Model building by Coset space dimensional reduction using eight-dimensional coset spaces

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Based on the work with K. Asai (ICRR), J. Sato (Yokohama National U.),
Y. Takanishi (Saitama U.), and M. J. S. Yang (Saitama U.),

Outline

1. Introduction 4 pages
2. Coset Space Dimensional Reduction 4 pages
3. Models obtained by CSDR using 8-dim cosets
   5 pages
4. Summary 1 page

Contents

https://higgstan.com/wp/wp-content/uploads/2013/04/parts-01.png

A naive question on the Higgs boson

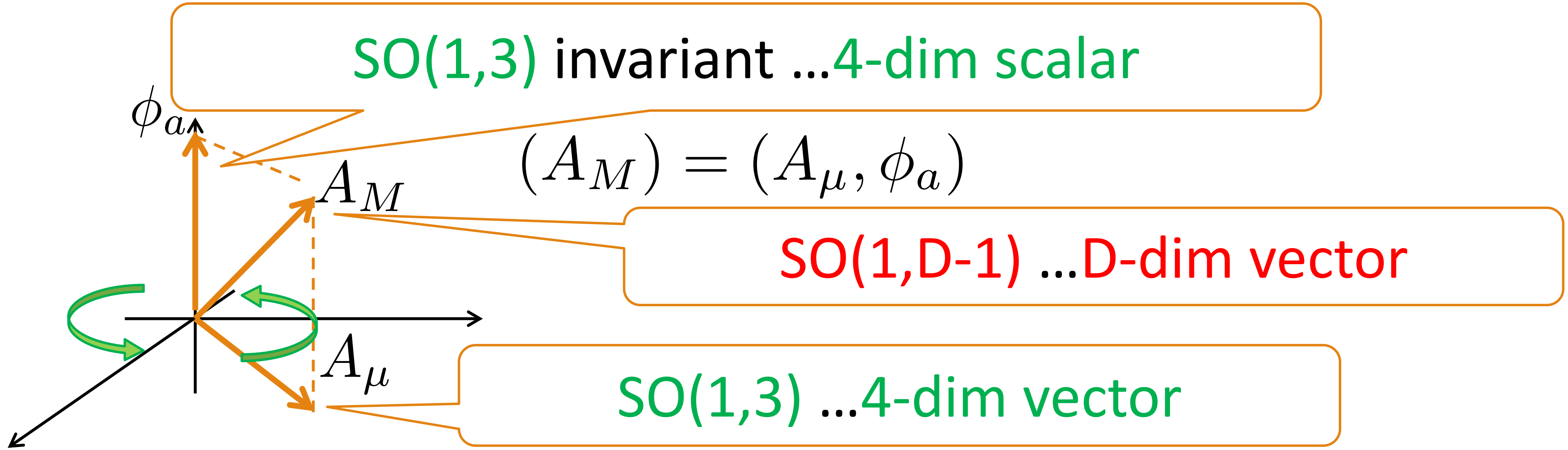
- Where does the Higgs come from?
  - No answer in the Standard model
    - Profile of the potential
    - Yukawa interactions
  - The origin of Higgs? ⇒ Beyond SM!!

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$(1, 2)\left(\tfrac{1}{2}\right)$$

4-dim scalar is the extra-dim components of D-dim gauge field

- <u>Gauge-Higgs Unification</u>
  - Higgs is unified with gauge field



$$(A_M) = (A_\mu, \phi_a)$$

Getting 4-dim theories from D-dim

- (naive) Dimensional Reduction
  - Mode expansion for extra-dim directions
  - Take only zero modes in low energy
- Exm) spacetime = $\mathbf{R}^{1,3} \times S^1$ (compactified to a circle) case

$$\Phi(x^\mu, y) = \Phi_0(x) + \sum_{n\neq 0} \Phi_n(x) e^{iny}$$

Zero mode

Kaluza-Klein modes (heavy)

  - Equivalent to requiring D-dim fields to be independent of extra coordinates

$$\Phi(x, y) = \Phi_0(x)$$

A special reduction scheme

- Assume the spacetime structure as:

$$M^D = \mathbb{R}^{1,3} \times S/R$$

"has the isometry group S"

D-dim Manifold

4-dim Minkowski

d=D-4dim compact extra space

- S/R: a coset space of a compact Lie group S by its subgroup R
- **Coset Space Dimensional Reduction (CSDR) is possible**

P. Forgacs and N. S. Manton, Commun. Math. Phys. 72 (1980), 15-35.
D. Kapetanakis and G. Zoupanos, Phys. Rept. 219(1992), 1 -76.

