Stability of the embedded string in the $SU(N) \times U(1)$ Higgs model and the application to some GUT breakings

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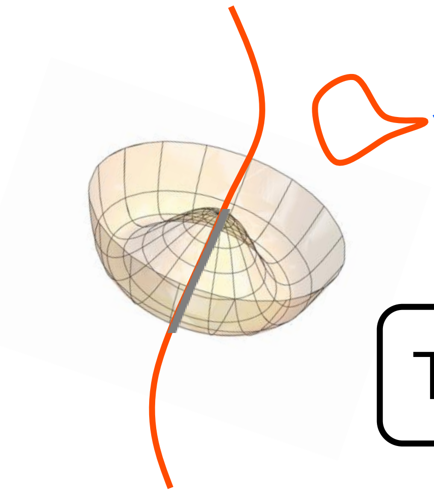
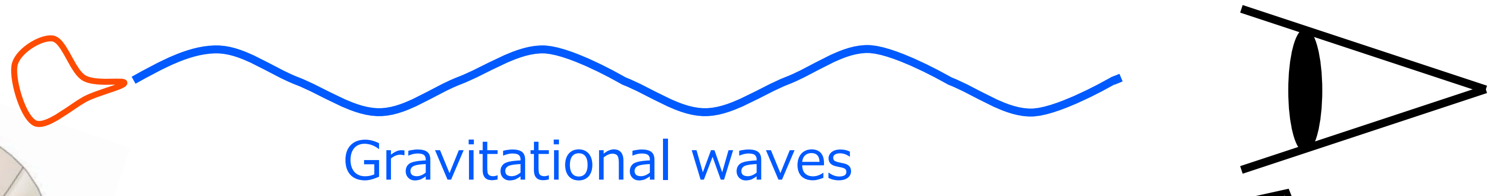
Introduction①

Cosmic strings





Gravitational Waves Observations

The energy scale of the symmetry breaking !

Candidates of beyond the standard model can be constrained by the GW observations if they predict a formation of cosmic strings after inflation.

Which symmetry breakings form cosmic strings?

Introduction②

- It is well known that cosmic strings are formed if nonzero winding numbers can be defined on the vacuum manifold (or mathematically, $\pi_1(G/H) \neq \{e\}$ when $G \to H$)

  Ex. When $U(1) \to \times$, Nielsen-Olesen strings are formed [H. Nielsen, P. Olesen (1973)]

- Such cosmic strings are stable because of a topological charge

  → Topological strings [T. Kibble (1976)]

- There are many studies of topological strings in some BSMs

  $U(1)_{B-L}$ breaking [W. Buchmuller, V. Domcke, H. Murayama, K. Schmitz (2020)],

  $SO(10)$ or E_6 GUT [G. Lazarides, R. Maji, Q. Shafi (2021)], and more

Actually, cosmic strings can be formed in some breakings also when $\pi_1(G/H) = \{e\}$

Embedded strings

[M. Barriola, T. Vachaspati, M. Bucher (1993), N. Lepora, A. Davis(1995)]

Introduction③

Embedded strings are also string like classical solutions.

→ Classically stable or not? (∵no nontrivial winding number)

⇔ Can be formed in the symmetry breaking?

Studied well only in $SU(2) \times U(1) \rightarrow U(1)$
(The classical stability depends on some parameters)

Electroweak breaking [Y. Nambu(1977), T. Vachaspati(1992)],

2HDM [M. Earnshaw, M. James (1993), L. Perivolaropoulos (1993), H. La (1993)]

How about other symmetry breakings?
If embedded string emits GW, some symmetry breakings can be suggested or rejected by future GW observations.

