Lepton Flavor Model with A4 symmetry and 3HDM

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Contents

1. Motivation

Quarks and leptons (SM particles) have the generation structure.

Quarks and leptons have mass differences and a flavor mixing.
Especially leptons have a large flavor mixing.

It can't be explained in SM.
Then we need new physics beyond the SM.

Altareli and Feruglio impose discrete symmetry(flavor symmetry) among generations. G. Altarelli and F. Feruglio, Nucl. Phys. B741 (2006), 215-235.

In this study, we choose A_4 symmetry as flavor symmetry.

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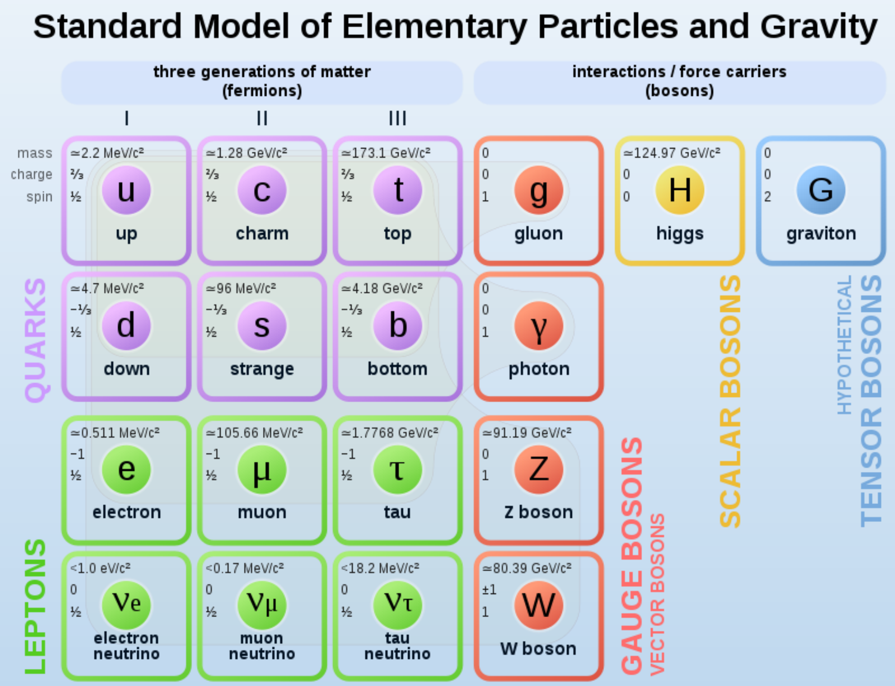
In addition, we suppose three Higgs doublets model(3HDM).

In general, the SM has one Higgs doublet.



https://www.wikiwand.com/

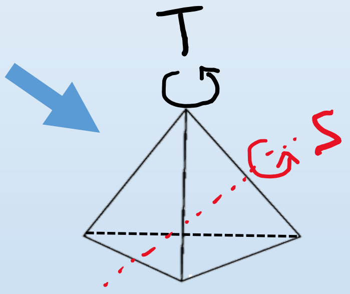
We built new flavor model and perform the analysis.

2. A_4symmetry

A_4 symmetry : Fourth order alternating group,

Smallest group containing triplet

Algebraic relation : $S^2 = (ST)^3 = T^3 = 1$ $(S, T$: generators$)$



regular tetrahedron

Representation : 1 : $S = 1, \quad T = 1$

$1'$: $S = 1, \quad T = e^{\frac{i4\pi}{3}} = \omega^2$

$1''$: $S = 1, \quad T = e^{\frac{i2\pi}{3}} = \omega$

3: $S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$

Multiplication rule : $1' \otimes 1' = 1''$, $1'' \otimes 1'' = 1'$, $1' \otimes 1'' = 1$, $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_S \oplus 3_A$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}_3 \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}_3 = (\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2)_1 \oplus (\alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1)_{1'} \oplus (\alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1)_{1''}$$

$$\oplus \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}_{3_S} \oplus \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}_{3_A}$$

3. 3HDM

Extend SM Higgs doublet to 3 (12 real scalar fields)

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \phi_3 = \begin{pmatrix} \phi_3^+ \\ \phi_3^0 \end{pmatrix}$$

Higgs Potential in 3HDM under $SU(2)_L \otimes U(1)_Y$

$$V = -\sum_{i,j=1}^{3} m_{ij}^2 \left(\phi_i^\dagger \phi_j\right) + \frac{1}{2} \sum_{i,j,k,l=1}^{3} \lambda_{ijkl} \left(\phi_i^\dagger \phi_j\right)\left(\phi_k^\dagger \phi_l\right)$$

Potential minimum conditions

$$\left(\frac{\partial V}{\partial \phi_1}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

$$\left(\frac{\partial V}{\partial \phi_2}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

$$\left(\frac{\partial V}{\partial \phi_3}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

⬇ Spontaneous symmetry breaking

3 degrees of freedom are eaten by W and Z bosons.

