

# Probing $\mu e \gamma \gamma$ contact interaction with $\mu \rightarrow e$ conversion

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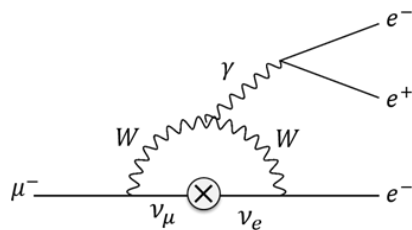
S. Davidson, Y. Kuno, Y. Uesaka, M.Y., PRD102 (2020)

# $\mu - e$ conversion $\mu^- + N(A, Z) \rightarrow e^- + N(A, Z)$

Lepton Flavor Violation (LFV):  
Clear evidence of new physics scenarios

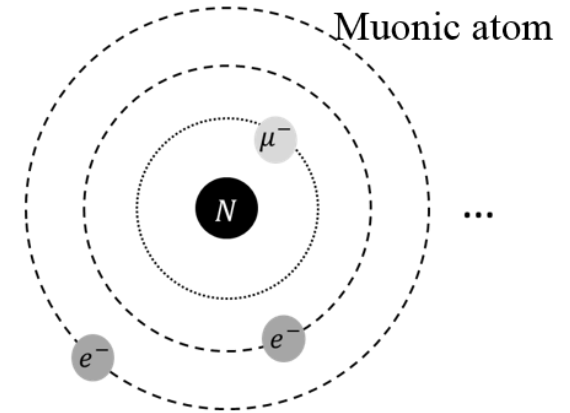
One of the most promising LFV process:  
 **$\mu \rightarrow e$  conversion in nuclei**

- ❑ Clean signal
- ❑ Versatile and sensitive probe
- ❑ Synergy with LHC/DIS experiments

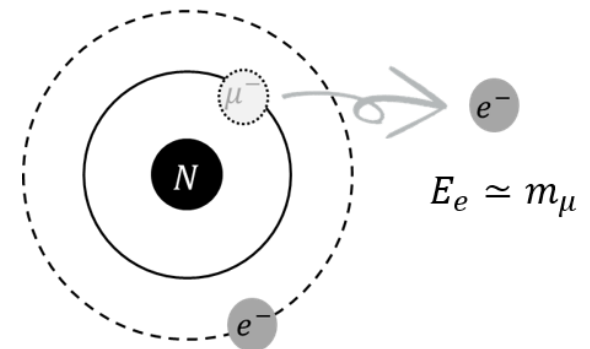


$$\text{BR} \approx 7.4 \times 10^{-55}$$

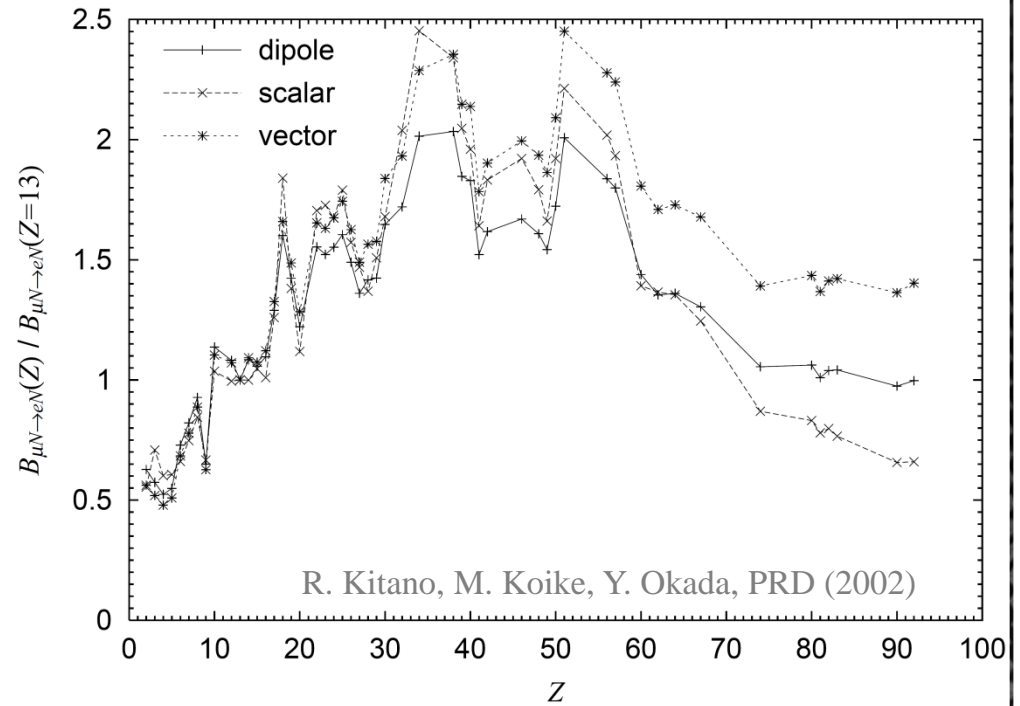
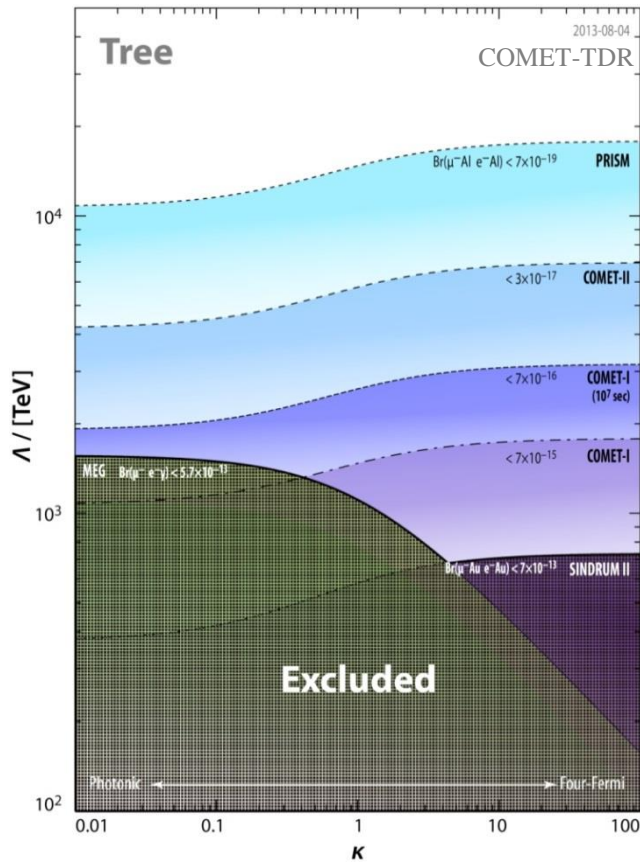
Too small to access  
experimentally



