Experimental testability of GUT model including the mirror particles

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Contents

Introduction

The current state of particle physics

Elementary particles are well described by the standard model(SM).

SM is based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries.

Strong force Weak force Electromagnetic force

But there are phenomena that cannot be explained by the SM.

- The origin of neutrino mass
- Dark matter
- Inflation etc.

Physics beyond the standard model (BSM) is needed!!!

Grand Unified Theory (GUT)

The theory of embedding $SU(3)_C \times SU(2)_L \times U(1)_Y$ into a large group.

Ex). Minimal SU(5) model

H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438

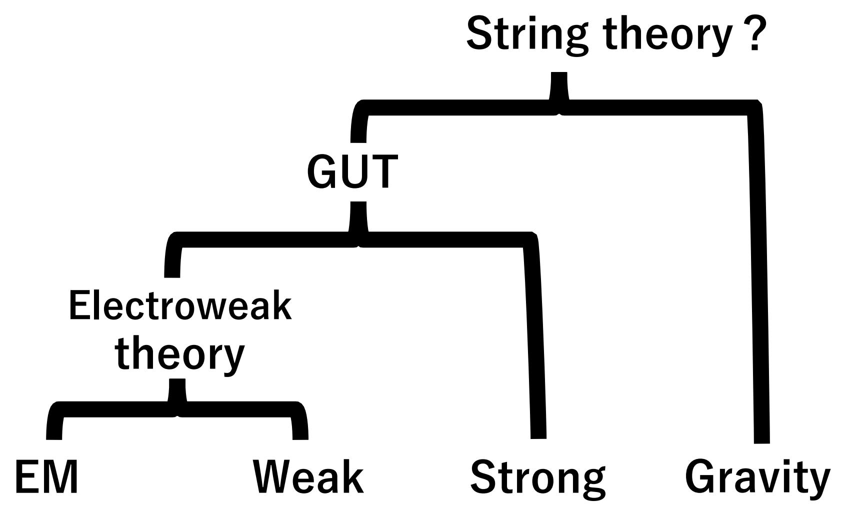
- Unify the SM gauge interactions → **Unification of strong, weak, and electromagnetic forces.**

$$A_\mu = \begin{pmatrix} G_\mu - \frac{1}{\sqrt{15}} B_\mu & V_\mu^\dagger \\ V_\mu & W_\mu + \frac{3}{2\sqrt{15}} B_\mu \end{pmatrix}$$

$G_\mu \quad W_\mu \quad B_\mu$

- Unification of quarks and leptons

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L , \quad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & e^c \\ d^1 & d^2 & d^3 & -e^c & 0 \end{pmatrix}_L$$



Proton decay

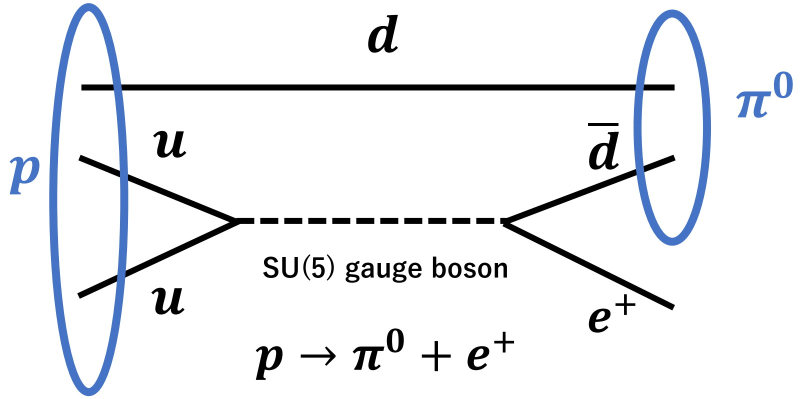
Q. Is it possible to test GUT experimentally? **A. YES**

In GUT, quarks and leptons are embedded into same representations.

→ The GUT predicts the existence of proton decay.

The GUT can be tested by proton decay search!!!

Current experimental results
Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$years
A. Takenaka et al. Phys. Rev. D 102, 112011 (2020)



The problems of GUT

➢ Inconsistency with experimental results

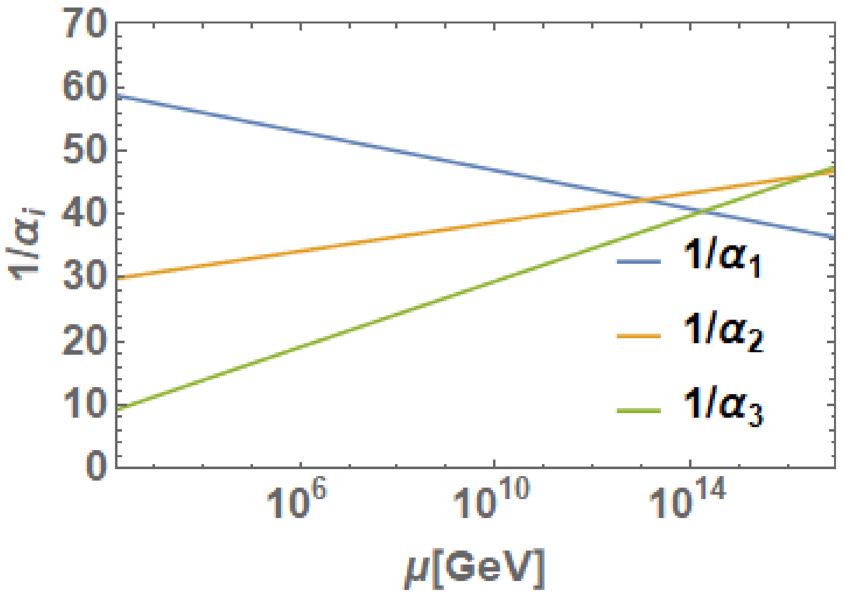
Minimal SU(5) model : $\tau_p(p \to \pi^0 e^+) \approx 10^{30} \sim 10^{31}$ years

