Axio-dilaton wormholes and their puzzling low-energy effects

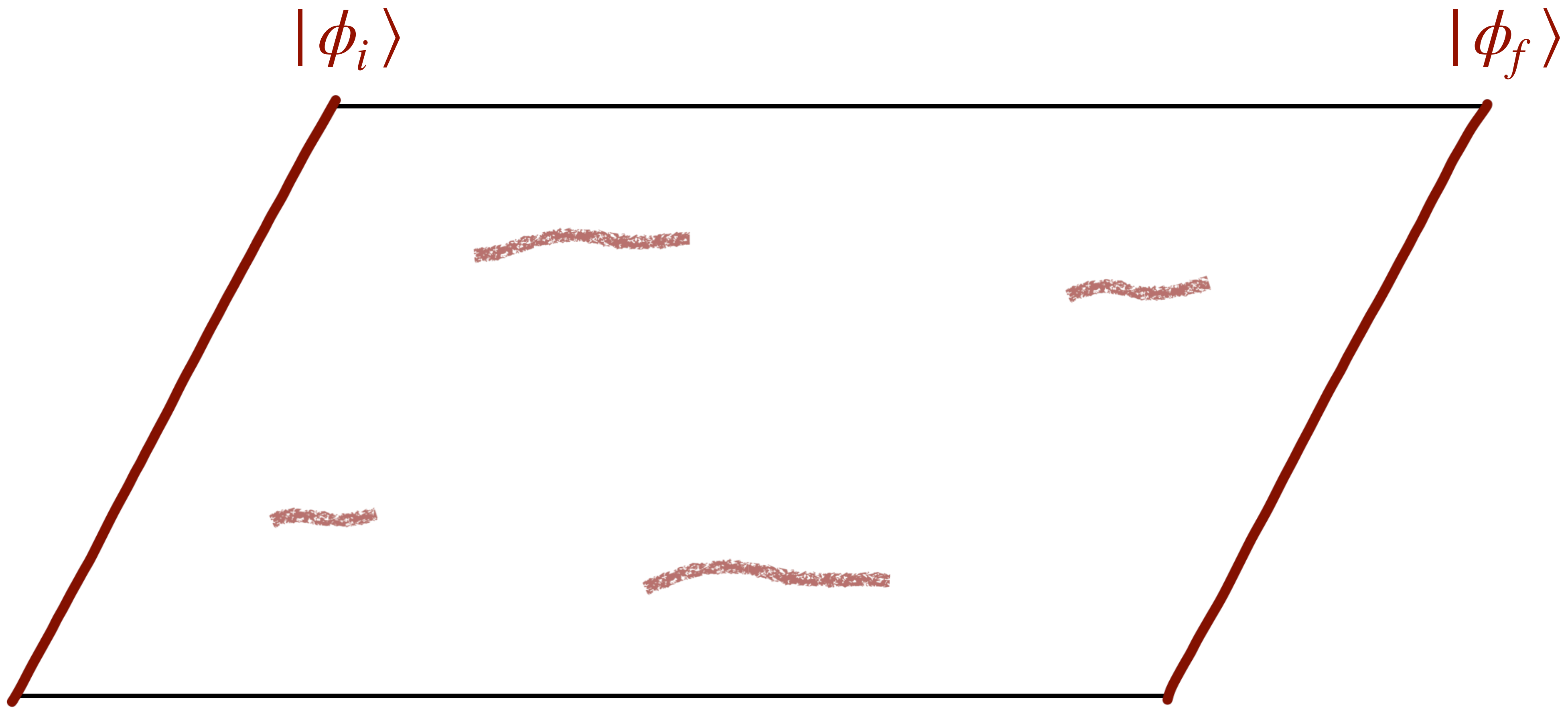
Pablo Soler
University of Oviedo

Work with S. Andriolo, G. Shiu, T. Van Riet - 2205.01119
(+ A. Hebecker, T. Mikhail - 1807.00824)

KEK-PH2022 meeting - Dec 22

Quantum sum over histories

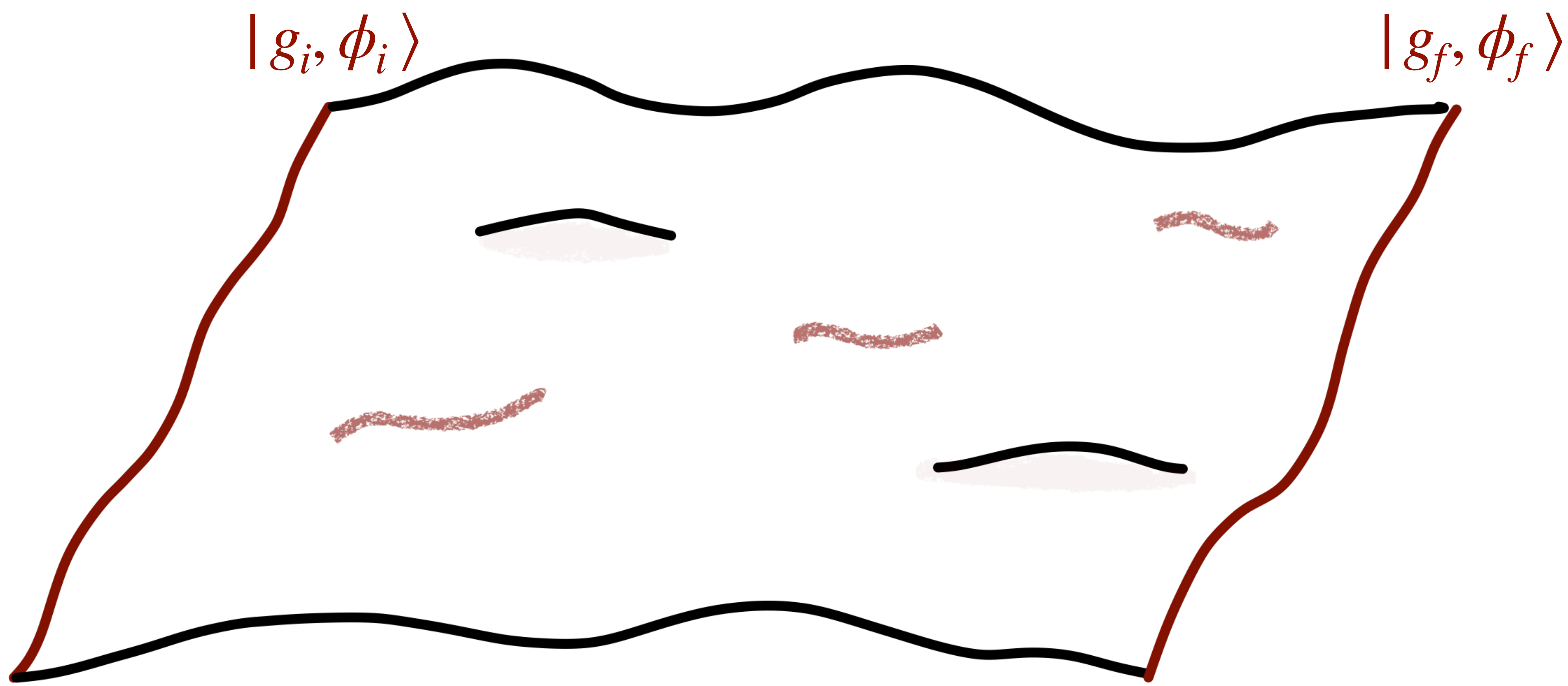
- Amplitudes in Quantum Field Theory:



