Revisiting Froggatt-Nielsen mechanism

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Work in progress

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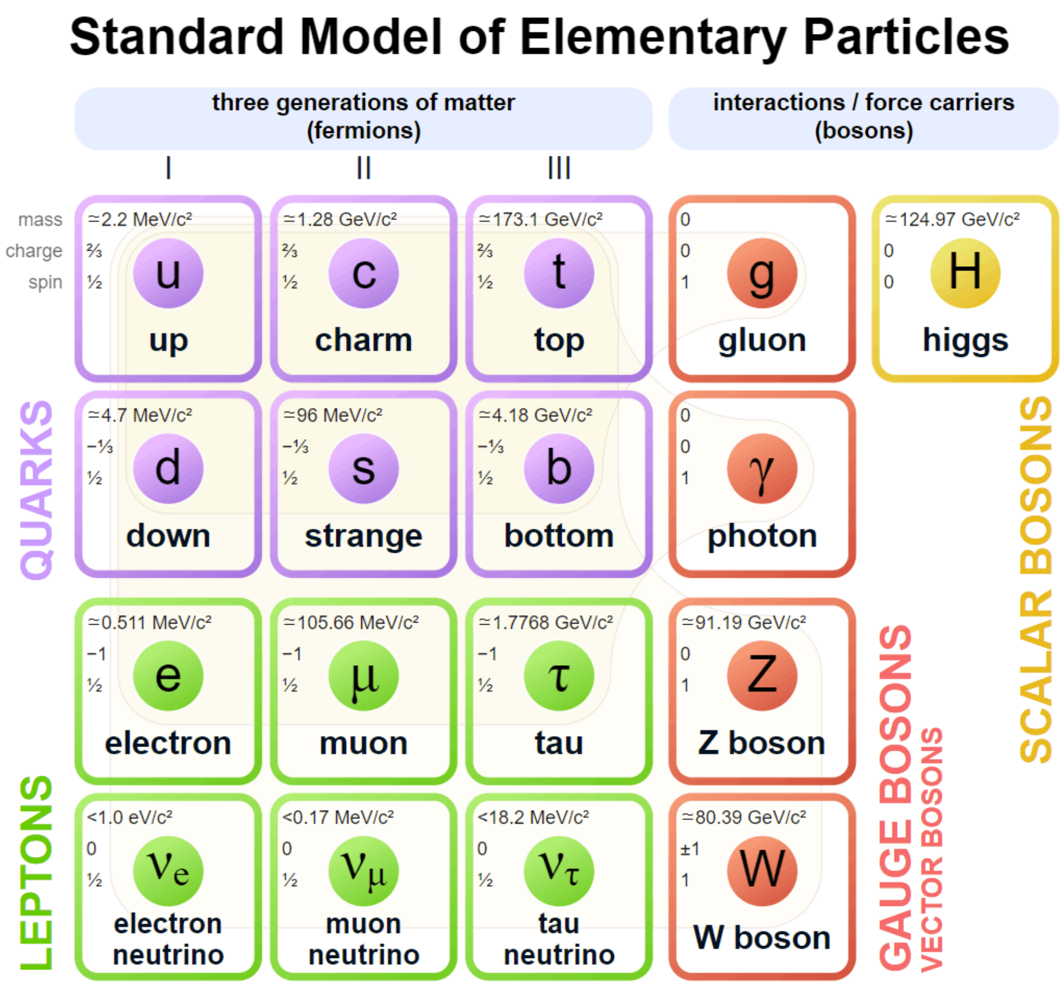
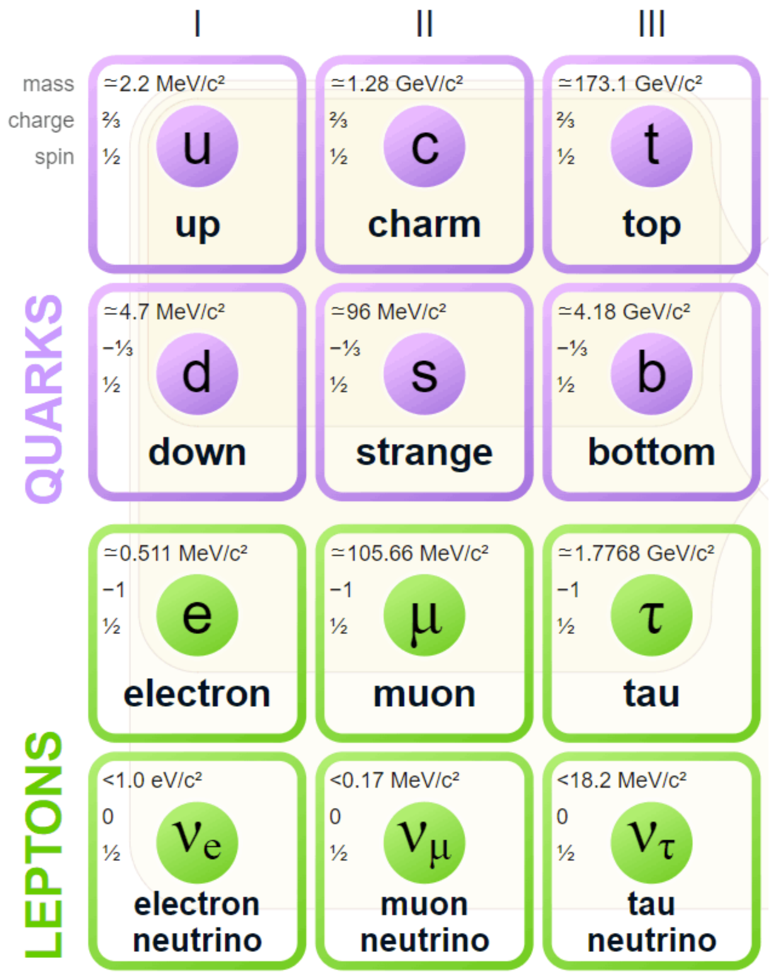
The standard model(SM)