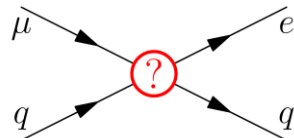
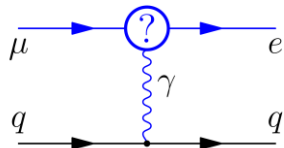
If lepton flavor is  
violated...



# Versatile and sensitive probe to LFV operators



$$\mathcal{L} = \frac{1}{1 + \kappa} \frac{m_\mu}{\Lambda^2} \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + \frac{\kappa}{1 + \kappa} \frac{1}{\Lambda^2} (\bar{\mu}_L \gamma^\mu e_L) (\bar{q}_L \gamma_\mu q_L)$$



It is not only a discovery channel,  
also has the potential to identify the  
types of LFV operator

# Unknown Z dependence

COMET/DeeMe/Mu2e experiments  
will search for  $\mu \rightarrow e$  conversion **with**  
**different targets**

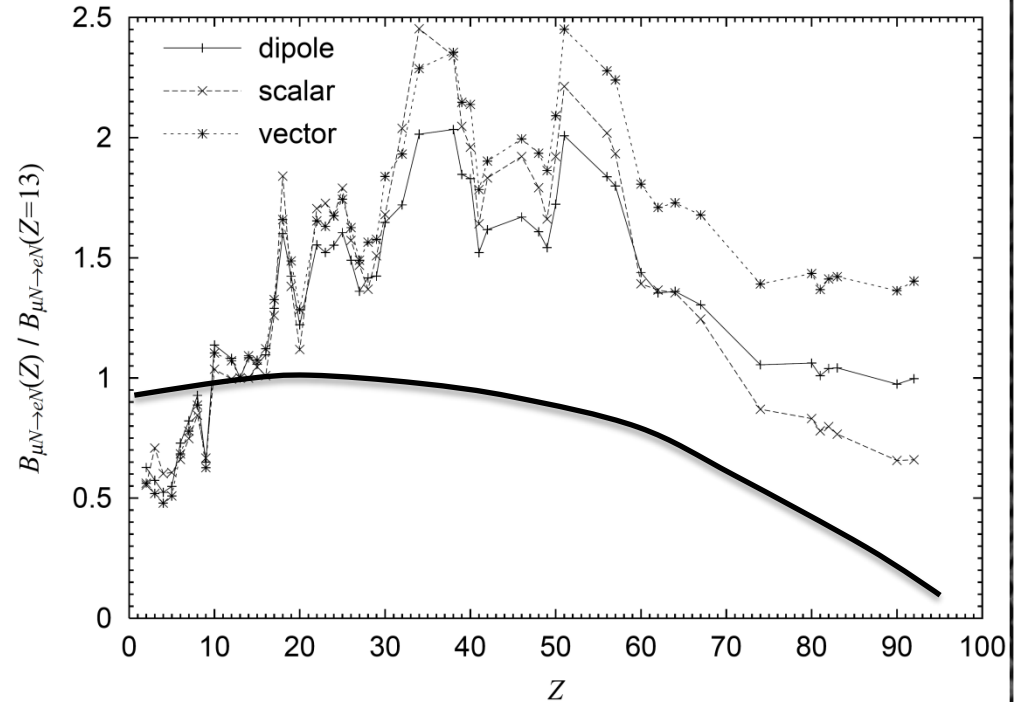
**If** : unknown Z dependence is observed

