# Unstable Nambu-Goldstone modes

Ryo Yokokura (KEK)

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and International Joint Workshop on the Standard Model and beyond

Based on Naoki Yamamoto & RY, Phys. Rev. D **106** 105004 [2203.02727]

# Two messages

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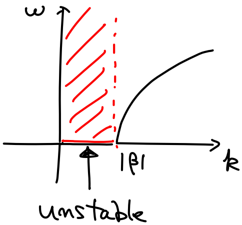
2: A class of instabilities can be universally understood by **symmetries**.

# Abstract



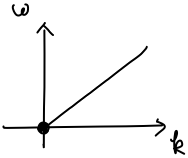

- Nambu-Goldstone (NG) modes can become unstable in the presence of background fields violating the Lorentz invariance.

  I give a general counting rule for "unstable NG modes" of ordinary and higher-form symmetries.

- Chiral plasma instability can be understood by an unstable NG mode for a 1-form symmetry.

# Contents

# Gapless modes = modes without energy (mass) gap




- Dispersion relation: $\omega = 0$ for $\boldsymbol{k} \to \mathbf{0}$ .
- Long wave excitation by infinitesimal energy $\to$ Dominating infrared (IR) physics
- Characterizing phase of matter: gapless phase

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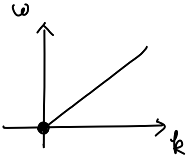


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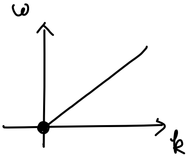


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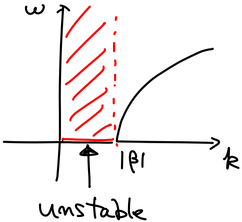
In the absence of Lorentz symmetry $\to$ IR corrections

- 1st order by $\omega$: $\omega^2 = \alpha\omega + k^2 \to$ gapped mode $\omega = \alpha + \frac{k^2}{\alpha}$
- 1st order by $k$: $\omega^2 = \beta k + k^2 \to$ **unstable mode**

# Unstable mode




- Dispersion relation $\omega = \sqrt{k^2 + \beta k}$
- For $\beta < 0$, there is instability $\omega = i\sqrt{|\beta k| - k^2}$ in finite region $0 < |k| < |\beta|$

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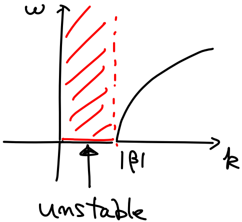


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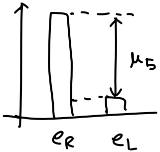
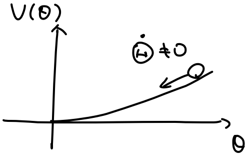
Such an instability arises in realistic systems!

# Example: chiral plasma instability in neutron stars and cosmology

[Carroll et al. '89; Joyce & Shaposhnikov '97; Akamatsu & Yamamoto '13]




**Maxwell + $\Theta$ term**

$$S = -\frac{1}{4}\int d^4x f_{\mu\nu} f^{\mu\nu} + \frac{1}{4}\int d^4x \Theta \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma}$$

$a_\mu$: $U(1)$ gauge field; $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$: field strength; $\Theta = \mu_5 t$; $\mu_5 > 0$: constant

- $\mu_5$ = chiral chemical potential, or $\Theta$ = time dependent axion
- 1st order of $k$: $\Theta \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} \sim \mu_5 \epsilon^{0ijk} a_i \partial_j a_k$

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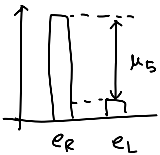


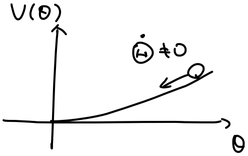


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Let us see the unstable mode explicitly.

# Instability for photon

EOM in momentum space

- For wave vector $\boldsymbol{k} = (k, 0, 0)$, ($a_0 = 0$ gauge)

$$(\omega^2 - k^2)\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = i\,k \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mu_5 \\ 0 & -\mu_5 & 0 \end{pmatrix}}_{\mu_5 \epsilon^{0ijl} k_l} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

- Dispersion relation: $\omega^2 = \pm|\mu_5 k| + k^2$ due to non-zero eigenvalues $\pm\mu_5$ of $(i\mu_5\epsilon^{01ij})$
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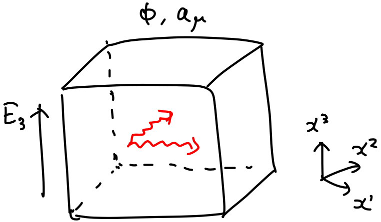
**Observation**

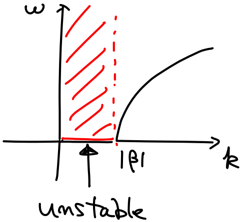
Number of unstable modes $= \frac{1}{2}\,\mathrm{rank}(\mu_5\epsilon^{01ij})$

The rest of the rank corresponds to dissipative modes $e^{-\sqrt{\mu_5 k - k^2}\,t}$

# Instability of gapless modes in background fields

In this talk, I consider this class of instability.