➡ ϕ is represented by the expansion of 9 (=12−3) real scalar fields

$$\langle\phi_i\rangle = \begin{pmatrix} \rho_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\chi_i) \end{pmatrix}, i = 1,2,3$$

Eigenstates of mass ➡

(i) Three CP-even scalar fields

(ii) Two CP-odd scalar fields

(iii) Four charged scalar fields

4

Higgs potential analysis

3HDM+A_4symmetry

Consider ϕ as A_4 triplet : $\phi = (\phi_1, \phi_2, \phi_3)$, $\left(\phi^\dagger = \left(\phi_1^\dagger, \phi_3^\dagger, \phi_2^\dagger\right)\right)$

Calculate Higgs potential $V = -\mu^2 \phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Multiplication rule of A_4

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}_3 \otimes \begin{pmatrix}\beta_1\\\beta_2\\\beta_3\end{pmatrix}_3 = (\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2)_1 \oplus (\alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1)_{1'} \oplus (\alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1)_{1''} \oplus \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}_{3_S} \oplus \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}_{3_A}$$

$$\phi^\dagger\phi = \begin{pmatrix}\phi_1^\dagger\\\phi_3^\dagger\\\phi_2^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3 = \left(\phi_1^\dagger\phi_1 + \phi_3^\dagger\phi_3 + \phi_2^\dagger\phi_2\right)_1 = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2$$

$$\left(\phi^\dagger\phi\right)^2 = \begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1^\dagger\\\phi_3^\dagger\\\phi_2^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1^\dagger\\\phi_3^\dagger\\\phi_2^\dagger\end{pmatrix}_3 = (\phi_1\phi_1 + 2\phi_2\phi_3)_1 \otimes \left(\phi_1^\dagger\phi_1^\dagger + 2\phi_2^\dagger\phi_3^\dagger\right)_1 + (\phi_2\phi_2 + 2\phi_3\phi_1)_{1''} \otimes \left(\phi_2^\dagger\phi_2^\dagger + 2\phi_3^\dagger\phi_1^\dagger\right)_{1'}$$

$$+ (\phi_3\phi_3 + 2\phi_1\phi_2)_{1'} \otimes \left(\phi_3^\dagger\phi_3^\dagger + 2\phi_1^\dagger\phi_2^\dagger\right)_{1''} + \frac{2}{3}\begin{pmatrix}\phi_1\phi_1 - \phi_2\phi_3\\\phi_3\phi_3 - \phi_1\phi_2\\\phi_2\phi_2 - \phi_3\phi_1\end{pmatrix}_3 \otimes \frac{2}{3}\begin{pmatrix}\phi_1^\dagger\phi_1^\dagger - \phi_2^\dagger\phi_3^\dagger\\\phi_2^\dagger\phi_2^\dagger - \phi_3^\dagger\phi_1^\dagger\\\phi_3^\dagger\phi_3^\dagger - \phi_1^\dagger\phi_2^\dagger\end{pmatrix}_3$$

$$= \left|\phi_1^2 + 2\phi_2\phi_3\right|^2 + \left|\phi_2^2 + 2\phi_3\phi_1\right|^2 + \left|\phi_3^2 + 2\phi_1\phi_2\right|^2 + \frac{4}{9}\left[\left|\phi_1^2 - \phi_2\phi_3\right|^2 + \left|\phi_2^2 - \phi_3\phi_1\right|^2 + \left|\phi_3^2 - \phi_1\phi_2\right|^2\right]$$

5

Vacuum structure

Potential

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$
$$= -\mu^2 (|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda_1 |\phi_1^2 + 2\phi_2\phi_3|^2 + \lambda_2 |\phi_2^2 + 2\phi_3\phi_1|^2 + \lambda_3 |\phi_3^2 + 2\phi_1\phi_2|^2$$
$$+\lambda_4 \left[|\phi_1^2 - \phi_2\phi_3|^2 + |\phi_2^2 - \phi_3\phi_1|^2 + |\phi_3^2 - \phi_1\phi_2|^2 \right]$$

Potential minimum conditions

$$\left(\frac{\partial V}{\partial \phi_i}\right)_{\phi_1 = \langle\phi_1\rangle, \phi_2 = \langle\phi_2\rangle, \phi_3 = \langle\phi_3\rangle} = 0, \quad i = 1,2,3$$

Local vacuum expectation values ($\lambda_1 \neq \lambda_4, 2\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \neq 0$)

$$\langle\phi_1\rangle = v_1$$

$$\langle\phi_2\rangle = -\frac{\lambda_2+\lambda_3-\lambda_4}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4} v_1 \pm \frac{\sqrt{\{-2\lambda_1^2+\lambda_2^2+\lambda_3^2-(\lambda_1-3\lambda_4)(\lambda_2+\lambda_3)-3\lambda_1\lambda_4\}v_1^2+\frac{1}{2}(2\lambda_1+\lambda_2+\lambda_3+\lambda_4)\mu^2}}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

$$\langle\phi_3\rangle = -\frac{\lambda_2+\lambda_3-\lambda_4}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4} v_1 \pm \frac{\sqrt{\{-2\lambda_1^2+\lambda_2^2+\lambda_3^2-(\lambda_1-3\lambda_4)(\lambda_2+\lambda_3)-3\lambda_1\lambda_4\}v_1^2+\frac{1}{2}(2\lambda_1+\lambda_2+\lambda_3+\lambda_4)\mu^2}}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