How to interpret the results?

Erroneous measurement?

Miscalculation??

or something???



**Complete all of types of LFV ope. to evade such confusion**

# LFV operator $\bar{e}\mu FF$

An LFV operator from new physics scenarios

L. Calibbi, D. Redigolo, R. Ziegler, J. Zupan, 2006.04795,  
J. Heeck and H. H. Patel, PRD100 (2019), etc.

$$\mathcal{L} = \frac{1}{v^3} (C_{FF,L} \bar{e} P_L \mu F_{\alpha\beta} F^{\alpha\beta} + C_{FF,R} \bar{e} P_R \mu F_{\alpha\beta} F^{\alpha\beta})$$
$$\propto C_{FF,L} \bar{e} P_L \mu \gamma\gamma + C_{FF,R} \bar{e} P_R \mu \gamma\gamma$$

**LFV process :  $\mu \rightarrow e\gamma\gamma$**

Experimental bound

$$\text{BR}(\mu \rightarrow e\gamma\gamma) \leq 7.2 \times 10^{-11}$$

Crystal Box Experiment, PRD38 (1988)

Bound on couplings

$$|C_{FF,L}|^2 + |C_{FF,R}|^2 \leq 2.2 \times 10^{-2}$$

J.D.Bowman, T.P.Cheng, L.F.Li, H.S.Matis, PRL41 (1978)

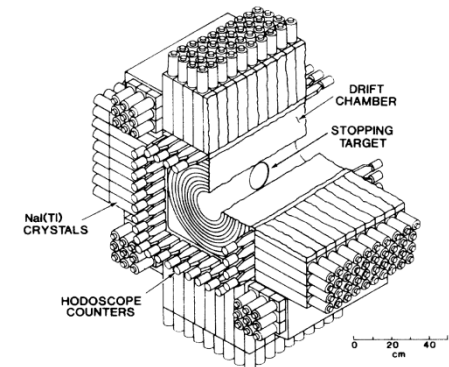
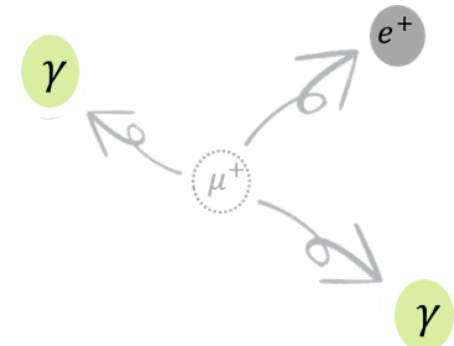


FIG. 2. A schematic cutaway diagram of the Crystal Box detector.

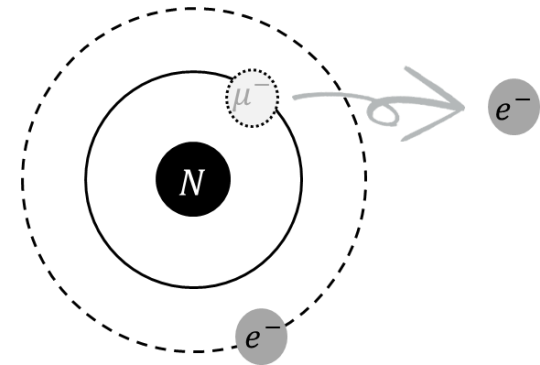
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**LFV process :  $\mu \rightarrow e$  conversion**



Experimental bound

$$\text{BR}(\mu \rightarrow e \text{ conv.}) \leq 7 \times 10^{-13}$$

SINDRUM-II, EPJC47 (2006)