Outline

CSDR
= spacetime ansatz × symmetry conditions

Requirements for the dependence of fields on extra-coordinates

- Naive (ordinary) reduction
  - Drop y-dependence of fields

$$\Phi(x,y) = \Phi(x)$$

- If extra space has isometry S
  - "Fields must be invariant under S"

$$\Phi^s(x,y) = \Phi(x,y)$$

N.S. Manton, Annals of Physics. 167 (1986), 328-353.

"Symmetry conditions"

- Gauge theories (CSDR)
  - "Must be invariant up to gauge trf."

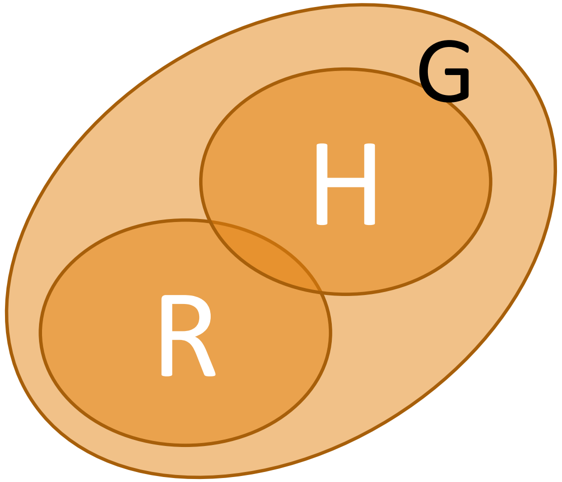
$$\Phi^s(x,y) = \Phi^g(x,y)$$

P. Forgacs and N. S. Manton, Commun. Math. Phys. 72 (1980), 15-35.

⇒ **Determines the gauge group and the field contents of the 4-dim theory**

Symmetry conditions ⇒ 4-dim theory

- R, the subgroup of isometry group S, is also a subgroup of the D-dim gauge group G
- H, the 4-dim gauge group (⊂G) commutes with R in G (centralizer $C_G(R)$)



- 4-dim gauge reps of the fields
  - Exm) 4-dim scalar from S/R vector A_α

$$SO(d) \supset R,$$

$$\boldsymbol{v} = \left(\bigoplus_i \rho_i \right)$$

SO(d) vector

$$G \supset R_G \times H,$$

$$\mathrm{ad}\, G = \bigoplus_j (r_j, h_j)$$

$$= (\mathrm{ad}\, R, 1) \oplus (1, \mathrm{ad}\, H) \oplus \dots$$

ρ_i and r_j are equivalent R reps

↓

A scalar in h_j rep of H appears in 4-dim

The Lagrangian of 4-dim theory

$$A^D \equiv \int d^4x\, d^{D-4}y \sqrt{-\tilde{g}} \left(-\frac{1}{8}\mathrm{Tr}(F_{MN}F^{MN}) + \frac{1}{2} i\bar{\psi}\Gamma^M D_M \psi \right)$$

D次元計量

≡ Yang-Mills sector ≡ Fermion sector

$$\overset{\text{CSDR}}{=} \int d^4x\, \mathcal{L}^4,$$

$$\mathcal{L}^4 = \mathrm{Vol}(S/R)$$

$$\times \left[-\frac{1}{4} F^s_{\mu\nu} F^{\mu\nu s} + \frac{1}{2} (D_\mu \phi_a)^s (D^\mu \phi^a)^s + V(\phi) + \frac{1}{2} i\bar{\psi}\gamma^\mu D_\mu \psi + \frac{1}{2} i\bar{\psi}\Gamma^a D_a \psi \right]$$

Gauge kinetic, Higgs kinetic, potential, Fermion kinetic, Yukawa

$$V(\phi) = -\frac{1}{8}\mathrm{Tr}\left[(f_{ab}{}^C \phi_C - [\phi_a, \phi_b])(f^{abD}\phi_D - [\phi^a, \phi^b]) \right]$$

Inputs and outputs of Coset Space Dimensional Reduction

INPUTS

- Spacetime dim D(extra dim d)
- Extra space S/R
- D-dim gauge group G
- Embedding $R \subset G$
- G rep F of D-dim spinor

CSDR

OUTPUTS

- 4-dim gauge group $H = C_G(R)$
- H reps of surviving scalars
- H reps of surviving fermions

Outline

CSDR works

- What we want in 4-dim: a chiral theory

… left- and right-handed fermions behave differently under the gauge group

- Conditions to be achieved
  - D=4n+2：real reps of G(D-dim gauge group) is allowed
    - D=6,10,14：previous studies

[T. Jittoh, M. Koike, T. Nomura, J. Sato and T. Shimomura, Prog. Theor. Phys., 120(2008), 1041-1063.]

  - D=4n：real reps of G is NOT allowed
    - Complex reps. In D-dim：had been less interesting… but
    - D=8：recent work … possibility of getting promising models??

