Quantum Effects on Neutrino Parameters From a Flavored Gauge Boson

Generating Neutrino Masses via RGE Running at the One-Loop Level

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Preface

- Based on Master's thesis at Technical University of Munich, Germany
- Paper in progress with Alejandro Ibarra (TUM)

Outline

Neutrino Physics

- Neutrinos massless in the Standard Model (SM), but...
- Oscillation experiments (two flavors - Gribov, Pontecorvo, 1968)

$$P_{\nu_a \longrightarrow \nu_b \neq \nu_a} \approx \sin^2(2\theta)\, \sin^2\left(\frac{\Delta m^2\, L}{4E}\right) \tag{1}$$

$\Longrightarrow$ neutrinos do have a mass!

- Flavor basis $\neq$ mass basis $\Longrightarrow$ mixing!
- Mass splittings (Esteban et al., 2020): $\left|\Delta m^2_{3\ell}\right| = \left|m^2_{\nu,\,3} - m^2_{\nu,\,1\,\text{or}\,2}\right| \sim 7 \times 10^{-3}\,\text{eV}^2$,

$$\left|\Delta m^2_{21}\right| = \left|m^2_{\nu,\,2} - m^2_{\nu,\,1}\right| \sim 3 \times 10^{-5}\,\text{eV}^2$$

The Weinberg-Operator

- Dimension 5 Weinberg-Operator $\mathcal{L}_\kappa \sim \kappa_{gf}\,(l^g \cdot \phi)\,(l^f \cdot \phi) + \mathrm{h.c.}$ gives neutrino masses after Electroweak Symmetry Breaking (Weinberg, 1979)

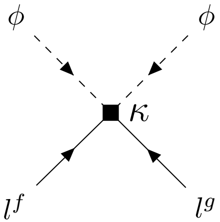


Figure 2 Feynman diagram of the Weinberg-Operator

- Eigenvalues $\sim$ masses
- Diagonalize $\kappa \longrightarrow U^*\,\kappa\,U^\dagger = \kappa_{\mathrm{diag}}$
- U: contains mixing angles

- Loop corrections to κ? Different scales of mass generation, production, measurement?
  $\Longrightarrow \beta_\kappa$, running of neutrino parameters

Renormalization

Part 1/2 - Renormalization Constants and RGEs

- Loop-diagrams divergent $\Longrightarrow$ need to regularize them
- Cancel divergences via counterterms: $\phi_{\mathrm{Bare}} = \sqrt{Z_\phi}\,\phi \approx \phi + \frac{1}{2}\,\delta Z_\phi\,\phi$
- Physical observables: dependence on renormalization scale? No!

$\longrightarrow$ Renormalization Group Equations (RGEs) (Callan, 1970; Symanzik, 1970)

$$0 \stackrel{!}{=} \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} R\left(Q^2/\mu^2,\, \alpha(\mu^2)\right) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \underbrace{\mu^2 \frac{\partial \alpha(\mu^2)}{\partial \mu^2}}_{\equiv \beta_\alpha} \frac{\partial}{\partial \alpha(\mu^2)}\right) R\left(Q^2/\mu^2,\, \alpha(\mu^2)\right) \tag{2}$$

Renormalization

Part 2/2 - β-Functions

- RGEs solved by reparametrization with running couplings in $R\left(1,\, \alpha(Q^2)\right)$ (Callan, 1970; Symanzik, 1970)
- RGEs of couplings, β-functions: define $t = \ln \mu^2/\mu_0^2$

$$\beta_\alpha = \frac{\mathrm{d}\alpha}{\mathrm{d}t} \Longrightarrow \alpha(t) \tag{3}$$

- To obtain β_α, solve

$$0 \overset{!}{=} \frac{\mathrm{d}}{\mathrm{d}t}\alpha_{\mathrm{Bare}} = \frac{\mathrm{d}}{\mathrm{d}t} f\left(\alpha,\, Z_\alpha,\, Z_{\phi_i}\right) \supset \beta_\alpha \tag{4}$$

- Loop corrections!

Outline

One-Loop Effects From a Flavored Gauge Boson

Part 1/3 - From SM to BSM

- Bare coupling $\kappa_B = Z_\phi^{-\frac{1}{2}} \left(Z_l^T\right)^{-\frac{1}{2}} (\kappa + \delta\kappa)\, Z_l^{-\frac{1}{2}}\, Z_\phi^{-\frac{1}{2}}$
- Form of 1-loop RGEs in SM (Babu et al., 1993; Casas et al., 2000; Antusch et al. 2001):

$$\beta_\kappa(t) = \frac{\mathrm{d}\kappa}{\mathrm{d}t} = \delta\kappa_{,1} - \frac{1}{2}\left(\delta Z_{\phi,1} + \delta Z_{l,1}\right)^T \kappa - \frac{1}{2}\kappa\left(\delta Z_{\phi,1} + \delta Z_{l,1}\right) \tag{5}$$

$$= \alpha\kappa + P^T\kappa + \kappa P \tag{6}$$

- Abelian extension of SM gauge group: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{L_\mu - L_\tau}$
- Effects on Weinberg-Operator's RGEs and neutrino parameters?

