Matter Parities from Finite Modular Symmetries

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Contents

Contents

- Introduction & Abstract
- Modular Transform
- Application for SUSY
- Summary and Future Works

SM and SUSY（1）

- Standard Model (SM)

  ：It describes interactions of particles in high accuracy.

- Problems：Neutrino mass, Dark matter,

  Hierarchy problem, Generations of matter, …)

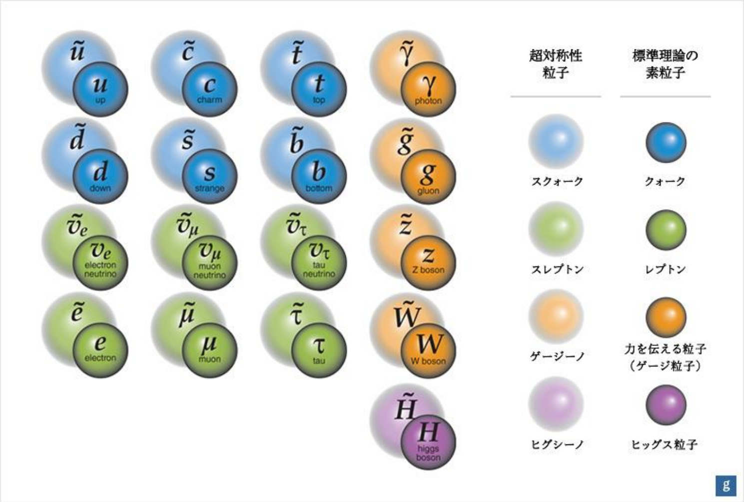
- “ What is Beyond Standard Model (BSM) ? ”

SM and SUSY（2）

- Supersymmetric Theory (SUSY)

  ：It has symmetry for interchange of Boson/Fermion.

  Researchers have expected that it is BSM.



https://www.icepp.s.u-tokyo.ac.jp/what/focus/03.html

SM and SUSY（2）

- Supersymmetric Theory (SUSY)

  ：It has symmetry for interchange of Boson/Fermion.

  Researchers have expected that it is BSM.

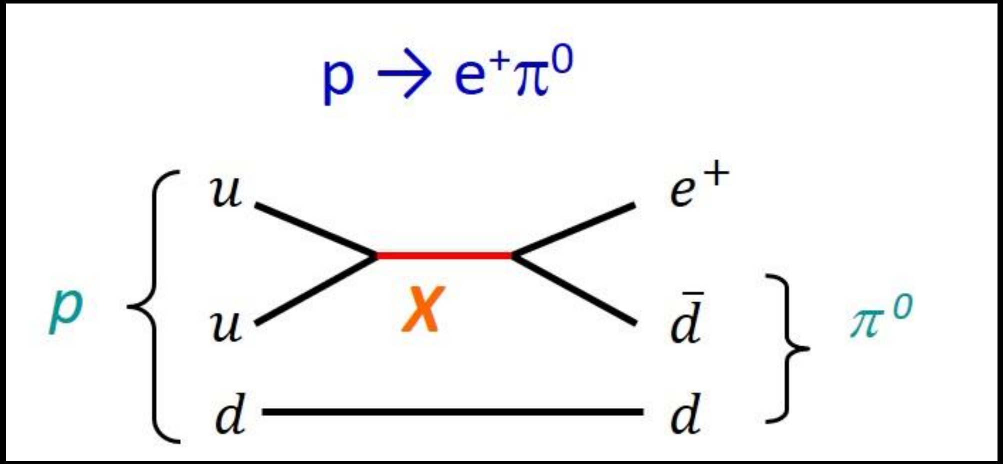
- Problem：No experiments find SUSY particles.

- It is important to build models,

  which describe the results, with SUSY’s advantage.

Abstract of this work

- SUSY extensions of SM (Minimal SUSY SM, etc.) have interactions breaking Baryon/Lepton numbers. Example) Proton decay



https://www-sk.icrr.u-tokyo.ac.jp/~hayato_s/protondecay.html

Abstract of this work

- SUSY extensions of SM (Minimal SUSY SM, etc.) have interactions breaking Baryon/Lepton numbers.
  Example) Proton decay

- We classified the extensions by discrete symmetries originated from finite modular symmetries.
  Then, we found limitations for the B/L num. breaking terms.

