Modular symmetry in the SMEFT

Hajime Otsuka

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References :

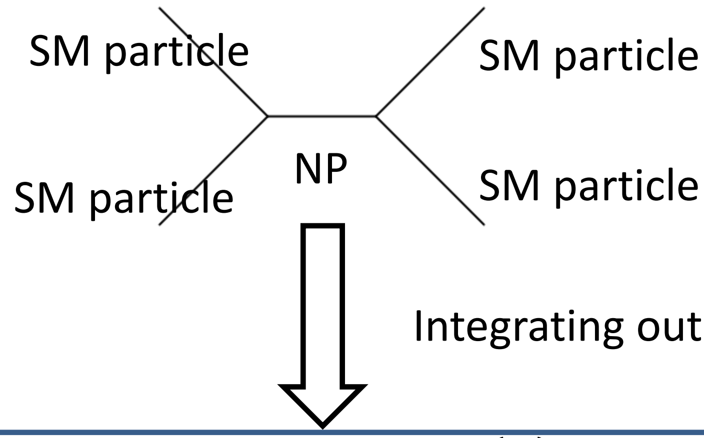
T. Kobayashi (Hokkaido U.), M. Tanimoto (Niigata U.),
K. Yamamoto (Hiroshima Inst. Tech.), arXiv: 2112.00493, 2204.12325

“KEK-PH 2022”@ KEK, 29 Nov. 2022

SMEFT (Standard Model Effective Field Theory)

— powerful bottom-up approach to find signatures of new physics

10^{17} [GeV] **String scale**



Integrating out heavy states

Λ_{NP} [GeV] **Cutoff scale**

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{c^{(5)}}{\Lambda_{\mathrm{NP}}}\mathcal{O}_5 + \frac{c^{(6)}}{\Lambda_{\mathrm{NP}}}\mathcal{O}_6 + \cdots$$

Higher-dimensional operators
—constrained by Lorentz, SM global/gauge sym.