One-Loop Effects From a Flavored Gauge Boson

Part 2/3 - Field Renormalization Contributions

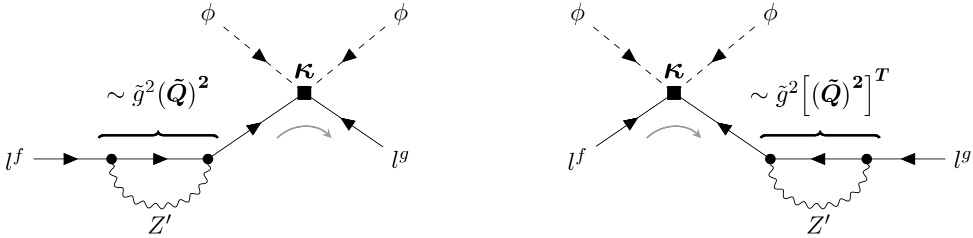


Figure 3 Structure of the contribution from the new gauge boson, Z', to the Weinberg-Operator via field renormalization of lepton-doublets

- Two conjugate diagrams, structure corresponds to $\beta_{\boldsymbol{\kappa}} \supset P^T \kappa + \kappa P$
- Expected, fits SM structure $\beta_{\boldsymbol{\kappa}}(t) = \alpha\kappa + P^T \kappa + \kappa P$

One-Loop Effects From a Flavored Gauge Boson

Part 3/3 - Vertex Correction Contribution and the New Quantum Effect $G^T \kappa G$

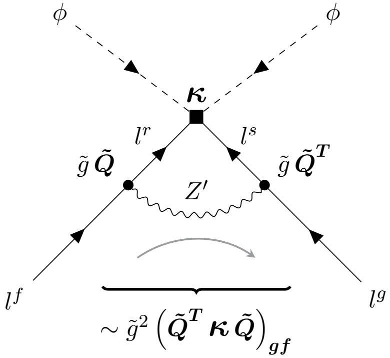


Figure 4 Structure of the Z' contribution to the Weinberg-Operator via vertex correction

- One diagram, contribution of form $\beta_{\mathcal{K}} \supset G^T \kappa G$
- Structure does not appear in SM $\beta_{\mathcal{K}}$-function!
- Origin: flavor-dependent gauge interaction!

$$G \propto \tilde{g}\,\tilde{Q} = \tilde{g}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The Extended β_κ-Function

Part 1/6 - The New Quantum Effect $G^T \kappa\, G$

- In SM, flavor-universal gauge couplings:

$$G_{SM} \propto \mathbb{1} \implies G_{SM}^T \,\kappa\, G_{SM} \;\propto\; \mathbb{1}^T \kappa\, \mathbb{1} \;=\; \kappa \tag{7}$$

$$\implies \beta_\kappa \;\supset\; G_{SM}^T \,\kappa\, G_{SM} \sim \alpha\,\kappa \quad ! \tag{8}$$

$$\beta_\kappa \;=\; \underbrace{\alpha\,\kappa + P^T \kappa + \kappa\, P}_{\text{Standard Model + } U(1)_{L_\mu - L_\tau}} \;+\; \underbrace{\boldsymbol{G^T \kappa\, G}}_{\boldsymbol{U(1)_{L_\mu - L_\tau}}} \quad ! \tag{9}$$

The Extended $\beta_{\boldsymbol{\kappa}}$-Function

Part 2/6 - Generating Neutrino Masses via RGEs 1/3

- Diagonalize $\beta_{\boldsymbol{\kappa}}$-function with leptonic mixing matrix U

$$\frac{\mathrm{d}\boldsymbol{\kappa_i}}{\mathrm{d}t} = \underbrace{\alpha\,\boldsymbol{\kappa_i} + 2\left(U\,P\,U^T\right)_{ii}\boldsymbol{\kappa_i}}_{\text{Standard Model + } U(1)_{L_\mu - L_\tau}} + \underbrace{\sum_{k=1}^{3}\left[\left(U\,G\,U^T\right)_{ki}\right]^2\boldsymbol{\kappa_k}}_{U(1)_{L_\mu - L_\tau}} \tag{10}$$

- Can raise rank of mass matrix at 1-loop! In SM only at 2-loops!

The Extended β_{κ}-Function

Part 3/6 - Generating Neutrino Masses via RGEs 2/3

- From just one non-zero mass at scale $\Lambda \sim 10^{14}$ GeV, generate two others via RGEs!

