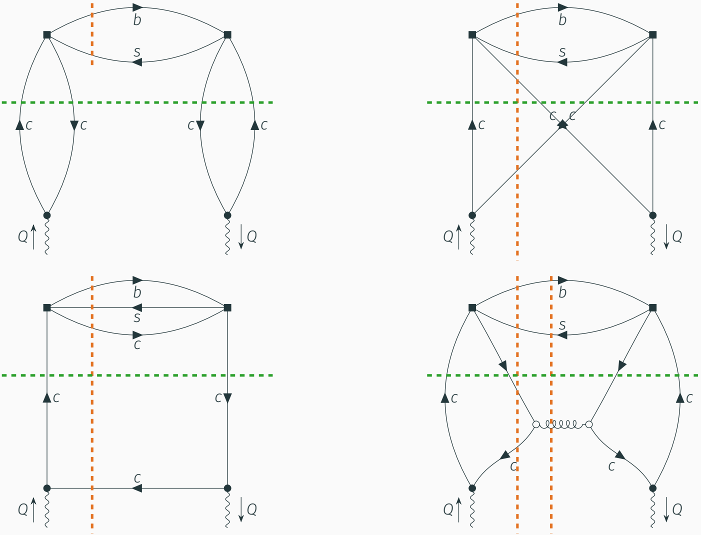





- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)

# Extrapolate Cuts of Π






- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, …)
- from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, …)

# Dispersion relation for Π



dispersive representation of the $b\bar{s}$ contribution to derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi}\int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\,\Pi(s)}{s-Q^2}$$

positive definite for $Q^2 < 0$

- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be computed in the local OPE
  $\rightarrow$ yields $\chi^{\mathrm{OPE}}(Q^2)$
- OPE result indicates that two derivatives are needed for convergence of dispersive integral
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be expressed in terms of the matrix elements $\mathcal{H}_\lambda$
  $\rightarrow$ yields $\chi^{\mathrm{had}}(Q^2)$
- global quark hadron duality suggests that both $\chi^{\mathrm{had}}$ and $\chi^{\mathrm{OPE}}$ are equal
  $\rightarrow$ yields a dispersive bound

the hadronic represenation reads schematically:

$$\chi^{\mathrm{OPE}}(Q^2) \geq \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \int_{(m_b+m_s)^2}^{\infty} ds \sum_{\lambda} \frac{\omega_\lambda(s)\,|\mathcal{H}_\lambda(s)|^2}{s-Q^2}$$

- aim: diagonalize this expression

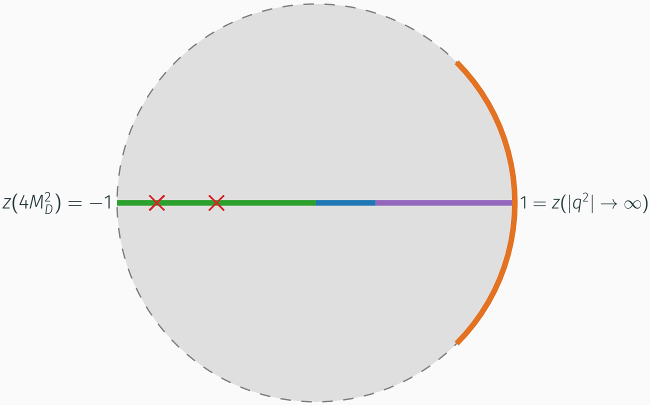
Ansatz:

$$\hat{\mathcal{H}}_\lambda(q^2) \equiv P(q^2) \times \phi_\lambda(q^2) \times \mathcal{H}_\lambda(q^2) \equiv \sum_n a_{\lambda,n} f_n(q^2)$$

- Blaschke factors $P$ remove narrow charmonia poles
- outer functions $\phi_\lambda$ account for weight function $\omega_\lambda$ and Cauchy integration kernel
- orthonormal functions $f_n$ diagonalizes remainder of the expression

normalisation to $\chi^{\mathrm{OPE}}$ leads to a diagonal bound

$$1 \geq \sum_\lambda \sum_n |a_{\lambda,n}|^2$$




light-cone OPE

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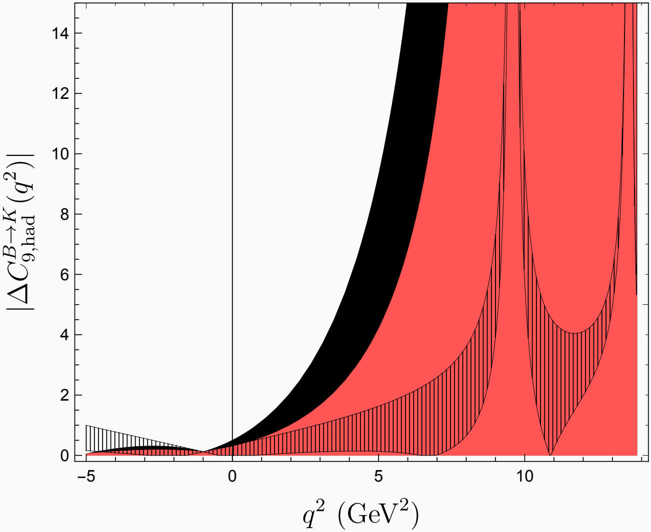
local OPE

int. domain

simple exercise: bound on the shift to $C_9$ from nonlocal form factors, assuming only two data points at negative $q^2$




$$\frac{1}{2} > \sum_n |a_n|^2$$

$$\frac{1}{11} > \sum_n |a_n|^2$$

- drawback: basis of orthonormal polynomials $f_n(z)$ behaves pathologically for $\mathrm{Re}\, z < 0$
  - $|f_n(-1)| \sim C^n$ with $C \geq 1$
  - can be partially alleviated by chosing free parameter $t_0$ in definition of $z$
- we do not currently claim control of the truncation error, rather, only a handle
- actively looking into alternative formulations of the dispersive bound that evade the pathological behaviour

# Summary

# Summary and Outlook



- anomalies make exclusive $b \to s\ell\ell$ decays an exciting research topic
- tensions mandate hightened scrutiny of theory assumptions and inputs
  - nonlocal form factors contribute the single-largest systematic uncertainty in exclusive $b \to s\ell\ell$ decays
  - I think there is a clear road toward a reliable description of these objects, but much work still needs to be done
- key is a combined theory + data driven approach
  - new developments show path in this direction
- looking forward to both upcoming phenomenological applications and upcoming experimental results