Todays talk

- We examine the condition of embedded strings formation for $SU(N) \times U(1) \rightarrow SU(N-1) \times U(1)$ and its supersymmetric extension
- We apply the condition for some GUT breakings
  → Representation of Higgs is constrained to form embedded string

1. Introduction

2. Z-string and its stability

3. Embedded strings in $SU(N) \times U(1)$

4. Applications to GUT breakings

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Z-string

$$SU(2) \times U(1) \xrightarrow{H:\ (\mathbf{2}, 1/2)} U(1) \quad \text{with } V(H) = \lambda \left(H^\dagger H - v^2\right)^2$$

$$\begin{array}{ccc} SU(2) \times U(1) & \longrightarrow & U(1) \\ \cup & & \cup \\ U(1) & \longrightarrow & \times \end{array}$$

related to
the Z boson

Consider
Nielsen-Olesen string

$\pi_1\big(SU(2) \times U(1)/U(1)\big) \simeq \{e\}$

No Nielsen-Olesen strings
are formed

Z-string solutions [Y. Nambu (1977), T. Vachaspati (1992)]

$$H = \begin{pmatrix} 0 \\ f(r)e^{in\theta} \end{pmatrix}, \qquad \vec{Z} = -\frac{nz(r)}{r}\,\vec{e}_\theta, \qquad (\text{others}) = 0$$

in cylindrical coordinate

$(f(0) = z(0) = 0, f(\infty) = v, z(\infty) = 2/\alpha \ \ (\alpha^2 = g_1^2 + g_2^2)), (n \in \mathbb{Z} \setminus \{0\})$

$f(r)e^{in\theta}$ and $\frac{nz(r)}{r}\,\vec{e}_\theta$ are the same as the Nielsen-Olesen string

Stability of the Z-string

Consider perturbations from the Z-string

$$H=\begin{pmatrix}\phi(x)\\ f(r)ve^{i\theta}+\delta h(x)\end{pmatrix},\ \vec{Z}=-\frac{z(r)}{\alpha r}\,\vec{e}_\theta+\delta\vec{Z}(x),\overrightarrow{W}^{\pm}(x),\vec{A}(x)$$

Calculate the variations of the energy and find modes decreasing it

Taking up to the quadratic terms, there are divided into two parts

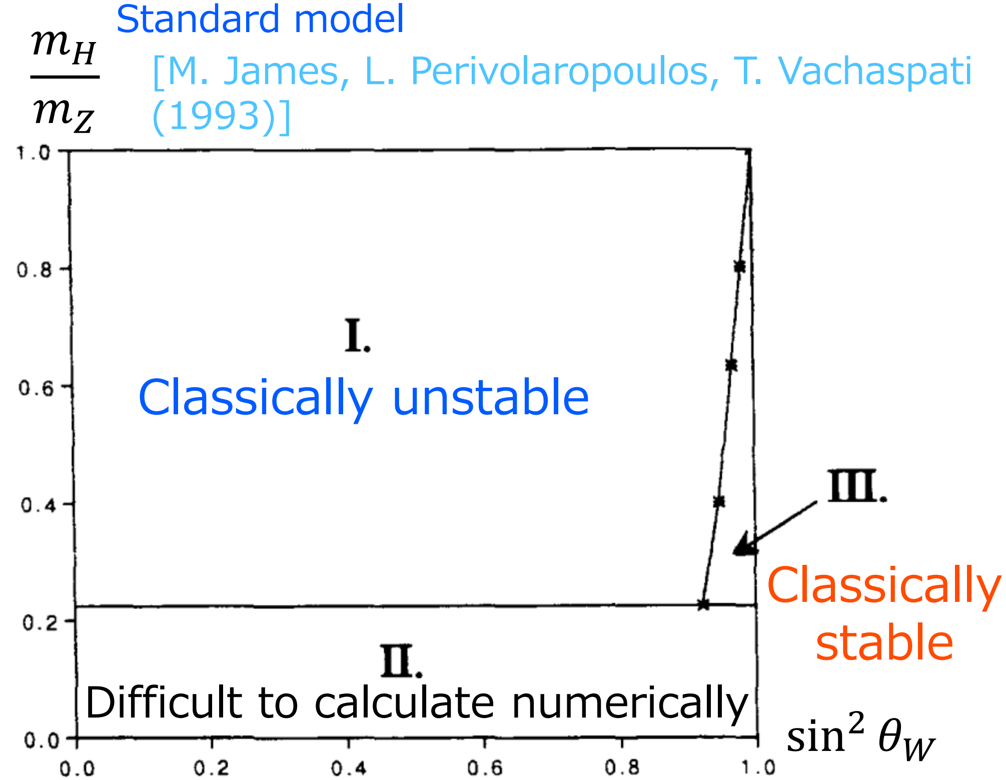
$\phi(x),\overrightarrow{W}^{\pm}(x)$ — Charged