- Very successful

https://en.wikipedia.org/wiki/Standard_Model

The standard model(SM)



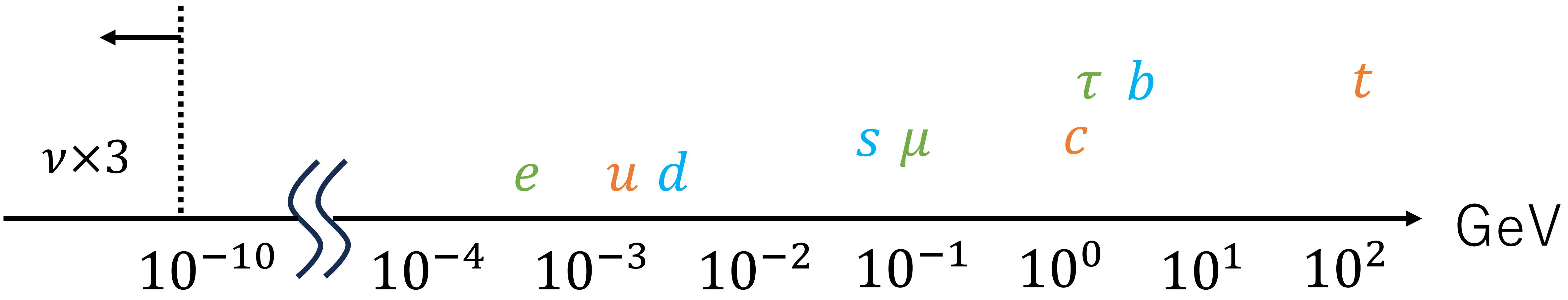
- Very successful
- Repetitive structure of quarks and leptons

https://en.wikipedia.org/wiki/Standard_Model

Situation

There exist flavor puzzles in the SM !