[T. Jittoh, M. Koike, T. Nomura, J. Sato and Y. Toyama, Physics Letters B, 675(2009), 450-454.]

What about D=12 (d=8) ?… ⇒ Try it

Requirements for inputs

- For a chiral theory in 4-dim
  - Rank R = Rank S [R. Bott, Princeton University Press(1965), 167-186.]
  - D=4n ⇒ G, R must accommodate complex reps
    Weyl fermions in D-dim (take 32 of SO(1,11))

- As ansatz
  - Only U(1) factor in R
    - ➢U(1) in R→U(1) in H：we want to avoid too many U(1)s
  - G reps of 12-dim fermion F： dim F $\lesssim$ 1000

⇨ **Search for H=SU(3)×SU(2)×U(1), SU(5), SO(10), E_6, and their U(1) extensions**

8-dim cosets of interest

S/R	vector under R	spinors under R
$SU(5)/SU(4)\times U(1)$	$\mathbf{8}_v = \mathbf{4}(1)+\overline{\mathbf{4}}(-1)$	$\mathbf{8}_c = \mathbf{4}(-1)+\overline{\mathbf{4}}(1)$ $\mathbf{8}_s = \mathbf{6}(0)+\mathbf{1}(2)+\mathbf{1}(-2)$
$SU(4)/SU(2)\times SU(2)\times U(1)$	$\mathbf{8}_v = (\mathbf{2},\mathbf{2})(1)+(\mathbf{2},\mathbf{2})(-1)$	$\mathbf{8}_c = (\mathbf{3},\mathbf{1})(0)+(\mathbf{1},\mathbf{3})(0)+(\mathbf{1},\mathbf{1})(2)+(\mathbf{1},\mathbf{1})(-2)$ $\mathbf{8}_s = (\mathbf{2},\mathbf{2})(1)+(\mathbf{2},\mathbf{2})(-1)$
$SO(7)/SO(6)$ $\times$ $SU(2)/U(1)$	$\mathbf{8}_v = \mathbf{6}(0)+\mathbf{1}(2)+\mathbf{1}(-2)$	$\mathbf{8}_c = \mathbf{4}(1)+\overline{\mathbf{4}}(-1)$ $\mathbf{8}_s = \mathbf{4}(-1)+\overline{\mathbf{4}}(1)$
$G_2/SU(3)$ $\times$ $SU(2)/U(1)$	$\mathbf{8}_v = \mathbf{3}(0)+\overline{\mathbf{3}}(0)+\mathbf{1}(2)+\mathbf{1}(-2)$	$\mathbf{8}_c = \mathbf{3}(-1)+\overline{\mathbf{3}}(1)+\mathbf{1}(-1)+\mathbf{1}(1)$ $\mathbf{8}_s = \mathbf{3}(1)+\overline{\mathbf{3}}(-1)+\mathbf{1}(1)+\mathbf{1}(-1)$
$Sp(4)/SU(2)\times SU(2)$ $\times$ $SU(3)/SU(2)\times U(1)$	$\mathbf{8}_v = (\mathbf{2},\mathbf{1},\mathbf{1})(2)+(\mathbf{1},\mathbf{2},\mathbf{2})(0)+(\mathbf{2},\mathbf{1},\mathbf{1})(-2)$	$\mathbf{8}_c = (\mathbf{1},\mathbf{2},\mathbf{1})(2)+(\mathbf{2},\mathbf{1},\mathbf{2})(0)+(\mathbf{1},\mathbf{2},\mathbf{1})(-2)$ $\mathbf{8}_s = (\mathbf{1},\mathbf{1},\mathbf{2})(2)+(\mathbf{2},\mathbf{2},\mathbf{1})(0)+(\mathbf{1},\mathbf{1},\mathbf{2})(-2)$

12-dim gauge group G

$\hat{S}/R$	G	H	result
$SU(5)/SU(4)\times U(1)$	$SU(9)$	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\times(1)$
	$SU(9)$	$SU(5)\times U(1)$	$\times(2)$
	$SO(18)$	$SO(10)\times U(1)$	✓
$SU(4)/SU(2)\times SU(2)\times U(1)$	$SO(14)$	$SU(5)\times U(1)$	$\times(2)$
$[SO(7)/SO(6)]\times[SU(2)/U(1)]$	$SU(9)$	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\times(3)$
	$SU(9)$	$SU(5)\times U(1)$	$\times(3)$
	$SO(18)$	$SO(10)\times U(1)$	✓
$[G_2/SU(3)]\times[SU(2)/U(1)]$	E_6	$SU(3)\times SU(2)\times U(1)$	$\times(4)$
	$SU(8)$	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\times(1)$
	$SU(8)$	$SU(5)\times U(1)$	$\times(3)$
$[Sp(4)/SU(2)\times SU(2)]\times[SU(3)/SU(2)\times U(1)]$	$SO(14)$	$SU(3)\times SU(2)\times U(1)$	$\times(4)$
	$SO(18)$	$SO(10)\times U(1)$	✓

(1) No generation of SM fermions can be obtained.
(2) If scalars are obtained, fermions are not, and if fermions, no scalars.
(3) No scalars are obtained. (4) U(1) charges of SM fermions are not realized.