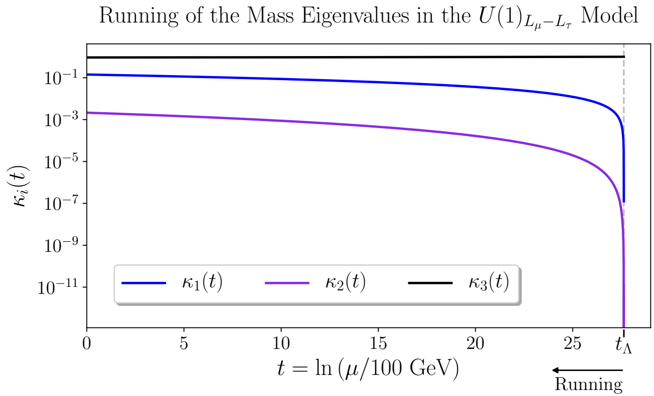


Figure 5 Running of the eigenvalues κ_i of κ due to the $G^T\,\kappa\,G$ term as a function of t; random, real leptonic mixing matrix $U(t_\Lambda)$ and $\tilde{g} = 0.5$; one non-zero eigenvalue at Λ

- Most running happens at the beginning

The Extended β_κ-Function

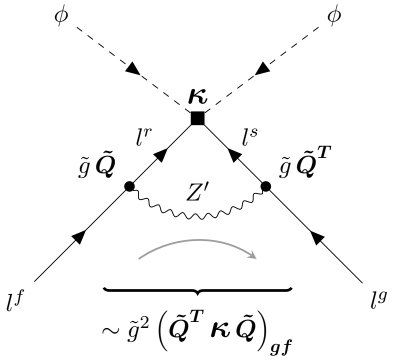
Part 4/6 - Generating Neutrino Masses via RGEs 3/3

	$m_{\nu,3}$	$m_{\nu,2}$	$m_{\nu,1}$
Mass Values Before Running	$8 \times 10^{-2}\,\mathrm{eV}$	$0\,\mathrm{eV}$	$0\,\mathrm{eV}$
Mass Values After Running	$\sim 8 \times 10^{-2}\,\mathrm{eV}$	$\sim 8 \times 10^{-3}\,\mathrm{eV}$	$\sim 8 \times 10^{-5}\,\mathrm{eV}$
Mass Splittings	$\boldsymbol{\Delta m^2_{3\ell} \sim 6 \times 10^{-3}\,\mathrm{eV}^2}$		$\boldsymbol{\Delta m^2_{21} \sim 6 \times 10^{-5}\,\mathrm{eV}^2}$
Measured Mass Splittings	$\boldsymbol{\Delta m^2_{3\ell} \sim 7 \times 10^{-3}\,\mathrm{eV}^2}$		$\boldsymbol{\Delta m^2_{21} \sim 3 \times 10^{-5}\,\mathrm{eV}^2}$

Table 1 Neutrino mass values before and after RGE running following fig. (5), and the resulting mass splittings; starting value inspired by cosmological constraint on mass sum, $\Sigma m_\nu < 0.111$ eV (eBoss, 2020)

The Extended $\beta_{\boldsymbol{\kappa}}$-Function

Part 5/6 - $G^T \boldsymbol{\kappa}\, G$ From Vector Interactions



- $\mathcal{L} \sim g_{ij}\,(\,\overline{l^i}\,\gamma^\mu\,l^j\,)\,V_\mu$ interaction only possibility for $G^T\,\kappa\,G$ term at 1-loop!
- $\Longrightarrow$ Only present in flavor-nonuniversal gauge theories & flavor gauge theories

The Extended $\beta_{\boldsymbol{\kappa}}$-Function

Part 6/6 - Most General $\beta_{\boldsymbol{\kappa}}$-Function

- Most general, symmetry-allowed RGEs for any number of lepton generations at 1-loop:

$$\frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha\kappa + P^T\kappa + \kappa P + G^T\kappa G + \frac{1}{2}\left[G_+^T\kappa G_- + G_-^T\kappa G_+\right] \tag{11}$$

$$U$$

$$\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \alpha\,\kappa_i + 2\,\tilde{P}_{ii}\,\kappa_i + \sum_{k=1}^{n}\mathrm{Re}\Big\{\Big[(\tilde{G}_{ki})^2\Big] + \Big[\tilde{G}_{+,\,ki}\,\tilde{G}_{-,\,ki}\Big]\Big\}\,\kappa_k \tag{12}$$

- Enhanced running in non-abelian gauge extensions!
- If no flow-reversing interactions: at any loop order!

