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Time evolution of Lepton family Number
carried by Majorana neutrinos
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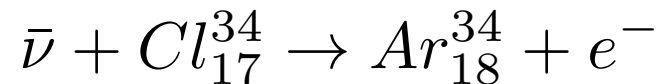
1 Introduction and Motivation

The properties of neutrinos are not fully understood yet.

- One central question is that whether neutrino is Majorana or Dirac particle ?
- How one can discriminate Majorana and Dirac ?
- To discriminate them,
one needs to go to very low momentum $|q| < m_\nu$.
- Temperature of CNB (Cosmic Neutrino Background) is
 $T_{CNB} \simeq 2K \sim 2 \times 10^{-4} \text{ (eV)} < m_\nu = 10^{-3} \text{ (eV)}$.
- The formula for the time evolution of lepton family number is derived for Majorana neutrinos.
- It is valid for relativistic and non-relativistic case.

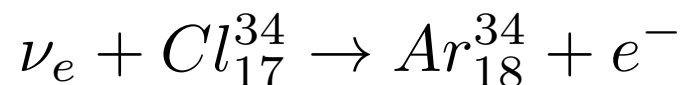
2 Background(<https://www.nobelprize.org/prizes/physics/2002/davis/facts/>)
See also Jose Bernabeu , Symmetry 2020,12,1316, S.M. Bilenky,
arXiv:Physics/0603039v3)

- Bruno Pontecorvo (1946) suggested the following process:



Raymond Davis Jr tried to observe the process and failed to detect it.

- If it would occur, this indicates the neutrino is a Majorana particle through $\bar{\nu}_e \rightarrow \nu_e$ oscillation.



3 Our set up

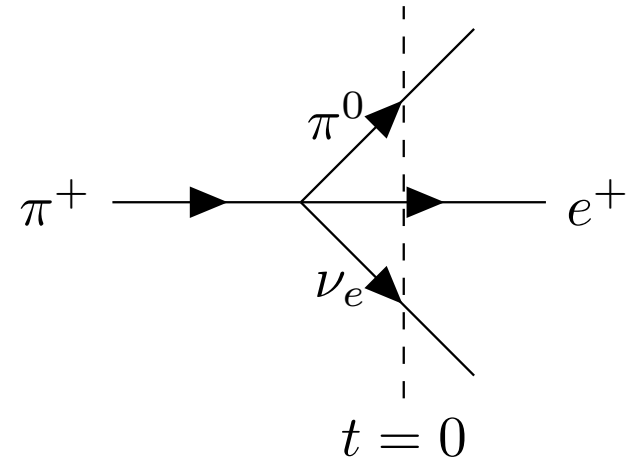
To investigate the Lepton Number under the presence of Majorana mass term, we consider the situation that the Majorana mass term is turned on at $t = 0$.

For $t < 0$: massless, $t > 0$: massive.

The situation corresponds to the

following Lagrangian: ($\alpha = e, \mu, \tau, i = 1, 2, 3$).

$$\begin{aligned} \mathcal{L} &= \overline{\nu_{\alpha L}} i \gamma^\mu \partial_\mu \nu_{\alpha L} - \theta(t) \frac{m_{\alpha\beta}}{2} \left(\overline{(\nu_{\alpha L})^c} \nu_{\beta L} + h.c. \right) \\ &= \frac{1}{2} \overline{\psi_i} i \gamma^\mu \partial_\mu \psi_i - \theta(t) \frac{m_i}{2} \overline{\psi_i} \psi_i. \end{aligned}$$



where the second line is written in terms of the mass eigenstate for the Majorana field, $\psi_i^c = \psi_i$,

$$\begin{aligned} \psi_i &= \nu_{iL} + (\nu_{iL})^c, \quad \nu_{\alpha L} = V_{\alpha i} \nu_{iL}, \\ m_i \delta_{ij} &= (V^T)_{i\alpha} m_{\alpha\beta} V_{\beta j}. \end{aligned}$$

4 Continuity Condition

The field operator is continuous at $t = 0$.

Massless

Massive

$$\nu_{L\alpha}(t = 0_- < 0, \mathbf{x}) = V_{\alpha i} P_L \psi_i(t = 0_+ > 0, \mathbf{x}), P_L = \frac{1 - \gamma_5}{2}.$$

DOF: Massless neutrino $a_\alpha(\mathbf{p})$ (LH neutrino) $b_\alpha(\mathbf{p})$ (RH anti-neutrino).