"Interesting" models

S/R	G	F	H	scalars	fermions
$\mathrm{SU}(5)/\mathrm{SU}(4)\times\mathrm{U}(1)$	$\mathrm{SO}(18)$	$\mathbf{256}$	$\mathrm{SO}(10)\times\mathrm{U}(1)$	$\mathbf{10}(1)+\overline{\mathbf{10}}(-1)$	$\mathbf{16}(2)+\mathbf{16}(0)+\mathbf{16}(-2)$ $+\mathbf{16}(1)+\mathbf{16}(-1)$
$\mathrm{SO}(7)/\mathrm{SO}(6)\times\mathrm{SU}(2)/\mathrm{U}(1)$	$\mathrm{SO}(18)$	$\mathbf{256}$	$\mathrm{SO}(10)\times\mathrm{U}(1)$	$\mathbf{10}(2)+\mathbf{10}(0)+\mathbf{10}(-2)$	$\mathbf{16}(1)+\mathbf{16}(-1)$ $+\mathbf{16}(1)+\mathbf{16}(-1)$
$\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1)$	$\mathrm{SO}(18)$	$\mathbf{256}$	$\mathrm{SO}(10)\times\mathrm{U}(1)$	$\mathbf{10}(2)+\mathbf{10}(0)+\mathbf{10}(-2)$	$\mathbf{16}(2)+\mathbf{16}(0)+\mathbf{16}(-2)$ $+\mathbf{16}(2)+\mathbf{16}(0)+\mathbf{16}(-2)$

- S/R=SU(5)/SU(4)×U(1), G=SO(18), F=256
  - 4-dim gauge group H=SO(10)×U(1)
  - 3(+2) generations of fermions
  - U(1) charges remind us Lμ-Lτ…??

Contents

Summary

- 12-dim gauge theory (extra space S/R, gauge group G)
+ symmetry conditions = 4-dim gauge theory with scalars

- A model with $S/R=SU(5)/SU(4)\times U(1)$, $G=SO(18)$, $F=256$
leads to $SO(10)\times U(1)$, 3(+2)-gen fermions

Backup Slides

Coset Space Dimensional Reduction (CSDR)

- Start with D-dim(G: gauge group) action A^D

$$A^D = \int d^4x\, d^{D-4}y\, \mathcal{L}^D(x,y)$$

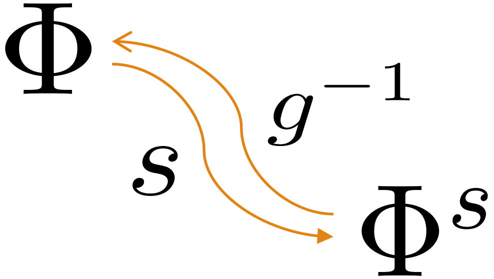
D-dim Lagrangian
x: 4-dim coordinates
y: extra-dim(S/R) coordinates

- A constraint eq. on the fields(next page) then
  - $\mathcal{L}^D$ loses its y dependence → 4-dim Lagrangian
  - 4-dim gauge group H、 reps of the 4-dim fields
- Integrate y → 4-dim action

Constraints: the symmetry conditions

- S transformation is canceled by G-gauge transformation
  - Symbolically

$$\Phi^s(x,y) = \Phi^g(x,y)$$



  - Infinitesimal form(CSDR constraints)