General Calculation of $\beta_{\boldsymbol{\kappa}}$

Part 1/1 - One-Loop $\beta_{\boldsymbol{\kappa}}$ and General Contributions from Gauge Bosons

- For any SM gauge extension

$$\beta_{\boldsymbol{\kappa}} \supset \delta\boldsymbol{\kappa}_{gf,1}\Big|_{V_\mu \text{ vertex}} = \frac{2}{16\pi^2} g_n^2 (3+\xi_n) \quad \left(T^T \boldsymbol{\kappa} T\right)_{gf}, \tag{13}$$

$$\beta_{\boldsymbol{\kappa}} \supset \delta\boldsymbol{\kappa}_{gf,1}\Big|_{V_\mu^\pm \text{ vertex}} = \frac{2}{16\pi^2} g_c^2 (3+\xi_c) \quad \frac{1}{2}\left(T_+^T \boldsymbol{\kappa} T_- + T_-^T \boldsymbol{\kappa} T_+\right)_{gf} \tag{14}$$

Outline

Summary - Standard Model vs. Flavor Gauge Theory

	Standard Model	$\boldsymbol{U(1)_{L_\mu - L_\tau}}$ **Gauge-Extension**
Interaction matrices in flavor-space	$Q_{SM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbb{1}$	$\tilde{Q} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \neq \mathbb{1}$
One-loop β_κ structure	$\alpha\,\kappa + P^T\,\kappa + \kappa\,P$	$\alpha\,\kappa + P^T\,\kappa + \kappa\,P + \boldsymbol{G^T\,\kappa\,G}$, with $G \propto \tilde{Q}$
One-loop $d\kappa_i/dt$ structure	$\propto \kappa_i$	$(\propto \kappa_i) + \boldsymbol{\sum_{j=1}^{3}}\,(\boldsymbol{\propto \kappa_j})$
Rank of κ throughout RGE evolution	constant	increases $\longrightarrow 3$
Mass splittings and mass hierarchy	Ad-hoc	Provides framework to explain and predict them

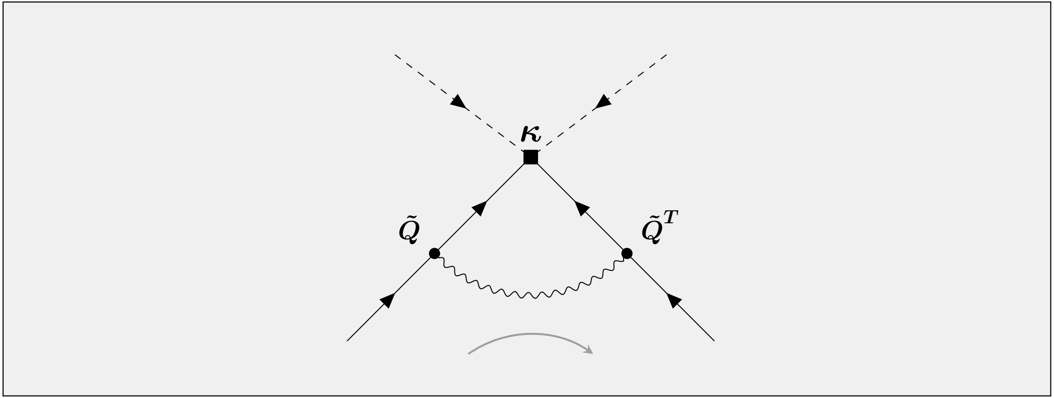
- Enhanced running in non-abelian flavor gauge theories
- Verify appearance via general contributions $\Longrightarrow$ “plug and play”

Outlook

- Wholistic non-abelian models
  - Symmetry breaking sequence
  - Charged lepton masses
  - Integrating out heavy gauge bosons
- In-depth phenomenological analysis
  - Mass running and splittings
  - Fixed points of mixing angles and phases
  - Experimental prospects

$\Longrightarrow$ Keep an eye out for the paper!

Thank you for your attention!



Outline

Backup Slides

Part 1/18 - Aim of UV Models

- Structure of neutrino mass matrices (Type-I Seesaw with right-handed neutrinos)

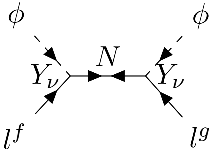


Figure 6 Feynman diagram for a Type-I Seesaw Mechanism

- Want: badly broken flavor gauge symmetry
  - New effect
  - No structure constraint $\Longrightarrow$ arbitrary $\kappa(t_\Lambda)$
  - Nice effects with fewer constraints!

- Symmetry, additional fields, and realization of Spontaneous Symmetry Breaking (SSB)

Backup Slides

Part 2/18 - The Six-Scalar $U(1)_{L_\mu - L_\tau}$ Model

- $U(1)_{L_\mu - L_\tau}$-symmetric phase $\kappa = \begin{pmatrix} \kappa_{11} & 0 & 0 \\ 0 & 0 & \kappa_{23} \\ 0 & \kappa_{23} & 0 \end{pmatrix}$
- Additional entries via SSB
- Six-scalar $U(1)_{L_\mu - L_\tau}$ model: right-handed neutrino mass matrix from