Massive Majorana $a_{Mi}(\mathbf{p}, \lambda = \pm 1)$ (λ : helicity).

$$LHS = \int' \frac{d^3 \mathbf{p}}{(2\pi)^3 2|\mathbf{p}|} \left(a_\alpha(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{x}} u_L(\mathbf{p}) + b_\alpha^\dagger(\mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{x}} v_L(\mathbf{p}) \right),$$

$$RHS = V_{\alpha i} P_L \int' \frac{\sum_{\lambda=\pm 1} d^3 \mathbf{p}}{(2\pi)^3 2E(\mathbf{p})} \left(a_{Mi}(\mathbf{p}, \lambda) u_i(\mathbf{p}, \lambda) e^{i\mathbf{p}\cdot\mathbf{x}} + a_{Mi}^\dagger(\mathbf{p}, \lambda) v_i(\mathbf{p}, \lambda) e^{-i\mathbf{p}\cdot\mathbf{x}} \right).$$

5 Relation between operators and time evolution

Relation bet. flavor operator and operator for mass eigenstate:

$$\frac{1}{\sqrt{2|\mathbf{p}|}} \begin{pmatrix} V_{\alpha j}^* a_{\alpha}(\mathbf{p}) \\ V_{\alpha j} a_{\alpha}^{\dagger}(-\mathbf{p}) \end{pmatrix} = \frac{\sqrt{N_j(\mathbf{p})}}{2E_j(\mathbf{p})} \begin{pmatrix} 1 & \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} \\ \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} & 1 \end{pmatrix} \begin{pmatrix} a_{Mj}(\mathbf{p}, -) \\ a_{Mj}^{\dagger}(-\mathbf{p}, -) \end{pmatrix},$$

$$N_{jp} = E_j(\mathbf{p}) + |\mathbf{p}|, E_j(\mathbf{p}) = \sqrt{|\mathbf{p}|^2 + m_j^2}. \quad (\alpha, \gamma = e, \mu, \tau, j = 1 \sim 3)$$

Time evolution of flavor operator:

$$a_{\alpha}(\mathbf{p}, t) = \sum_{j, \gamma} V_{\alpha j} V_{\gamma j}^* \left(\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}| \sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})} \right) a_{\gamma}(\mathbf{p}) \\ - \sum_{j, \gamma} V_{\alpha j} V_{\gamma j} \frac{m_j \sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})} a_{\gamma}^{\dagger}(-\mathbf{p}),$$

For an anti-neutrino operator $b_{\alpha}(\mathbf{p}, t)$, just replace $V \rightarrow V^*$.

6 Lepton family number operator

The countable lepton family number of neutrinos through charged current interaction: ($\alpha = e, \mu, \tau$)

$$L_\alpha(t) = \int' \frac{d^3 p}{(2\pi)^3 |2\mathbf{p}|} \left(a_\alpha^\dagger(\mathbf{p}, t) a_\alpha(\mathbf{p}, t) - b_\alpha^\dagger(\mathbf{p}, t) b_\alpha(\mathbf{p}, t) \right).$$

$$L_\alpha(0) = \int' \frac{d^3 p}{(2\pi)^3 |2\mathbf{p}|} \left(a_\alpha^\dagger(\mathbf{p}) a_\alpha(\mathbf{p}) - b_\alpha^\dagger(\mathbf{p}) b_\alpha(\mathbf{p}) \right).$$

7 Expectation value of the total lepton number

The expectation value with a state with a definite lepton number $\sigma \in (e, \mu, \tau)$

$$|\mathbf{q}, \sigma\rangle = n_q a_\sigma^\dagger(\mathbf{q})|0\rangle$$

$$(n_q \text{ normalization const. } \langle \mathbf{q}, \sigma | \mathbf{q}, \sigma \rangle = 1.).$$

$$\langle \mathbf{q}, \sigma | L(t) | \mathbf{q}, \sigma \rangle = \sum_{i=1}^3 |V_{\sigma i}|^2 \left(\frac{q^2 + m_i^2 \cos 2E_i t}{E_i^2} \right).$$

where $q = |\mathbf{q}|$. The expectation value of the total lepton number is in the region $[-1, 1]$.

$$-1 \leq \sum_i |V_{\sigma i}|^2 \frac{|\mathbf{q}|^2 - m_i^2}{|\mathbf{q}|^2 + m_i^2} \leq \langle \mathbf{q}, \sigma | L(t) | \mathbf{q}, \sigma \rangle \leq 1.$$

The lower bound can be negative for $q^2 < m_{\text{lightest}}^2$ and reaches to -1 for $\mathbf{q} = 0$.