$\delta h(x),\delta\vec{Z}(x),\vec{A}(x)$ — Neutral (Non-negative)

Some calculations

Standard model

[M. James, L. Perivolaropoulos, T. Vachaspati (1993)]



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Embedded string in $SU(N) \times U(1)$

$$SU(N) \times U(1) \xrightarrow{\phi:\left(\boldsymbol{N}, \frac{1}{2}\right)} SU(N-1) \times U(1)$$

with $V(\phi) = \lambda(|\phi|^2 - v^2)^2$ ⟵ $\pi_1(\mathcal{V}) \simeq \pi_1(S^{2N-1}) \simeq \{e\}$

<u>Embedded string solution</u>

G_μ^a, B_μ: $SU(N), U(1)$ gauge bosons

$$\phi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f(r)e^{in\theta} \end{pmatrix}, \quad \vec{\tilde{Z}} \equiv \sqrt{\frac{2(N-1)}{N}} \frac{g_N}{\alpha_N} \vec{G}^{N^2-1} - \frac{g_1}{\alpha_N} \vec{B} = -\frac{nz(r)}{r} \vec{e}_\theta,$$

$$\left(\alpha_N^2 \equiv \frac{2(N-1)}{N} g_N^2 + g_1^2\right)$$

(others) $= 0$

$$\left(T^{N^2-1} = \frac{1}{\sqrt{2N(N-1)}} \mathrm{diag}(1, \ldots, 1, 1-N)\right)$$

They are exactly same as the Z-string when $N = 2$

"Generalized Z-string"

Check the stability

Consider perturbations

Higgs: $\phi(x) = \begin{pmatrix} \boxed{\begin{matrix} \phi_1(x) \\ \vdots \\ \phi_{N-1}(x) \end{matrix}} \\ f(r)e^{in\theta} + \boxed{\delta\phi(x)} \end{pmatrix}$, Gauge boson: $\begin{pmatrix} \boxed{\vec{G}^a(x)} & \boxed{\vec{G}^+(x)} \\ \boxed{\vec{G}^-(x)} & \end{pmatrix}$,

$\vec{\tilde{Z}} = -\frac{z(r)}{\alpha_N r}\,\vec{e}_\theta + \boxed{\delta\vec{\tilde{Z}}(x)}$,

$\boxed{\vec{\tilde{A}}(x)} \equiv \frac{g_1}{\alpha_N}\vec{G}^{N^2-1} + \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}\vec{B}$