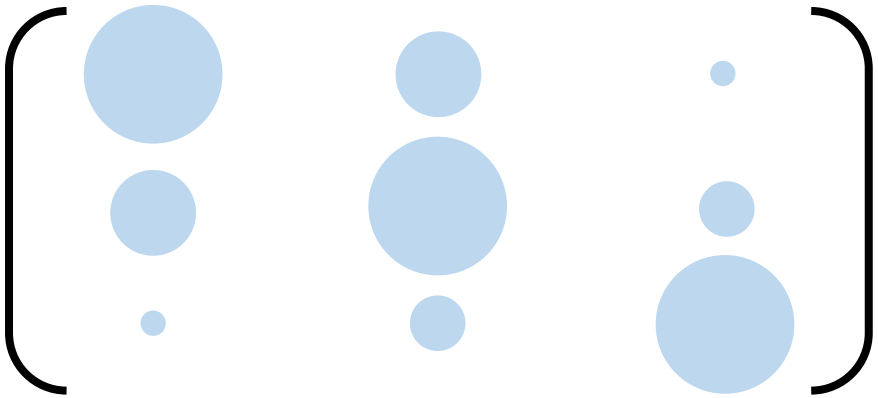
Fermion mass structure



There is hierarchical mass structure. e.g, $\frac{m_t}{m_u} = O(10^5)$

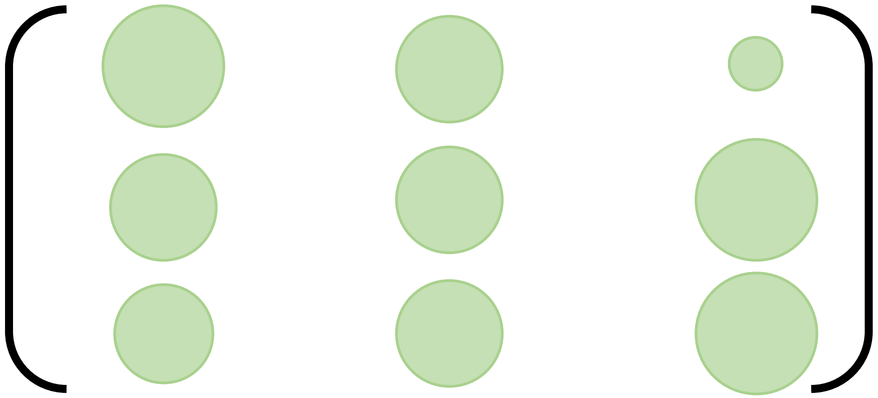
Structure of mixings

$|V_{\mathrm{CKM}}| =$



hierarchical

$|V_{\mathrm{PMNS}}| =$



anarchical

Mixing matrices have distinctive structures.

Froggatt-Nielsen (FN) mechanism

C.D.Froggatt and H.B.Nielsen, Nucl,Phys.B 147 (1979)

- SM fermions have charges under new U(1) symmetry

$$f_i : q(f_i), \qquad (\ i\text{: generation }\ )$$

- We cannot write ordinary Yukawa interactions

$$f_i\, f_j\, H \longrightarrow \cancel{f_i\, f_j\, H}$$

- If we introduce new scalar $S : -1$, new operators can be written

$$\kappa_{ij}\, f_i\, f_j\, H \left(\frac{S}{M_{\mathrm{Pl}}}\right)^{|q(f_i)+q(f_j)|} \qquad \kappa_{ij} = O(1)$$

FN mechanism

- If S obtains VEV, Yukawa interactions arise,

$$\kappa_{ij}\, f_i\, f_j\, H \left(\frac{S}{M_{\mathrm{Pl}}}\right)^{|q(f_i)+q(f_j)|} \xrightarrow[S \to \langle S \rangle]{} \kappa_{ij}\, f_i\, f_j\, H \left(\frac{\langle S \rangle}{M_{\mathrm{Pl}}}\right)^{|q(f_i)+q(f_j)|},$$

$$y_{ij} = \kappa_{ij} \times \epsilon^{|q(f_i)+q(f_j)|}, \qquad \epsilon = \langle S \rangle / M_{\mathrm{Pl}} < 1$$

- **Hierarchy is realized naturally by this mechanism.**

Contents

Many studies

Hierarchy of Quark Masses, Cabibbo Angles and CP Violation

C.D. Froggatt (CERN and Glasgow U.), Holger Bech Nielsen (Bohr Inst.)