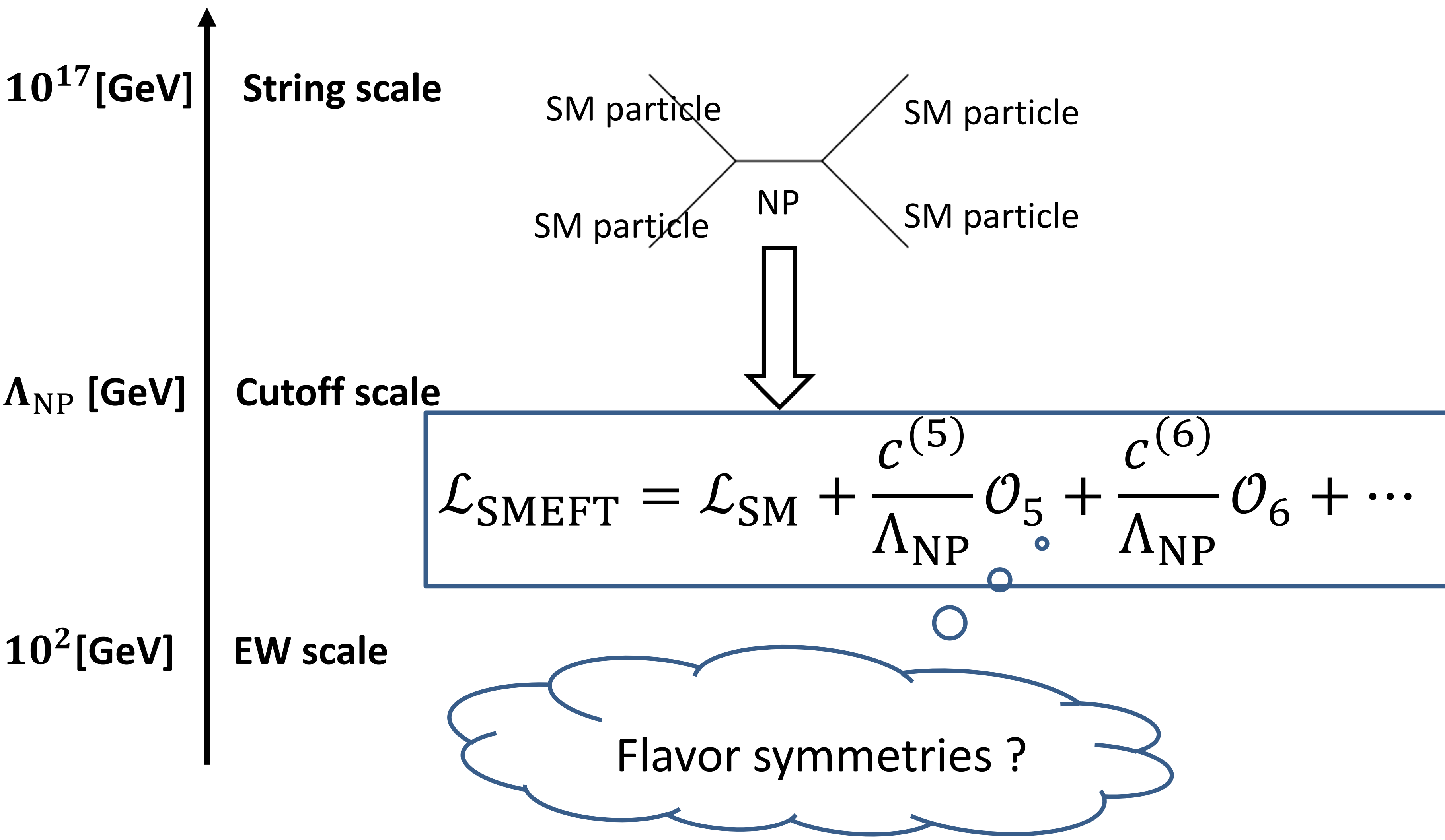
10^{2} [GeV] **EW scale**

— 2499 dim-6 op. in the SMEFT in the absence of any flavor sym.

e.g., $\Delta F = 1$ semi-leptonic operators : $(\bar{L}\gamma^{\mu}L)(\bar{Q}\gamma_{\mu}Q)$, …

Grzadkowski-Iskrzynski-Misiak-Rosiek, 1008.4884

Flavor symmetries from UV physics (string theory) ?



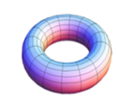
$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{c^{(5)}}{\Lambda_{\rm NP}}\mathcal{O}_5 + \frac{c^{(6)}}{\Lambda_{\rm NP}}\mathcal{O}_6 + \cdots$$

Modular symmetry

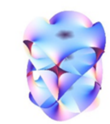
- Absence of UV divergence in superstring theory
- Symmetries in the string EFT

10^{17}[GeV] | String scale

Compactifications :
- Torus



- Calabi-Yau



String EFT with modular symmetry

- Flavor symmetries ⊂ Modular symmetry

F. Feruglio, 1706.08749

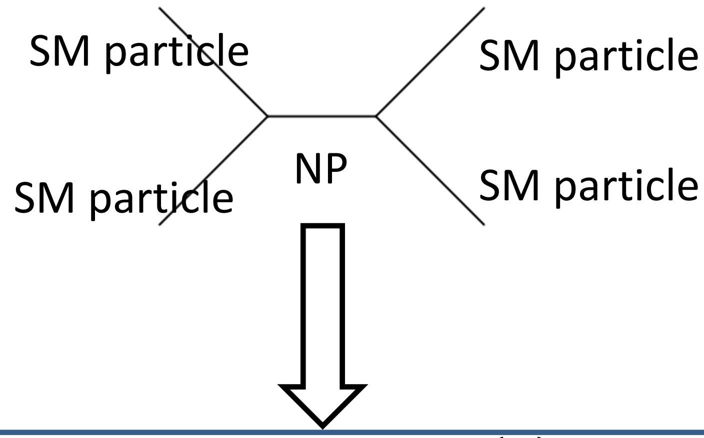
- Modular sym. ⋊ CP ⊂ Generalized modular symmetry

e.g., $SL(2,\mathbb{Z}) \rtimes \mathbb{Z}_2^{\mathrm{CP}} \simeq GL(2,\mathbb{Z})$

1. Torus : Baur-Nilles-Trautner-Vaudrevange, ('19), Novichkov-Penedo-Petcov-Titov ('19)
2. Multi-moduli (Calabi-Yau) : Ishiguro-Kobayashi-Otsuka, ('20)

Today's talk : Modular symmetry in the SMEFT

- 10^{17}[GeV] **String scale**
- $\Lambda_{\rm NP}$ [GeV] **Cutoff scale**
- 10^{2}[GeV] **EW scale**



$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{c^{(5)}}{\Lambda_{\rm NP}}\mathcal{O}_5 + \frac{c^{(6)}}{\Lambda_{\rm NP}}\mathcal{O}_6 + \cdots$$

Higher-dim. operators
with modular sym.