Rewrite VEV with v and β

$$\langle\phi\rangle = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}} v \sin\beta \\ \frac{1}{\sqrt{2}} v \sin\beta \end{pmatrix}$$

v : Higgs VEV
β : free parameter

4. Flavor model

	$\bar{l}=(\bar{l_e},\bar{l_\mu},\bar{l_\tau})$	e_R	μ_R	τ_R	ν_R	$\phi=(\phi_1,\phi_2,\phi_3)$
$SU(2)_L$	2	1	1	1	1	2
A_4	3	1	$1''$	$1'$	3	3

SM gauge and A_4invariant Lagrangian mass term： $L_Y = L_l + L_D + L_M + h.c.$

(1) Mass terms of charged leptons： $L_l = y_e\bar{l}\phi e_R + y_\mu\bar{l}\phi\,\mu_R + y_\tau\bar{l}\phi\tau_R$
(2) Mass term of Dirac neutrino： $L_D = y_D\bar{l}\tilde{\phi}\nu_R$
(3) Mass term of right-handed Majorana neutrino： $L_M = M\bar{\nu}_R^c\nu_R$

↓

Calculate mass matrices of charged leptons and left-handed Majorana neutrino

Calculation of mass matrices

Multiplication rule of A_4

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}_3 \otimes \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}_3$$

$$= (\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2)_1 \oplus (\alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1)_{1'} \oplus (\alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1)_{1''}$$

$$\oplus \begin{pmatrix}2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2\\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1\\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1\end{pmatrix}_{3_S} \oplus \begin{pmatrix}\alpha_2\beta_3 - \alpha_3\beta_2\\ \alpha_1\beta_2 - \alpha_2\beta_1\\ \alpha_3\beta_1 - \alpha_1\beta_3\end{pmatrix}_{3_A}$$

(1) Mass terms of charged leptons

$$L_l = y_e\bar{l}\phi e_R + y_\mu\bar{l}\phi\,\mu_R + y_\tau\bar{l}\phi\tau_R$$

$$y_e\begin{pmatrix}\bar{l}_e\\ \bar{l}_\mu\\ \bar{l}_\tau\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3 \otimes (e_R)_1 = y_e(\bar{l}_e\phi_1 + \bar{l}_\mu\phi_2 + \bar{l}_\tau\phi_3)e_R$$

1

$< \phi >= (v_1, v_2, v_3)$

$$y_e(\bar{e}_L v_1 + \bar{\mu}_L v_2 + \bar{\tau}_L v_3)e_R$$

$$\rightarrow y_e(\bar{e}_L v_1 + \bar{\mu}_L v_3 + \bar{\tau}_L v_2)e_R + y_\mu(\bar{\tau}_L v_3 + \bar{e}_L v_2 + \bar{\mu}_L v_1)\mu_R + y_\tau(\bar{\mu}_L v_2 + \bar{\tau}_L v_1 + \bar{e}_L v_3)\tau_R$$

$$= (y_e v_1)\bar{e}_L e_R + (y_\mu v_2)\bar{e}_L\mu_R + (y_\tau v_3)\bar{e}_L\tau_R$$
$$+(y_e v_3)\bar{\mu}_L e_R + (y_\mu v_1)\bar{\mu}_L\mu_R + (y_\tau v_2)\bar{\mu}_L\tau_R$$
$$+(y_e v_2)\bar{\tau}_L e_R + (y_\mu v_3)\bar{\tau}_L\mu_R + (y_\tau v_1)\bar{\tau}_L\tau_R$$

Mass matrix of charged leptons

$$M_l = \begin{pmatrix}y_e v_1 & y_\mu v_2 & y_\tau v_3\\ y_e v_3 & y_\mu v_1 & y_\tau v_2\\ y_e v_2 & y_\mu v_3 & y_\tau v_1\end{pmatrix}_{LR}$$

Calculation of mass matrices

(2) Mass term of Dirac neutrino

$$L_D = y_D \bar{l}\tilde{\phi}\nu_R$$

$$= y_D \underbrace{\begin{pmatrix} \bar{l}_e \\ \bar{l}_\mu \\ \bar{l}_\tau \end{pmatrix}_3 \otimes \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_3 \\ \tilde{\phi}_2 \end{pmatrix}_3}_{3} \otimes \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}_3 = \left(\frac{y_{DS}}{3} \begin{pmatrix} 2\bar{l}_1\tilde{\phi}_1 - \bar{l}_2\tilde{\phi}_2 - \bar{l}_3\tilde{\phi}_3 \\ 2\bar{l}_3\tilde{\phi}_2 - \bar{l}_1\tilde{\phi}_3 - \bar{l}_2\tilde{\phi}_1 \\ 2\bar{l}_2\tilde{\phi}_3 - \bar{l}_3\tilde{\phi}_1 - \bar{l}_1\tilde{\phi}_2 \end{pmatrix}_{3S} + \frac{y_{DA}}{2} \begin{pmatrix} \bar{l}_2\tilde{\phi}_2 - \bar{l}_3\tilde{\phi}_3 \\ \bar{l}_1\tilde{\phi}_3 - \bar{l}_2\tilde{\phi}_1 \\ \bar{l}_3\tilde{\phi}_1 - \bar{l}_1\tilde{\phi}_2 \end{pmatrix}_{3A} \right) \otimes \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}_3$$