Jun, 1978

22 pages

Published in: *Nucl.Phys.B* 147 (1979) 277-298

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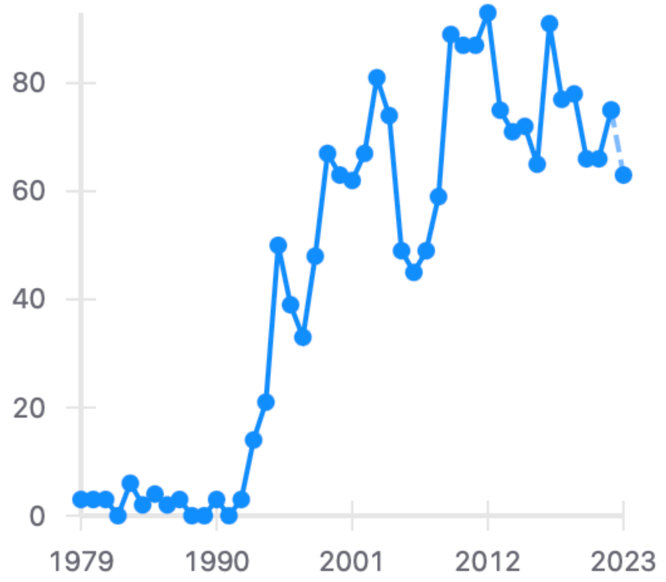


How to find good charges

- Compare the plausibility of multiple different FN charge assignments

- We adopt **the Bayes factor**

(J. Bergstrom, D. Meloni and L. Merlo, PRD 89 (2014) 9, 093021)

Bayes theorem

$$\mathrm{P}(M_i|Data) \propto \mathrm{P}(Data|M_i) \times \mathrm{P}(M_i)$$

$\mathrm{P}(M_i|Data)$: posterior probability of models

$\mathrm{P}(M_i)$: prior probability of models

$\mathrm{P}(Data|M_i)$: marginal likelihood

Bayes theorem

$$\mathrm{P}(M_i|Data) \propto \mathrm{P}(Data|M_i) \times \mathrm{P}(M_i)$$

$$\mathrm{P}(Data|M_i) = \int d\theta\, \mathrm{P}(Data|\theta, M_i) \times \mathrm{P}(\theta|M_i)$$

$\mathrm{P}(\theta|M_i)$: prior distribution of model parameters

$\mathrm{P}(Data|\theta, M_i)$: likelihood function

Bayes factor

$$\frac{\mathrm{P}(M_i|Data)}{\mathrm{P}(M_j|Data)} \propto \boxed{\frac{\mathrm{P}(Data|M_i)}{\mathrm{P}(Data|M_j)}} \times \frac{\mathrm{P}(M_i)}{\mathrm{P}(M_j)}$$

Bayes factor: Comparison of the plausibility of two models

Set up(SM)

SM part

$$-\mathcal{L}_{\mathrm{Y}} = y_{ij}^{(u)} Q_i\, \bar{u}_j\, H + y_{ij}^{(d)} Q_i\, \bar{d}_j\, H^{\dagger} + y_{ij}^{(e)} L_i\, \bar{e}_j\, H^{\dagger} + h.c.,$$

Set up(ν masses)

dim-5 operators

$$-\mathcal{L}_{\rm Y} = y_{ij}^{(\nu)} \frac{(L_i H)(L_j H)}{2\Lambda_N} + h.c., \qquad m_{ij}^{(\nu)} = y_{ij}^{(\nu)} \frac{v_{\rm EW}^2}{\Lambda_N}$$

seesaw$\left(q(\bar{N}_a) = 0\right)$

$$-\mathcal{L}_{\rm Y} = y_{ia}^{(D)} L_i \, \bar{N}_a \, H + \frac{1}{2} \, y_{ab}^{(R)} \, M_R \, \bar{N}_a \, \bar{N}_b + h.c.,$$

$$m_{ij}^{(\nu)} = (y^{(D)} \left(y^{(R)}\right)^{-1} \left(y^{(D)}\right)^T)_{ij} \frac{v_{\rm EW}^2}{M_R}$$

Concrete function forms

$$\mathrm{P}(M_i|Data) = \int d\theta\, \mathrm{P}(Data|\theta, M_i) \times \mathrm{P}(\theta|M_i) \times \mathrm{P}(M_i)$$

$$\mathrm{P}(M_i) = \mathrm{P}(M_j) \ \text{ for } \ i \neq j$$

$$\mathrm{P}(\theta = \kappa_{ij}, \epsilon | M_i) \propto \exp(-|\kappa_{ij}|^2), \quad (0.1 \leq \epsilon \leq 0.3)$$

$$\mathrm{P}(Data = x_a|\theta, M_i) \propto \delta(x_a(\theta) - x_a^{\mathrm{obs}})$$