8 limit (I) Ultrarelativistic $q^2 \gg m_i m_j$

(I) The ultrarelativistic limit of $\langle \sigma | L_\alpha(t) | \sigma \rangle$ leads to the probability of neutrino oscillation. $0 \leq P_{\sigma \rightarrow \alpha}(t) \leq 1$.

$$\langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle \rightarrow P_{\sigma \rightarrow \alpha}(t)$$

$$P_{\sigma \rightarrow \alpha}(t) = \delta_{\sigma\alpha} - 4 \sum_{(i,j) \text{ cyclic}} \text{Re.}(V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*) \sin^2 \frac{\Delta m_{ij}^2 t}{4q}$$

$$- 2 \text{Im}(V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*) \sum_{(i,j) \text{ cyclic}} \sin \frac{\Delta m_{ij}^2 t}{2q} \quad (E_i - E_j \rightarrow \frac{\Delta m_{ij}^2}{2q}.)$$

V is PMNS matrix for Majorana neutrinos. (CPV: Two Majorana phases α_{31} , α_{21} , one KM type (Dirac) phase δ). In the limit (I), it only depends on δ .

$$V_{\sigma i} =$$

$$\begin{pmatrix} c_{12}c_{13} & (s_{12}c_{13})e^{i\frac{\alpha_{21}}{2}} & (s_{13}e^{-i\delta})e^{i\frac{\alpha_{31}}{2}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta})e^{i\frac{\alpha_{21}}{2}} & (s_{23}c_{13})e^{i\frac{\alpha_{31}}{2}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & (-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta})e^{i\frac{\alpha_{21}}{2}} & (c_{23}c_{13})e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

9 limit (II) Non-relativistic $q^2 \ll m_i m_j$

(II) zero momentum limit: $q \rightarrow 0$ (dependent on the Majorana phases)

$$\begin{aligned}
 \langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle &= \sum_{i=1}^3 |V_{\alpha i}|^2 |V_{\sigma i}|^2 m_i^2 \cos 2m_i t \\
 &+ \sum_{(i,j) \text{ cyclic}} \text{Re}(V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*) \left\{ (\cos(m_i - m_j)t) \left(1 - \text{Re}\left(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right)\right) \right. \\
 &\quad \left. + (\cos(m_i + m_j)t) \left(1 + \text{Re}\left(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right)\right) \right\} + \text{Im}(V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*) \\
 &\times \sum_{(i,j) \text{ cyclic}} (\cos(m_i - m_j)t - \cos(m_i + m_j)t) \text{Im}\left(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right).
 \end{aligned}$$

For $\sigma = e$, the following combinations of PMNS elements directly related to Majorana phases.

$$\frac{V_{e1}^* V_{e2}}{V_{e1} V_{e2}^*} = e^{i\alpha_{21}}, \quad \frac{V_{e1}^* V_{e3}}{V_{e1} V_{e3}^*} = e^{i(\alpha_{31} - 2\delta)}.$$

10 Numerical calculation

Input of neutrino parameters: (NuFIT 5.0(2020), I. Esteban et.al.).

	mass ² differences	s_{12} s_{23}	s_{13} δ	$(\alpha_{21}, \alpha_{31} - 2\delta)$ $m_{ee}^{max}, m_{ee}^{min}$
normal	$\Delta m_{21}^2 = 7.4 \times 10^{-5}$ $\Delta m_{31}^2 = 2.52 \times 10^{-3}$	0.55 0.76	0.15 1.09π	$(0, 0), (\pi, \pi)$ 0.012, 0.0018
inverted	$\Delta m_{21}^2 = 7.4 \times 10^{-5}$ $\Delta m_{23}^2 = 2.50 \times 10^{-3}$	0.55 0.76	0.15 1.57π	$(0, 0), (\pi, \pi)$ 0.050, 0.018

We assume the lightest neutrino mass to be 0.01[eV]. From that choice of the lightest neutrino mass each mass eigenvalue m_i [eV] is given by,

$$m_1 = 0.0100, \quad m_2 = 0.0132, \quad m_3 = 0.0512, \quad (\text{Normal}),$$

$$m_1 = 0.0502, \quad m_2 = 0.0510, \quad m_3 = 0.0100, \quad (\text{Inverted}).$$