H. Georgi, H.R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974)

Current experimental results

Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$ years

A. Takenaka et al. Phys. Rev. D 102, 112011 (2020)

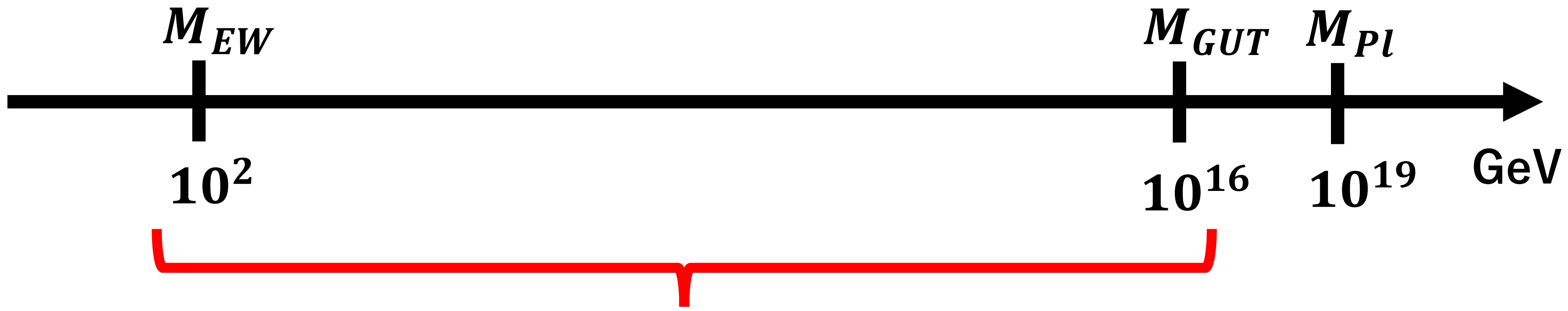


Blue line : U(1) gauge coupling
Orange line : SU(2) gauge coupling
Green line : SU(3) gauge coupling

➢ No unification of the SM gauge couplings successfully

The GUT is needed some extensions!

Hierarchy



There is a large energy gap between the electroweak scale M_{EW} and the GUT scale M_{GUT}.

→ The possibility of new physics in intermediate scale.

Ex). New particles, New symmetries

The new physics contribute to RGE and proton lifetime.

Today's talk

We build a new GUT model based on previous study.

N. Okada, D. Raut and Q. Shafi, Phys. Rev. D 104, no. 5, 055041 (2021)

Our model : $SU(5) \times U(1)_X \times U(1)_{PQ}$ model

Previous study : Add one family of the same representation of the SM particles and one mirror family.

Our study : Add only three mirror families.

We discuss the problems of GUT in terms of mass relations.

Mirror family : The conjugate of the representation of the SM particles

$$\tilde{\psi}_5 = D^c(3, 1, -1/3) \oplus L(1, 2, -1/2)$$

$$\tilde{\psi}_{\overline{10}} = U^c(3, 1, -2/3) \oplus Q(3^*, 2^*, 1/6) \oplus E^c(1, 1, 1)$$

Our model

Symmetries

- $U(1)_{PQ}$ symmetry R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977) R. D. Peccei, Lect. Notes Phys. 741, 3 (2008)
  - ➢ Solve the strong CP problem.
  - ➢ The candidate of dark matter. N. Okada, D. Raut and Q. Shafi, Phys. Rev. D 104, no. 5, 055041 (2021)
- $U(1)_X$ symmetry T. Appelquist, B. A. Dobrescu and A. R. Hopper, Phys. Rev. D 68, 035012 (2003)

The definition of $U(1)_X$ charge: $Q_X = x_H Q_Y + Q_{B-L}$

Q_Y : Hyper charge
Q_{B-L} : B−L(Baryon-Lepton) charge
x_H : free parameter

In $x_H = -4/5$, we can assign the $U(1)_X$ charge to $\bar{5}$ representation and 10 representation successfully.

Matter contents of our model

$SU(5)\times U(1)_X\times U(1)_{PQ}$ model

	SU(5)	$U(1)_X$	$U(1)_{PQ}$
$\psi^i_{\bar{5}}$	$\bar{\mathbf{5}}$	$-3/5$	0
ψ^i_{10}	$\mathbf{10}$	$+1/5$	0
$\tilde{\psi}^i_5$	$\mathbf{5}$	$+3/5$	1
$\tilde{\psi}^i_{\overline{10}}$	$\overline{\mathbf{10}}$	$-1/5$	1
$(N^c)^j$	$\mathbf{1}$	$+1$	0
Σ	$\mathbf{24}$	0	-1
χ	$\mathbf{45}$	$-2/5$	0
Φ	$\mathbf{1}$	-2	0
H	$\mathbf{5}$	$-2/5$	0

$\boldsymbol{\psi^i_{\bar{5}(10)}}(\boldsymbol{i=1\sim3})$: the SM particles

$\boldsymbol{\tilde{\psi}^i_{5(\overline{10})}}(\boldsymbol{i=1\sim3})$: the mirror particles

$\boldsymbol{(N^c)^j(j=1\sim3)}$: Majorana neutrinos

$\boldsymbol{\Sigma,\chi,\Phi,H}$: complex scalar fields

Scenario of symmetry breaking

Four complex scalar fields Σ, χ, Φ, H break symmetry just below.

$$SU(5) \times U(1)_X \times U(1)_{PQ} \xrightarrow{<\Sigma>} SU(3)_C \times SU(2)_L \times U(1)_X \times U(1)_Y$$

$$\xrightarrow{<\Phi>} SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\xrightarrow{<H>,<\chi>} SU(3)_C \times U(1)_{em}$$

Yukawa interactions

In our model, there are three types of Yukawa interactions.