Other examples

- Massless axion + photon in background electric field [Ooguri & Oshikawa '11]
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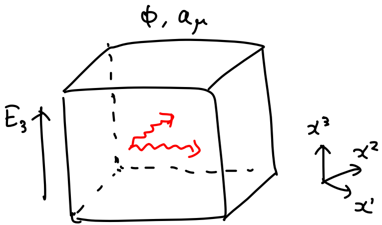


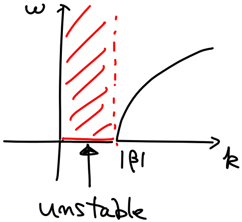


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Q: Can we understand the unstable modes universally?

# Hint: Counting rule of gapped modes for $\omega^2 = \alpha\omega + k^2$

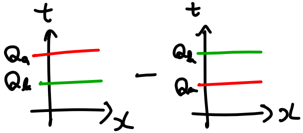
[Watanabe & Murayama '12; Hidaka '12]

Assumptions (rough)

- Gapless modes are NG modes
- Dispersion relation: $\omega^2 = \alpha\omega + k^2$, some of NG modes becomes gapped

**Number of gapped modes is determined by symmetry**

$$(\#\text{ of gapped modes}) = \frac{1}{2}\,\text{rank}\langle [Q_a, Q_b] \rangle$$




$Q_a$ broken symmetry generator

Virtue: Dispersion relation can be determined universally without details of models.

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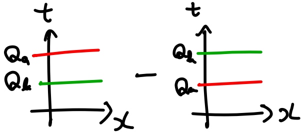


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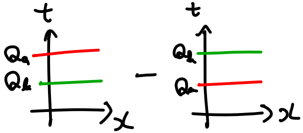


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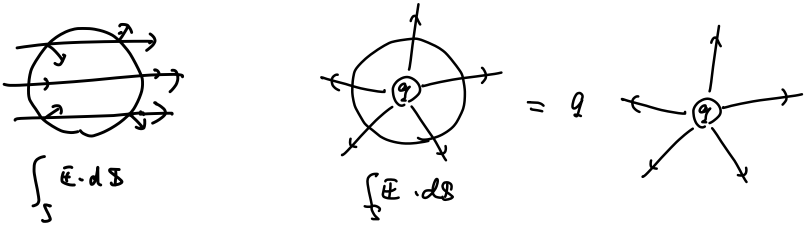
Is the photon a NG mode?

# Photon is NG mode of 1-form symmetry [Gaiotto et al. '14; Lake '18]

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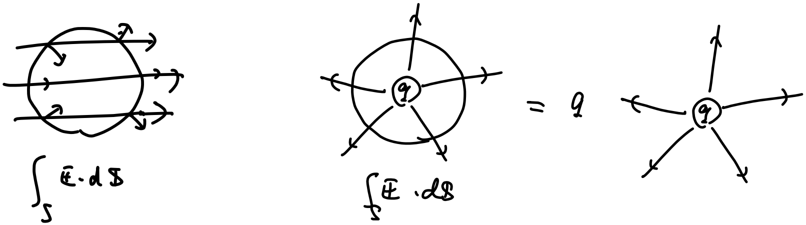


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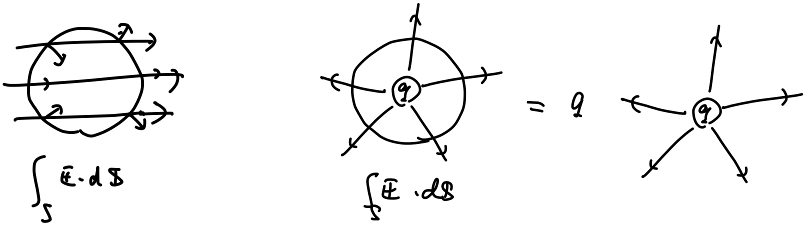

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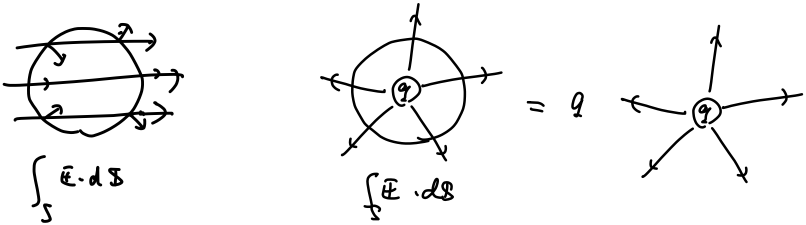