  $-\mathcal{L} \supset \frac{1}{2} M_{ij} \overline{N_i^C} N_j + \frac{1}{2} \lambda_{ij} \overline{N_i^C} S_{ij} N_j$

$$M_R = \begin{pmatrix} M_{ee} + \lambda_{ee} \langle S_{ee} \rangle & \lambda_{e\mu} \langle S_{e\mu} \rangle & \lambda_{e\tau} \langle S_{e\tau} \rangle \\ \lambda_{e\mu} \langle S_{e\mu} \rangle & \lambda_{\mu\mu} \langle S_{\mu\mu} \rangle & M_{\mu\tau} + \lambda_{\mu\tau} \langle S_{\mu\tau} \rangle \\ \lambda_{e\tau} \langle S_{e\tau} \rangle & M_{\mu\tau} + \lambda_{\mu\tau} \langle S_{\mu\tau} \rangle & \lambda_{\tau\tau} \langle S_{\tau\tau} \rangle \end{pmatrix} \tag{15}$$

Backup Slides

Part 3/18 - Non-Abelian Flavor Gauge Extensions

- Flavor gauge groups $\Gamma \supset U(1)_{L_\mu - L_\tau}$? $\implies SU(2)_{\mu\tau}$, $SU(3)_{\mu\tau}$
- $SU(3)$: N_i triplet representation, scalar anti-sextet; $-\mathcal{L} \supset \cancelto{0}{\frac{1}{2} M_{ij} \overline{N_i^C} N_j} + \frac{1}{2} \lambda \overline{N^C} S N$

$$M_R = \frac{\lambda}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\,\langle S_{11}\rangle & \langle S_{12}\rangle & \langle S_{13}\rangle \\ \langle S_{12}\rangle & \sqrt{2}\,\langle S_{22}\rangle & \langle S_{23}\rangle \\ \langle S_{13}\rangle & \langle S_{23}\rangle & \sqrt{2}\,\langle S_{33}\rangle \end{pmatrix} \tag{16}$$

- Flavor-charged gauge bosons: $G_\pm \implies$ stronger running, flavor-changing interactions
- Anti-sextet breaks $SU(3)_{\mu\tau} \longrightarrow SU(2)_{\mu\tau} \longrightarrow \emptyset$, or $SU(3)_{\mu\tau} \longrightarrow \emptyset$

Backup Slides

Part 4/18 - Degenerate Mass Generation

- From two degenerate masses at scale Λ, generate one other, and splitting via RGEs!

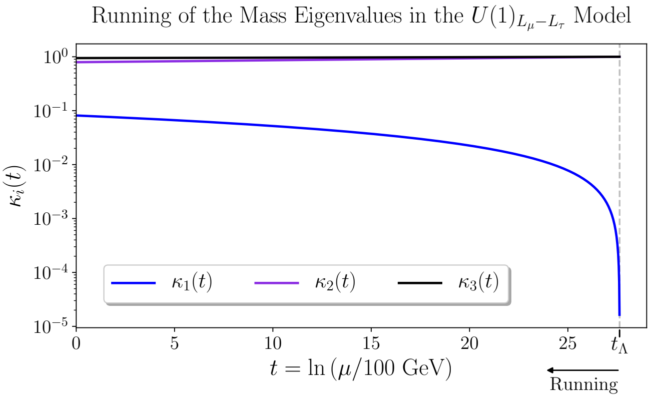


Figure 7 Running of the eigenvalues κ_i of κ due to the $G^T\,\kappa\,G$ term as a function of t; random, real leptonic mixing matrix $U(t_\Lambda)$; two degenerate eigenvalues at Λ

- Inverted hierarchy!

Backup Slides

Part 5/18 - Two Neutrino $U(1)_{L_\mu - L_\tau}$ Model

- Two neutrino $U(1)_{L_\mu - L_\tau}$ model:

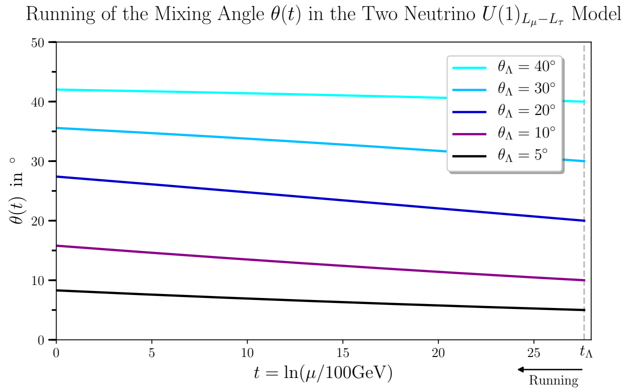


Figure 8 Running of the mixing angle $\theta(t)$ due to the vertex renormalization effects of Z', as a function of t;

- Analytical solution for $\theta(t)$
- Always increases
- If $\theta_\Lambda = 0$, stays 0

Backup Slides

Part 6/18 - Weinberg-Operator and RGE Contributions

- Explicit form of Weinberg-Operator: $\mathcal{L}_{\kappa} = \frac{1}{4}\,\kappa_{gf}\,\overline{l^C}^g_c\,\epsilon^{cd}\,\phi_d\,l^f_b\,\epsilon^{ba}\,\phi_a + \text{h.c.}$
- Mass term: $\mathcal{L}_{\kappa,\nu} = \frac{1}{4}\,\kappa_{gf}\,\overline{\nu^C}^g\,\nu^f\,\phi^0\,\phi^0 + \text{h.c.} \xrightarrow{\text{EWSB}} \frac{\kappa_{gf}\,v_{\text{EW}}^2}{4}\,\frac{1}{2}\,\overline{\nu^C}^g\,\nu^f + \text{h.c.}$
- SM diagrams:

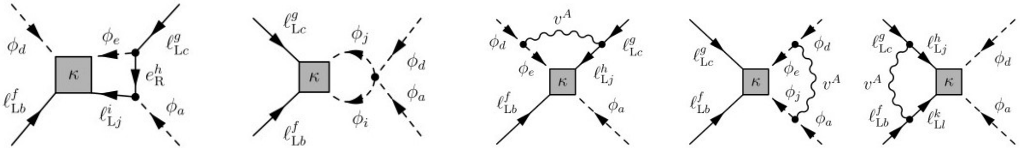


Figure 9 SM vertex correction contributions to κ RGEs at 1-loop (taken from Antusch, 2003)

Backup Slides

Part 7/18 - RGE Contributions

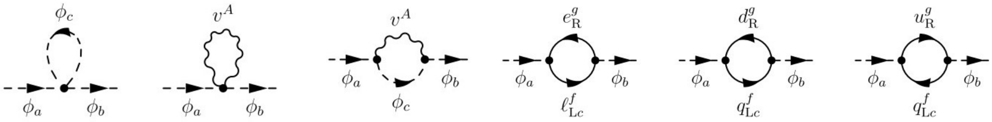


Figure 10 SM Higgs doublet wave function renormalization contributions to κ RGEs at 1-loop (taken from Antusch, 2003)

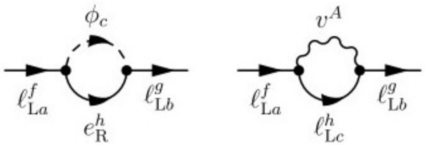


Figure 11 SM lepton doublet wave function renormalization contributions to κ RGEs at 1-loop (taken from Antusch, 2003)

Backup Slides

Part 8/18 - Explicit RGEs

- RGEs for U(1)$_{L_\mu - L_\tau}$ (SM: Babu et al., 1993; Casas et al., 2000; Antusch, 2003; $U(1)_{L_\mu - L_\tau}$: LT, 2023):

$$\beta_\kappa(t) = \frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha\kappa + P^T\kappa + \kappa P + G^T\kappa G, \tag{17}$$

$$16\pi^2\ \alpha = \lambda - 3g_2^2 + 2\mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e), \tag{18}$$

$$16\pi^2\ P = -\frac{3}{2}Y_e^\dagger Y_e\,, \tag{19}$$

$$\sqrt{16\pi^2}\, G = \sqrt{6}\tilde{g}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \Big| \quad G^T\,\kappa\,G = \frac{6\tilde{g}^2}{16\pi^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \kappa_{22} & -\kappa_{23} \\ 0 & -\kappa_{23} & \kappa_{33} \end{pmatrix} \tag{20}$$

Backup Slides

Part 9/18 - Charged Gauge Bosons

- Charged generators:

$$T_\pm \equiv T_1 \pm i\, T_2\,, \tag{21}$$

- Charged gauge bosons:

$$X_1\, T_1 + X_2\, T_2 \stackrel{!}{=} \frac{1}{\sqrt{2}}\, X_+\, T_+ + \frac{1}{\sqrt{2}}\, X_-\, T_- +\,, \tag{22}$$

$$X_+\, X_- = \frac{1}{2}\, X_1^2 + \frac{1}{2}\, X_2^2 \tag{23}$$

Backup Slides

Part 10/18 - Flavor-Charged Gauge Boson Vertex Corrections

- Flavor-charged gauge boson contributions

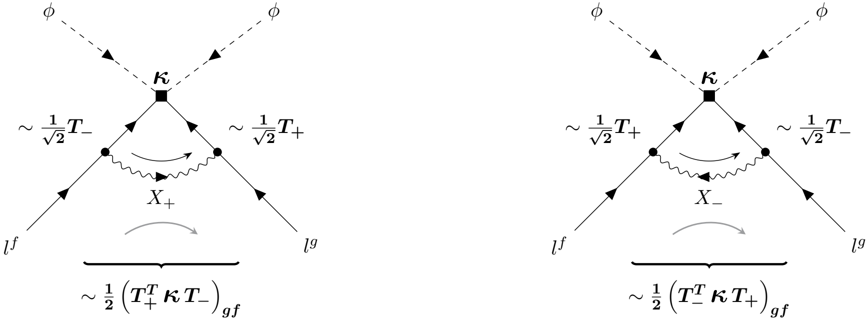


Figure 12 Structure of $X\pm$ to the Weinberg-Operator via vertex correction; the gray arrow denotes fermion flow