$$\langle \phi_f | \phi_i \rangle = \int_{\phi_i}^{\phi_f} \mathscr{D}\phi \, e^{-S[\phi]}$$

Quantum sum over histories

- Amplitudes in Quantum Gravity:



$$\langle g_f, \phi_f | g_i, \phi_i \rangle = \int_{g_i,\phi_i}^{g_f,\phi_f} \mathscr{D}g\, \mathscr{D}\phi\, e^{-S[g,\phi]}$$

Quantum sum over histories

- Amplitudes in Quantum Gravity:

$$\langle g_f, \phi_f | g_i, \phi_i \rangle = \sum_{top} \int_{g_i, \phi_i}^{g_f, \phi_f} \mathcal{D}g \, \mathcal{D}\phi \, e^{-S[g,\phi]}$$

How do non-trivial topologies affect the
Euclidean path integral of Quantum Gravity at low energies?

Outline

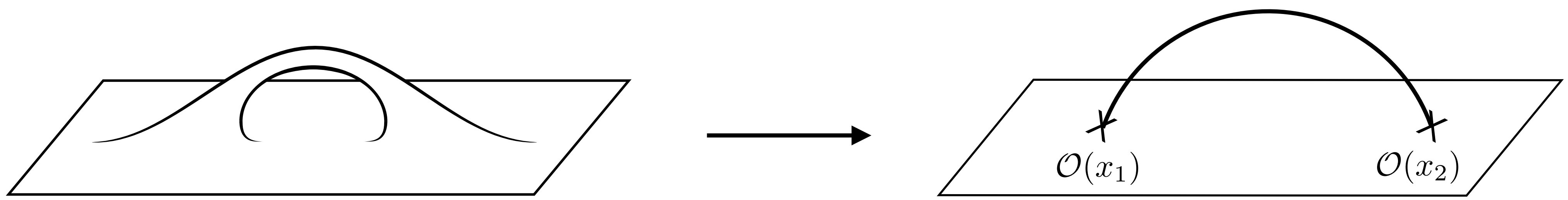
- The effects (and puzzles) of wormholes:
  - Coleman α-parameters
- Axionic Wormholes:
  - Axion potentials
  - The axion WGC
- Axionic wormholes with massive dilatons
- Conclusions & open questions

Wormhole effects on EFT

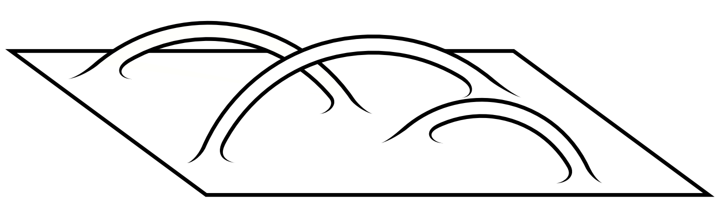
For review: Hebecker, Mikhail, PS ‘18

Wormholes & α-parameters

- In the IR, wormholes look like joined instanton/anti-instanton pairs which couple to local operators $\mathcal{O}(x)$:



- In the dilute gas approximation, their contribution can be exponentiated, leading to a **bi-local effective action**:



$N = 3$

$$Z = \int [\mathcal{D}\Phi]\, e^{-S_0[\Phi]} \sum_{N=0}^{\infty} \frac{1}{N!} \left(e^{-S_{wh}} \int dx_1 \mathcal{O}(x_1) \int dx_1 \mathcal{O}(x_2) \right)^N$$

$$= \int [\mathcal{D}\Phi]\, e^{-S_0[\Phi]} \exp \left(e^{-S_{wh}} \int_{x_1} \int_{x_2} \mathcal{O}(x_1)\mathcal{O}(x_2) \right)$$

Coleman '88, Preskill '89…

Wormholes & α-parameters

- A bi-local effective action can be recast in a local form with the help of **α-parameters**:

$$e^{-\Delta S} \sim \exp\left(e^{-S_{wh}} \int_{x_1} \int_{x_2} \mathcal{O}(x_1)\mathcal{O}(x_2)\right) \sim \int d\alpha \, \exp\left(-\alpha^2 + \alpha\, e^{-S_{wh}/2} \int_x \mathcal{O}(x)\right)$$

Euclidean wormholes induce non-perturbative corrections to the effective action with (Gaussian) **random couplings!**