Figure1 normal hierarchy

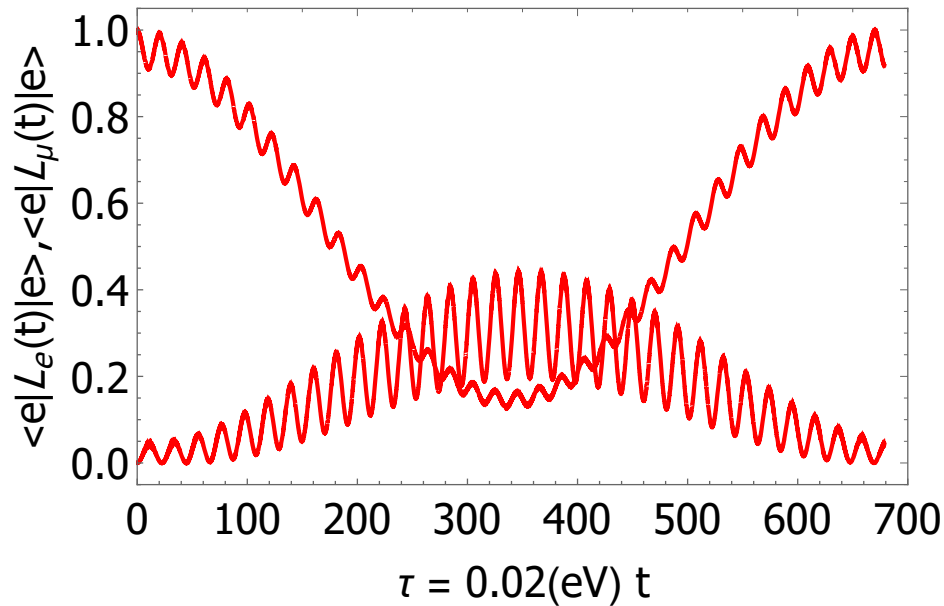
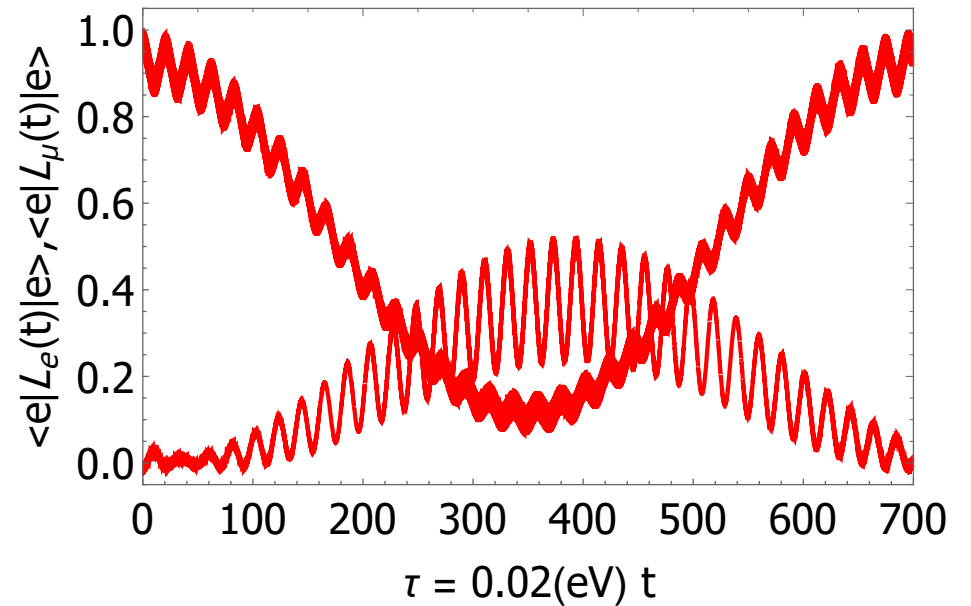