Outline

1. Introduction

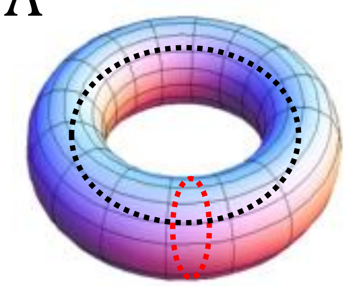
2. **Modular symmetry in the SMEFT**

3. Conclusion

T^2 torus

- $SL(2,\mathbb{Z})$ geometric (modular) symmetry

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$



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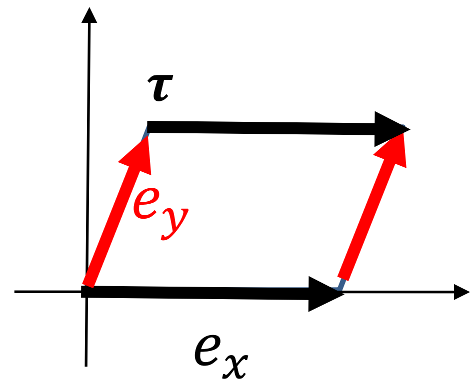


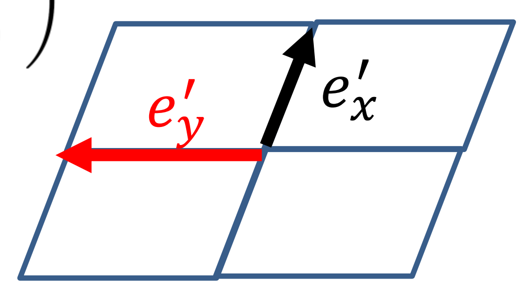
- Lattice vectors are related under $SL(2,\mathbb{Z})$ modular transformation:

$$\begin{pmatrix} e'_y \\ e'_x \end{pmatrix} = \begin{pmatrix} p & q \\ s & t \end{pmatrix} \begin{pmatrix} e_y \\ e_x \end{pmatrix}$$

$p, q, s, t \in \mathbb{Z}$ satisfying $pt - qs = 1$

Two generators : S and T

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



T^2 torus

- $SL(2,\mathbb{Z})$ geometric (modular) symmetry

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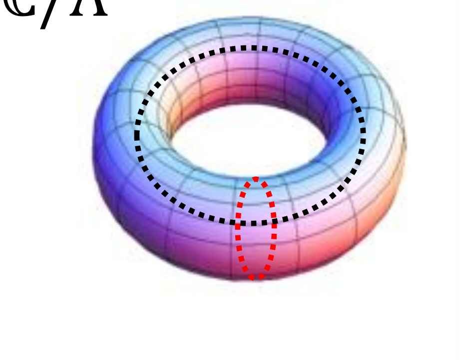

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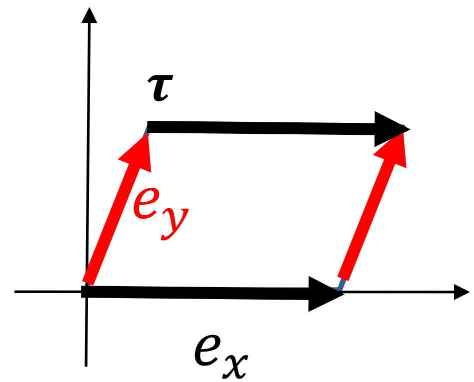

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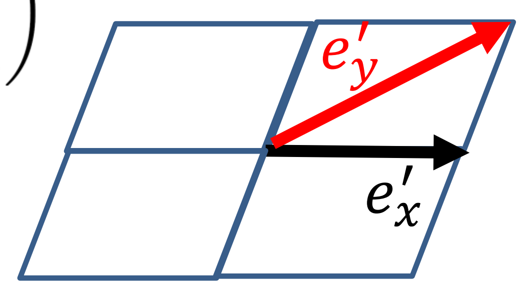
Two generators : S and T