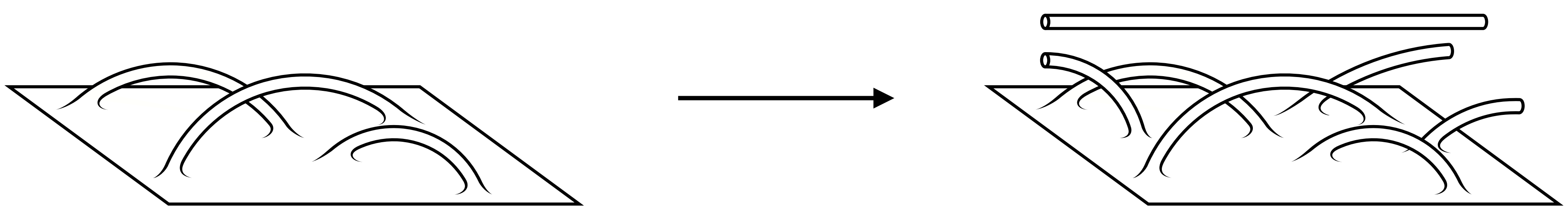
$$Z \longrightarrow \int [\mathcal{D}\Phi] \, d\alpha \, e^{-\alpha^2} \, \mathbf{exp}\left(-\int_x \mathcal{L}_0[\Phi] + \alpha\, e^{-S_{wh}/2} \mathcal{O}(x)\right)$$

- How should we interpret α-parameters?

Coleman '88, Preskill '89...

Wormholes & α-parameters

- Once wormholes are introduced, one needs to consider "baby-universe states"

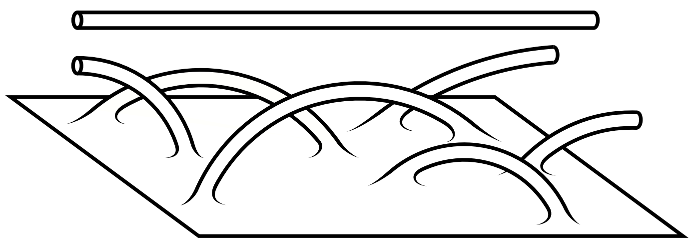


$|\Phi_i\rangle \otimes |0\rangle$ $|\Phi_f\rangle \otimes |0\rangle$ $|\Phi_i\rangle \otimes |2\rangle$ $|\Phi_f\rangle \otimes |3\rangle$

  - Hilbert space enlarged: $\mathscr{H} = \mathscr{H}_\Phi \otimes \mathscr{H}_{b.u.}$
  - Introduce baby-universe creation/annihilation ops: $[a, a^\dagger] = 1$
  - $\mathscr{H}_{b.u.}$ spanned by $\{|n\rangle\}$, where $|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$

Wormholes & α-parameters

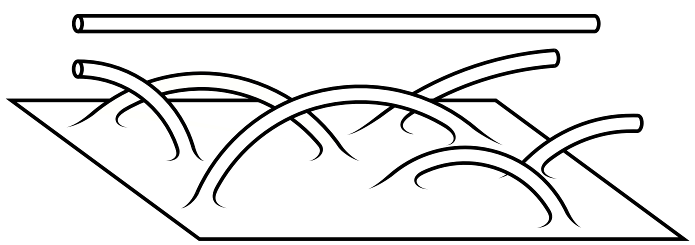
- The effect of wormhole insertions on $\mathscr{H}_{bu}$ is given by

$\sum_{top}$  $\longrightarrow \exp\left[e^{-S_{wh}}\left(a+a^{\dagger}\right)\int_x \mathcal{O}(x)\right]$

Wormholes & α-parameters

- The effect of wormhole insertions on $\mathscr{H}_{bu}$ is given by

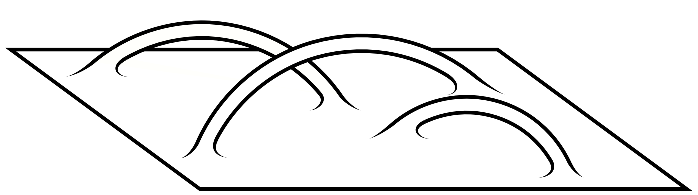
$$\sum_{top}$$



$$\longrightarrow \quad \exp\left[e^{-S_{wh}}\left(a+a^{\dagger}\right)\int_x \mathcal{O}(x)\right]$$

For example

$$\sum_{top} \langle 0|$$

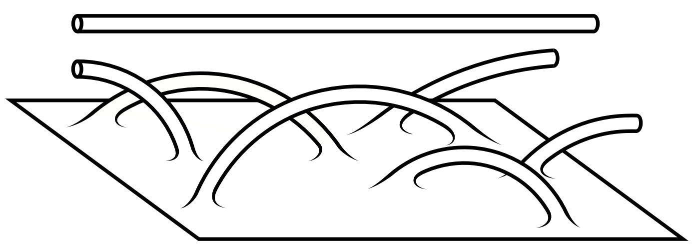


$$|0\rangle \quad \longrightarrow \quad \langle 0|\exp\left[e^{-S_{wh}}\left(a+a^{\dagger}\right)\int_x \mathcal{O}(x)\right]|0\rangle$$

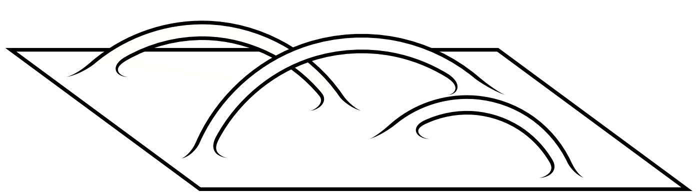
$$= \int d\alpha\, e^{-\alpha^2}\exp\left[\alpha\, e^{-S_{wh}}\int_x \mathcal{O}(x)\right]$$

Wormholes & α-parameters

- The effect of wormhole insertions on $\mathcal{H}_{bu}$ is given by

$\sum_{top}$  $\longrightarrow \quad \exp\left[e^{-S_{wh}}\left(a+a^{\dagger}\right)\int_x \mathcal{O}(x)\right]$

For example

$\sum_{top} \langle 0|$  $|0\rangle \longrightarrow \langle 0|\exp\left[e^{-S_{wh}}\left(a+a^{\dagger}\right)\int_x \mathcal{O}(x)\right]|0\rangle$

$$= \int d\alpha\, e^{-\alpha^2} \exp\left[\alpha\, e^{-S_{wh}}\int_x \mathcal{O}(x)\right]$$

But we should consider more general in/out states!