Figure2 inverted hierarchy

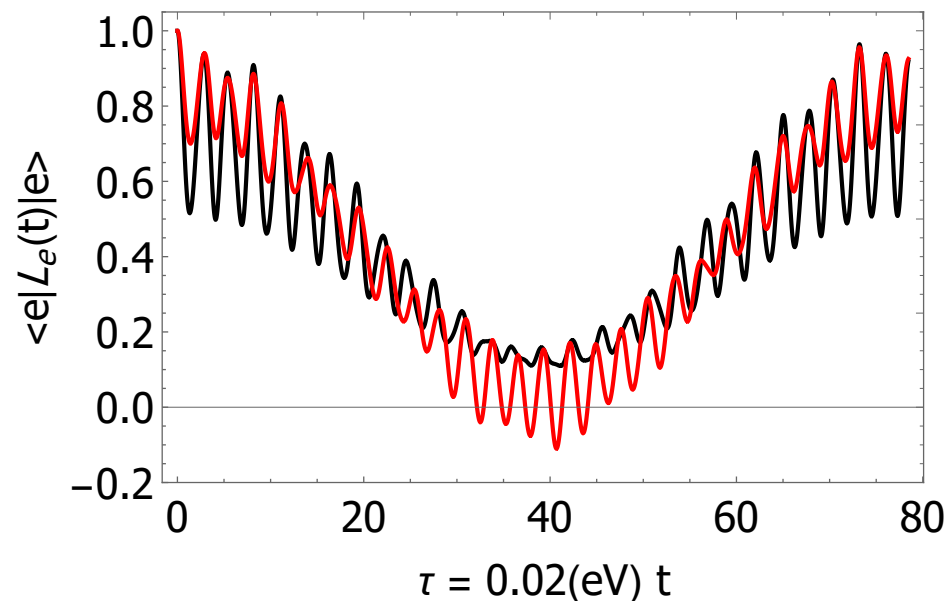


- (1) The long period $\tau_L = \frac{2\pi}{E_2 - E_1} \simeq 680 \sim 700$, $\tau_{S_{1,2}} = 20.3, 20.9 = \frac{2\pi}{E_3 - E_{1,2}}$.
- (2) The beat like behavior for muon number with $\tau_{beat} = \frac{\tau_{S_1} \tau_{S_2}}{\tau_{S_2} - \tau_{S_1}} = \tau_L$.
- (3) The lepton family numbers are always within the range $[0, 1]$.

12 Numerical calculation II

$(m_1 \sim m_2 \sim 0.01 < q = 0.02 < m_3 \sim 0.03 \text{ eV})$: Normal Hierarchy

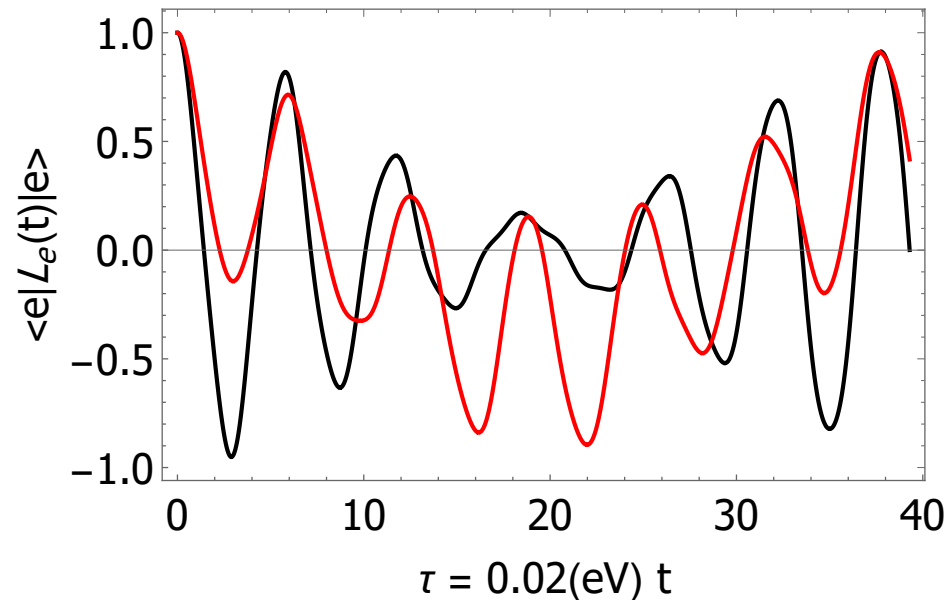
Figure3 $\langle e|L_e(t)|e\rangle$ electron number.



The strong dependence on Majorana phases for the electron number. The black(red)curve shows the case for the Majorana phase such that $|m_{ee}|$ is maximum (minimum). $\langle e|L_e(t \sim 0)|e\rangle \simeq 1 - (|m_{ee}|^2 + (m^\dagger m)_{ee})t^2$.

13 Numerical calculation III ($q = 2.0 \times 10^{-4} < m_1 = 0.01$)

Figure4 $\langle e|L_e(t)|e\rangle$: NH



Reducing the momentum to the level of CNB temperature, the dependencies on the Majorana phases are more pronounced. ($m_{\pm} = m_2 \pm m_1$.)

$$\langle e|L_e(t)|e\rangle = \begin{cases} \cos(m_+ t) \cos^2\left(\frac{m_- t}{2}\right) : (\alpha_{21} = 0) \\ \cos(m_- t) \cos^2\left(\frac{m_+ t}{2}\right) : (\alpha_{21} = \pi) \end{cases}$$