Parameters

Fermion Yukawa $y_u, y_c, y_t, y_d, y_s, y_b, y_e, y_\mu, y_\tau$

CKM and PMNS parameters $s_{12}^{\mathrm{CKM}}, s_{23}^{\mathrm{CKM}}, s_{13}^{\mathrm{CKM}}, \delta^{\mathrm{CKM}}, s_{12}^{\mathrm{PMNS}}, s_{23}^{\mathrm{PMNS}}, s_{13}^{\mathrm{PMNS}}$

Neutrino mass ratio $\Delta m_{12}^2/\Delta m_{13}^2 = (m_1^2 - m_2^2)/(m_1^2 - m_3^2)$

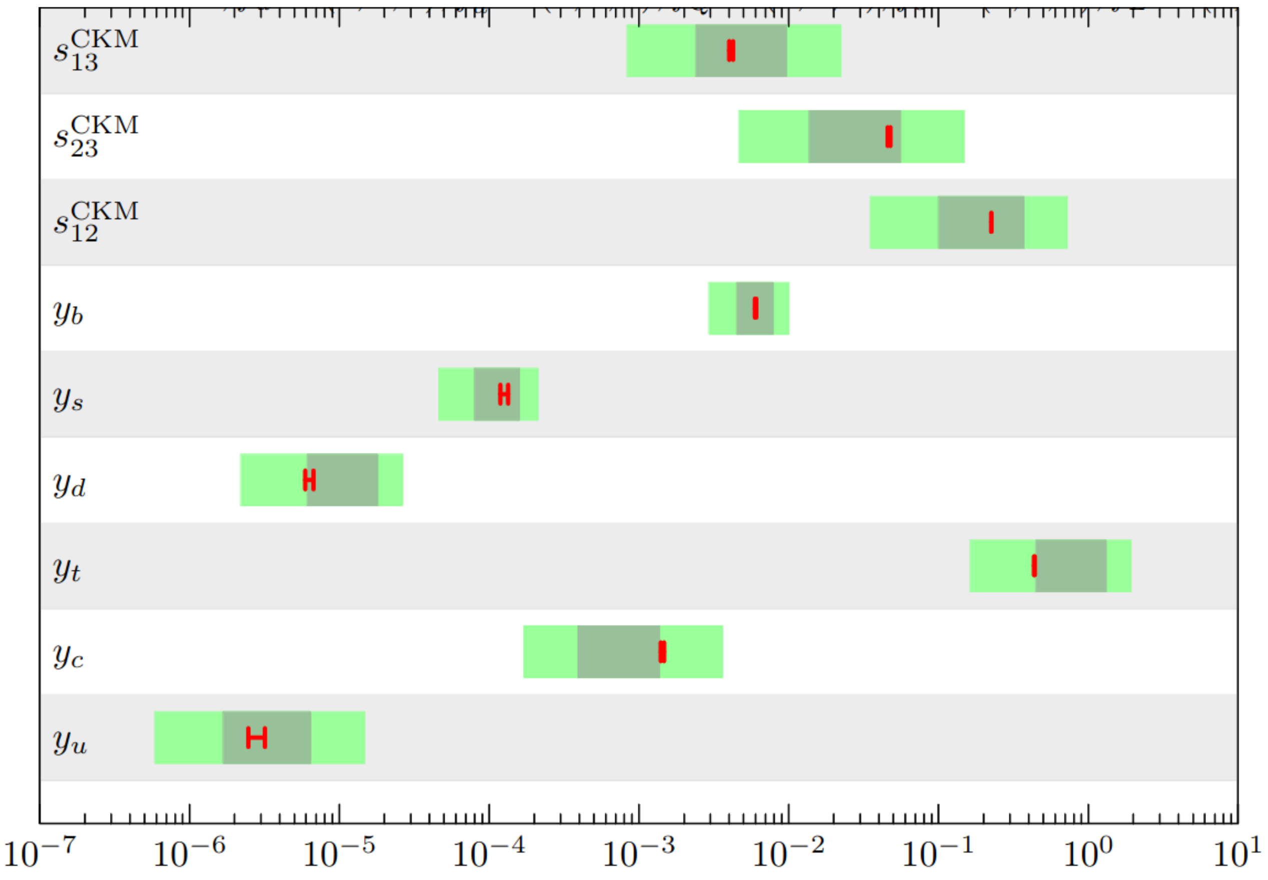
Results(quarks)

$$\epsilon = 0.14, \quad q(Q) = (3,2,0), \quad q(\bar{u}) = (4,2,0), \quad q(\bar{d}) = (3,3,3)$$