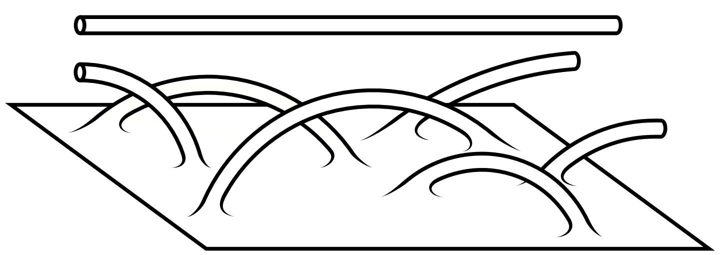
Wormholes & α-parameters

- In particular **α-eigenstates** are eigenstates of $\left(a + a^\dagger\right)$:

$$\left(a + a^\dagger\right) |\, \alpha \rangle = \alpha \,|\, \alpha \rangle$$

α-eigenstate propagation yields:

$$\langle \alpha |\, \sum_{top}$$



$$|\, \alpha \rangle \quad \longrightarrow \quad \langle \alpha \,|\, \exp \left[e^{-S_{wh}} \left(a + a^\dagger\right) \int_x \mathcal{O}(x) \right] |\, \alpha \rangle$$

$$\sim \quad \exp \left[\alpha \, e^{-S_{wh}} \int_x \mathcal{O}(x) \right]$$

- Wormholes induce effective action with **unknown α-couplings:**

$$\Delta S_{eff} = \alpha \, e^{-S_{wh}} \int d^4x \, \mathcal{O}(x)$$

Conceptual issues

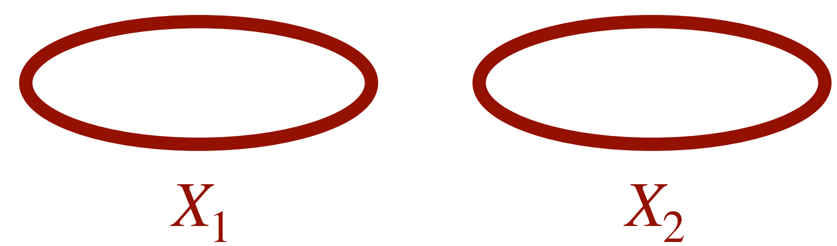
Wormholes and holography

Wormholes & AdS/CFT

- The presence of random couplings is extremely puzzling from the perspective of a fundamental theory
- Problems are manifest in holography (AdS/CFT):

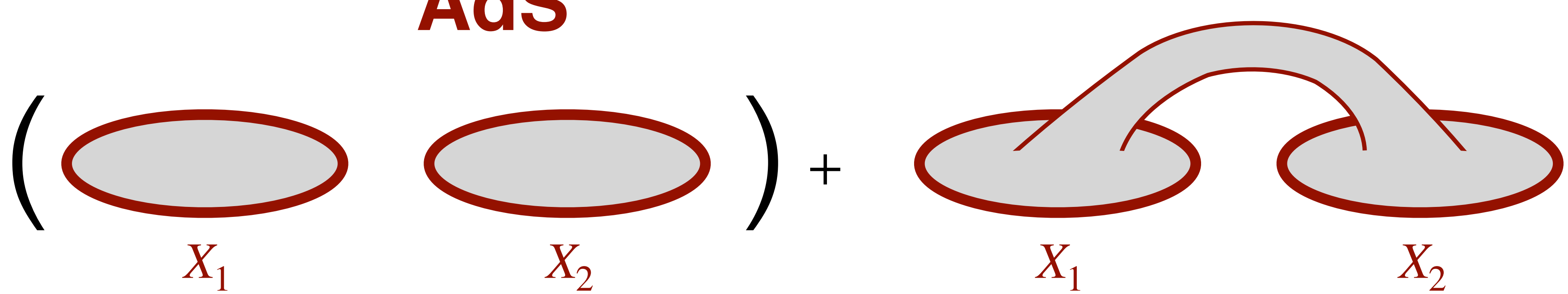
CFT

$$Z_{CFT}(X_1 \sqcup X_2) = Z_{CFT}(X_1)\, Z_{CFT}(X_2) =$$



AdS

$$Z_{AdS}(X_1 \sqcup X_2) =$$



Partition functions do not factorize!

Maldacena, Maoz '04; Arkani-Hamed, Orgera, Polchinski '07, Hertog, Trigiante, Van Riet '17, Marolf, Santos '21…

Conceptual questions

- Are wormhole effects and α-parameters real?
- Can they be embedded in string theory?
- How are they compatible with AdS/CFT?

Let us study a specific setup with wormhole solutions
with important **phenomenological implications**

Axionic wormholes

Wormholes & axions

"axion" = ALP

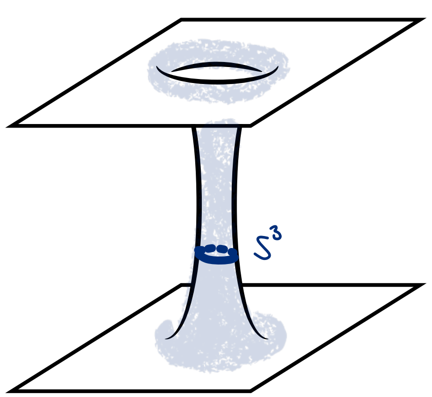
- Let's consider Euclidean Einstein-Axion theory:

$$\mathcal{L} \sim -\mathcal{R} + f^2|\partial\phi|^2 \qquad \text{with} \qquad \phi \equiv \phi + 2\pi$$

- This can be dualized $(H = dB_2 \equiv f^2 * d\phi)$ to give:

$$\mathcal{L} \sim -\mathcal{R} + \frac{1}{f^2}|dB|^2$$

- This theory admits smooth solutions



$$\int_{S^3} H_3 = n$$

The throat is a three-sphere of minimum radius R, supported by n units of H-flux:

$$M_P^2 R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad R^2 \sim \frac{n}{f M_P} \gg \Lambda_{UV}^{-2}$$