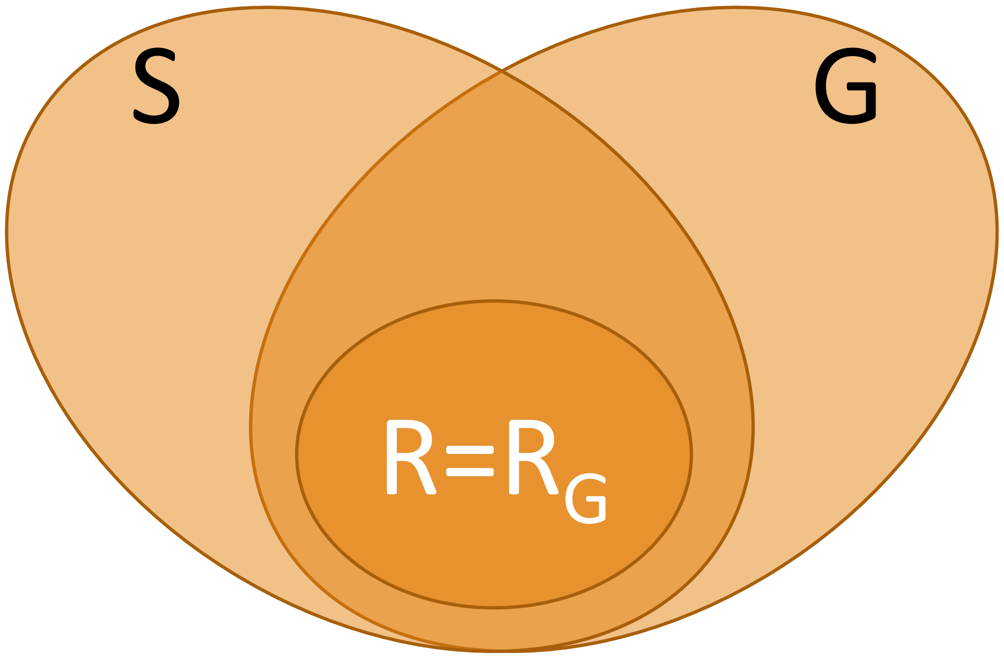
$$\delta_A \Phi = D_\Phi(W_A)\Phi$$

transformation: $y \to y + \epsilon\xi_A$
Corresponding gauge trf: $\exp(W_A(y))$
where $W_A(y) \in \mathrm{Lie}(G)$

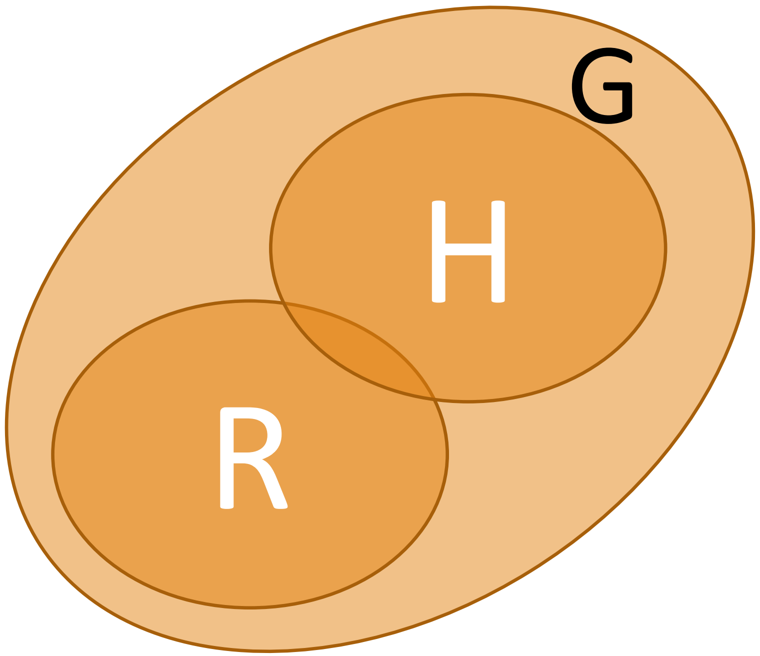
➢ G-invariance of D-dim Lagrangian⇒S invariance
➢ Fields satisfying this condition survive in 4-dim **(≡matching)**