$$\to \frac{y_{DS}}{3}\left[(2\bar{\nu}_e v_1 - \bar{\nu}_\mu v_2 - \bar{\nu}_\tau v_3)\nu_{R1} + (2\bar{\nu}_\tau v_2 - \bar{\nu}_e v_3 - \bar{\nu}_\mu v_1)\nu_{R3} + (2\bar{\nu}_\mu v_3 - \bar{\nu}_\tau v_1 - \bar{\nu}_e v_2)\nu_{R2}\right]$$

$$+ \frac{y_{DA}}{2}\left[(\bar{\nu}_\mu v_2 - \bar{\nu}_\tau v_3)\nu_{R1} + (\bar{\nu}_e v_3 - \bar{\nu}_\mu v_1)\nu_{R3} + (\bar{\nu}_\tau v_1 - \bar{\nu}_e v_2)\nu_{R2}\right]$$

Mass matrix of Dirac neutrino

$$M_D = y_{DS}\begin{pmatrix} \frac{2}{3}v_1 & -\frac{1}{3}v_2 & -\frac{1}{3}v_3 \\ -\frac{1}{3}v_2 & \frac{2}{3}v_3 & -\frac{1}{3}v_1 \\ -\frac{1}{3}v_3 & -\frac{1}{3}v_1 & \frac{2}{3}v_2 \end{pmatrix} + y_{DA}\begin{pmatrix} 0 & -\frac{y_{DA}}{2}v_2 & \frac{y_{DA}}{2}v_3 \\ \frac{y_{DA}}{2}v_2 & 0 & -\frac{y_{DA}}{2}v_1 \\ -\frac{y_{DA}}{2}v_3 & \frac{y_{DA}}{2}v_1 & 0 \end{pmatrix}$$

Multiplication rule of A_4

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}_3 \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}_3$$

$$= (\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2)_1 \oplus (\alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1)_{1'} \oplus (\alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1)_{1''}$$

$$\oplus \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}_{3_S} \oplus \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}_{3_A}$$

Calculation of mass matrices

(3) Mass term of right-handed Majorana neutrino

$$L_M = \frac{1}{2} M \bar{\nu}_R^c \nu_R$$

$$= \frac{1}{2} M \begin{pmatrix} \bar{\nu}_{R1}^c \\ \bar{\nu}_{R2}^c \\ \bar{\nu}_{R3}^c \end{pmatrix}_{\underline{3}} \otimes \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}_{\underline{3}} = \frac{1}{2} M \left(\overline{\nu_1^c} \nu_1 + \overline{\nu_2^c} \nu_3 + \overline{\nu_3^c} \nu_2 \right)$$

Mass matrix of right-handed Majorana neutrino

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}$$

Calculate mass matrix of left-handed Majorana neutrino by using type-I seesaw mechanism

Minkowski '77; Gell-Mann, Ramond, Slansky; Yanagida; Glashow; Mohapatra, Senjanovic '79

$$m_\nu = -M_D M_R^{-1} M_D^\dagger$$

$$= \begin{pmatrix} \frac{-4y_{DS}^2(2v_1^2 + v_2 v_3) + 9y_{DA}^2 v_2 v_3}{18M} & \frac{4y_{DS}(2y_{DS} - 3y_{DA})v_3^2 + (4y_{DS}^2 - 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_2}{36M} & \frac{4y_{DS}(2y_{DS} + 3y_{DA})v_2^2 + (4y_{DS}^2 + 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_3}{36M} \\ \frac{4y_{DS}(2y_{DS} - 3y_{DA})v_3^2 + (4y_{DS}^2 - 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_2}{36M} & \frac{-(2y_{DS} - 3y_{DA})^2 v_2^2 + 8y_{DS}(2y_{DS} + 3y_{DA})v_1 v_3}{36M} & \frac{-4y_{DS}^2(v_1^2 + 5v_2 v_3) + 9y_{DA}^2(v_1^2 + v_2 v_3)}{36M} \\ \frac{4y_{DS}(2y_{DS} + 3y_{DA})v_2^2 + (4y_{DS}^2 + 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_3}{36M} & \frac{-4y_{DS}^2(v_1^2 + 5v_2 v_3) + 9y_{DA}^2(v_1^2 + v_2 v_3)}{36M} & \frac{8y_{DS}(2y_{DS} - 3y_{DA})v_1 v_2 - 9(2y_{DS} - 3y_{DA})^2 v_3^2}{36M} \end{pmatrix}$$

Calculation of Yukawa couplings

①Calculate $|y_e|^2, |y_\mu|^2, |y_\tau|^2$

Denote $h_e \equiv |y_e|^2, h_\mu \equiv |y_\mu|^2, h_\tau \equiv |y_\tau|^2$

Mass matrix of charged leptons

$$M_l = \begin{pmatrix} y_e v_1 & y_\mu v_2 & y_\tau v_3 \\ y_e v_3 & y_\mu v_1 & y_\tau v_2 \\ y_e v_2 & y_\mu v_3 & y_\tau v_1 \end{pmatrix}_{LR},$$