Giddings, Strominger '88; Coleman, Lee '89…

Wormholes & axions

"axion" = ALP

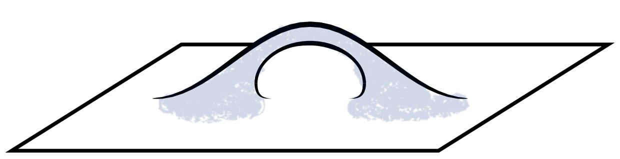
- Let's consider Euclidean Einstein-Axion theory:

$$\mathcal{L} \sim -\mathcal{R} + f^2 |\partial\phi|^2 \qquad \text{with} \qquad \phi \equiv \phi + 2\pi$$

- This can be dualized $(H = dB_2 \equiv f^2 * d\phi)$ to give:

$$\mathcal{L} \sim -\mathcal{R} + \frac{1}{f^2}|dB|^2$$

- This theory admits smooth solutions



The throat is a three-sphere of minimum radius R, supported by n units of H-flux:

$$M_P^2 R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad R^2 \sim \frac{n}{f\, M_P} \gg \Lambda_{UV}^{-2}$$

Giddings, Strominger '88; Coleman, Lee '89...

Wormholes & axions

“axion” = ALP

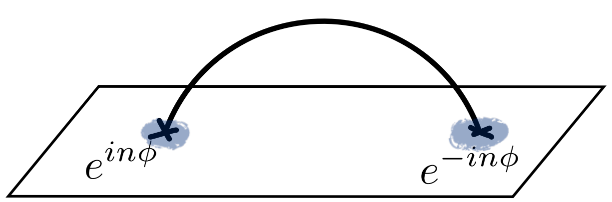
- Let’s consider Euclidean Einstein-Axion theory:

$$\mathcal{L} \sim -\mathcal{R} + f^2 |\partial\phi|^2 \qquad \text{with} \qquad \phi \equiv \phi + 2\pi$$

- This can be dualized $(H = dB_2 \equiv f^2 * d\phi)$ to give:

$$\mathcal{L} \sim -\mathcal{R} + \frac{1}{f^2} |dB|^2$$

- This theory admits smooth solutions



The throat is a three-sphere of minimum radius R, supported by n units of H-flux:

$$M_P^2 R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad R^2 \sim \frac{n}{f M_P} \gg \Lambda_{UV}^{-2}$$

Giddings, Strominger ’88; Coleman, Lee ’89…

Wormholes & axions

- These wormholes couple to axions and yield

$$\Delta V_{eff} \sim \alpha\, e^{-S_{wh}} \cos\left(\frac{n\phi}{f}\right)$$

  - Non-perturbative axion potential: $S_{wh} \sim \frac{n}{f} M_P$
  - Quantum gravity breaks global symmetry: $\phi \to \phi + c$.
  - Undetermined coefficient α (conceptually problematic).
  - Interesting for pheno? Highly suppressed: $S_{wh} \gg \frac{M_P^2}{\Lambda_{UV}^2}$

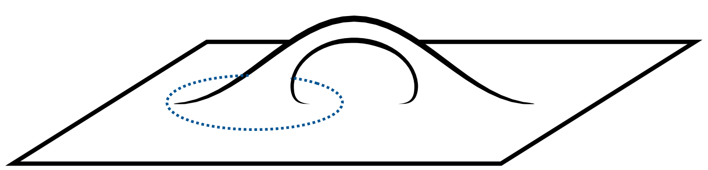
Weak Gravity Conjecture (WGC)

"In consistent theories of Quantum Gravity,
gravity is the weakest force"

Wormholes & WGC

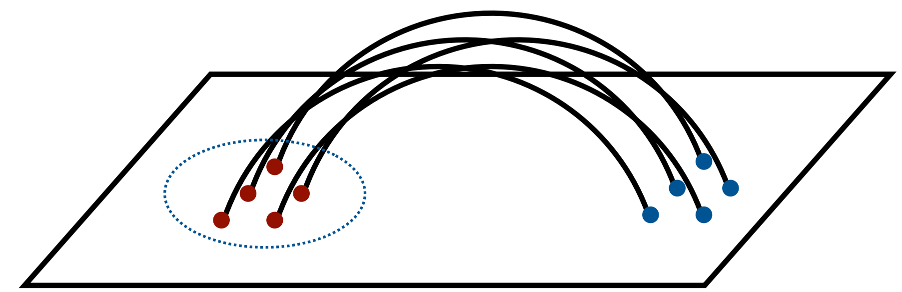
- WGC: There should exist “microscopic” effects that dominate over macroscopic wormholes

Option 1: microscopic **wormholes** dominate



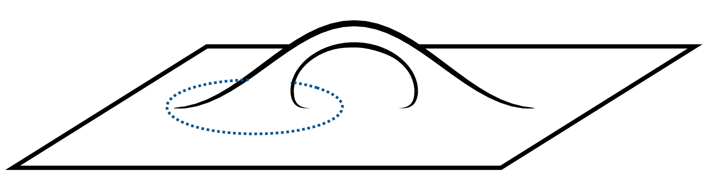
$\int_{S^3} H_3 = n$

$S_{wh}^{(n)} \geq n\, S_{micro}$ →



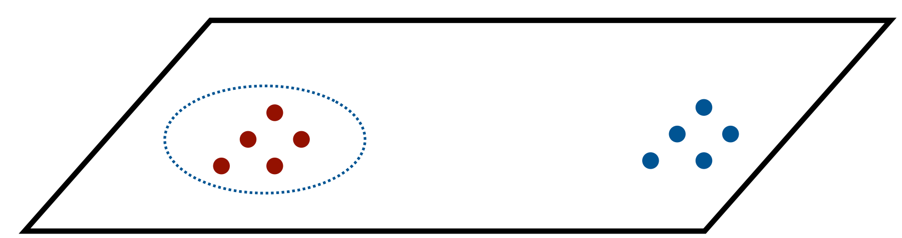
$\int_{S^3} H_3 = n$

Option 2: microscopic **instantons** dominate



$\int_{S^3} H_3 = n$

$S_{wh}^{(n)} \geq 2\, n\, S_{micro}$ →



$\int_{S^3} H_3 = n$

(No α–parameters)