Symmetry condition
⇒4-dim gauge group H

R(⊂S) satisfies R⊂G



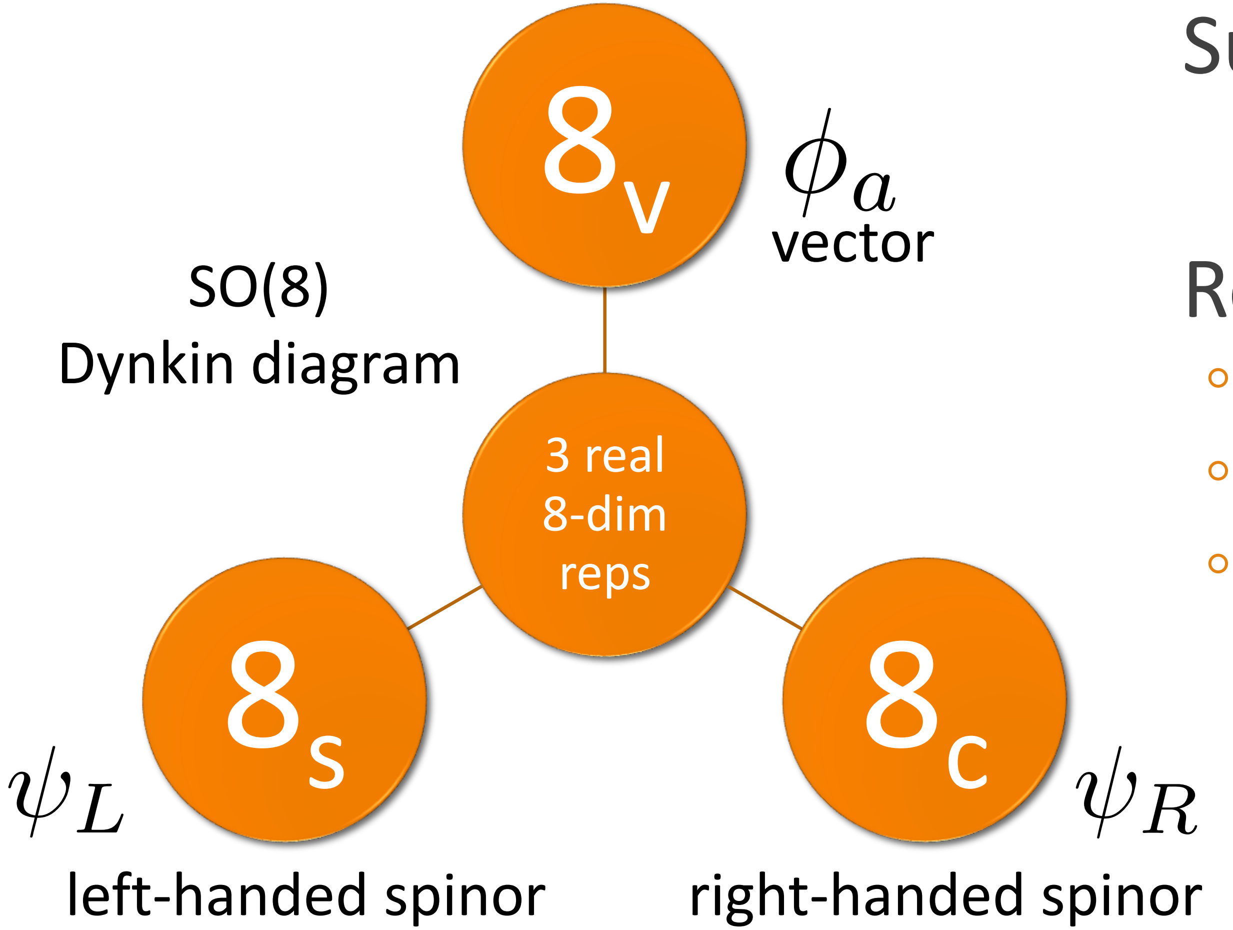
H is the centralizer of R in G



$$\delta\Phi = D_\Phi \Phi$$

R⊂SO(8) R⊂G

Notes on SO(d=8)



Surviving H reps in 4-dim...

Refer to the decompositions

- For scalars : 8_v
- For LH-fermions : 8_s
- For RH-fermions : 8_c

under R and those of G reps under R×H

In this talk：CSDR in 4+8=12 dim

D=4+8 dim spacetime $\mathbf{R}^{1+3} \times S/R$

Gauge theory with the gauge group G

(gauge boson A＋spinor ψ)

Coset Space Dimensional Reduction

4-dim spacetime $\mathbf{R}^{1,3}$

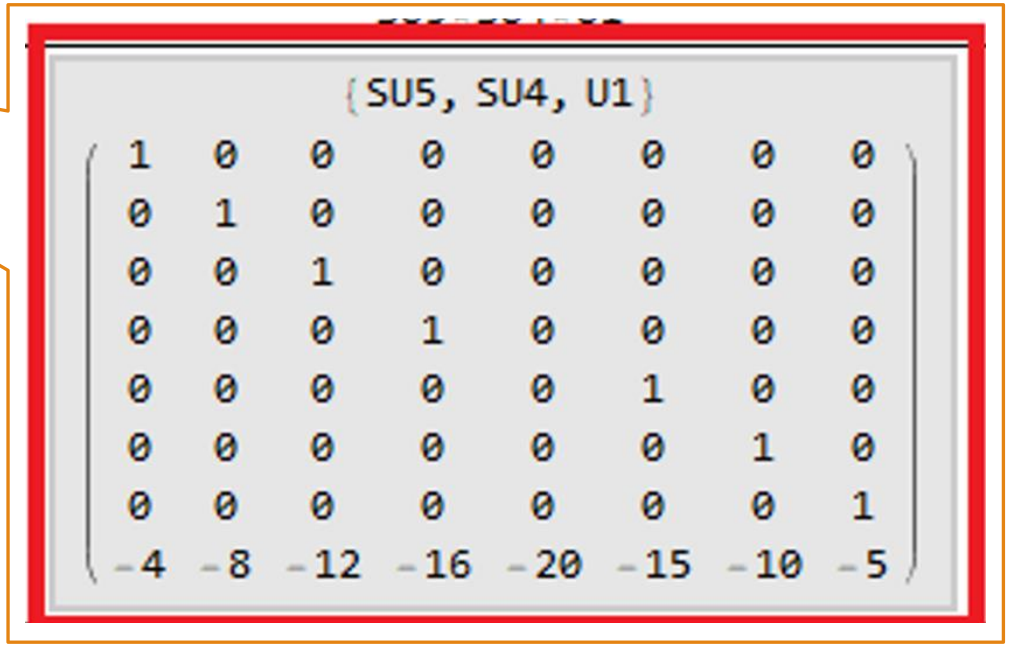
Gauge theory with the gauge group $H(=G_{SM},G_{GUT})$