14 Numerical calculation IV; Normal vs Inverted for L_τ

Figure5 L_τ for NH: $q = 0.0002$

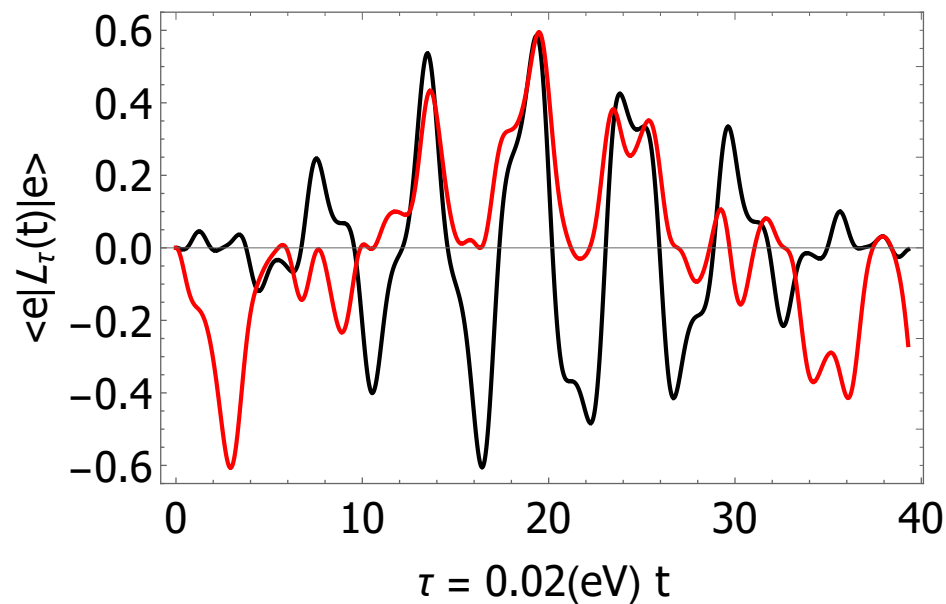
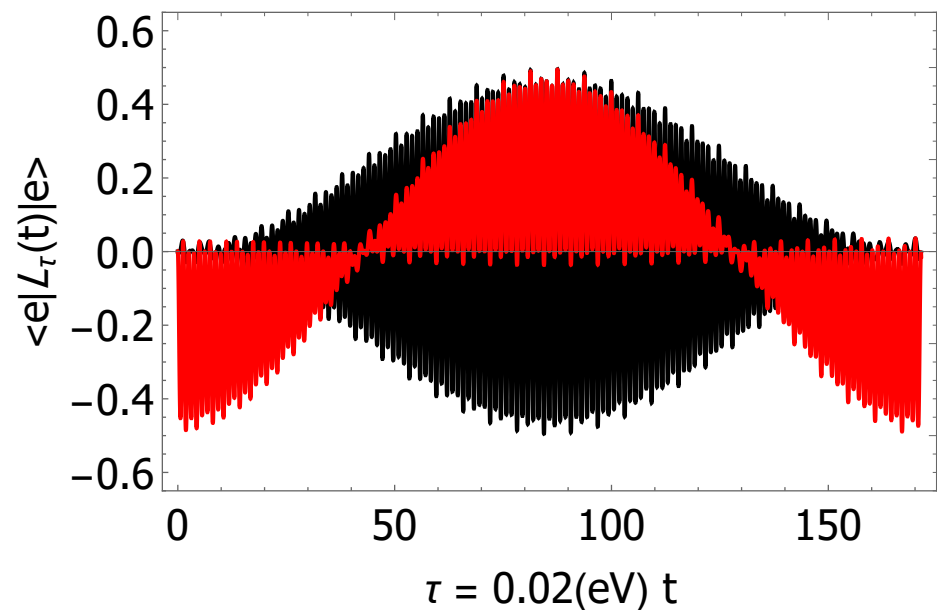


Figure6 L_τ for IH: $q = 0.0002$



$$\langle e|L_\tau(t)|e \rangle = -(2s_{12}c_{12}s_{23})^2 \begin{cases} \cos(m_+ t) \sin^2\left(\frac{m_- t}{2}\right) : (\alpha_{21} = 0) \\ \cos(m_- t) \sin^2\left(\frac{m_+ t}{2}\right) : (\alpha_{21} = \pi) \end{cases}$$

$$m_+|_{\text{inverted}} \simeq 5 \times m_+|_{\text{normal}}, \quad m_-|_{\text{inverted}} \simeq \frac{1}{5} \times m_-|_{\text{normal}}.$$

15 Numerical calculation V: The momentum dependence

As the momentum q decreases from the larger ($q = 20(\text{eV})$) to the smaller ($q = 2 \times 10^{-5}(\text{eV})$), the electron number at $\tau = 40$ changes from $+1$ to -1 . This indicates the transition from ν_e to $\bar{\nu}_e$ due to Majorana mass is more pronounced for the low momentum $q < m_\nu$.