Wormholes & WGC

- WGC: There should exist "microscopic" effects that dominate over macroscopic wormholes

$$\Delta V_{eff} \sim \alpha\, e^{-S_{wh}} \cos\left(\frac{n\,\phi}{f}\right) + e^{-S_{micro}} \cos\left(\frac{\phi}{f}\right)$$

WGC predicts:

$$S_{micro} \lesssim \frac{S_{wh}}{n} \sim \frac{M_P}{f} \qquad \Longrightarrow \qquad \boxed{f\, S_{micro} \lesssim M_P}$$

Arkani-Hamed, Motl, Nicolis, Vafa '06;
Brown, Cottrell, Shiu, PS '15
Heidenreich, Reece, Rudelius '15

- Important consequences for phenomenology (axion inflation, monodromy/relaxion, axion (fuzzy) Dark Matter, strong CP,…)

c.f. Hebecker, Mikhail, PS '18
(see also Kanazawa's talk)

Axio/dilaton wormholes

Axio-dilaton wormholes

$$S_E = \int d^4x \sqrt{g} \, \left(-\frac{1}{2}\mathscr{R} + \frac{1}{2}(\nabla\chi)^2 + \frac{1}{2f^2}\, e^{-\beta\chi}\, H_3^2 \right)$$

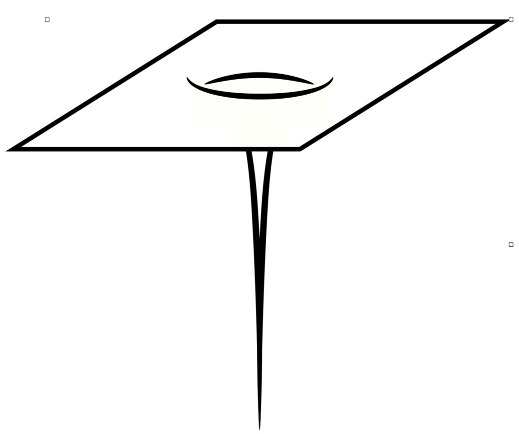
Axio-dilaton wormholes

$$S_E = \int d^4x\sqrt{g}\,\left(-\frac{1}{2}\mathscr{R} + \frac{1}{2}(\nabla\chi)^2 + \frac{1}{2f^2}\,e^{-\beta\chi}\,H_3^2\right)$$

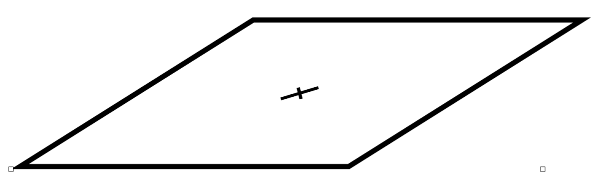
- Three types of O(4) symmetric solutions possible:

$$ds^2 = \left(1 + k\frac{a_0^4}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2\,, \qquad H = \frac{n}{2\pi^2}\,dVol_{S^3}\,, \qquad \chi = \chi(r)$$