VEV

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}} v \sin\beta \\ \frac{1}{\sqrt{2}} v \sin\beta \end{pmatrix}$$

$$M_l M_l^\dagger = \begin{pmatrix} h_e v^2\cos^2(\beta) + \frac{1}{2}(h_\mu + h_\tau)v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_e + h_\mu)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\tau v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_e + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\mu v^2\sin^2(\beta) \\ -\frac{1}{\sqrt{2}}(h_e + h_\mu)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\tau v^2\sin^2(\beta) & h_\mu v^2\cos^2(\beta) + \frac{1}{2}(h_e + h_\tau)v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_\mu + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_e v^2\sin^2(\beta) \\ -\frac{1}{\sqrt{2}}(h_e + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\mu v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_\mu + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_e v^2\sin^2(\beta) & h_\tau v^2\cos^2(\beta) + \frac{1}{2}(h_e + h_\mu)v^2\sin^2(\beta) \end{pmatrix}$$

Diagonalize $M_l M_l^\dagger$ with unitary matrix V_l

$$V_l^\dagger M_l M_l^\dagger V_l = \begin{pmatrix} m_e^2 & & \\ & m_\mu^2 & \\ & & m_\tau^2 \end{pmatrix}$$

$m_e = 0.51099$ MeV
$m_\mu = 105.658$ MeV
$m_\tau = 1776.86$ MeV

https://pdg.lbl.gov

Solve the eigenvalues equation

$$\begin{cases} \mathrm{Tr}\left(M_l M_l^\dagger\right) = m_e^2 + m_\mu^2 + m_\tau^2 \\ \det\left(M_l M_l^\dagger\right) = m_e^2 m_\mu^2 m_\tau^2 \\ \left[\mathrm{Tr}\left(M_l M_l^\dagger\right)\right]^2 - \mathrm{Tr}\left(M_l M_l^\dagger M_l M_l^\dagger\right) = 2\left(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2\right) \end{cases}$$

We get $h_e = |y_e|^2, h_\mu = |y_\mu|^2, h_\tau = |y_\tau|^2$

11

Calculation of physical quantity

②Calculate unitary matrix V_l

Substitute the obtained $|y_e|^2, |y_\mu|^2, |y_\tau|^2$ into $M_l M_l^\dagger$ ➡ Calculate unitary matrix V_l

③Consider the same for neutrinos and find the unitary matrix V_ν that diagonalizes $m_\nu m_\nu^\dagger$

④Calculate $U_{PMNS} = V_l^\dagger V_\nu$ which parameterized by mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

⑤Calculate δ_{CP}

Jarlskog invariant： $J_{CP} = \mathrm{Im}\left[U_{\alpha i}U_{\beta i}^* U_{\alpha j}^* U_{\beta j}\right] \Rightarrow \mathrm{Im}\left[U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}\right] = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta_{CP}$

$$\therefore \sin\delta_{CP} = \frac{\mathrm{Im}\left[U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}\right]}{s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2}$$

and $|U_{\tau 1}|^2 = s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2 s_{12}s_{23}c_{12}s_{13}c_{23}\cos\delta_{CP}$ ➡ $\cos\delta_{CP} = \dfrac{s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - |U_{\tau 1}|^2}{2 s_{12}s_{23}c_{12}s_{13}c_{23}}$

Calculate δ_{CP} from $\sin\delta_{CP}$ and $\cos\delta_{CP}$

⑥Calculate effective mass m_{ee} in neutrinoless double beta $(0\nu\beta\beta)$ decay experiment and Majorana phases η_1, η_2

Numeric calculation

⑦Use the data from PDG(2021) and NuFIT 5.1.

e

$J = \frac{1}{2}$

Mass $m = (548.579909070 \pm 0.000000016) \times 10^{-6}$ u
Mass $m = 0.5109989461 \pm 0.0000000031$ MeV
$|m_{e^+} - m_{e^-}|/m < 8 \times 10^{-9}$, CL = 90%
$|q_{e^+} + q_{e^-}|/e < 4 \times 10^{-8}$
Magnetic moment anomaly
$(g-2)/2 = (1159.65218091 \pm 0.00000026) \times 10^{-6}$
$(g_{e^+} - g_{e^-}) / g_{\text{average}} = (-0.5 \pm 2.1) \times 10^{-12}$
Electric dipole moment $d < 0.11 \times 10^{-28}$ $e$cm, CL = 90%
Mean life $\tau > 6.6 \times 10^{28}$ yr, CL = 90% [a]

μ

$J = \frac{1}{2}$

Mass $m = 0.1134289257 \pm 0.0000000025$ u
Mass $m = 105.6583745 \pm 0.0000024$ MeV