(gauge boson A＋spinor ψ＋scalar ϕ)

Key points

- What kind of model?
- SM or GUTs with Higgs？
- Field contents？

Embedding R×H⊂G



↑projection matrix…describes the embedding
as weight mapping
S/R=SU(5)/SU(4)U(1), G=SU(9), H=SU(4)U(1)

Investigating G rep F

27	$\{3\otimes1\otimes1\otimes-2,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ 1\otimes3\otimes1\otimes2,\ 1\otimes3\otimes2\otimes-1,\ 3\otimes1\otimes2\otimes1\}$
351	$\{8\otimes3\otimes1\otimes2,\ 1\otimes6\otimes1\otimes2,\ 1\otimes3\otimes3\otimes2,\ 1\otimes3\otimes1\otimes2,\ \bar{6}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{6}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes3\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes2\otimes3,\ 6\otimes1\otimes1\otimes-2,\ 3\otimes8\otimes1\otimes-2,\ 3\otimes1\otimes3\otimes-2,\ 3\otimes1\otimes1\otimes-2,\ 6\otimes1\otimes2\otimes1,\ 3\otimes8\otimes2\otimes1,\ 3\otimes1\otimes2\otimes1,\ 3\otimes1\otimes2\otimes1,\ 3\otimes1\otimes1\otimes4,\ 8\otimes3\otimes2\otimes-1,\ 1\otimes6\otimes2\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ \bar{3}\otimes\bar{3}\otimes2\otimes-3,\ 1\otimes3\otimes1\otimes-4\}$
351'	$\{8\otimes3\otimes1\otimes2,\ 1\otimes6\otimes3\otimes2,\ 1\otimes3\otimes1\otimes2,\ \bar{6}\otimes\bar{6}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes3\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes2\otimes3,\ 6\otimes1\otimes3\otimes-2,\ 3\otimes8\otimes1\otimes-2,\ 3\otimes1\otimes1\otimes-2,\ 6\otimes1\otimes2\otimes1,\ 3\otimes8\otimes2\otimes1,\ 3\otimes1\otimes2\otimes1,\ 6\otimes1\otimes1\otimes4,\ 8\otimes3\otimes2\otimes-1,\ 1\otimes6\otimes2\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ \bar{3}\otimes\bar{3}\otimes2\otimes-3,\ 1\otimes6\otimes1\otimes-4\}$
1728	$\{8\otimes3\otimes1\otimes-4,\ 1\otimes3\otimes3\otimes-4,\ 1\otimes3\otimes1\otimes-4,\ 15\otimes1\otimes1\otimes-2,\ 6\otimes8\otimes1\otimes-2,\ 6\otimes1\otimes1\otimes-2,\ 3\otimes8\otimes3\otimes-2,\ 3\otimes8\otimes1\otimes-2,\ 3\otimes8\otimes1\otimes-2,\ 3\otimes1\otimes3\otimes-2,\ 3\otimes1\otimes3\otimes-2,\ 3\otimes1\otimes1\otimes-2,\ 3\otimes1\otimes1\otimes-2,\ 3\otimes1\otimes1\otimes-2,\ \overline{15}\otimes\bar{3}\otimes1\otimes0,\ \bar{6}\otimes\bar{3}\otimes3\otimes0,\ \bar{6}\otimes\bar{3}\otimes1\otimes0,\ \bar{6}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\overline{15}\otimes1\otimes0,\ \bar{3}\otimes\bar{6}\otimes3\otimes0,\ \bar{3}\otimes\bar{6}\otimes1\otimes0,\ \bar{3}\otimes\bar{6}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes3\otimes0,\ \bar{3}\otimes\bar{3}\otimes3\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{3}\otimes\bar{3}\otimes1\otimes0,\ \bar{6}\otimes\bar{3}\otimes2\otimes-3,\ \bar{3}\otimes\bar{6}\otimes2\otimes-3,\ \bar{3}\otimes\bar{3}\otimes2\otimes-3,\ \bar{3}\otimes\bar{3}\otimes2\otimes-3,\ 8\otimes6\otimes1\otimes2,\ 8\otimes3\otimes3\otimes2,\ 8\otimes3\otimes1\otimes2,\ 8\otimes3\otimes1\otimes2,\ 1\otimes15\otimes1\otimes2,\ 1\otimes6\otimes1\otimes2,\ 1\otimes3\otimes3\otimes2,\ 1\otimes3\otimes3\otimes2,\ 1\otimes3\otimes1\otimes2,\ 1\otimes3\otimes1\otimes2,\ 1\otimes3\otimes1\otimes2,\ 8\otimes6\otimes2\otimes-1,\ 8\otimes3\otimes2\otimes-1,\ 8\otimes3\otimes2\otimes-1,\ 8\otimes3\otimes2\otimes-1,\ 1\otimes15\otimes2\otimes-1,\ 1\otimes6\otimes2\otimes-1,\ 1\otimes3\otimes4\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ 1\otimes3\otimes2\otimes-1,\ 3\otimes1\otimes2\otimes-5,\ 15\otimes1\otimes2\otimes1,\ 6\otimes8\otimes2\otimes1,\ 6\otimes1\otimes2\otimes1,\ 3\otimes8\otimes2\otimes1,\ 3\otimes8\otimes2\otimes1,\ 3\otimes8\otimes2\otimes1,\ 3\otimes1\otimes4\otimes1,\ 3\otimes1\otimes2\otimes1,\ 3\otimes1\otimes2\otimes1,\ 3\otimes1\otimes2\otimes1,\ 3\otimes1\otimes2\otimes1,\ 3\otimes8\otimes1\otimes4,\ 3\otimes1\otimes3\otimes4,\ 3\otimes1\otimes1\otimes4,\ \bar{6}\otimes\bar{3}\otimes2\otimes3,\ \bar{3}\otimes\bar{6}\otimes2\otimes3,\ \bar{3}\otimes\bar{3}\otimes2\otimes3,\ \bar{3}\otimes\bar{3}\otimes2\otimes3,\ 1\otimes3\otimes2\otimes5\}$