Diagonal part

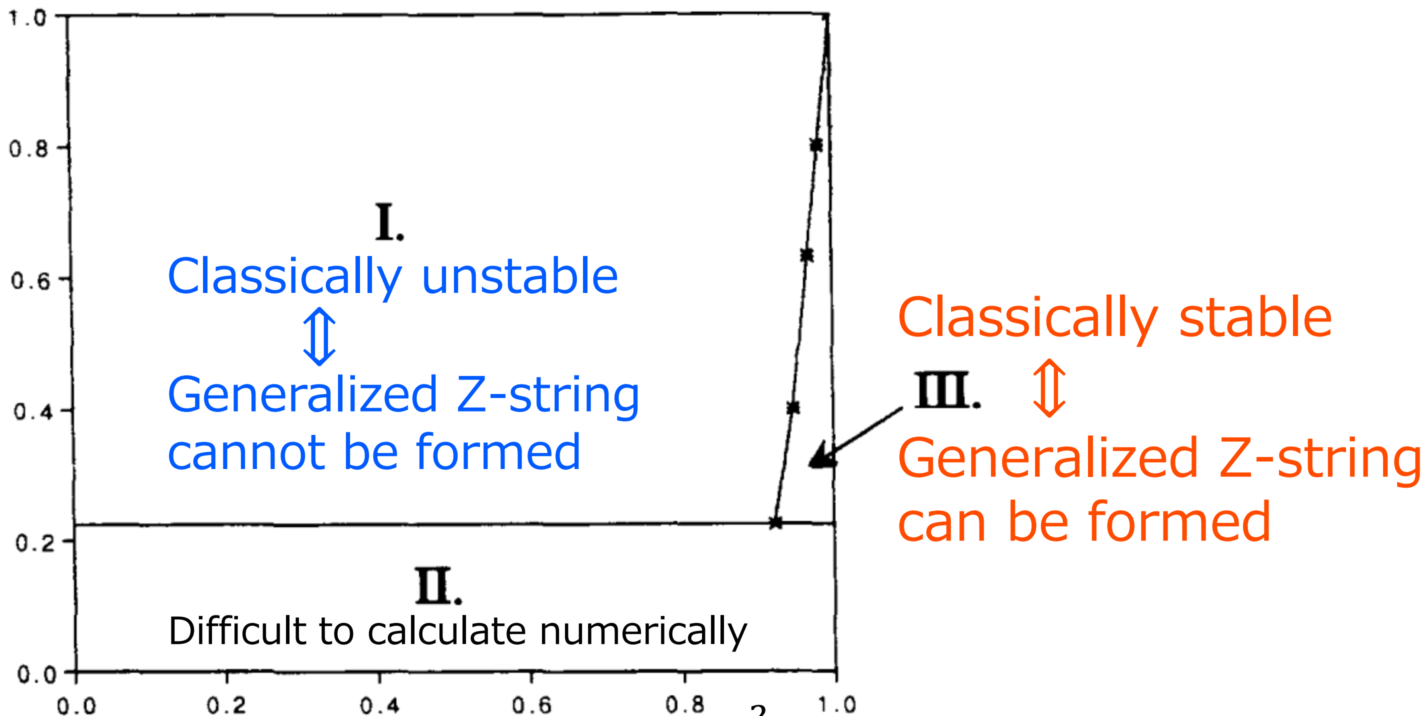
(light blue): $SU(N-1)$ adjoint, (blue): $SU(N-1)$ singlet,
(orange): $SU(N-1)$ findamental

Take variations of the energy up to quadratic terms

$\vec{G}^a(x)\quad (a = 1, \dots, (N-1)^2 - 1)$	$\phi_k(x)\quad (k = 1, \dots, N-1)$, $\vec{G}^+(x), \vec{G}^-(x)$	$\delta\phi(x), \delta\vec{\tilde{Z}}(x)$, $\vec{\tilde{A}}(x)$
Non-negative ($\because$ only squared terms)	Equivalent to the charged part of the Z-string by replacing $\cos\theta_W$ with g_N/α_N	Non-negative ($\because$ same as N-O string)

Parameter region

$$\frac{m_\phi}{m_{\tilde{Z}}} = \sqrt{\frac{8\lambda}{\alpha_N}}$$



$\cancel{\sin^2\theta_W} \Rightarrow 1 - \frac{g_N^2}{\alpha_N^2}$

We can approximately evaluate the stable region as

$$\frac{m_\phi}{m_{\tilde{Z}}} \lesssim 1 - \frac{8g_N^2}{\alpha_N^2} \Leftrightarrow g_1 \gtrsim \sqrt{\left|\frac{8}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}\right|}\, g_N$$