Backup Slides

Part 11/18 - Wave Function Renormalization and $SU(N)$

- $SU(2)$ triplet representation in $(\mu,\, e,\, \tau)$ basis:

$$T_3^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T_+^{(3)} = \sqrt{2}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_-^{(3)} = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{24}$$

- Resulting vertex contribution in $(e,\, \mu,\, \tau)$ basis:

$$\delta\kappa_{,1} = \frac{2}{16\pi^2}\,\tilde{g}^2\,(3+\xi_X)\begin{pmatrix} 2\kappa_{23} & \kappa_{12} & \kappa_{13} \\ \kappa_{12} & \kappa_{22} & \kappa_{11}-\kappa_{23} \\ \kappa_{13} & \kappa_{11}-\kappa_{23} & \kappa_{33} \end{pmatrix} \tag{25}$$

Backup Slides

Part 12/18 - Mass Generation in $SU(2)_{\mu\tau}$ 1/2

- $SU(2)_{\mu\tau}$ with just one mass at Λ:

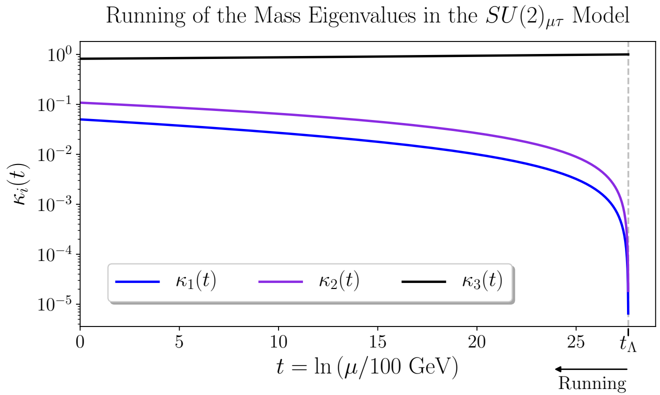


Figure 13 Running of the eigenvalues κ_i of κ due to the $G^T_{(\pm)}\,\kappa\,G_{(\mp)}$ terms in $SU(2)_{\mu\tau}$ as a function of t; random, real leptonic mixing matrix $U(t_\Lambda)$; one nonzero eigenvalue at Λ

- Larger generated masses!

Backup Slides

Part 13/18 - Mass Generation in $SU(2)_{\mu\tau}$ 2/2

- $SU(2)_{\mu\tau}$ with two degenerate masses at Λ:

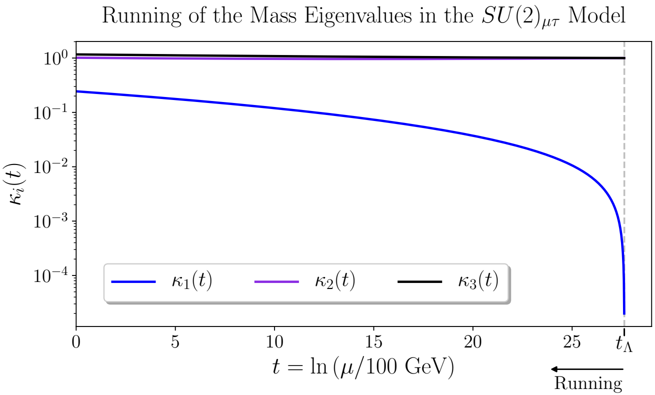


Figure 14 Running of the eigenvalues κ_i of κ due to the $G^T_{(\pm)}\,\kappa\,G_{(\mp)}$ terms in $SU(2)_{\mu\tau}$ as a function of t; random, real leptonic mixing matrix $U(t_\Lambda)$; two degenerate eigenvalues at Λ

- Larger generated mass!

Backup Slides

Part 14/18 - Wave Function Renormalization, and $SU(N)$

- General wave function renormalization

$$\delta Z_{l,\,gf,\,1}\,\Big|_{lV_\mu} = -\frac{2}{16\pi^2}\,g_n^2\,\xi_n\,\left(\boldsymbol{T^2}\right)_{\boldsymbol{gf}}\,, \tag{26}$$

$$\delta Z_{l,\,gf,\,1}\,\Big|_{lV_\mu^\pm} = -\frac{2}{16\pi^2}\,g_n^2\,\xi_n\,\frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{T}_+^\dagger\,\boldsymbol{T}_+ + \boldsymbol{T}_-^\dagger\,\boldsymbol{T}_-\right)_{\boldsymbol{gf}} \tag{27}$$

- All $SU(N)$ gauge bosons active (fundamental representation):

$$\sum_{A=1}^{N^2-1}\left[\left(T^A\right)^T\kappa\,T^A\right]_{jl} = \sum_{A=1}^{N^2-1} T^A_{ij}\,\kappa_{ik}\,T^A_{kl} = \kappa_{ik}\,\frac{1}{2}\left(\delta_{il}\,\delta_{jk} - \frac{1}{N}\,\delta_{ij}\,\delta_{kl}\right) = \frac{N-1}{2N}\,\kappa_{jl} \tag{28}$$