Contents

Modular Transform（1）

- SL(2,Z) Transform

$$\tau \to \gamma\tau = \frac{a\tau+b}{c\tau+d} \quad \left(\tau \in \mathbb{C}, \quad \mathrm{Im}\tau > 0, \quad ad - bc = 1\right)$$

Modular Transform（1）

- SL(2,Z) Transform

$$\tau \to \gamma\tau = \frac{a\tau+b}{c\tau+d} \quad \left(\tau \in \mathrm{C}, \quad \mathrm{Im}\tau > 0, \quad ad - bc = 1\right)$$

- Generators

$$\text{S trans.}: \tau \to -\frac{1}{\tau} \;,\; \text{T trans.}: \tau \to \tau + 1$$

$$S^2 = -1 \quad , \quad (ST)^3 = 1$$

Modular Transform（2）

- Principal Congruence Subgroups

$$\Gamma(N) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL(2,Z); \begin{bmatrix} a & b \\ c & d \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \bmod N \right\}$$

Modular Transform（2）

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- $PSL(2,Z) = SL(2,Z)/Z_2$, $\bar{\Gamma}(N) = \Gamma(N)/Z_2$

$\rightarrow \Gamma_N = PSL(2,Z)/\bar{\Gamma}(N)$

Modular Transform（2）

- Principal Congruence Subgroups

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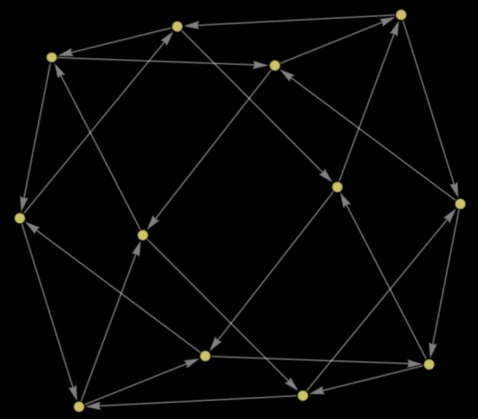
- $PSL(2,Z) = SL(2,Z)/Z_2$, $\bar{\Gamma}(N) = \Gamma(N)/Z_2$

$\rightarrow \Gamma_N = PSL(2,Z)/\bar{\Gamma}(N)$

Example) $\Gamma_2 \simeq S_3$ (Symmetric group of degree 3)

$\Gamma_3 \simeq A_4$ (Alternating group of degree 4)



Modular Forms (1)

- Using unitary matrix ρ, modular form f is defined by

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau)$$

k is called as "weight".

Modular Forms（1）

- Using unitary matrix ρ, modular form f is defined by

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau)$$

k is called as "weight".

- Example) A_4 triplet with $k = 2$ $(q = e^{2\pi i\tau},\ \mathrm{Im}\tau \gg 1)$

$$\begin{pmatrix} f_1(\tau) \\ f_2(\tau) \\ f_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \cdots \\ -6q^{1/3}(1 + 7q + 8q^2 + \cdots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \cdots) \end{pmatrix}$$

Modular Forms（2）

- Using unitary matrix ρ, modular form f is defined by

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau)$$

  k is called as "weight".

- Especially for S transform,

$$f_i(S^2\tau) = (-1)^k \rho_{ij}(S^2) f_j(\tau)$$

  This means that k is even number in Γ_N.

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Application for SUSY (1)

In 4d N=1 SUSY,

Super potential

$$W = \sum_n Y_{I_1\cdots I_n}(\tau)\,\Phi_{I_1}\cdots\Phi_{I_N}$$

Kahler potential

$$K = \sum_I \frac{|\Phi_I|^2}{\left(i(\bar{\tau}-\tau)\right)^{k_I}}$$

Application for SUSY（2）

$$W = \sum_n Y_{I_1 \cdots I_n}(\tau)\, \Phi_{I_1} \cdots \Phi_{I_N}$$

$$K = \sum_I \frac{|\Phi_I|^2}{\left(i(\bar{\tau} - \tau)\right)^{k_I}}$$

We assigned modular weights

for Φ_I (chiral supermultiplets) & $Y_{I_1 \cdots I_n}(\tau)$ (couplings)

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We assigned modular weights

for Φ_I (chiral supermultiplets) & $Y_{I_1 \cdots I_n}(\tau)$ (couplings)

"Those potentials have modular symmetry."