Supersymmetric extension

Super potential $W = \lambda_s S(\Phi_2^\top \Phi_1 - v^2)$ (Φ_1: $\left(\boldsymbol{N}, \frac{1}{2}\right)$, Φ_2: $\left(\bar{\boldsymbol{N}}, -\frac{1}{2}\right)$ S: gauge singlet)

Scalar part: ϕ_1, ϕ_2, s

$$V(\phi_1, \phi_2, s) = \lambda_s^2 \left|\phi_2^\top \phi_1 - v^2\right|^2 + \lambda_s^2 |s| \left(|\phi_1|^2 + |\phi_2|^2\right) + \frac{g_1^2}{8}\left(|\phi_1|^2 - |\phi_2|^2\right) + \frac{g_N^2}{2}\left(\phi_1^\dagger T^a \phi_1 - \phi_2^\top T^a \phi_2^*\right)^2$$

Symmetric for $\phi_1 \leftrightarrow \phi_2^*$ $\Rightarrow$ $\phi_1 = \phi_2^*$ in the embedded string

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2^* \end{pmatrix} =: \begin{pmatrix} \phi_0 \\ \phi \end{pmatrix}$$

Zero VEV mode ← It does not affect the instability of the embedded string

Non-zero VEV mode

$$V(\phi) = \underline{\frac{\lambda_s^2}{4}(|\phi|^2 - 2v^2)^2} + \cancel{\lambda_s^2 |s|^2 |\phi|^2} \quad (\because\ \langle s \rangle = 0)$$

We can evaluate the stability as same as the non-SUSY case by replacing λ with $\lambda_s^2/4$ (The vertical axis also corresponds to $m_\phi/m_{\tilde{Z}}$)

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The case of groups unified

When $SU(N)$ and $U(1)$ were unified to a simple group G,

$$G \to \cdots \to SU(N) \times U(1) \times H \to SU(N-1) \times U(1) \times H$$

$$g_N = {g_1}' \qquad g_N = \eta_{RG} {g_1}' = \frac{\eta_{RG}}{2q} g_1$$

$$\phi \supset (\boldsymbol{N}, q, \boldsymbol{1})\Big|_{{g_1}'} = (\boldsymbol{N}, 1/2, \boldsymbol{1})\Big|_{g_1}$$

(η_{RG}: renormalization group effect)