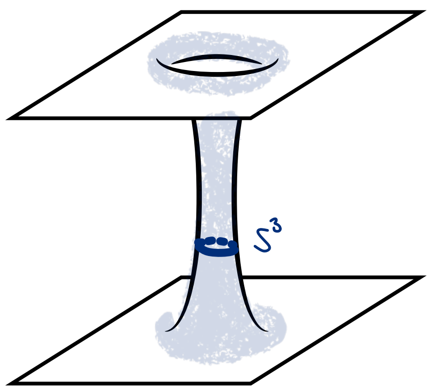
Cored ***instantons***
$(k = +1)$



Flat (extremal) ***instanton***
$(k = 0)$



Axion ***wormhole***
$(k = -1)$

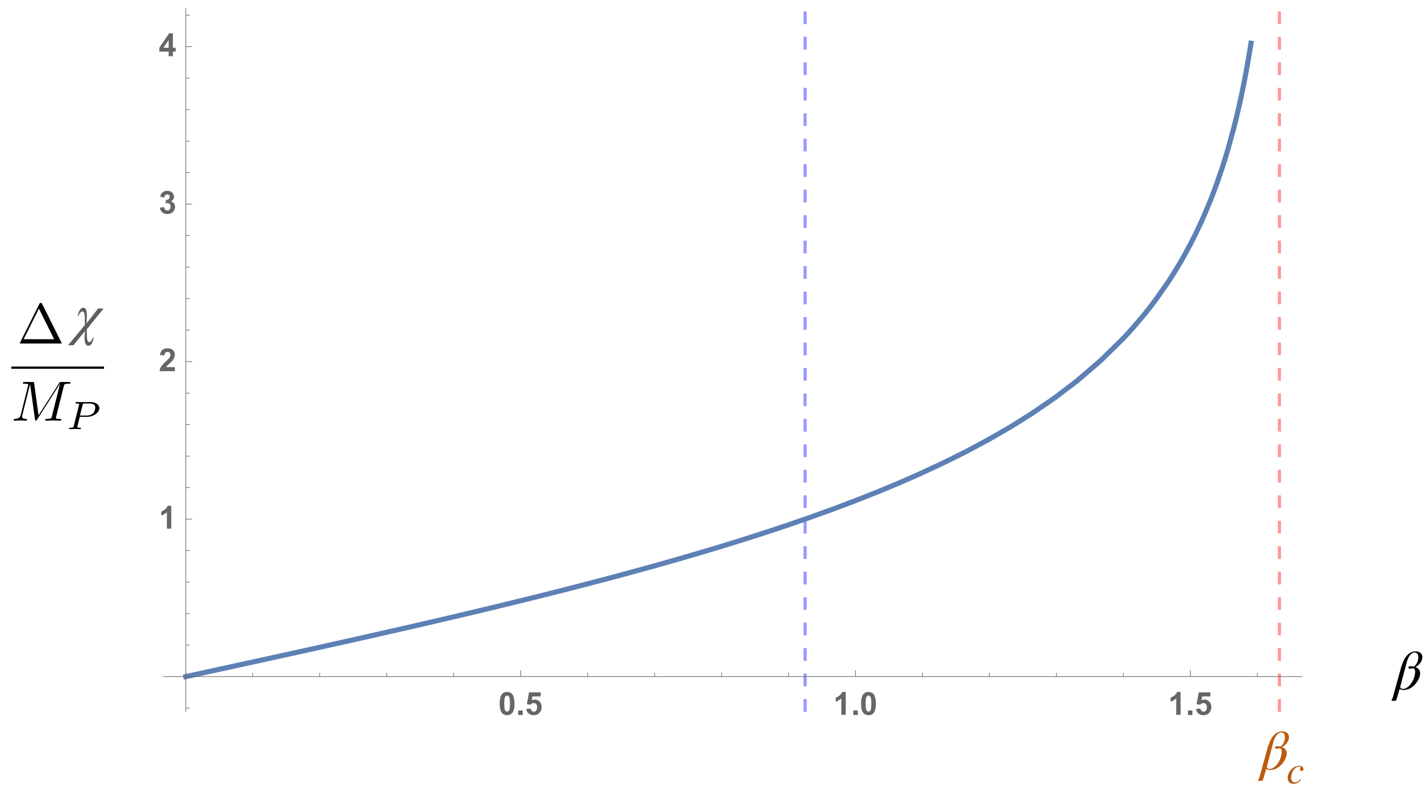


E. Bergshoeff, A. Collinucci, U. Gran, D. Roest and S. Vandoren, ‘04

Axio-dilaton wormholes

- Wormholes and large distances:

$$e^{\beta\,\Delta\chi} = \cos^{-2}\left(\frac{\pi}{2}\frac{\beta}{\beta_c}\right)$$



Dilaton displacement blows up at $\beta = \beta_c = \frac{4}{\sqrt{6}}$

Hebecker, Mangat, Theisen, Witkowski '16

Axio-dilaton wormholes

- For $\beta \geq \beta_c$:

  Wormholes do not exist! (No conceptual problems)

  Only cored & flat **instantons**: $\Delta S_{eff} \sim e^{-S_{inst}} \cos(\phi/f)$

  Typical setup in UV theories/string theory

- For $\beta < \beta_c$:

  All solutions (**wormhole** & **instantons**) coexist:

$$\Delta S_{eff} \sim \alpha\, e^{-S_{inst}} \cos(\theta/f)$$

Axionic wormholes with massive dilatons

A toy model for the Weak Gravity Conjecture

Andriolo, Shiu, PS, Van Riet '22

Wormholes with massive dilaton

- We study wormhole solutions when we include a dilaton mass

$$S_E = \int d^4x \sqrt{g} \left[-\frac{R}{2} + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2f^2} e^{-\beta\phi} H^2 + m^2\phi^2 \right] \qquad (\beta < \beta_c)$$

- Solutions interpolate between (large) axionic wormholes and (small) axio-dilaton wormholes

  - Large $R^2 \gg 1/m^2$: dilaton freezes and decouples (same as $\beta \to 0$)

    Axionic wormholes

  - Small $R^2 \ll 1/m^2$: dilaton is effectively massless

    Axio-dilaton wormholes

Wormholes & axionic WGC

- Our main interest is in the action-to-charge ratio of wormholes

$$s_n \equiv \frac{S_{wh}\, f}{n}$$

  - Large wormholes ($R^2 m^2 \sim \frac{m^2 q}{f} \gg 1$): perturbative expansion

$$s_q = \sqrt{\frac{3}{2}}\pi^3 \left(1 - 4\pi\sqrt{3}\, \frac{\beta^2 f M_P}{q\, m^2} \right) + \dots$$

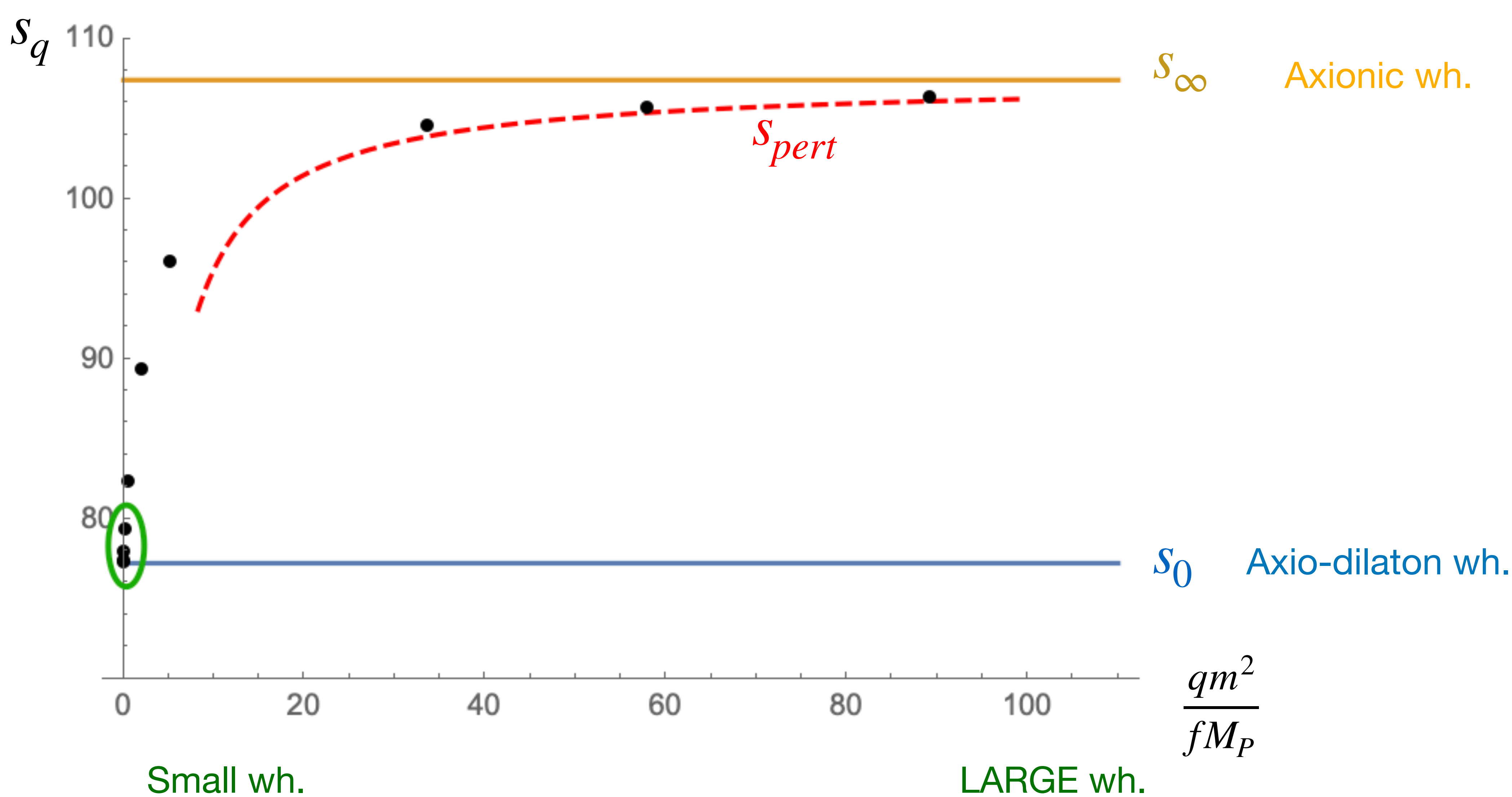
c.f. Andriolo, Huang, Noumi, Ooguri, Shiu '20