- Involved only the SM particles

$$\mathcal{L}_{SM} \supset \sum_{i,j=1}^{3} \left[(Y_1^{ij} H + Y_2^{ij} \chi) \psi_{10}^i \psi_{10}^j \right] + \sum_{i,j=1}^{3} \left[\left(Y_3^{ij} H^* + Y_4^{ij} \chi^* \right) \psi_{\bar{5}}^i \psi_{10}^j \right] + \text{h.c.}$$

- Involved the mirror particles

$$\mathcal{L}_{mirror} \supset \sum_{i,j=1}^{3} \tilde{Y}_5^{ij} \Sigma \psi_{\bar{5}}^i \tilde{\psi}_5^j + \sum_{i,j=1}^{3} \tilde{Y}_{10}^{ij} \Sigma \psi_{10}^i \tilde{\psi}_{\overline{10}}^j + \text{h.c.}$$

- Involved the Majorana neutrinos

$$\mathcal{L}_{neutrino} \supset - \sum_{i,j=1}^{3} Y_D^{ij} H \psi_{\bar{5}}^i (N^c)^j - \left(\frac{1}{2} \sum_{\beta=1}^{3} Y_M^{\beta} \Phi (N^c)^{\beta} (N^c)^{\beta} + \text{h.c.} \right)$$

Derivation of mass eigenvalues

We assume the diagonal matrices for computing up-type quark, down-type quark, and charged lepton mass eigenvalues for simplicity.

$$
\begin{array}{c|ccc|ccc}
 & & \mathrm{SM}_R & & & \mathrm{Mirror}_R & \\
\hline
 & m_{11} & & & m_{14} & & \\
\mathrm{SM}_L & & m_{22} & & & m_{25} & \\
 & & & m_{33} & & & m_{36} \\
\hline
 & m_{41} & & & & & \\
\mathrm{Mirror}_L & & m_{52} & & & & \\
 & & & m_{63} & & & \\
\end{array}
$$

Mass relations

We get the mass relations between up-type quark mass m_{ui} and down-type quark mass m_{di}, charged lepton mass m_{ei} $(i = 1\sim6)$.

$$m_{d1}^2 m_{d2}^2 = \frac{m_{u1} m_{u2} (v_\Sigma \tilde{Y}_5^{11})^2}{240}、 \quad m_{e1}^2 m_{e2}^2 = \frac{27 m_{u1} m_{u2} (v_\Sigma \tilde{Y}_5^{11})^2}{80}$$

$$m_{d3}^2 m_{d4}^2 = \frac{m_{u3} m_{u4} (v_\Sigma \tilde{Y}_5^{22})^2}{240}、 \quad m_{e3}^2 m_{e4}^2 = \frac{27 m_{u3} m_{u4} (v_\Sigma \tilde{Y}_5^{22})^2}{80}$$

$$m_{d5}^2 m_{d6}^2 = \frac{m_{u5} m_{u6} (v_\Sigma \tilde{Y}_5^{33})^2}{240}、 \quad m_{e5}^2 m_{e6}^2 = \frac{27 m_{u5} m_{u6} (v_\Sigma \tilde{Y}_5^{33})^2}{80}$$

We identify the mass eigenvalues to satisfy the mass relations.

Neutrino mass eigenvalues

In neutrino case, we also assume the diagonal matrix for simplicity.

$$
\begin{array}{c}
\mathrm{SM_L} \\ \\ \\ \mathrm{Mirror_L} \\ \\ \\ \mathrm{majorana} \\ \\ \\
\end{array}
\begin{pmatrix}
 & & & nu_{14} & & & nu_{17} & & \\
 & & & & nu_{25} & & & nu_{28} & \\
 & & & & & nu_{36} & & & nu_{39} \\
nu_{41} & & & & & & & & \\
 & nu_{52} & & & & & & & \\
 & & nu_{63} & & & & & & \\
nu_{71} & & & & & & nu_{77} & & \\
 & nu_{82} & & & & & & nu_{88} & \\
 & & nu_{93} & & & & & & nu_{99}
\end{pmatrix}
$$

Type- I
Seesaw mechanism

$$ (6\times3) \times \left(3\times3 \right)^{-1} \times (3\times6) $$

Minkowski '77; Gell-Mann, Ramond, Slansky; Yanagida; Glashow; Mohapatra, Senjanovic '79

$$
\begin{pmatrix}
nu_{11} & & & nu_{14} & & & & & \\
 & nu_{22} & & & nu_{25} & & & & \\
 & & nu_{33} & & & nu_{36} & & & \\
nu_{41} & & & & & & & & \\
 & nu_{52} & & & & & & & \\
 & & nu_{63} & & & & & & \\
 & & & & & & nu'_{77} & & \\
 & & & & & & & nu'_{88} & \\
 & & & & & & & & nu'_{99}
\end{pmatrix}
$$

Get the neutrino mass eigenvalues.

Neutrino mass relations

We get the mass relations between neutrino mass m_i and up-type quark mass m_{ui}, down-type quark mass m_{di} $(i = 1\sim6)$.