Then, we got conditions for weight sum.

Application for SUSY (3)

Minimal Supersymmetric Standard Model (MSSM)

$$W = y_{ij}^{u} Q_i H_u \bar{U}_j + y_{ij}^{d} Q_i H_d \bar{D}_j + y_{ij}^{l} L_i H_d \bar{E}_j + y_{ij}^{n} L_i H_u N_j + m_{ij}^{n} N_i N_j + \mu H_u H_d$$

By even/odd combinations of integer

$$\left(k_Q, k_{U,D}, k_L, k_{E,N}, k_{\mathrm{Higgs}}\right)$$

we classified MSSM.

Application for SUSY（4）

$$W = y^u_{ij} Q_i H_u \bar{U}_j + y^d_{ij} Q_i H_d \bar{D}_j + y^l_{ij} L_i H_d \bar{E}_j + y^n_{ij} L_i H_u N_j + m^n_{ij} N_i N_j + \mu H_u H_d$$

$m^n_{ij} \to \underline{2k_N}$ $\mu \to \underline{k_{Hu} + k_{Hd}}$

Application for SUSY（4）

$$W = y^u_{ij} Q_i H_u \overline{U}_j + y^d_{ij} Q_i H_d \overline{D}_j + y^l_{ij} L_i H_d \bar{E}_j + y^n_{ij} L_i H_u N_j + m^n_{ij} N_i N_j + \mu H_u H_d$$

y^l_{ij} → $\underline{k_L + k_{Hd} + k_E}$

y^n_{ij} → $\underline{k_L + k_{Hu} + k_N}$

m^n_{ij} → $\underline{2k_N}$

μ → $\underline{k_{Hu} + k_{Hd}}$

Application for SUSY (4)

$$W = y^u_{ij} Q_i H_u \bar{U}_j + y^d_{ij} Q_i H_d \bar{D}_j + y^l_{ij} L_i H_d \bar{E}_j + y^n_{ij} L_i H_u N_j + m^n_{ij} N_i N_j + \mu H_u H_d$$

$\underline{k_Q + k_{Hu} + k_U}$ $\underline{k_Q + k_{Hd} + k_D}$ $\underline{k_L + k_{Hd} + k_E}$ $\underline{k_L + k_{Hu} + k_N}$ $\underline{2k_N}$ $\underline{k_{Hu} + k_{Hd}}$

MSSM → 4 types

Integer weights（1）

Example) When k_{Higgs} is even weight,

(i) The others are even

→ B/L num. breaking terms are allowed

including proton decay.

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
$H_u H_d$	✓	✓	✓	✓
LH_u	✓	✓		
$LL\bar{E}$	✓	✓		
$LQ\bar{D}$	✓	✓		
$\bar{U}\bar{D}\bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U}\bar{U}\bar{D}\bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q\bar{U}\bar{E}H_d$	✓	✓		
LH_uLH_u	✓	✓	✓	✓
$LH_uH_dH_u$	✓	✓		
$\bar{U}\bar{D}^*\bar{E}$	✓	✓		
$H_u{}^*H_d\bar{E}$	✓	✓		
$Q\bar{U}L^*$	✓	✓		
$QQ\bar{D}^*$	✓		✓	

Integer weights（2）

Example) When k_{Higgs} is even weight,

(ii) $(k_Q, k_{U,D})$ is odd, $(k_L, k_{E,N})$ is even

→ Baryon num. breaking terms are prohibited.

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
$H_u H_d$	✓	✓	✓	✓
$L H_u$	✓	✓		
$L L \bar{E}$	✓	✓		
$L Q \bar{D}$	✓	✓		
$\bar{U} \bar{D} \bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U} \bar{U} \bar{D} \bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q \bar{U} \bar{E} H_d$	✓	✓		
$L H_u L H_u$	✓	✓	✓	✓
$L H_u H_d H_u$	✓	✓		
$\bar{U} \bar{D}^* \bar{E}$	✓	✓		
$H_u^* H_d \bar{E}$	✓	✓		
$Q \bar{U} L^*$	✓	✓		
$QQ \bar{D}^*$	✓		✓	

Integer weights（2）

Example) When k_{Higgs} is even weight,

(ii) $(k_Q, k_{U,D})$ is odd, $(k_L, k_{E,N})$ is even

→ Baryon num. breaking terms are prohibited.