s_{ij}^{CKM}

$y^{(d,s,b)}$

$y^{(u,c,t)}$



Results(quarks)

f_Q	$f_{\bar{u}}$	$f_{\bar{d}}$	$\log_{10}(Z/Z_0)$	ϵ
$3, 2, 0$	$4, 2, 0$	$3, 3, 3$	96.34 ± 0.10	0.141 ± 0.005
$4, 2, 0$	$6, 3, 1$	$6, 4, 4$	96.03 ± 0.12	0.253 ± 0.006
$5, 3, 0$	$7, 3, 0$	$5, 5, 5$	95.94 ± 0.12	0.302 ± 0.007
$5, 3, 0$	$6, 3, 0$	$5, 5, 4$	95.86 ± 0.13	0.285 ± 0.007
$4, 3, 0$	$7, 3, 1$	$6, 5, 4$	95.82 ± 0.10	0.291 ± 0.007
$3, 2, 0$	$4, 2, 0$	$-9, 3, 3$	95.81 ± 0.11	0.145 ± 0.006
$4, 3, 0$	$6, 3, 0$	$5, 4, 4$	95.76 ± 0.13	0.255 ± 0.007
$5, 3, 0$	$6, 3, 0$	$5, 5, 5$	95.73 ± 0.12	0.297 ± 0.007
$4, 2, 0$	$6, 3, 1$	$5, 4, 4$	95.69 ± 0.11	0.248 ± 0.007
$4, 2, 0$	$5, 3, 1$	$6, 4, 4$	95.68 ± 0.13	0.247 ± 0.006

Results(leptons)