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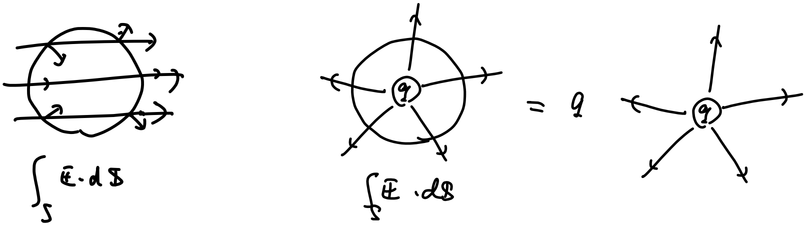

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Chiral plasma instability = the instability of NG mode in bg. field $\Theta$

This talk: Counting the number of instability by symmetry

# Counting unstable NG modes in chiral plasma instability

# Idea: replacing "time" by "space"

Gapped modes $\omega^2 = \alpha\omega + k^2$

- Correction by 1st order of **frequency** $\omega$

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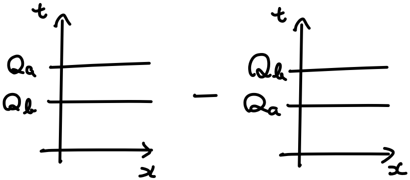

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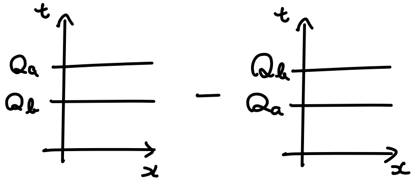


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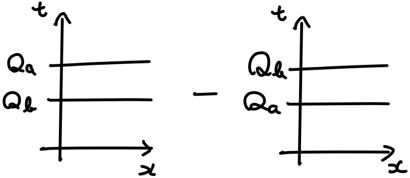


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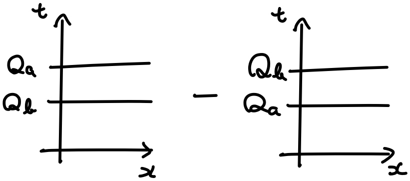


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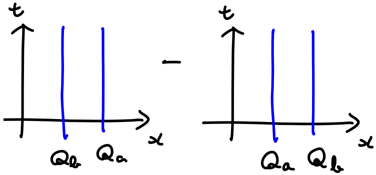
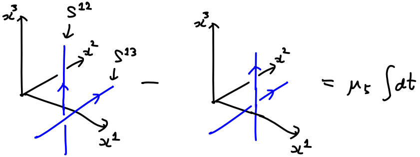


# Counting unstable NG mode in chiral plasma instability




**Counting rule**

$$(\#\text{ of unstable NG modes along } x^l\text{-dir.}) = \frac{1}{2}\,\mathrm{rank}(\epsilon^{0lij}\mu_5) = \frac{1}{2}\,\mathrm{rank}\langle [Q(S^{li}), Q(S^{lj})]_{x^l}\rangle$$

$S^{li}$: worldsheet perpendicular to $x^l, x^i$-directions; $[A, B]_{x^l}$: commutator of $A, B$ along $x^l$-direction

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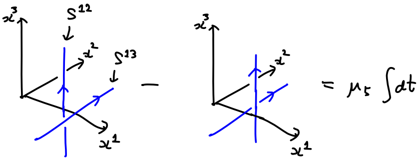


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The counting rule can be derived using methods of ordinary QFT.

Here, I will consider unstable mode along $x^1$ direction for concreteness.

# Extract $\epsilon_{01ij}\mu_5$ from symmetry generator

**Method**

Use of Schwinger-Dyson eq. $\langle \frac{\delta \mathcal{O}}{\delta a} \rangle \sim \langle \frac{\delta S}{\delta a} \mathcal{O} \rangle$

- EOM: $\frac{\delta S}{\delta a_\nu} = \partial_\mu (f^{\mu\nu} - \Theta \tilde{f}^{\mu\nu}) = 0 \rightarrow$ conserved current: $j^{\mu\nu} = f^{\mu\nu} - \Theta \tilde{f}^{\mu\nu}$

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- Use of Schwinger-Dyson eq.

$$\epsilon^{01ij}\mu_5 \delta^4(x) \sim \left\langle \frac{\delta j^{1i}}{\delta a_j} \right\rangle \sim \left\langle \frac{\delta S}{\delta a_j} j^{1i} \right\rangle$$

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**Method**

Use of Schwinger-Dyson eq. $\langle \frac{\delta \mathcal{O}}{\delta a} \rangle \sim \langle \frac{\delta S}{\delta a} \mathcal{O} \rangle$