Bound on couplings

$$|C_{FF,L}|^2 + |C_{FF,R}|^2 \leq ???$$

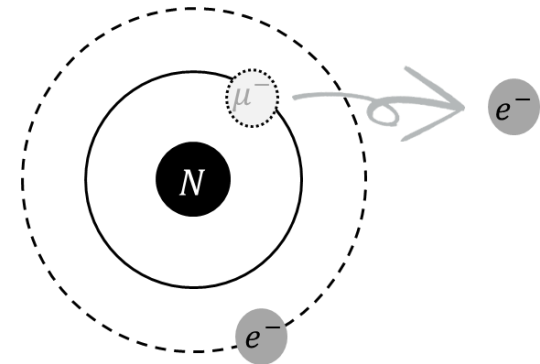
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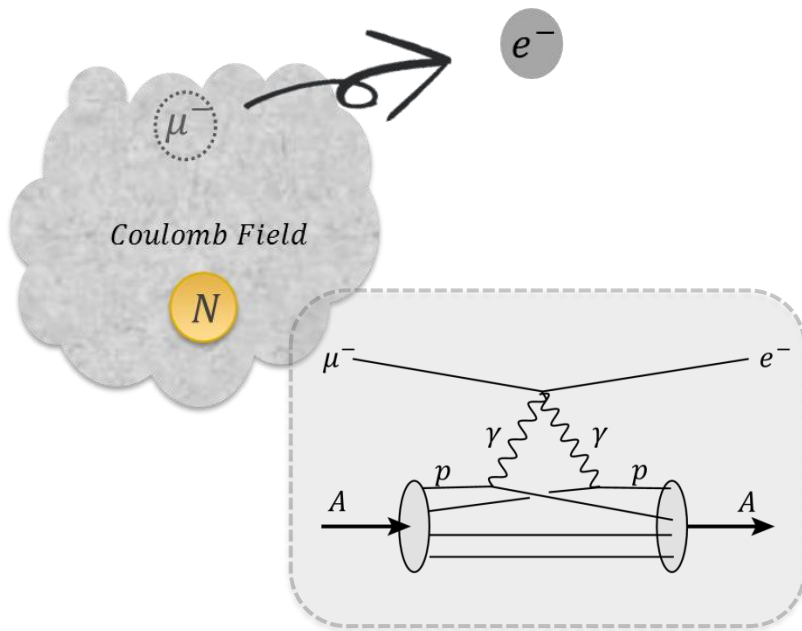
$$|C_{FF,L}|^2 + |C_{FF,R}|^2 \leq ???$$

Aim of this work

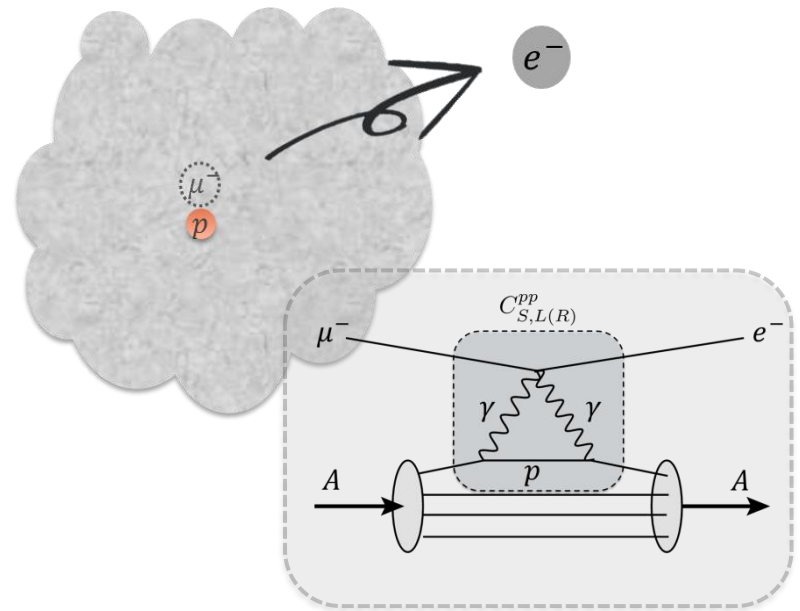
- ❑ How sensitive to  $\bar{e}\mu FF$  is  $\mu \rightarrow e$  conv.?
- ❑ What is target dependence of conv. rate?

# Subprocesses of $\mu \rightarrow e$ conversion via $\bar{e}\mu FF$

Conversion in the classical electric field  
@ momentum transfer  $\sim m_\mu$



Conversion via the effective scalar op.  
@ momentum transfer  $\gtrsim m_p$



**Rate  $\propto$  (overlap of  $\mu$ ,  $e$ , and  $E^2$ )  $\propto Z^2$**

**Rate  $\propto$  (overlap of  $\mu$ ,  $e$ , and  $p$ )  $\propto Z$**



# $\mu \rightarrow e$ conversion in the classical electric field

## LFV operator

$$\mathcal{L} = \frac{1}{v^3} (C_{FF,L} \bar{e} P_L \mu F_{\alpha\beta} F^{\alpha\beta} + C_{FF,R} \bar{e} P_R \mu F_{\alpha\beta} F^{\alpha\beta})$$