dim-5 operators

$\epsilon = 0.26, \quad q(L) = (5,4,4),$
$q(\bar{e}) = (5,2,0)$

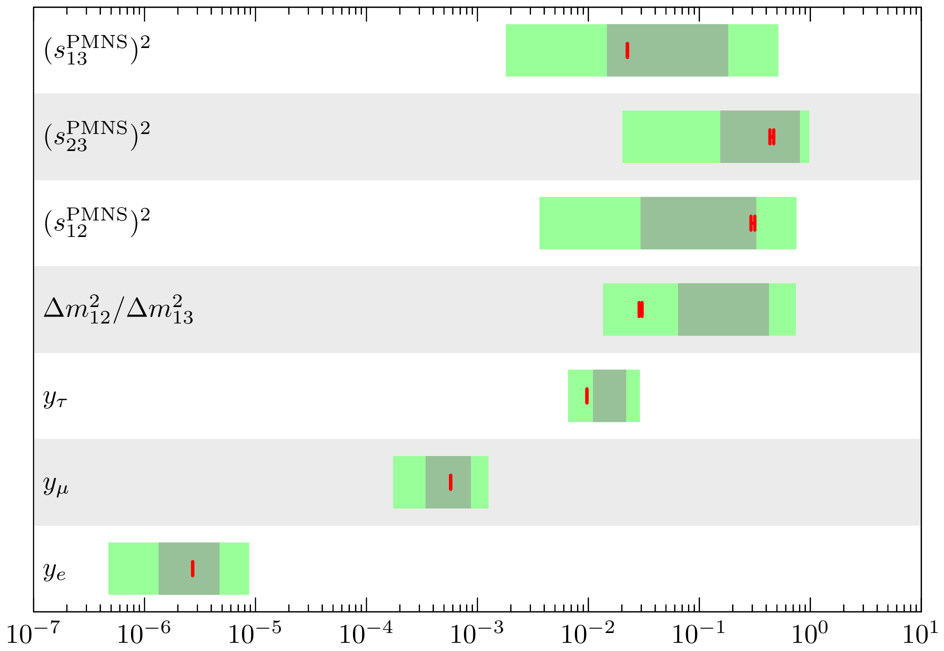
seesaw

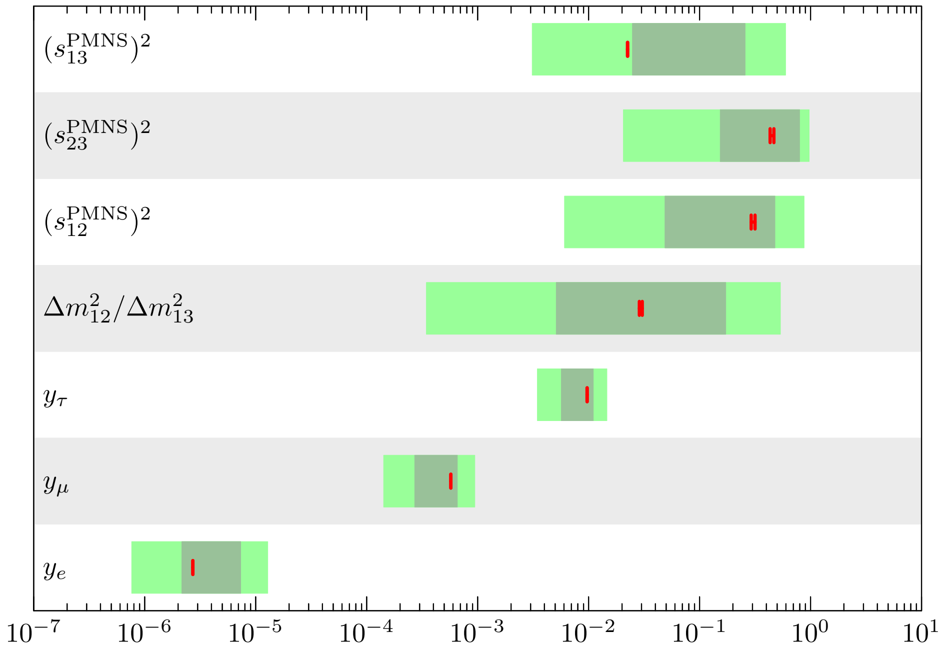
$\epsilon = 0.28, \quad q(L) = (5,4,4),$
$q(\bar{e}) = (5,2,0)$

$\left(s_{ij}^{\mathrm{PMNS}}\right)^2$

$\Delta m_{12}^2/\Delta m_{13}^2$

$y^{(e,\mu,\tau)}$





Results(leptons)

dim-5 operators

f_L	$f_{\bar{e}}$	$\log_{10}(Z/Z_0)$	ϵ
$5, 4, 4$	$5, 2, 0$	57.22 ± 0.13	0.264 ± 0.010
$7, 6, 6$	$3, -2, 0$	57.00 ± 0.15	0.264 ± 0.009
$6, 6, 5$	$3, -2, 0$	56.95 ± 0.15	0.231 ± 0.010
$5, 5, 4$	$6, 2, 0$	56.84 ± 0.12	0.300 ± 0.010
$5, 5, 4$	$5, 2, 0$	56.77 ± 0.20	0.290 ± 0.011
$7, 6, 6$	$4, -2, 0$	56.66 ± 0.14	0.284 ± 0.009
$4, 3, 3$	$4, 2, 0$	56.65 ± 0.22	0.193 ± 0.009
$7, 7, 6$	$3, -2, 0$	56.59 ± 0.22	0.291 ± 0.011
$7, 7, 6$	$4, -2, 0$	56.58 ± 0.08	0.300 ± 0.010
$6, 5, 5$	$2, -2, 0$	56.54 ± 0.20	0.194 ± 0.010

seesaw

f_L	$f_{\bar{e}}$	$\log_{10}(Z/Z_0)$	ϵ
$5, 4, 4$	$5, 2, 0$	51.18 ± 0.22	0.282 ± 0.011
$4, 4, 4$	$6, 2, 0$	50.96 ± 0.19	0.278 ± 0.010
$5, 4, 4$	$6, 2, 0$	50.83 ± 0.26	0.298 ± 0.010
$4, 3, 3$	$5, 2, 0$	50.80 ± 0.18	0.224 ± 0.009
$4, 3, 3$	$4, 2, 0$	50.54 ± 0.22	0.208 ± 0.010
$4, 4, 4$	$5, 2, 0$	50.42 ± 0.22	0.268 ± 0.011
$4, 4, 3$	$5, 2, 0$	50.41 ± 0.12	0.245 ± 0.010
$5, 4, 4$	$4, 2, 0$	50.38 ± 0.19	0.274 ± 0.012
$3, 3, 3$	$5, 2, 0$	50.27 ± 0.20	0.204 ± 0.009
$5, 5, 5$	$3, -2, 0$	50.23 ± 0.26	0.204 ± 0.009

Results(quark + lepton)

non-GUT

$$\epsilon = 0.26, \quad q(Q) = (4,3,0), \quad q(\bar{u}) = (6,3,0), \quad q(\bar{d}) = (5,4,4)$$
$$q(L) = (5,4,4), \quad q(\bar{e}) = (5,2,0)$$

SU(5)-GUT

$$\epsilon = 0.26, \quad q(10) = (5,3,0), \quad q(\bar{5}) = (5,4,3),$$

Contents

Summary

- The FN mechanism is a promising mechanism to explain the flavor puzzles

- We are searching for good FN charge assignments which can explain the flavor puzzles

Thank you for your attention!