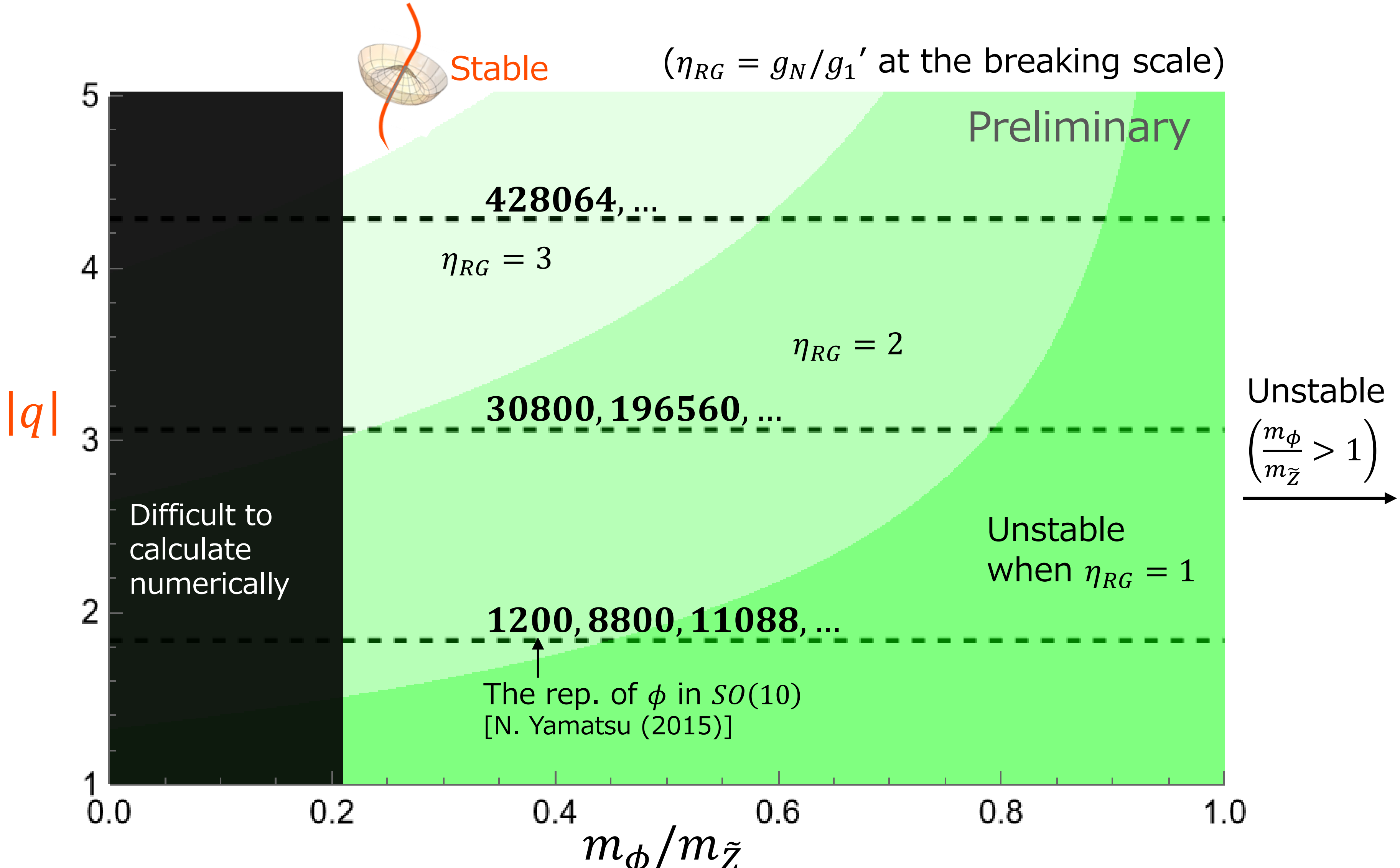
The generalized Z-strings are formed when g_N and g_1 satisfy

$$g_1 \gtrsim \sqrt{\left|\frac{8}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}\right|}\, g_N \Rightarrow \boxed{|q| \gtrsim \frac{\eta_{RG}}{2}\sqrt{\frac{8}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}}$$

The condition for $|q|$ can be regarded as the conditions for representations of ϕ in G