## Amplitude

$$\mathcal{M} = \frac{1}{v^3} \int d^3r \bar{\psi}_e (C_{FF,L} P_L + C_{FF,R} P_R) \psi_\mu^{1s} \langle N | F_{\alpha\beta} F^{\alpha\beta} | N \rangle$$

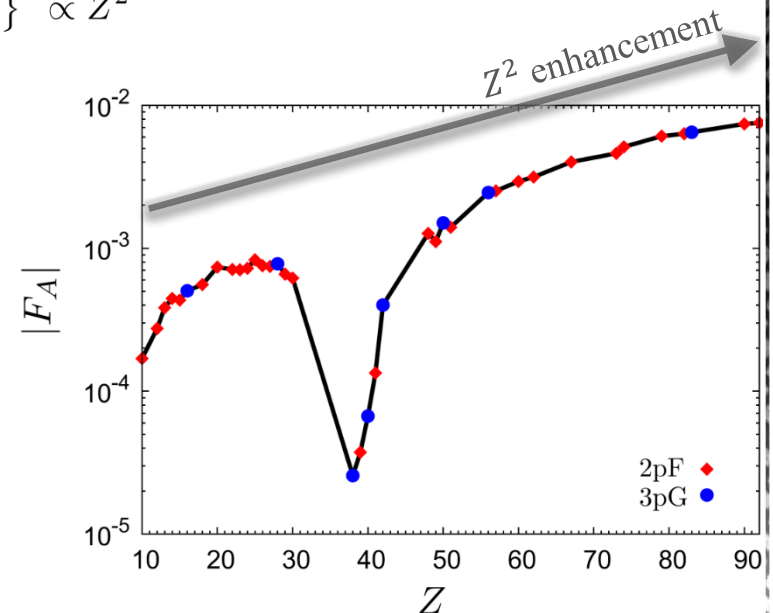
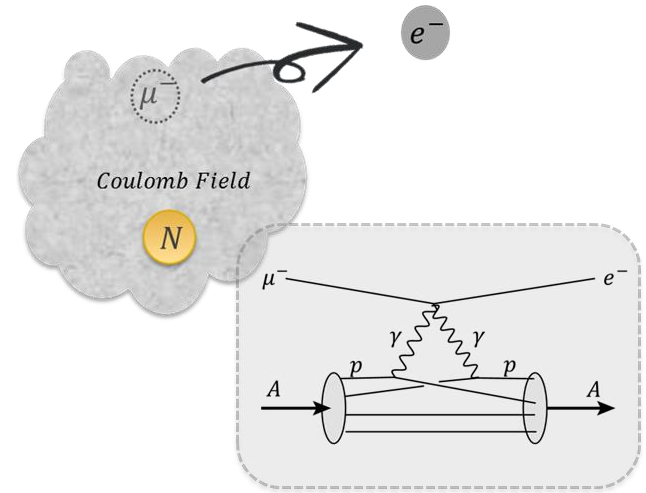
$$\propto -2 \{E(r)\}^2 \propto Z^2$$

## Conversion probability

$$\Gamma_{\text{conv}} = 16 G_F^2 m_\mu^5 \underbrace{|F_A|^2}_{\text{Overlap of wave functions and electric field}} \left(\frac{m_\mu}{v}\right)^2$$

$$\times \left( |C_{FF,L} + C_{FF,R}|^2 + |C_{FF,L} - C_{FF,R}|^2 \right)$$

Overlap of wave functions  
and electric field



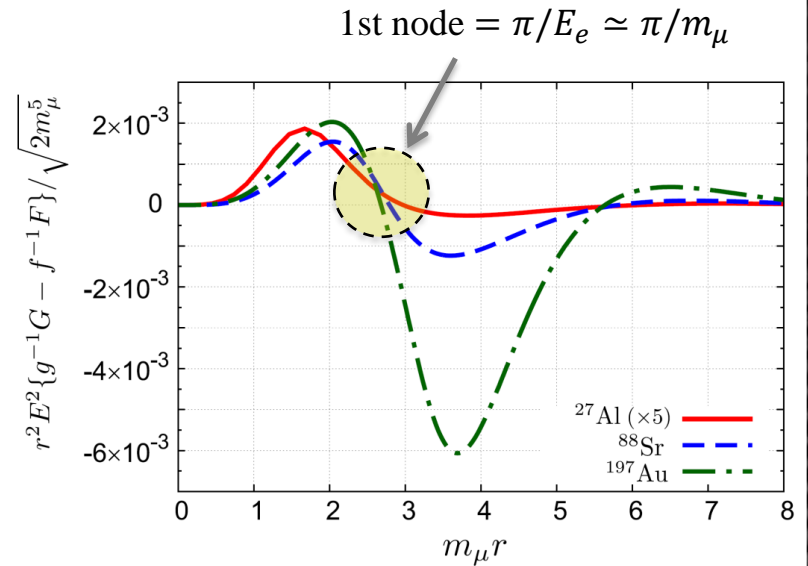
# $\mu \rightarrow e$ conversion in the classical electric field

$$F_A = \frac{1}{\sqrt{2m_\mu^7}} \int dr r^2 E^2(r) \left[ g^{-1}(r)G(r) - f^{-1}(r)F(r) \right]$$