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



Finite subgroups of modular group

Modular group

$$\bar{\Gamma} \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

Finite subgroups

$$\Gamma_N = \bar{\Gamma}/\bar{\Gamma}(N)$$

$\bar{\Gamma}(N)$: congruence subgroup

Non-abelian discrete symmetries : $\Gamma_2 \simeq S_3,\ \Gamma_3 \simeq A_4, \Gamma_4 \simeq S_4, \Gamma_5 \simeq A_5, \dots.$

identified with the flavor symmetries of quarks/leptons

F. Feruglio, 1706.08749

Finite subgroups of modular group

$SL(2,\mathbb{Z})$

$$\{S,T \mid S^2=-\mathbb{I}, S^4=(ST)^3=\mathbb{I}\}$$

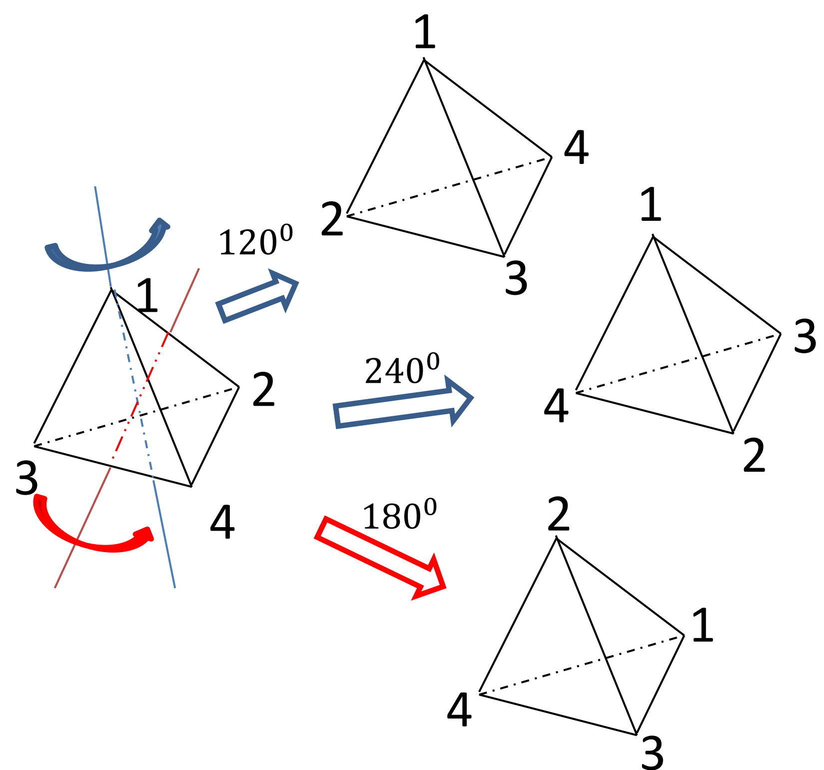
Finite subgroups :

$$\Gamma_N=\{S,T \mid S^2=(ST)^3=T^N=\mathbb{I}\}$$

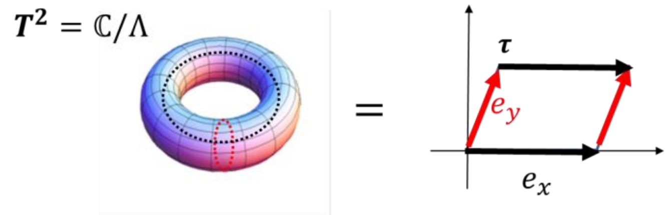
$\Gamma_3 = A_4$: Tetrahedral symmetry

Generators : S and T

$$S^2=(ST)^3=T^3=1$$



$SL(2,Z)$Modular trf.

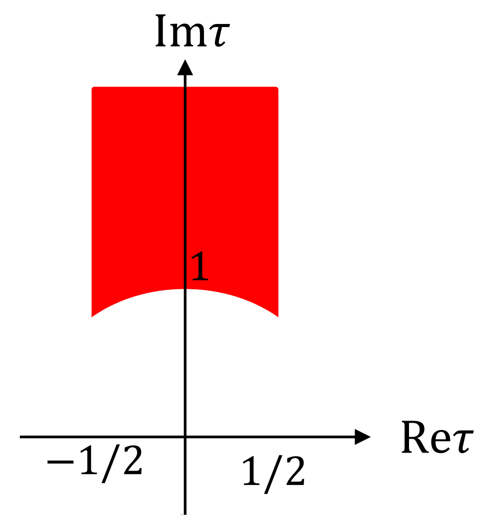


$$\tau \equiv \frac{e_y}{e_x} \to \tau' \equiv \frac{e'_y}{e'_x} = \frac{p\tau + q}{s\tau + t} = R(\tau)$$

Two generators :

$$S:\ \tau \to -1/\tau$$

$$T:\ \tau \to \tau + 1$$



Let us consider 4D effective theory which is invariant under $SL(2,Z)$ modular trf.