Backup Slides

Part 15/18 - Neutrino Mass Matrices (Type-I Seesaw) (Heeck et al., 2011)

- Type-I Seesaw Mechanism: Diagonalizing mass matrix $M_{\nu,tot} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$ from $-\mathcal{L} \supset Y_{\nu,ij}\, \bar{l}^i\, \epsilon\, \phi^*\, N^j + \frac{1}{2}\, M_{R,ij}\, \overline{N^C}^i\, N^j + \text{h.c.}$ gives: $M_\nu \approx -M_D\, M_R^{-1}\, M_D^T$
- If U(1)$_{L_\mu - L_\tau}$ exact: $M_R = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & 0 & M_{\mu\tau} \\ 0 & M_{\mu\tau} & 0 \end{pmatrix}$, and with $M_D = \begin{pmatrix} m_{\nu_e} & 0 & 0 \\ 0 & m_{\nu_\mu} & 0 \\ 0 & 0 & m_{\nu_\tau} \end{pmatrix}$

$$\longrightarrow M_\nu = -\begin{pmatrix} \frac{m_{\nu_e}^2}{M_{ee}} & 0 & 0 \\ 0 & 0 & \frac{m_{\nu_\mu} m_{\nu_\tau}}{M_{\mu\tau}} \\ 0 & \frac{m_{\nu_\mu} m_{\nu_\tau}}{M_{\mu\tau}} & 0 \end{pmatrix}$$

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Part 16/18 - Dark Matter from the Proposed Models

- Dark matter in $U(1)_{L_\mu - L_\tau}$ six-scalar model? N_i or S_{ij} possible
- S_{ij}:
  - vevs $\langle S_{ij} \rangle = \sqrt{\frac{\mu_{ij}^2}{\lambda_{ij}^S}}$
  - Massive components with masses μ_{ij}
  - N_i masses $\sim \lambda_{ij} \langle S_{ij} \rangle = \lambda_{ij} \sqrt{\frac{\mu_{ij}^2}{\lambda_{ij}^S}}$

    $\implies S_{ij}$ viable dark matter candidates if $\lambda_{ij} \gtrsim \sqrt{\lambda_{ij}^S}$
- Could use massive S_{ij} remnants as dark matter, N_i for leptogenesis (CP-violating decay to ϕ and l^f)

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Part 17/18 - Gauge-Independence 1/2

Gauge-invariance via Nambu-Goldstone bosons (one generation), otherwise

SSB-generated Weinberg-Operator in unitary gauge!

$$\mathcal{L}_{\kappa} \sim \kappa\,(l \cdot \phi)\,(l \cdot \phi)\;\; e^{i\,(-2q_l)\,\chi\,/\,M_{Z'}} + \text{h.c.} \tag{29}$$

$$\mathcal{L}_{Z'+\text{GF}} = -\frac{1}{4}\,F'_{\mu\nu}\,F'^{\mu\nu} + \frac{1}{2}\,M_{Z'}^2\left(Z'_\mu + \frac{1}{M_{Z'}}\,\partial_\mu\,\chi\right)^2 - \frac{1}{2\xi_{Z'}}\,\left(\partial_\mu Z'^\mu - \xi_{Z'}\,\tilde{g}\,\chi\right)^2 \tag{30}$$

$$\beta_{\kappa}\,\Big|_{Z',\,\xi_{Z'}} = \underbrace{2 \cdot \frac{1}{16\pi^2}\,\xi_{Z'}\,\tilde{g}^2\,q_l^2\,\kappa}_{\text{from }\delta Z_l} + \underbrace{\frac{1}{16\pi^2}\,2\,\xi_{Z'}\,\tilde{g}^2\,q_l^2\,\kappa}_{\text{from }\delta\kappa} = 4\,\frac{1}{16\pi^2}\,\xi_{Z'}\,\tilde{g}^2\,q_l^2\,\kappa \tag{31}$$

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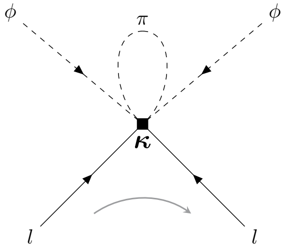
Part 18/18 - Gauge-Independence 2/2

- Tadpole diagram from quadratic term of exponential (dim. 7):



$$\propto \quad \frac{\kappa}{M_{Z'}^2}\,\frac{i}{16\pi^2}\,\frac{1}{4-d}\,\underbrace{\xi_{Z'}\,\tilde{g}^2\,M_{Z'}^2\,(-2q_l)^2}_{m_\chi^2}$$

- Suppression scale cancels with tadpole mass $\delta\kappa_{\text{tadpole},1} = -\frac{1}{16\pi^2}\,\xi_{Z'}\,\tilde{g}^2\,(-2q_l)^2\,\kappa$
- Gauge-dependence cancels: ✓