Radial wave func. of  $\mu$

Radial wave func. of  $e$

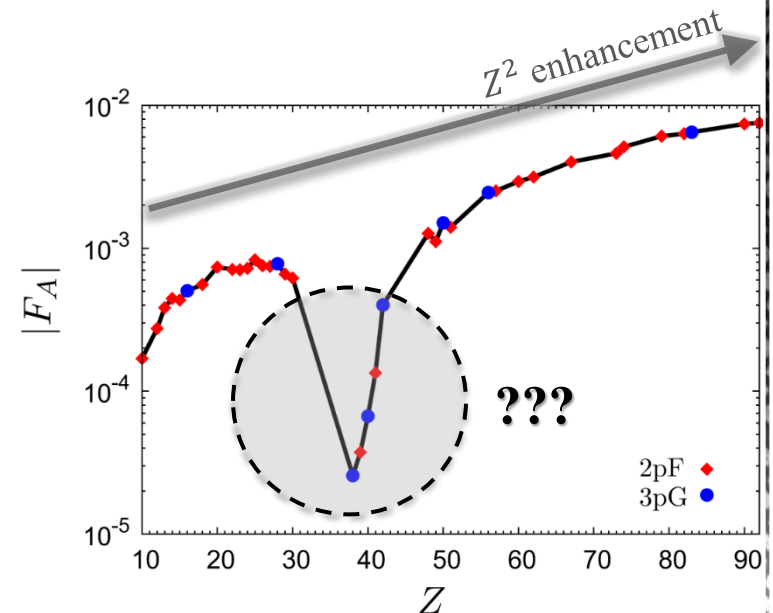
Accidental cancellation between  **$E^2$ -boosted wave functions inside electron 1st node** and **wave functions outside the 1st node**



## Conversion probability

$$\Gamma_{\text{conv}} = 16G_F^2 m_\mu^5 \underbrace{|F_A|^2}_{\text{Overlap of wave functions and electric field}} \left( \frac{m_\mu}{v} \right)^2 \times \left( |C_{FF,L} + C_{FF,R}|^2 + |C_{FF,L} - C_{FF,R}|^2 \right)$$

Overlap of wave functions and electric field



# $\mu \rightarrow e$ conversion via the effective scalar operator

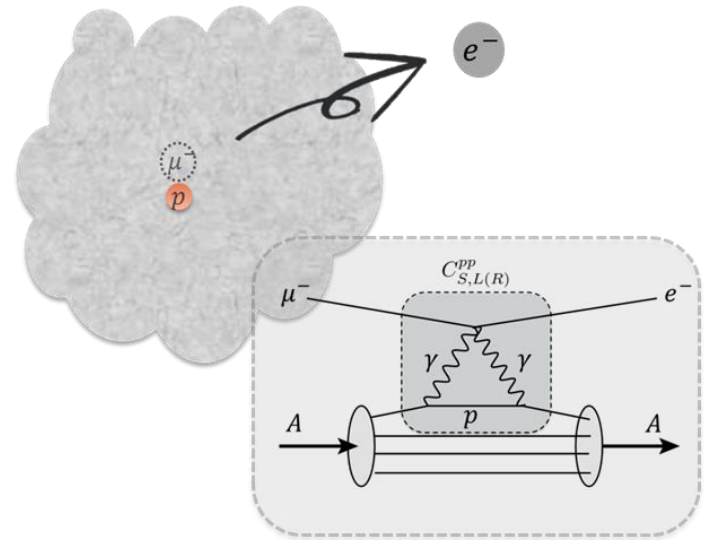
Off-shell photon @ momentum transfer  $\gtrsim m_p$

Effective scalar operator

$$\mathcal{O}_{S,L(R)}^{pp} = C_{S,L(R)} (\bar{e} P_{L(R)} \mu) (\bar{\psi}_p P_{L(R)} \psi_p)$$

Effective coefficient in the RGE of QED

$$\begin{aligned} C_{S,L(R)}^{pp} &= -\frac{6\alpha_{\text{em}} m_p}{\pi v} \ln\left(\frac{2 \text{ GeV}}{m_\mu}\right) C_{FF,L(R)} \\ &= -2.26 \times 10^{-4} C_{FF,L(R)} \end{aligned}$$



# Total branching ratio for $\bar{e}\mu FF$ operator

BR for the  $\bar{e}\mu FF$  operator

$$\frac{BR(\mu A \rightarrow e A)}{|C_{FF,L}|^2 + |C_{FF,R}|^2} = \begin{cases} 6.6 \times 10^{-9} | +1 + 15 |^2 & \text{for } ^{27}\text{Al} \\ 9.1 \times 10^{-8} | -1 + 3.8 |^2 & \text{for } ^{197}\text{Au} \end{cases}$$

(Loop contribution) > (Tree contribution)???