Modular symmetric 4D EFT

- In models with modular symmetry,

  Yukawa/Higher-dim. ops.

  = Functions of modular forms with modular weights k_Y ($\in 2\mathbb{Z}$)

$$\sum_n Y_{i_1 \ldots i_n}(\tau)\, \phi_{i_1} \cdots \phi_{i_n}$$

Modular trf. :	$\tau \to \gamma\tau = \dfrac{p\tau + q}{s\tau + t}$
Couplings :	$Y(\tau) \to (s\tau + t)^{k_Y} \rho(\gamma) Y(\tau)$
Matters ϕ_i : (modular weight (k_i))	$\phi_i \to (s\tau + t)^{-k_i} \rho_i(\gamma) \phi_i$

$\rho(\gamma) \in \Gamma_{\mathrm{N}} \subset SL(2, Z)$

When $k_Y = \sum_i k_i$, this action is invariant under discrete modular group Γ

(We don't need the flavon.)

Modular forms

- Couplings are described by the modular function (modular forms)

E.g., $\underline{A_4 \text{ triplet of modular function with } k = 2}$

η : Dedekind eta-function

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right) \\
Y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
Y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),
\end{aligned}
$$

F. Feruglio, 1706.08749

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$$Y_3(\tau) = \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),$$

F. Feruglio, 1706.08749

$$Y_1(\tau) = 1 + 12q + 36q^2 + 12q^3 + \cdots, \quad q = e^{2\pi i\tau}$$

$$Y_2(\tau) = -6q^{1/3}(1 + 7q + 8q^2 + \cdots),$$

$$Y_3(\tau) = -18q^{2/3}(1 + 2q + 5q^2 + \cdots).$$

$\mathrm{Im}\tau \gg 1$

Modular forms

- Modulus-dependent Yukawa couplings would lead to

(1) Mass hierarchy of charged lepton masses
(2) Differences of neutrino masses squared and mixing angles

- $k = 4{,}6$ modular forms : constructed by tensor products of $k = 2$

J. T. Penedo, S. T. Petcov, 1806.11040

$$
\begin{aligned}
Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \cdots , \\
Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \cdots), \\
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\end{aligned}
$$

$q = e^{2\pi i\tau}$

$\mathrm{Im}\tau \gg 1$

Modular A_4 models (Lepton sector)

Kobayashi-Omoto-Shimizu-Takagi-Tanimoto-Tatsuishi, 1808.03012

Okada-Tanimoto, 1905.13421,…Ding-King-Liu, 1907.11714

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{3}$
k	2	(0, 0, 0)	0	2

A_4 singlets : $\mathbf{1} : (S=1, T=1), \quad \mathbf{1}' : (S=1, T=w^2), \quad \mathbf{1}'' : (S=1, T=w)$

A_4 triplet : $S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & w^2 & 0 \\ 0 & 0 & w \end{pmatrix}$

$[\bar{L}_R L_L]$

A_4 : $\{1, 1'', 1'\} \otimes 3$

k_I : 0 $\quad -2$

⇒

modular form

$[\bar{L}_R Y(\tau_q) L_L]$

$\{1, 1'', 1'\} \otimes 3 \otimes 3$

0 $\quad$ 2 $\quad -2$

$$[\bar{L}_R L_L]$$

$A_4 :\quad \{1, 1'', 1'\} \otimes 3$

$k_I :\quad 0 \quad -2$

⇒

modular form

$$[\bar{L}_R Y(\tau_q) L_L]$$

$\{1, 1'', 1'\} \otimes 3 \otimes 3$

$0 \quad 2 \quad -2$

In interaction basis,

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$
$$+ \gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$
$$= (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