$m_i (i = 1\sim6)$: neutrino mass eigenvalues

$$m_1 m_2 = \frac{9m_{d1}^2 m_{d2}^2}{m_{u1} m_{u2}}、\quad m_3 m_4 = \frac{9m_{d3}^2 m_{d4}^2}{m_{u3} m_{u4}}、\quad m_5 m_6 = \frac{9m_{d5}^2 m_{d6}^2}{m_{u5} m_{u6}}$$

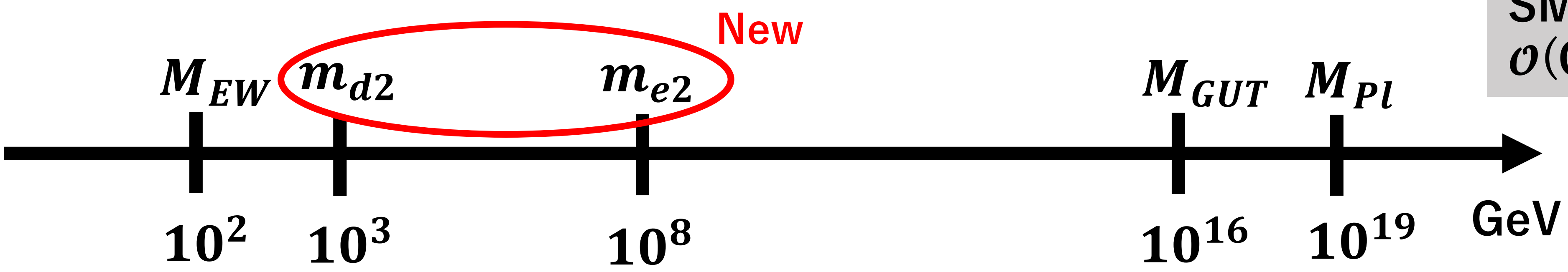
We can estimate the mirror neutrino masses using these neutrino mass relations.

Numerical analysis

The identification of Fermion masses

We identify the fermion masses to satisfy the mass relations and to unify the SM gauge couplings successfully.

m_{u1}	SM	m_{u2}	$\mathcal{O}$(GUT)	m_{u3}	SM	m_{u4}	$\mathcal{O}$(GUT)	m_{u5}	SM	m_{u6}	$\mathcal{O}$(GUT)
m_{d1}	bottom quark	m_{d2}	$\mathcal{O}(10^3)$GeV	m_{d3}	SM	m_{d4}	$\mathcal{O}$(GUT)	m_{d5}	SM	m_{d6}	$\mathcal{O}$(GUT)
m_{e1}	electron	m_{e2}	$7.36 \times 10^4 \times m_{d2}$	m_{e3}	SM	m_{e4}	$\mathcal{O}$(GUT)	m_{e5}	SM	m_{e6}	$\mathcal{O}$(GUT)



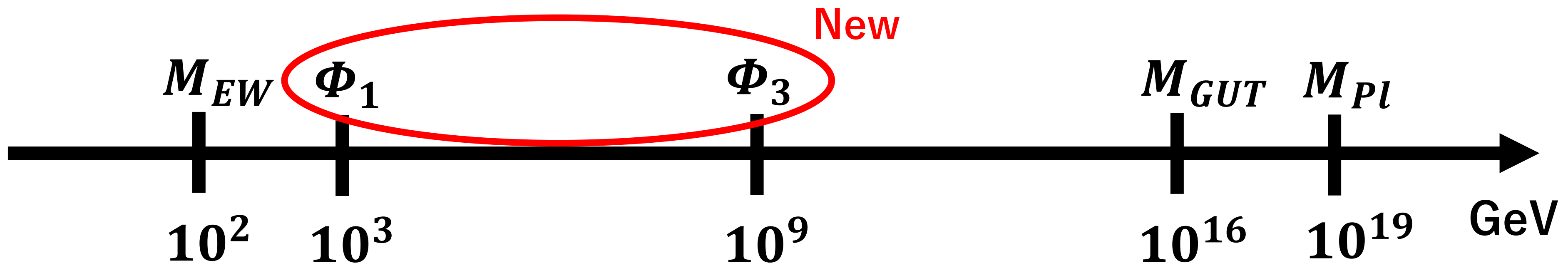
SM : the SM particles
$\mathcal{O}$(GUT) : GUT scale

The identification of scalar masses

The 45 representation Higgs χ is composed just below.

$$\chi_{45} \sim \underbrace{\left(8, 2, \frac{1}{2}\right)}_{\Phi_1} \oplus \underbrace{\left(\overline{6}, 1, -\frac{1}{3}\right)}_{\Phi_2} \oplus \underbrace{\left(3, 3, -\frac{1}{3}\right)}_{\Phi_3} \oplus \underbrace{(\overline{3}, 2, -7/6)}_{\Phi_4} \oplus \underbrace{(3, 1, -1/3)}_{\Phi_5} \oplus \underbrace{(\overline{3}, 1, 4/3)}_{\Phi_6} \oplus \underbrace{(1, 2, 1/2)}_{H_2}$$