τ

$J = \frac{1}{2}$

Mass $m = 1776.86 \pm 0.12$ MeV

https://pdg.lbl.gov

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
		bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
	$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
	$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

http://www.nu-fit.org/

⑧Take β at random

VEV of Higgs

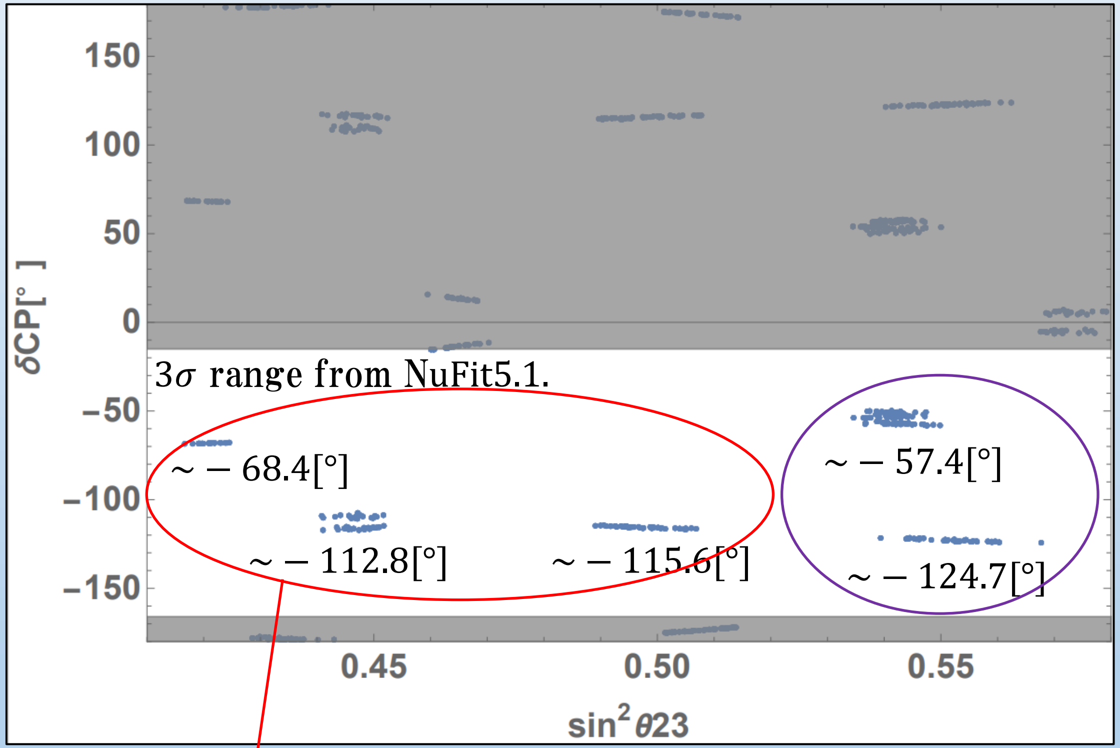
$$\langle\phi\rangle = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \end{pmatrix}$$

$$\beta: -\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$$
$$v = 173\ \text{Gev}$$

5. Result

Numerical result (1)

Prediction of δ_{CP} and $\sin^2\theta_{23}$



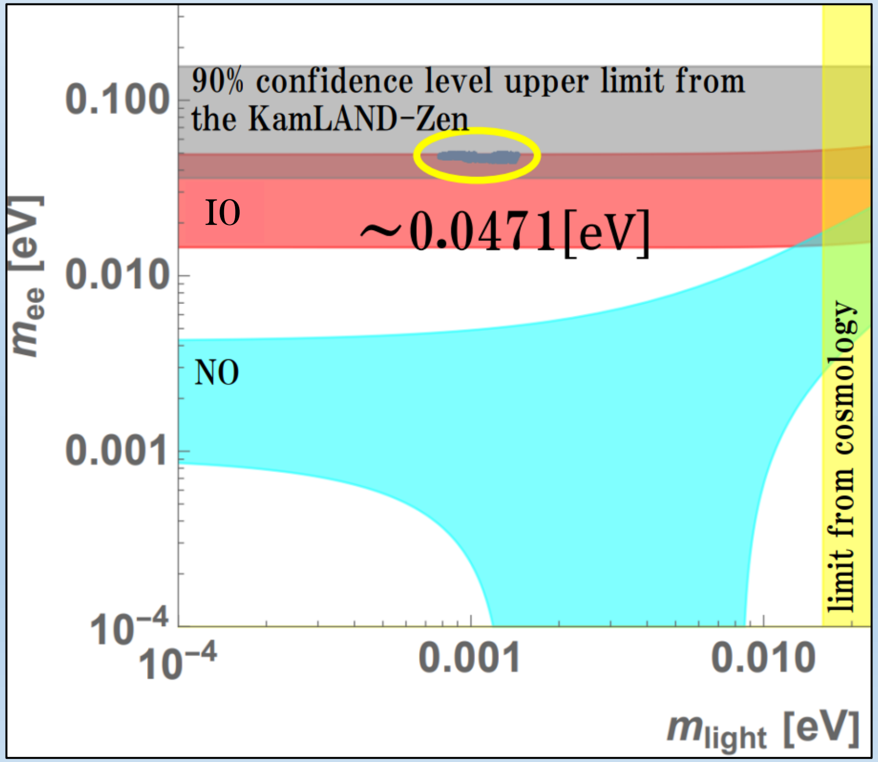
Strong prediction of δ_{CP}

Numerical result (2)