$\bar{R}L$ $\bar{L}R$	$\bar{\mu}_R \Gamma \tau_L$ $\bar{\mu}_L \Gamma \tau_R$	$\bar{e}_R \Gamma \tau_L$ $\bar{e}_L \Gamma \tau_R$	$\bar{e}_R \Gamma \mu_L$ $\bar{e}_L \Gamma \mu_R$
Coeff.	$\beta_e\, Y_3(\tau_e)$ $\gamma_e\, Y_2^*(\tau_e)$	$\alpha_e\, Y_2(\tau_e)$ $\gamma_e\, Y_3^*(\tau_e)$	$\alpha_e\, Y_3(\tau_e)$ $\beta_e\, Y_2^*(\tau_e)$

$$[\bar{L}_R L_L] \quad \Rightarrow \quad [\bar{L}_R Y(\tau_q) L_L]$$

modular form

$$A_4: \quad \{1, 1'', 1'\} \otimes 3 \qquad\qquad \{1, 1'', 1'\} \otimes 3 \otimes 3$$

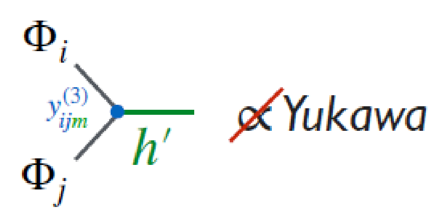
$$k_I: \quad 0 \quad -2 \qquad\qquad 0 \quad 2 \quad -2$$

In interaction basis,

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$
$$+ \gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$
$$= (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

Flavor and mass eigenstates are slightly different

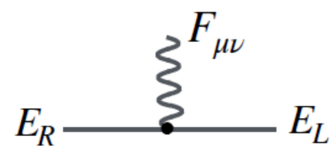
If there are additional unknown modes (e.g., multi Higgs)



SMEFT with modular symmetry

Lepton Flavor Violations (LFV)

Dipole operator



$$\mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2}\left(\mathcal{C}'_{\substack{e\gamma \\ LR}}\mathcal{O}_{\substack{e\gamma \\ LR}} + \mathcal{C}'_{\substack{e\gamma \\ RL}}\mathcal{O}_{\substack{e\gamma \\ RL}}\right)$$

$$\mathcal{O}_{\substack{e\gamma \\ LR}} = \frac{v}{\sqrt{2}}\overline{E}_L\sigma^{\mu\nu}E_R F_{\mu\nu}$$

- Lepton mass matrices are well fitt
  at $\underline{\tau = i + \epsilon}$ ($\underline{|\epsilon| \simeq O(10^{-2})}$)

- Modular A_4 $(\subset SL(2,Z))$ case

T. Kobayashi, H.O., M. Tanimoto, K. Yamamoto, 2204.12325, 2112.00493

$$\text{BR}(\tau \to \mu\gamma) \gg \text{BR}(\mu \to e\gamma) \sim \text{BR}(\tau \to e\gamma)$$

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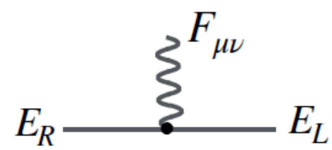

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T. Kobayashi, H.O., M. Tanimoto, K. Yamamoto, 2204.12325, 2112.00493

$$\mathrm{BR}(\tau \to \mu\gamma) \gg \mathrm{BR}(\mu \to e\gamma) \sim \mathrm{BR}(\tau \to e\gamma)$$

- U(2) case

$$\mathrm{BR}(\tau \to \mu\gamma) \gg \mathrm{BR}(\mu \to e\gamma) \gg \mathrm{BR}(\tau \to e\gamma)$$

G. Isidori, J. Pages and F. Wilsch, 2111.13724

Conclusion

10^{18}[GeV] **Quantum gravity**

10^{17}[GeV] **Superstring theory**

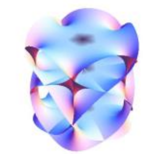
10^{16}[GeV] **GUT**

10^{2}[GeV] **EW scale**

Modular symmetries of 6D space

Torus :

- $SL(2,\mathbb{Z})$ modular group



Calabi-Yau :

- $Sp(2h+2,\mathbb{Z})$ modular group

Strong constraints on the string EFT
（Yukawa couplings/Higher-dim. operators
：function of moduli fields）

Phenomenology :

- LFV in the SMEFT
- B/L-num. violating operators
  (Talk by S. Nishimura)