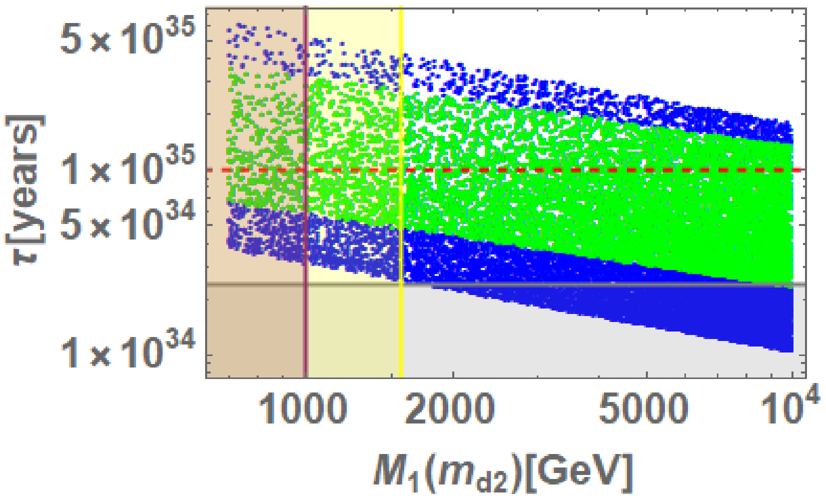
To unify the SM gauge couplings successfully, we assume that Φ_1 has mass $M_1 = \mathcal{O}(10^3)$GeV and Φ_3 has mass $M_3 = \mathcal{O}(10^9)$GeV. (The other scalars have GUT scale masses.)



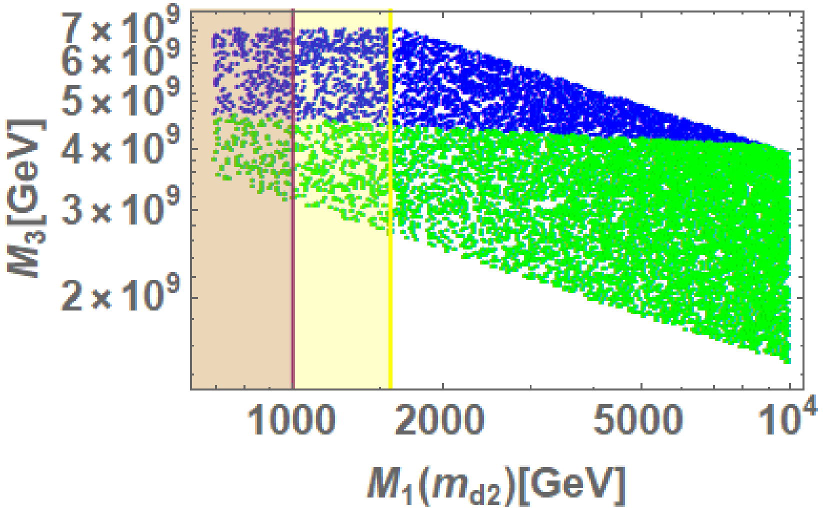
The relation of new particles

We assume m_{d2} and M_1 have the same mass.

- The relation between $M_1(m_{d2})$ and proton lifetime



- The relation between $M_1(m_{d2})$ and M_3



Green : Accuracy of unification 1%

Blue : Accuracy of unification 3%

Purple : Excluded regions of M_1
(M_1 >1TeV)
V. Miralles and A. Pich, arXiv : 1910.07947

Yellow : Excluded regions of m_{d2}
(m_{d2}>1570GeV)
R.L. Workman et al, Particle Data Group(2022)

Gray : Super-Kamiokande
$\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$years
A. Takenaka et al. Phys. Rev. D 102, 112011 (2020)

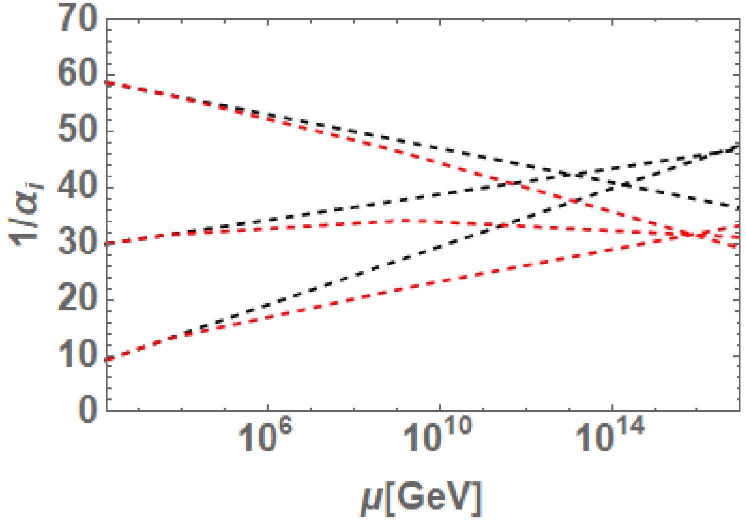
Red : Hyper-Kamiokande
$\tau_p(p \to \pi^0 e^+) \lesssim 1.0 \times 10^{35}$years

HYPER-KAMIOKANDE collaboration (2019)

Unification of the SM gauge couplings

Black : SM
Red : Our model

Benchmark point :
$m_{d2} = M_1$ =5TeV、
m_{e2}=7.36 × 10^4 × m_{d2}=3.7 × 10^8GeV、
M_3=2 × 10^9GeV