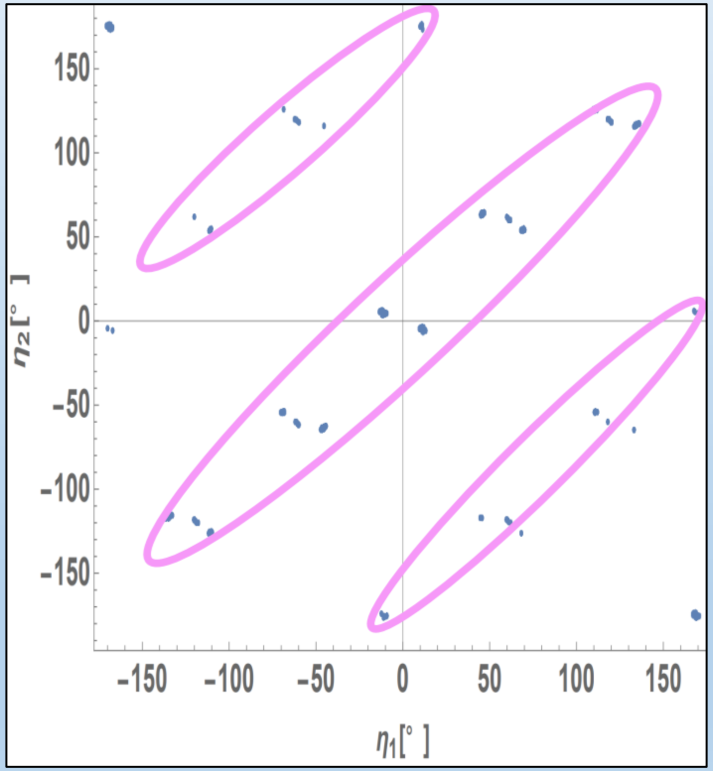
Prediction of the effective mass m_{ee} of the electron neutrino in the $0\nu\beta\beta$ decay experiment and the lightest neutrino mass m_{light}

Effective mass of electron neutrino

$$m_{ee} = \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 \right|$$



Prediction of Majorana phases η_1, η_2



6. Conclusion

We consider A_4 symmetry as Flavor symmetry.
We consider Higgs field ϕ as A_4 triplet.
→ We perform Higgs potential analysis and obtain local VEV.

$$\langle\phi\rangle = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \end{pmatrix}$$

We build new flavor model by using 3HDM and A_4 symmetry.
We calculate mass matrices of charged leptons and neutrinos under new flavor model.

Mass matrix of charged leptons

$$M_l = \begin{pmatrix} y_e v_1 & y_\mu v_2 & y_\tau v_3 \\ y_e v_3 & y_\mu v_1 & y_\tau v_2 \\ y_e v_2 & y_\mu v_3 & y_\tau v_1 \end{pmatrix}_{LR}$$

Mass matrix of left-handed Majorana neutrinos

$$m_\nu = -M_D M_R^{-1} M_D^\dagger \quad \left(M_D = y_{DS}\begin{pmatrix} \frac{2}{3}v_1 & -\frac{1}{3}v_2 & -\frac{1}{3}v_3 \\ -\frac{1}{3}v_2 & \frac{2}{3}v_3 & -\frac{1}{3}v_1 \\ -\frac{1}{3}v_3 & -\frac{1}{3}v_1 & \frac{2}{3}v_2 \end{pmatrix} + y_{DA}\begin{pmatrix} 0 & -\frac{y_{DA}}{2}v_2 & \frac{y_{DA}}{2}v_3 \\ \frac{y_{DA}}{2}v_2 & 0 & -\frac{y_{DA}}{2}v_1 \\ -\frac{y_{DA}}{2}v_3 & \frac{y_{DA}}{2}v_1 & 0 \end{pmatrix},\ M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix} \right)$$

We perform numerical analysis and calculate δ_{CP} , effective mass m_{ee} and Majorana phases η_1, η_2.
We obtain strong predictions of δ_{CP} and m_{ee} ($m_{ee} \approx 0.0471$[eV]).
→This flavor model is confirmed by neutrino experiments in near future.

Alternating group

Alternating group：the set of even permutations of the symmetric group
置換

symmetric group(permutation group)： set of n-dimensional permutation

even permutation：permutation expressed as a product of even number of transposition
互換

parameter

SM　Yukawa coupling→3∗3∗2=18

Flavor structure　Yukawa coupling→$y_e, y_\mu, y_\tau, y_{DS}, y_{DA}, \phi_{DA}$→ **6**

Model parameter

$$y_e, y_\mu, y_\tau, y_{DS}, y_{DA}, \phi_{DA}, \beta$$

Physical quantity

$$m_e, m_\mu, m_\tau, \Delta m^2_{21}, \Delta m^2_{32}, \theta_{12}, \theta_{23}, \theta_{13}, \delta_{CP}, m_{ee}, \eta_1, \eta_2$$

数値計算

④ V_lを求める

求めた $|y_e|^2, |y_\mu|^2, |y_\tau|^2$ を$M_l M_l^\dagger$へ代入し、対角化するユニタリー行列V_lを求める

⑤ニュートリノについても同様に考え、V_νを求める

左巻きマヨラナニュートリノの質量行列m_ν について、ユニタリー行列V_νにより対角化されるとすると、

$$V_\nu^\dagger m_\nu m_\nu^\dagger V_\nu = \begin{pmatrix} m_3^2 & & \\ & m_1^2 & \\ & & m_2^2 \end{pmatrix}$$ を得る（inverted order ）