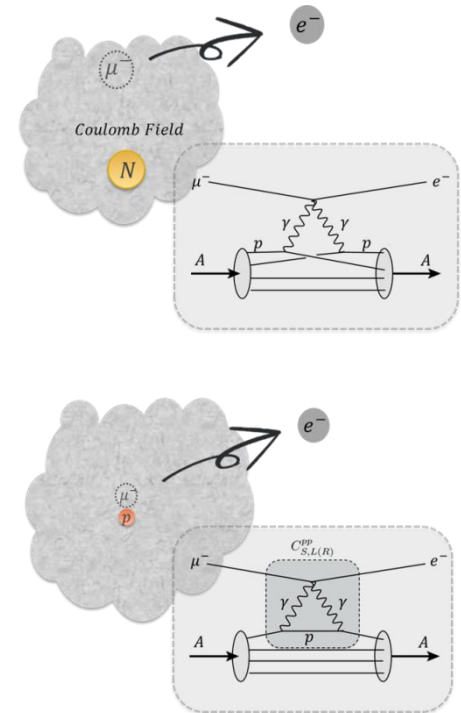
Reason : (overlap with proton  $S_A^{(p)}$ ) > (overlap with electric field  $F_A$ )

$$\left| \frac{F_A}{S_A^{(p)}} \right| \simeq \frac{2m_\mu^{-1} [Ze/(4\pi R^2)]^2}{Z(4\pi R^3/3)^{-1}} = \frac{2Z\alpha}{3m_\mu R} \simeq \begin{cases} 0.02 & \text{for } ^{27}\text{Al} \\ 0.06 & \text{for } ^{197}\text{Au} \end{cases}$$

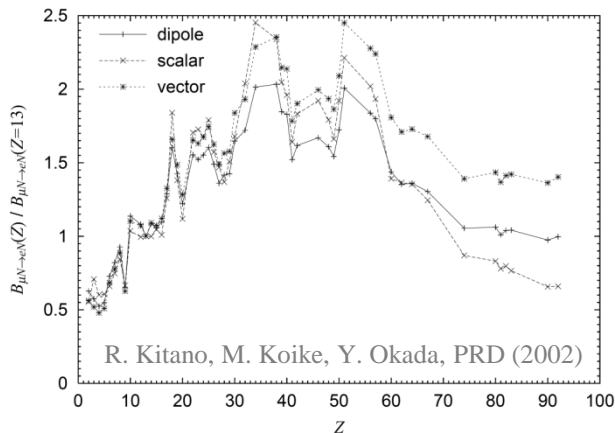
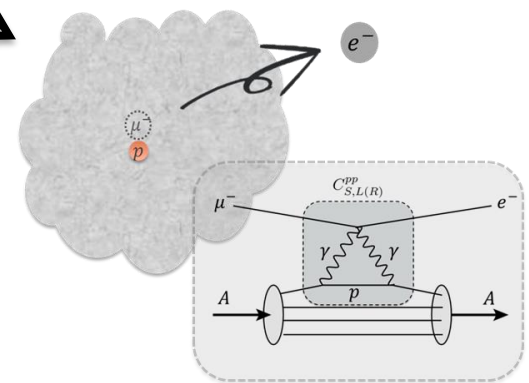
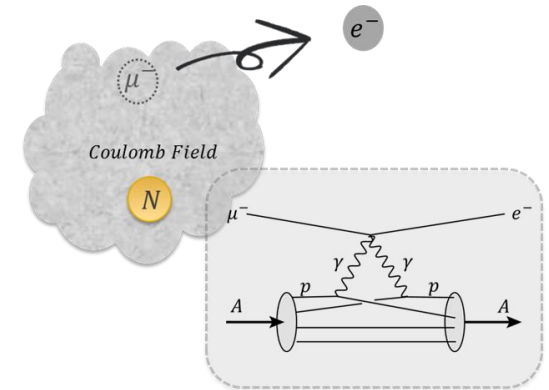
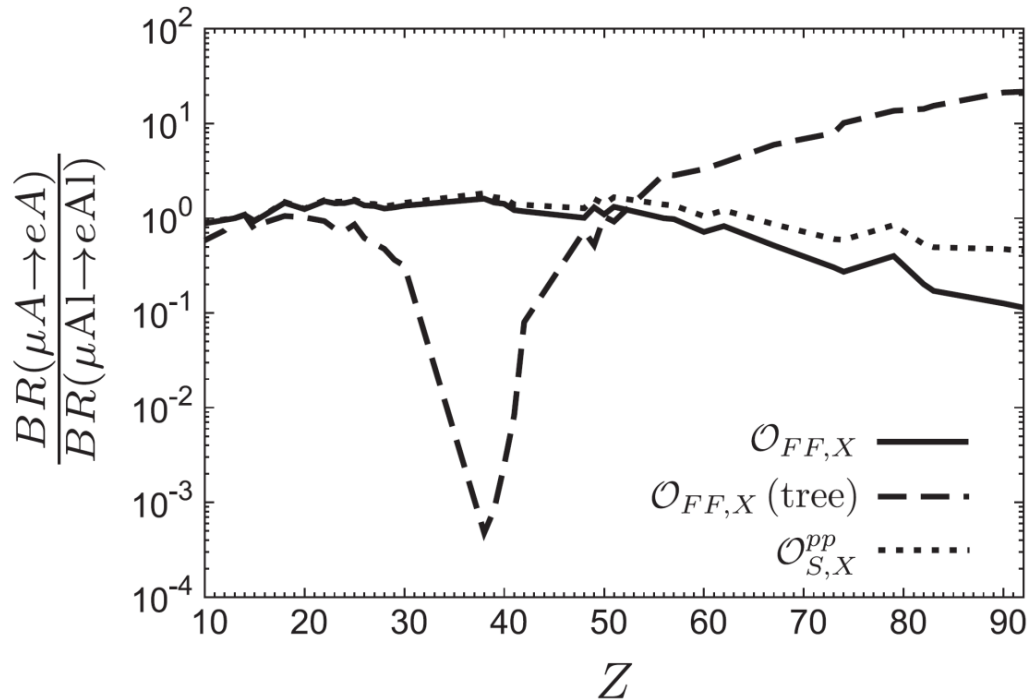
Approximation