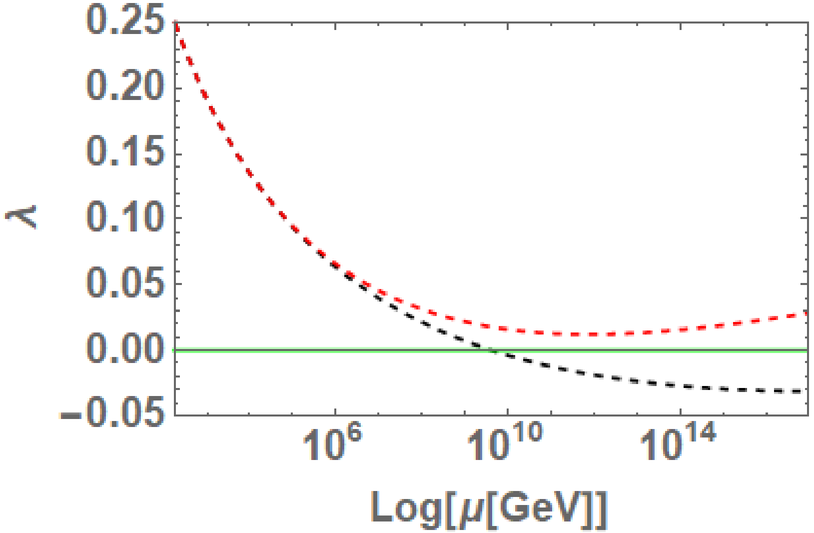


M_{GUT} ≈7.9 × 10^{15}GeV

$\alpha_{\mathrm{GUT}} = \alpha_1 = \alpha_2 = \alpha_3$ ≈1/31.6

$$\tau_p(p \to \pi^0 e^+) \approx \frac{1}{\alpha_{\mathrm{GUT}}^2} \frac{M_{\mathrm{GUT}}^4}{m_p^5} \approx 8.86 \times 10^{34}\text{years}$$

P. Nath and P. Fileviez Perez, Phys. Rept. 441, 191 (2007)



Heavy neutrino masses

Using benchmark point($m_{u2} = M_{GUT} = 7.9 \times 10^{15}$GeV、$m_{d1} = 4.18$GeV、$m_{d2} = 5$TeV), we estimate heavy neutrino masses.

R.L. Workman et al, Particle Data Group(2022)

Ex). $m_{u1} = 172.69$GeV(top quark)

$$m_1 m_2 = 2.88 \times 10^9 (\text{eV})^2$$

$$m_1 m_2 = \frac{9 m_{d1}^2 m_{d2}^2}{m_{u1} m_{u2}}$$

Substitute the upper bound of the lightest neutrino mass.

Planck (2018)
NuFIT v5.1

Normal ordering : $m_{lightest} < 0.03$eV

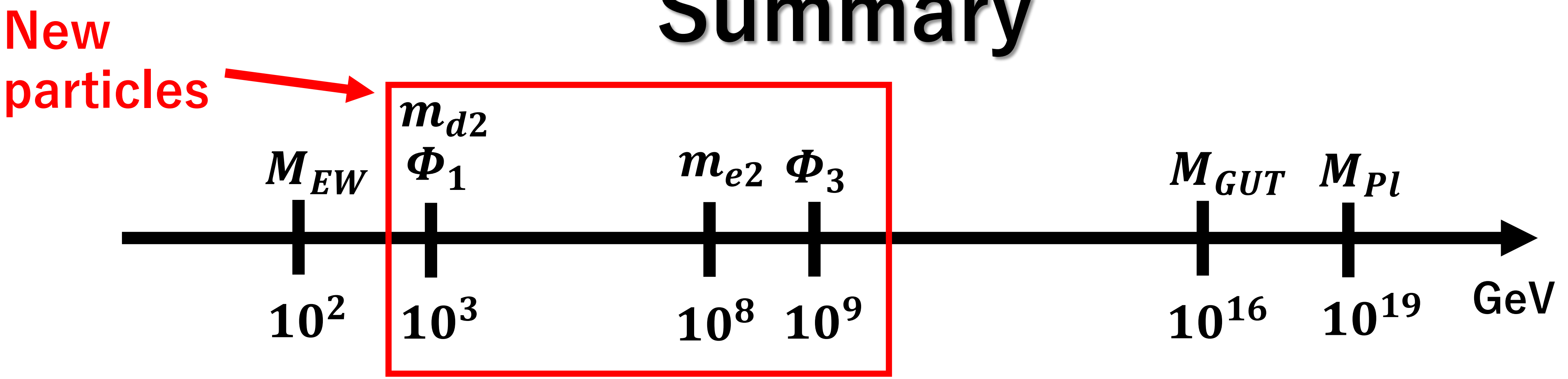
→ $m_{heavy} > 96.1$GeV

Inverted ordering : $m_{lightest} < 0.016$eV

→ $m_{heavy} > 180.13$GeV

Summary

Summary



- We assume new particles in intermediate scale from mass relations. Then, the SM gauge couplings unify successfully at high energy and our model can be tested by Hyper-Kamiokande experiment.

- We estimate heavy neutrino masses from neutrino mass relations.

- In future work, we discuss dark matter and inflation etc.