  - Small wormholes ($\frac{m^2 q}{f} \lesssim \mathcal{O}(1)$): numerical computation…

c.f. Kallosh, Linde, Linde, Susskind '95

Wormholes & axionic WGC

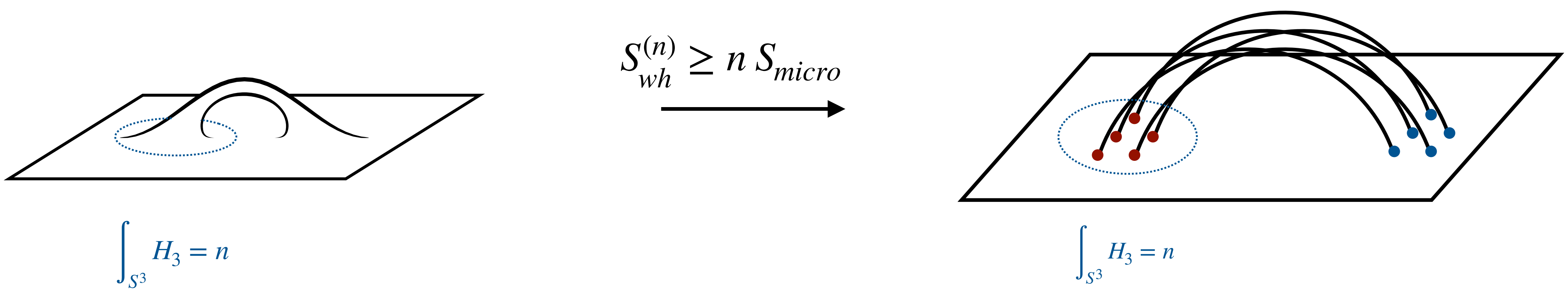
- Action-to-charge ratio: $s_0 < s_q < s_\infty$



Andriolo, Shiu, PS, Van Riet '22

Wormholes & axionic WGC

- This result, for $\beta < \beta_c$ (were solutions are under control), confirms the WGC expectation

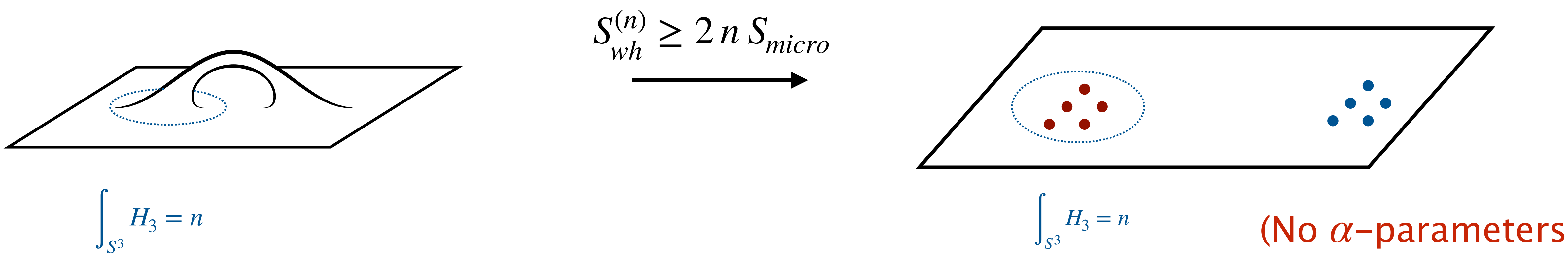


Smaller (“microscopic”) wormholes dominate

Andriolo, Shiu, PS, Van Riet ‘22

Wormholes & axionic WGC

- It suggests that if $\beta \geq \beta_c$ (as is typical in string theory), larger wormholes would be subdominant ('decay') to instantons



Smaller ("microscopic") **instantons** dominate

- A possible solution to conceptual issues with α-parameters, but:

  Treatment in EFT not possible, need UV completion (string theory)

  Even the, α-contributions subdominant, but maybe not absent

Andriolo, Shiu, PS, Van Riet '22

Conclusions

Conclusions

- Wormholes and instantons are crucial ingredients in axion pheno.
  - Wormholes break shift symmetry, but are highly suppressed
  - WGC: stronger effects through “microscopic” objects (Pheno!)
- Wormholes pose deep puzzles (α-parameters & AdS/CFT)
- Toy UV model: axions & massive dilatons ($\beta < \beta_c$)
  - The ‘mild’ (wormhole) WGC is satisfied: $S_{wh}^{(n)} \geq n\, S_{micro}$
  - Ultimate fate of α-parameters still unclear.
- Task: clean string models with all β and dilaton masses
  - Explore wormhole extremality/stability & holographic avatars