Nuclear electric field  $|\vec{E}(r)| \simeq \frac{Ze}{4\pi R^2}$   $\because E(r)$  is maximized at  $r \sim R$  ( $R$ : nuclear radius)

Proton density  $\rho \simeq Z\left(\frac{4\pi R^3}{3}\right)^{-1}$   $\because$  Assuming a uniform distribution



# Target dependence of BR



- ◆ Conversion in the electric field  $\propto Z^2$
- ◆ Two subprocesses destructively contribute at high- $Z$
- ◆ Different  $Z$  dependence from other types of operators

# Constraint on the $\bar{e}\mu FF$ interaction

Current constraint on  $\bar{e}\mu FF$  coupling

Coefficient	Constraint	Process
$ C_{FF,X} + im_\mu C_{VFF,Y}/(4v) $	$< 2.2 \times 10^{-2}$	$\text{BR}(\mu \rightarrow e\gamma\gamma) < 7.2 \times 10^{-11}$
$ C_{F\tilde{F},X} + im_\mu C_{VF\tilde{F},Y}/(4v) $	$< 2.2 \times 10^{-2}$	$\text{BR}(\mu \rightarrow e\gamma\gamma) < 7.2 \times 10^{-11}$
$ \dots + C_{FF,X} $	$< 1.0 \times 10^{-3}$	$\text{BR}(\mu\text{Au} \rightarrow e\text{Au}) < 7 \times 10^{-13}$

◆  $\mu \rightarrow e$  conversion set the most stringent constraint!

◆ COMET/Mu2e sensitivity

$$C_{FF,L(R)} \leq 7.6 \times 10^{-6} \left( \frac{\text{BR}(\mu\text{Al} \rightarrow e\text{Al})}{10^{-16}} \right)^{1/2}$$

◆ Sensitive to the LFV mediator which dominantly couples with heavy flavors, like Higgs

# Summary

- ❑  $\mu \rightarrow e$  conversion: Not only a discovery channel, also has the potential to identify the LFV operator
- ❑ Complete all of types of LFV ope. to avoid the confusion from unknown  $Z$  dependence of  $\mu \rightarrow e$  conversion rate
- ❑  **$\mu \rightarrow e$  conversion via  $\bar{e}\mu\gamma\gamma$  operator**
- ❑  $\mu \rightarrow e$  conversion set the most stringent constraint on the  $\bar{e}\mu\gamma\gamma$  operator
- ❑ Different  $Z$  dependence from other types of LFV operators