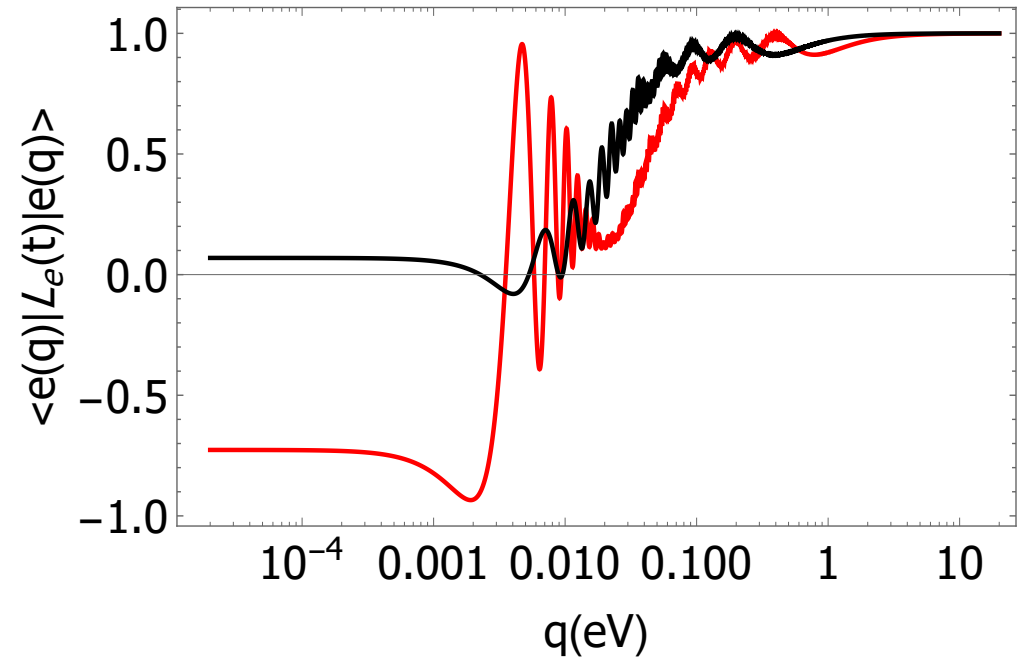


Figure 7 q dependence of $\langle e(q) | L_e(t) | e(q) \rangle$ for fixed time $\tau = 40$ (red) and $\tau = 20$ (black)

16 Implication on CNB and summary

- In the early universe, neutrinos' momentum is large and the lepton family numbers are approximately conserved. While the universe is cooled down, the momentum carried by the neutrinos is red-shifted and they become non-relativistic. Then the effects of Majorana mass term is turned on, and the lepton number may oscillate and alternate its sign . These effects strongly depend on the Majorana phases, neutrino masses, and hierarchies as we have seen.
- We investigated the time evolution of lepton (family) number under the presence of Majorana mass terms.
- With the lepton family number, the neutrino flavor transition and the neutrino and anti-neutrino transition effects is included in a single framework and the formula for lepton numbers is valid for the broad range of the momentum.
- The effect of Majorana mass is more significant at the low momentum region $q < m_\nu$.

17 Set up for the production and detection of electron (anti-) neutrino at $t = 0$ and $t > 0$. $L_e(t)$: the electron number carried by (anti-) neutrino.