行列とその固有値に関する方程式

$$\begin{cases} \mathrm{Tr}\left(m_\nu m_\nu^\dagger\right) = m_1^2 + m_2^2 + m_3^2 \\ \det\left(m_\nu m_\nu^\dagger\right) = m_1^2 m_2^2 m_3^2 \\ \left[\mathrm{Tr}\left(m_\nu m_\nu^\dagger\right)\right]^2 - \mathrm{Tr}\left(m_\nu m_\nu^\dagger m_\nu m_\nu^\dagger\right) = 2\left(m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2\right) \end{cases}$$

$$\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.42(\mathrm{eV})^2$$
$$\Delta m_{23}^2 = m_2^2 - m_3^2 = 2.49(\mathrm{eV})^2$$

m_3, Mを振る

$|y_{DS}|, |y_{DA}|$, 位相$\alpha(\equiv \mathrm{Arg}(y_{DA}/y_{DS}))$について解く

求めた $|y_{DS}|, |y_{DA}|$, 位相$\alpha(\equiv \mathrm{Arg}(y_{DA}/y_{DS}))$を$m_\nu m_\nu^\dagger$へ代入し、対角化するユニタリー行列$V_\nu$を求める

数値計算

⑥混合角$\theta_{12}, \theta_{23}, \theta_{13}$を求める

$V_l^{\dagger}V_\nu$を数値計算する　ここで、

$$V_l^{\dagger}V_\nu = U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

$$\theta_{12} = \tan^{-1}\left(\left|\frac{\left(V_l^{\dagger}V_\nu\right)_{12}}{\left(V_l^{\dagger}V_\nu\right)_{11}}\right|\right), \theta_{23} = \tan^{-1}\left(\left|\frac{\left(V_l^{\dagger}V_\nu\right)_{23}}{\left(V_l^{\dagger}V_\nu\right)_{33}}\right|\right), \theta_{13} = \sin^{-1}\left|\left(V_l^{\dagger}V_\nu\right)_{13}\right|$$

であるため、数値計算で求めた$V_l^{\dagger}V_\nu$から、混合角$\theta_{12}, \theta_{23}, \theta_{13}$を求める

$V_l^{\dagger}V_\nu$から求めた混合角$\hat{\theta}_{12}, \hat{\theta}_{23}, \hat{\theta}_{13}$と実験値を比較する

$0.546 \leq \theta_{12} \leq 0.620$
$0.695 \leq \theta_{23} \leq 0.900$
$0.144 \leq \theta_{12} \leq 0.157$

範囲内にある$\hat{\theta}_{12}, \hat{\theta}_{23}, \hat{\theta}_{13}$を採用し、
そうではない場合は値を振り直し
再び最初から行う

数値計算

⑦δ_{CP}を求める

Jarlskog 不変量：$J_{CP} = \mathrm{Im}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right] \Rightarrow = \mathrm{Im}\left[U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}\right] = s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}\sin\delta_{CP}$

$$\therefore \sin\delta_{CP} = \frac{\mathrm{Im}\left[U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}\right]}{s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}}$$

また　$|U_{\tau 1}|^2 = s^2_{12}s^2_{23} + c^2_{12}s^2_{13}c^2_{23} - 2s_{12}s_{23}c_{12}s_{13}c_{23}\cos\delta_{CP}$ より、　$\cos\delta_{CP} = \dfrac{s^2_{12}s^2_{23} + c^2_{12}s^2_{13}c^2_{23} - |U_{\tau 1}|^2}{2s_{12}s_{23}c_{12}s_{13}c_{23}}$

これらの値からδ_{CP}を求める

⑧マヨラナ位相を求める

ニュートリノがマヨラナ粒子である場合、 U_{PMNS}にマヨラナ位相η_1, η_2が加わる

$$U'_{PMNS} = U_{PMNS} \cdot P = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$$

⑨有効質量$|m_{ee}|$を求める

ニュートリノを放出しない二重β崩壊（$0\nu\beta\beta$）実験での有効質量$|m_{ee}|$を計算する　　$m_{ee} = m_1U^2_{e1} + m_2U^2_{e2} + m_3U^2_{e3}$

数値計算(補遺)

⑧マヨラナ位相を求める

ニュートリノがマヨラナ粒子である場合、 U_{PMNS}にマヨラナ位相η_1, η_2が加わる

$$U'_{PMNS} = U_{PMNS} \cdot P = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$$

マヨラナ位相をニュートリノの質量を対角化するユニタリー行列に加えて、$U_{PMNS} \cdot P = V_l V_\nu \cdot P = V_l V'_\nu$ とする

そしてM_νはV_νを用いて複素対角化される

$$V_\nu^\dagger M_\nu V_\nu^* = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & e^{i\gamma} \end{pmatrix} = D_m P_m$$

P_Mは数値計算から求まる→Pも求めることができる

$$V_\nu'^\dagger M_\nu V_\nu'^* = P^\dagger V_\nu^\dagger M_\nu V_\nu^* P^* = D_m P_m P^{*2} = D_m$$

$$\because P_m P^{*2} = 1$$