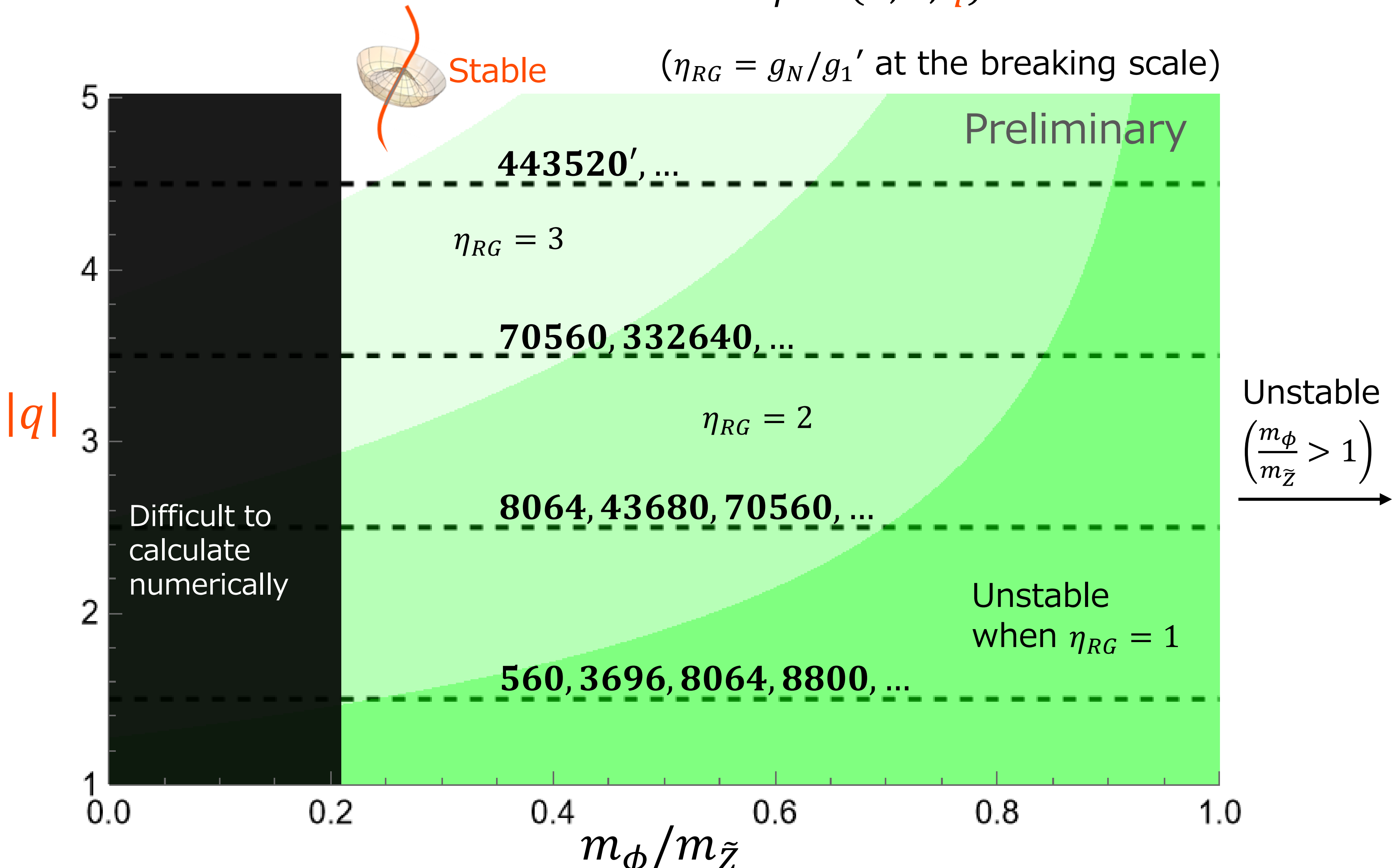
① $SO(10)$
$\rightarrow SU(3) \times SU(2) \times SU(2) \times U(1) \xrightarrow{\phi \supset (\mathbf{1},\mathbf{1},\mathbf{2},q)} SU(3) \times SU(2) \times U(1)$

$(\eta_{RG} = g_N/g_1'$ at the breaking scale)



② $SO(10) \to SU(4) \times SU(2) \times U(1) \to SU(3) \times SU(2) \times U(1)$

$\phi \supset (\mathbf{4}, \mathbf{1}, q)$



Summary and outlook

- We consider embedded strings in $SU(N) \times U(1)$ Higgs model, and find the condition to form them.
  It can be applied to the supersymmetric case similarly.

- We find if the embedded string is formed in GUT, high dimensional representation Higgs is needed.
  If groups are not unified, the $U(1)$ charge of Higgs are constrained to form embedded strings.

As future works,

- We study about the precise gravitational waves spectrum from embedded strings.
- We examine the GUT models which have high dimensional representation Higgs.