✓ : $k_L + k_Q + k_D = even + odd + odd = even$

× : $k_U + k_D + k_D = odd + odd + odd = odd$

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
H_uH_d	✓	✓	✓	✓
LH_u	✓	✓		
$LL\bar{E}$	✓	✓		
$LQ\bar{D}$	✓	✓		
$\bar{U}\bar{D}\bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U}\bar{U}\bar{D}\bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q\bar{U}\bar{E}H_d$	✓	✓		
LH_uLH_u	✓	✓	✓	✓
$LH_uH_dH_u$	✓	✓		
$\bar{U}\bar{D}^*\bar{E}$	✓	✓		
$H_u^*H_d\bar{E}$	✓	✓		
$Q\bar{U}L^*$	✓	✓		
$QQ\bar{D}^*$	✓		✓	

Integer weights (3)

Example) When k_{Higgs} is even weight,

(iii) $(k_Q, k_{U,D})$ is even, $(k_L, k_{E,N})$ is odd

→ Lepton num. breaking terms are prohibited.

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
$H_u H_d$	✓	✓	✓	✓
$L H_u$	✓	✓		
$L L \bar{E}$	✓	✓		
$L Q \bar{D}$	✓	✓		
$\bar{U} \bar{D} \bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U} \bar{U} \bar{D} \bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q \bar{U} \bar{E} H_d$	✓	✓		
$L H_u L H_u$	✓	✓	✓	✓
$L H_u H_d H_u$	✓	✓		
$\bar{U} \bar{D}^* \bar{E}$	✓	✓		
$H_u{}^* H_d \bar{E}$	✓	✓		
$Q \bar{U} L^*$	✓	✓		
$QQ \bar{D}^*$	✓		✓	

Integer weights（3）

Example) When k_{Higgs} is even weight,

(iii) $(k_Q, k_{U,D})$ is even, $(k_L, k_{E,N})$ is odd

→ Lepton num. breaking terms are prohibited.

(iv) The others are odd

→ B/L num. breaking terms are prohibited.

: R-parity (Z2 parity)

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
$H_u H_d$	✓	✓	✓	✓
$L H_u$	✓	✓		
$L L \bar{E}$	✓	✓		
$L Q \bar{D}$	✓	✓		
$\bar{U} \bar{D} \bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U} \bar{U} \bar{D} \bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q \bar{U} \bar{E} H_d$	✓	✓		
$L H_u L H_u$	✓	✓	✓	✓
$L H_u H_d H_u$	✓	✓		
$\bar{U} \bar{D}^* \bar{E}$	✓	✓		
$H_u{}^* H_d \bar{E}$	✓	✓		
$Q \bar{U} L^*$	✓	✓		
$QQ\bar{D}^*$	✓		✓	

Rational weights（1）

（II） k_N is even weight

→ Baryon triality (Z3 parity)

（III） k_N is odd weight

→ Proton hexality (Z6 parity)

B/L num. breaking terms are prohibited more strictly.

	(II)	(III)
Yukawa	✓	✓
$H_u H_d$	✓	✓
LH_u	✓	
$LL\bar{E}$	✓	
$LQ\bar{D}$	✓	
$\bar{U}\bar{D}\bar{D}$		
$QQQL$		
$\bar{U}\bar{U}\bar{D}\bar{E}$		
$QQQH_d$		
$Q\bar{U}\bar{E}H_d$	✓	
LH_uLH_u	✓	✓
$LH_uH_dH_u$	✓	
$\bar{U}\bar{D}^*\bar{E}$	✓	
$H_u^* H_d\bar{E}$	✓	
$Q\bar{U}L^*$	✓	
$QQ\bar{D}^*$		