Backup

Accuracy of unification

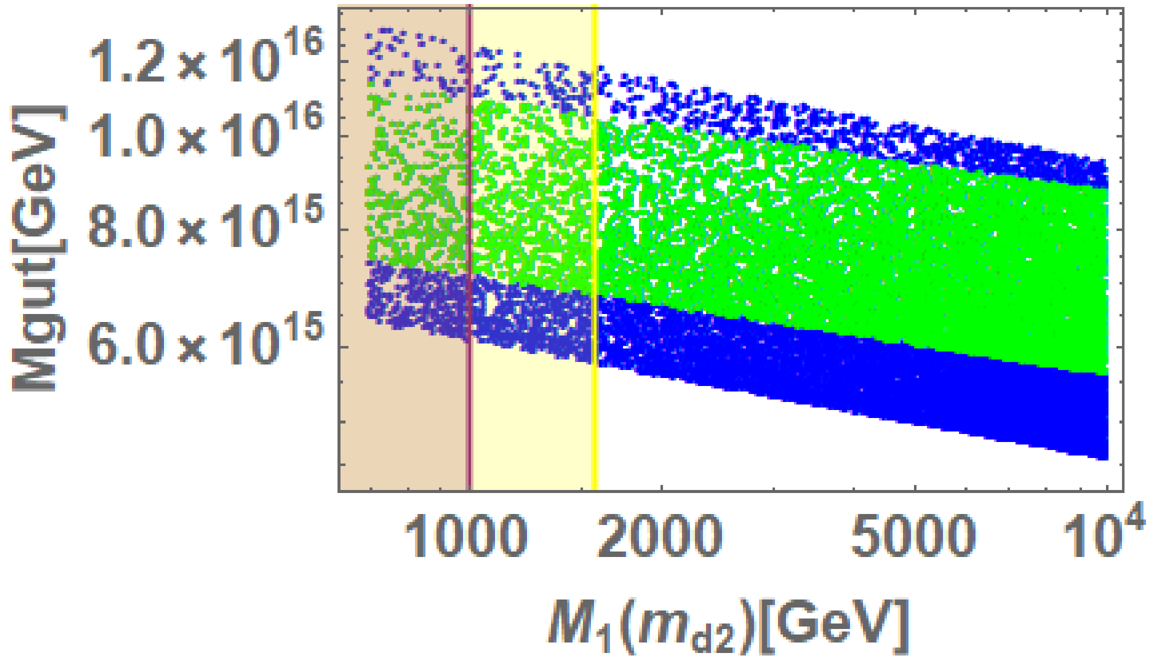
The SM gauge couplings α_1, α_2, and α_3

We assume $r_{12} = \frac{\alpha_2}{\alpha_1}$、 $r_{23} = \frac{\alpha_3}{\alpha_2}$.

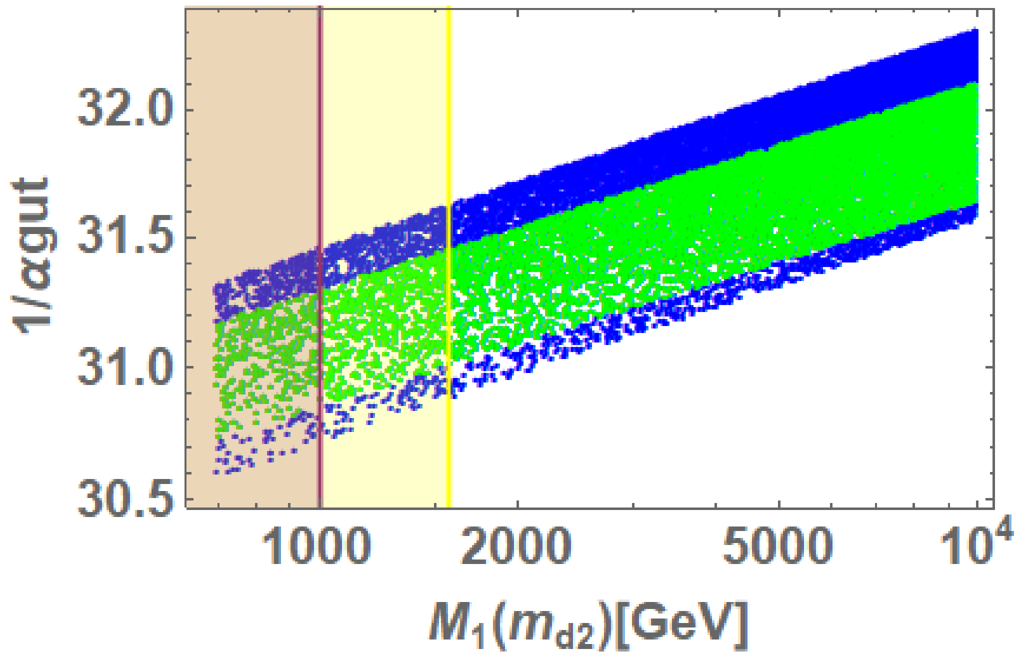
If $0.99 < \frac{r_{23}}{r_{12}} < 1.01$, accuracy of unification is 1% or less.
If $0.97 < \frac{r_{23}}{r_{12}} < 1.03$, accuracy of unification is 3% or less.

The comparison about $M_{\rm GUT}$ and $\alpha_{\rm GUT}$

- The relation between $M_1(m_{d2})$ and $M_{\rm GUT}$



- The relation between $M_1(m_{d2})$ and $\alpha_{\rm GUT}$



The calculation of beta function

The contributions of new particles are added from the mass of each particle.

Beta coefficient : $b_i = -\frac{11}{3}N + \frac{2}{3}T(R_f)N_f^c + \frac{1}{3}T(R_s)N_s \quad (i = 1 \sim 3)$

N : N of SU(N) group
N_f^c : the number of chiral fermion
N_s : the number of complex scalar

$$T(R) = \mathrm{Tr}[L^i L^j] = \begin{cases} \frac{1}{2}\delta_{ij} & (R : \text{basic representation}) \\ N\delta_{ij} & (R : \text{adjoint representation}) \end{cases}$$

Ex). One SU(3) triplet chiral fermion case is $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Vacuum Expectation Value(VEV)

- The VEV of 5 representation Higgs: $<H>=(0,0,0,0,\frac{v}{\sqrt{2}})$

- The VEV of 24 representation Higgs : $<\Sigma>=\frac{v_\Sigma}{2\sqrt{15}}$ Diag $(-2,-2,-2,3,3)$