Integral measure

$$A = U_L \Sigma U_R^\dagger \qquad \Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3)$$

$$\prod_{i,j} d\,\mathrm{Re}(A_{ij})\, d\,\mathrm{Im}(A_{ij})$$
$$= (\sigma_1^2 - \sigma_2^2)^2(\sigma_2^2 - \sigma_3^2)^2(\sigma_3^2 - \sigma_1^2)^2 d\sigma_1^2 d\sigma_2^2\, d\sigma_3^2 \frac{dU_L dU_R}{d\phi_1 d\phi_2 d\phi_3}$$

Prior function

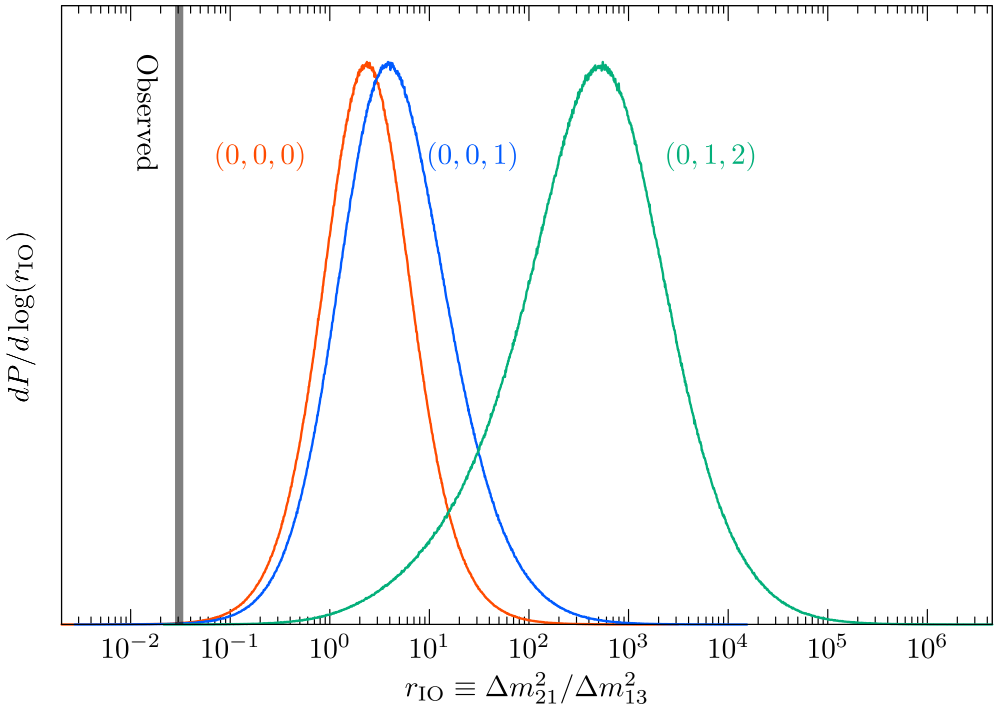
$$\pi(\kappa_u) = \pi\left(y_u \circ \delta^{-|q_i+u_j|}\right)$$
$$= \exp(-\,\mathrm{tr}[\left(y_u \circ \delta^{-|q_i+u_j|}\right)^{\dagger} (y_u \circ \delta^{-|q_i+u_j|})]/2)$$

$$y_u = U_{uL}\, \Sigma_u\, U_{uR}^{\dagger}, \qquad \Sigma = \mathrm{diag}(\sigma_{u1}, \sigma_{u2}, \sigma_{u3})$$

ν masses (Inverted ordering)

prior distributions of $r_{\mathrm{IO}} = \Delta m^2_{21}/\Delta m^2_{13}$

dim-5 operators



seesaw