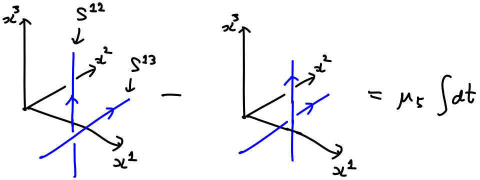
- EOM: $\frac{\delta S}{\delta a_\nu} = \partial_\mu (f^{\mu\nu} - \Theta \tilde{f}^{\mu\nu}) = 0 \to$ conserved current: $j^{\mu\nu} = f^{\mu\nu} - \Theta \tilde{f}^{\mu\nu}$
- $j^{1i}$ has $\epsilon^{01ij}\mu_5$: $j^{1i} \sim \epsilon^{01ij}\Theta\partial_0 a_j \sim \epsilon^{01ij}\mu_5 a_j$
- Use of Schwinger-Dyson eq.

$$\epsilon^{01ij}\mu_5 \delta^4(x) \sim \left\langle \frac{\delta j^{1i}}{\delta a_j} \right\rangle \sim \left\langle \frac{\delta S}{\delta a_j} j^{1i} \right\rangle$$

- EOM = conservation law (the same as ordinary NG mode)

$$\epsilon^{01ij}\mu_5 \delta^4(x) \sim \langle \partial_\mu j^{\mu j} j^{1i} \rangle$$

# Relation between $\epsilon^{01ij}\mu_5$ and symmetry generator




**$\epsilon^{01ij}\mu_5$ = commutator**

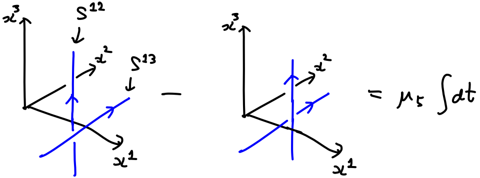
$$\epsilon^{01ij}\mu_5 \propto \langle [Q(S^{1i}), Q(S^{1j})]_{x^1} \rangle$$

Integrate $\epsilon^{01ij}\mu_5\delta^4(x) \sim \langle \partial_\mu j^{\mu j} j^{1i}(x) \rangle$

- Integral of $j^{1i}$: integral surface should be perpendicular to $j^{1i}$ (the same as in ordinary sym. gen. $Q = \int d^3\boldsymbol{x} j^0$)
- Integral of $\partial_\mu j^{\mu j} \to [Q(S^{1i}), Q(S^{1j})]_{x^1}$

(the same as ordinary Ward-Takahashi id. integral of $\langle \partial_\mu j^\mu \mathcal{O} \rangle \sim \langle \delta\mathcal{O} \rangle$ gives commutator $\langle [Q, \mathcal{O}] \rangle \sim \langle \delta\mathcal{O} \rangle$)

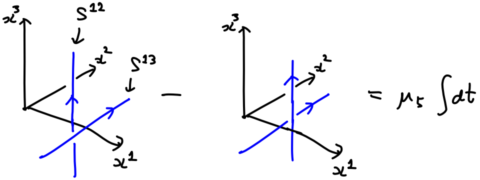
## Counting unstable NG mode by symmetry generator




**Number of unstable NG modes = commutator**

$$
\begin{aligned}
(\#\text{ of unstable NG modes along } x^1\text{-direction}) &= \frac{1}{2}\,\mathrm{rank}(\epsilon^{01ij}\mu_5) \\
&= \frac{1}{2}\,\mathrm{rank}(\langle [Q(S^{1i}), Q(S^{1j})]_{x^1}\rangle)
\end{aligned}
$$

# Summary of derivation



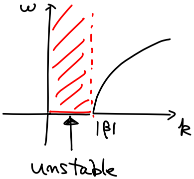

Instability $=$ 1st order correction by $k$

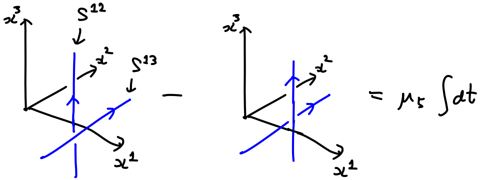
- Time dependent background field $\Theta \propto \mu_5 t$

Counting rule

- Symmetry generators are temporally extended, spatially localized.
- Commutator is taken along the direction of $\boldsymbol{k}$.

# Summary




- Chiral plasma instability can be understood by an unstable NG mode for a 1-form symmetry.
- # of unstable modes is counted in terms of symmetry

Our paper [2203.02727]

- A general counting rule for unstable NG modes of ordinary and higher-form symmetries is established.
- Instability of axion in electric field can also be described.
- A new example of an unstable NG mode for an ordinary symmetry is proposed.

Future work

- Fate of instability. Chiral plasma instability decreases by the dynamics of $\mu_5$ & unstable NG modes.

  [Yamamoto & RY, in preparation]

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