- The VEV of 1 representation Higgs : $<\Phi>=\frac{v_\Phi}{\sqrt{2}}$

- The 45 representation Higgs : $\chi_c^{ab}=-\chi_c^{ba}$、$\chi_a^{ab}=0(a,b,c=1\sim5)$

  its VEV: $<\chi>_1^{15}=<\chi>_2^{25}=<\chi>_3^{35}=\frac{v_\chi}{\sqrt{2}}$、 $<\chi>_4^{45}=-\frac{3v_\chi}{\sqrt{2}}$

Neutrino mass relations

$$m_1 m_2 = \frac{9 m_{d1}^2 m_{d2}^2}{m_{u1}^2 m_{u2}^2}$$

- $m_{u1} = 2.16\text{MeV}$(up quark)

$$m_1 m_2 = 2.3 \times 10^{14} (\text{eV})^2$$

- $m_{u1} = 1.27\text{GeV}$(charm quark)

$$m_1 m_2 = 3.92 \times 10^{11} (\text{eV})^2$$

- $m_{u1} = 172.69\text{GeV}$(top quark)

$$m_1 m_2 = 2.88 \times 10^{9} (\text{eV})^2$$

Heavy neutrino mass

Normal ordering : $m_{lightest} < 0.03\text{eV}$

- $m_{u1} = 2.16\text{MeV}$(up quark)

  $m_{heavy} > 7.68 \times 10^6\text{GeV}$

- $m_{u1} = 1.27\text{GeV}$(charm quark)

  $m_{heavy} > 1.31 \times 10^4\text{GeV}$

- $m_{u1} = 172.69\text{GeV}$(top quark)

  $m_{heavy} > 96.1\text{GeV}$

Inverted ordering : $m_{lightest} < 0.016\text{eV}$

- $m_{u1} = 2.16\text{MeV}$(up quark)

  $m_{heavy} > 1.44 \times 10^7\text{GeV}$

- $m_{u1} = 1.27\text{GeV}$(charm quark)

  $m_{heavy} > 2.45 \times 10^4\text{GeV}$

- $m_{u1} = 172.69\text{GeV}$(top quark)

  $m_{heavy} > 180.13\text{GeV}$

Mass eigenvalues

1 : − 2 : +

- Up-type quark

$$m_{u1,2} = \frac{1}{480}\left[17\left(v_\Sigma \tilde{Y}^{11}_{10}\right)^2 + 1920\left(v_H Y^{11}_1\right)^2 \pm \sqrt{-64 v_\Sigma^2 \left(\tilde{Y}^{11}_{10}\right)^4 + \left\{17\left(v_\Sigma \tilde{Y}^{11}_{10}\right)^2 + 1920\left(v_H Y^{11}_1\right)^2\right\}^2}\right]$$

- Down-type quark

$$m_{d1,2} = \frac{1}{120}\left[15 m_{u1} m_{u2} + 15\left(v_H Y^{11}_3 + 2 v_\chi Y^{11}_4\right)^2 + \left(v_\Sigma \tilde{Y}^{11}_5\right)^2 \pm \sqrt{-60 m_{u1} m_{u2} \left(v_\Sigma \tilde{Y}^{11}_5\right)^2 + \left\{15 m_{u1} m_{u2} + 15\left(v_H Y^{11}_3 + 2 v_\chi Y^{11}_4\right)^2 + \left(v_\Sigma \tilde{Y}^{11}_5\right)^2\right\}^2}\right]$$

- Charged lepton

$$m_{e1,2} = \frac{1}{160}\left[720 m_{u1} m_{u2} + 20\left(v_H Y^{11}_3 - 6 v_\chi Y^{11}_4\right)^2 + 3\left(v_\Sigma \tilde{Y}^{11}_5\right)^2 \pm \sqrt{-8640 m_{u1} m_{u2} \left(v_\Sigma \tilde{Y}^{11}_5\right)^2 + \left\{720 m_{u1} m_{u2} + 20\left(v_H Y^{11}_3 - 6 v_\chi Y^{11}_4\right)^2 + 3\left(v_\Sigma \tilde{Y}^{11}_5\right)^2\right\}^2}\right]$$

- Neutrino

$$m_{1,2} = \frac{1}{2\sqrt{2} v_\Phi Y^1_M}\left[\left(v_H Y^1_D\right)^2 \pm \sqrt{\left(v_H Y^1_D\right)^4 + \frac{72\left(m_{d1} m_{d2} v_\Phi Y^1_M\right)^2}{m_{u1} m_{u2}}}\right]$$

The Yukawa interactions of the SM particles

$$M_U = \frac{1}{\sqrt{2}}[2(Y_1 + Y_1^T)v_H - 4(Y_2 - Y_2^T)v_\chi]$$

$$M_D = \frac{1}{2}(Y_3^T v_H^* + 2Y_4^T v_\chi^*)$$

$$M_E = \frac{1}{2}(Y_3 v_H^* - 6Y_4 v_\chi^*)$$

The mass difference between M_D and M_E.

The product of SU(5) representations

$$\mathbf{5} \times \mathbf{5} = \mathbf{10} + \mathbf{15}$$

$$\overline{\mathbf{5}} \times \mathbf{10} = \mathbf{5} + \overline{\mathbf{45}}$$

$$\mathbf{10} \times \mathbf{10} = \overline{\mathbf{5}} + \mathbf{45} + \mathbf{50}$$

$$\mathbf{10} \times \overline{\mathbf{10}} = \mathbf{1} + \mathbf{24} + \mathbf{75}$$