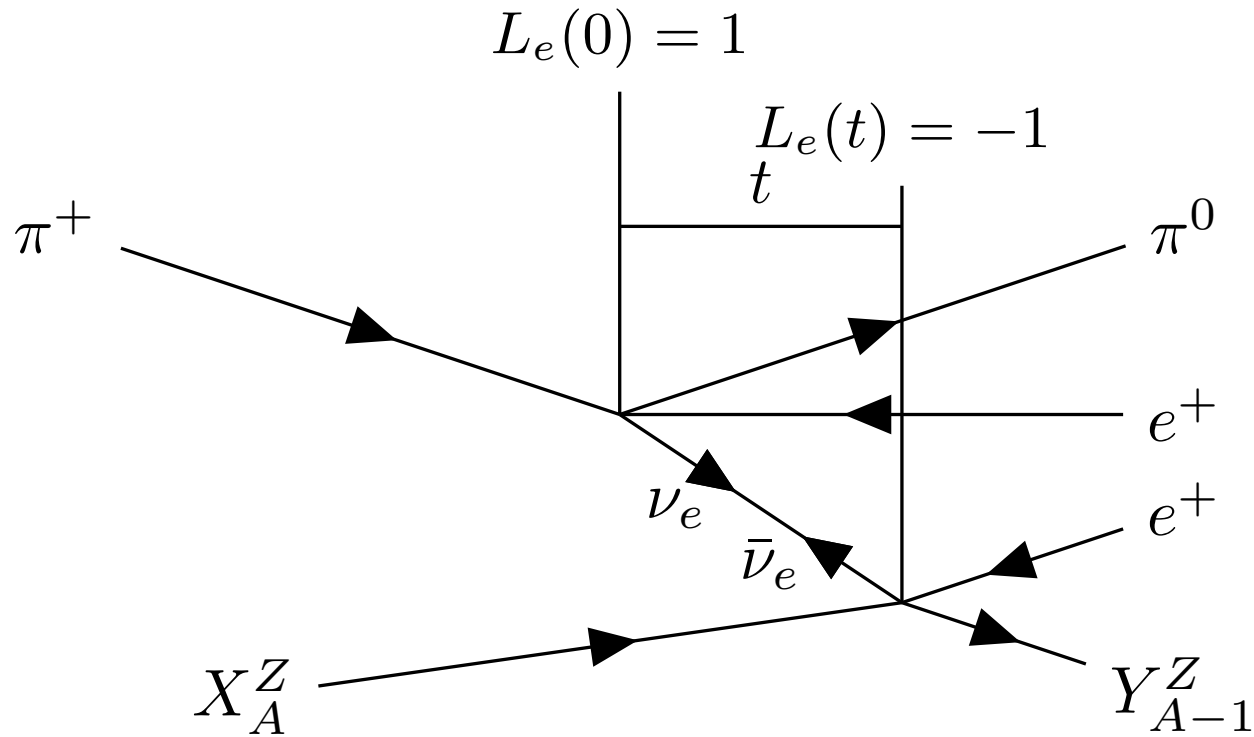


Figure8 $\pi^+ + X \rightarrow \pi^0 + e^+ + (\nu_e(0) \rightarrow \bar{\nu}_e(t)) + X \rightarrow \pi^0 + e^+ + Y + e^+$

18 Time evolution of lepton number operator $L(t) = \sum_{\alpha=e,\mu,\tau} L_\alpha(t)$

$$L(t) = \sum_{\alpha} L_\alpha(0) \leftarrow \text{Total lepton Number at } t = 0$$

$$\begin{aligned}
 & - \int' \frac{d^3 \mathbf{p}}{(2\pi)^3 |2\mathbf{p}|} \frac{2m_i^2 \sin^2(E_i(\mathbf{p})t)}{E_i^2(\mathbf{p})} \left(V_{\beta i} V_{\gamma i}^* a_\beta^\dagger(\mathbf{p}) a_\gamma(\mathbf{p}) - V_{\beta i}^* V_{\gamma i} b_\beta^\dagger(\mathbf{p}) b_\gamma(\mathbf{p}) \right) \\
 & + \int'_{\mathbf{p} \in A} \frac{m_i \sin(2E_i(\mathbf{p})t)}{E_i(\mathbf{p})} \left(V_{\beta i} V_{\gamma i} a_\beta^\dagger(\mathbf{p}) a_\gamma^\dagger(-\mathbf{p}) - V_{\beta i}^* V_{\gamma i}^* b_\beta^\dagger(\mathbf{p}) b_\gamma^\dagger(-\mathbf{p}) + h.c. \right) \\
 & + 2i \int'_{\mathbf{p} \in A} \frac{m_i |\mathbf{p}| \sin^2(E_i(\mathbf{p})t)}{E_i^2(\mathbf{p})} \left(V_{\beta i} V_{\gamma i} a_\beta^\dagger(\mathbf{p}) a_\gamma^\dagger(-\mathbf{p}) - V_{\beta i}^* V_{\gamma i}^* b_\beta^\dagger(\mathbf{p}) b_\gamma^\dagger(-\mathbf{p}) - h.c. \right).
 \end{aligned}$$

$$\int'_{\mathbf{p} \in A} = \int'_{\mathbf{p} \in A} \frac{d^3 p}{(2\pi)^3 2|\mathbf{p}|} \text{ and } A \text{ is a hemisphere region of } \mathbf{p} \neq 0.$$

$$\begin{aligned}
\langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle &= \sum_{i=1}^3 |V_{\alpha i}|^2 |V_{\sigma i}|^2 \frac{q^2 + m_i^2 \cos 2E_i t}{E_i^2} \\
+ \sum_{(i,j) \text{ cyclic}} \text{Re}(V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*) &\{ \cos(E_i - E_j)t \times \\
&\left(1 + \frac{q^2 - m_i m_j \text{Re}\left(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right)}{E_i E_j}\right) + \cos(E_i + E_j)t \times \\
&\left(1 - \frac{q^2 - m_i m_j \text{Re}\left(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right)}{E_i E_j}\right) \} + \text{Im}(V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*) \times \\
&\sum_{(i,j) \text{ cyclic}} \left\{ (\cos(E_i - E_j)t - \cos(E_i + E_j)t) \frac{m_i m_j}{E_i E_j} \text{Im}\left(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right) \right. \\
&\left. - \left(\left(\frac{q}{E_i} - \frac{q}{E_j}\right) \sin(E_i + E_j)t + \left(\frac{q}{E_i} + \frac{q}{E_j}\right) \sin(E_i - E_j)t \right) \right\}.
\end{aligned}$$