S/R = G2/SU(3) × SU(2)/U(1), G=E6, R=U(3)U(1), H=SU(3)SU(2)U(1),
R×H decompositions of F=27,351,351’(,1728) do not reproduce 5 kinds of U(1) charges (which the SM has), so these cases are excluded.

U(1) linear combinations

1-rep

990-rep

(SU(4),SU(3),SU(2),U(1),U(1)) ⇐R and H

80	$\{4\otimes 1\otimes 2\otimes (3\,(3x+y))\otimes (-3\sqrt{\tfrac{3}{2}}\,n\,(x-2y)),$
	$1\otimes 8\otimes 1\otimes 0\otimes 0,\ 1\otimes 1\otimes 3\otimes 0\otimes 0,\ 1\otimes 1\otimes 1\otimes 0\otimes 0,\ 1$
	$1\otimes \bar{3}\otimes 2\otimes (-9x+4y)\otimes (-\sqrt{6}\,n\,(2x+3y)),\ \bar{4}$

8_v
$4(1)+\bar{4}(-1)$

S/R=SU(5)/SU(4)U(1), G=SU(9), H=SU(3)SU(2)U(1)U(1)

Two U(1)'s ⇒ choice of embedding from linear combinations

Set coefficient x so that scalar survives in 4-dim, and for each F we search for some y which realize surviving fermions

Symmetry condition
$\Rightarrow \mathcal{L}^D$ is invariant under S trf.

- Sym. Cond.(☆) and gauge inv.(♡) give

$$\mathcal{L}^{D^s}(x,y) \overset{☆}{=} \mathcal{L}^{D^g}(x,y) \overset{♡}{=} \underline{\mathcal{L}^D(x,y)}$$

- Meanwhile, L^D is defined as scalar under S:

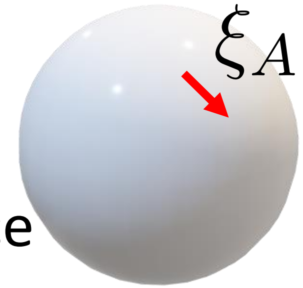
$$\mathcal{L}^{D^s}(x,y) = \underline{\mathcal{L}^D(x,y^{s^{-1}})}$$

- ∴ for an arbitrary S trf.

$$\mathcal{L}^D(x,y^s) = \mathcal{L}^D(x,y)$$

Translations on S/R

$y \to y + \epsilon\xi_A \quad (\forall \epsilon > 0)$ Inv. → no y-dependence



Future developments

- Relaxing the ansatzes
  - More U(1) factors in R
  - Diverse dimensions of F the rep of D-dim fermions
  - Hard work doing manually ⇒ automation!

- More phenomenological discussions
  - Analysis of scalar potentials … symmetry breaking
  - $L_\mu - L_\tau$ symmetry ?
  - Mass hierarchy of 3 generations ?