Rational weights（2）

$2k_N = even$ $\qquad$ $k_{Hu} + k_{Hd} = even$

（Ⅲ） $k_N = odd,\ k_Q = 0$

$$(k_{Hu}, k_{Hd}) = \begin{cases} (\text{Integer}, \text{Integer}) \\ \left(\dfrac{Mh \pm n}{M}, \dfrac{Mh \mp n}{M}\right) \end{cases}$$

	(II)	(III)
Yukawa	✓	✓
H_uH_d	✓	✓
LH_u	✓	
$LL\bar{E}$	✓	
$LQ\bar{D}$	✓	
$\bar{U}\bar{D}\bar{D}$		
$QQQL$		
$\bar{U}\bar{U}\bar{D}\bar{E}$		
$QQQH_d$		
$Q\bar{U}\bar{E}H_d$	✓	
LH_uLH_u	✓	✓
$LH_uH_dH_u$	✓	
$\bar{U}\bar{D}^*\bar{E}$	✓	
$H_u{}^*H_d\bar{E}$	✓	
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Rational weights（2）

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$$(k_{Hu}, k_{Hd}) = \begin{cases} (\mathrm{Integer}, \mathrm{Integer}) \\ \left(\frac{Mh \pm n}{M}, \frac{Mh \mp n}{M}\right) \end{cases}$$

$$k_U = k_{Hd} + even, \qquad k_D = k_{Hu} + even$$

$$k_L = -k_{Hd} + odd, \qquad k_E = \pm\frac{2n}{M} + odd$$

	(II)	(III)
Yukawa	✓	✓
$H_u H_d$	✓	✓
LH_u	✓	
$LL\bar{E}$	✓	
$LQ\bar{D}$	✓	
$\bar{U}\bar{D}\bar{D}$		
$QQQL$		
$\bar{U}\bar{U}\bar{D}\bar{E}$		
$QQQH_d$		
$Q\bar{U}\bar{E}H_d$	✓	
LH_uLH_u	✓	✓
$LH_uH_dH_u$	✓	
$\bar{U}\bar{D}^*\bar{E}$	✓	
$H_u^*H_d\bar{E}$	✓	
$Q\bar{U}L^*$	✓	
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Rational weights（3）

$2k_N = even$ $\qquad$ $k_{Hu} + k_{Hd} = even$

（Ⅲ） $k_N = odd,\ k_Q = 0$

$$(k_{Hu}, k_{Hd}) = \begin{cases} (\text{Integer}, \text{Integer}) \\ \left(\frac{Mh \pm n}{M}, \frac{Mh \mp n}{M}\right) \end{cases}$$

✓ ： $k_L + k_{Hu} + k_L + k_{Hu} = even$

× ： $k_U + k_D + k_D = odd$

	(II)	(III)
Yukawa	✓	✓
H_uH_d	✓	✓
LH_u	✓	
$LL\bar{E}$	✓	
$LQ\bar{D}$	✓	
$\bar{U}\bar{D}\bar{D}$		
$QQQL$		
$\bar{U}\bar{U}\bar{D}\bar{E}$		
$QQQH_d$		
$Q\bar{U}\bar{E}H_d$	✓	
LH_uLH_u	✓	✓
$LH_uH_dH_u$	✓	
$\bar{U}\bar{D}^*\bar{E}$	✓	
$H_u^*H_d\bar{E}$	✓	
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Summary

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- In our article, we also discussed
  SUSY SU(5) GUT, and SO(10) GUT.

- We have classified these models, and these have patterns
  which prohibit B/L num. breaking terms.

- Using integer weight, we found Z2 R-parity.
  Using rational weight, we found Z6 proton hexality.

Future works

- This work consider modular transforms in Γ_N.
  However, when we select double covering of Γ_N
  as modular group, allowed interactions
  will be changed in low-energy effective theory.

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  However, when we select double covering of Γ_N
  as modular group, allowed interactions
  will be changed in low-energy effective theory.

- When we consider CP transforms,
  modular group extends to GL(2,Z).

  We are going on first point.

backup

Modular Forms

- $S^2\tau = \tau$ in Γ_N, so $(-1)^k \rho_{ij}(S^2) = I$

- k is even : $\rho_{ij}(S^2) = I$
- k is odd : $\rho_{ij}(S^2) = -I$