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S_4 flavor model with 3HDM

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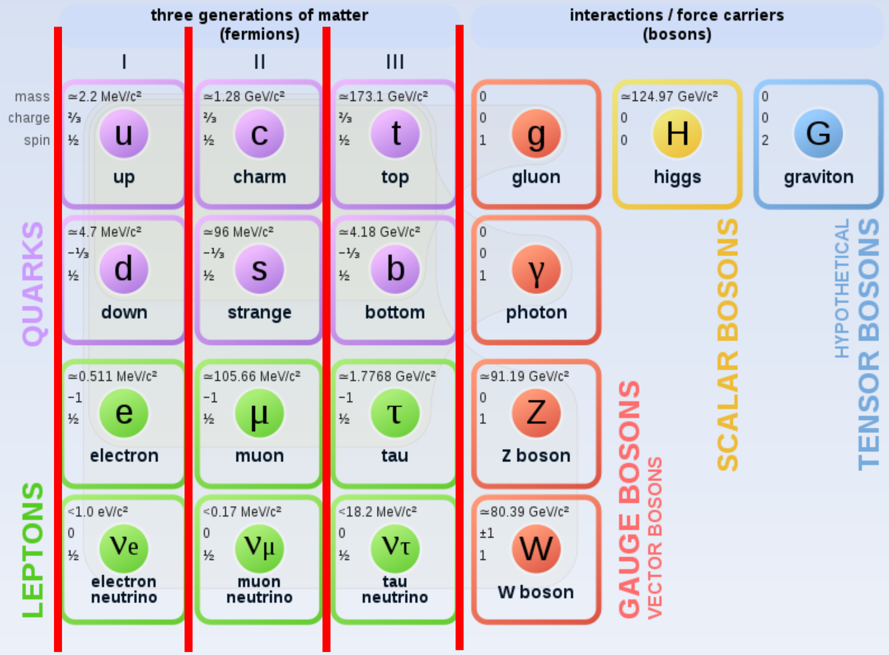
1. Introduction

Fundamental forces
(strong, weak, electromagnetic and gravitational interactions)

Standard model(SM)
$SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry

Quarks and leptons (SM particles)
→ generation structure
(mass differences and flavor mixing)
Especially leptons → large flavor mixing

Standard Model of Elementary Particles and Gravity



https://www.wikiwand.com/

It can't be explained in SM. (SM only assign parameter.)

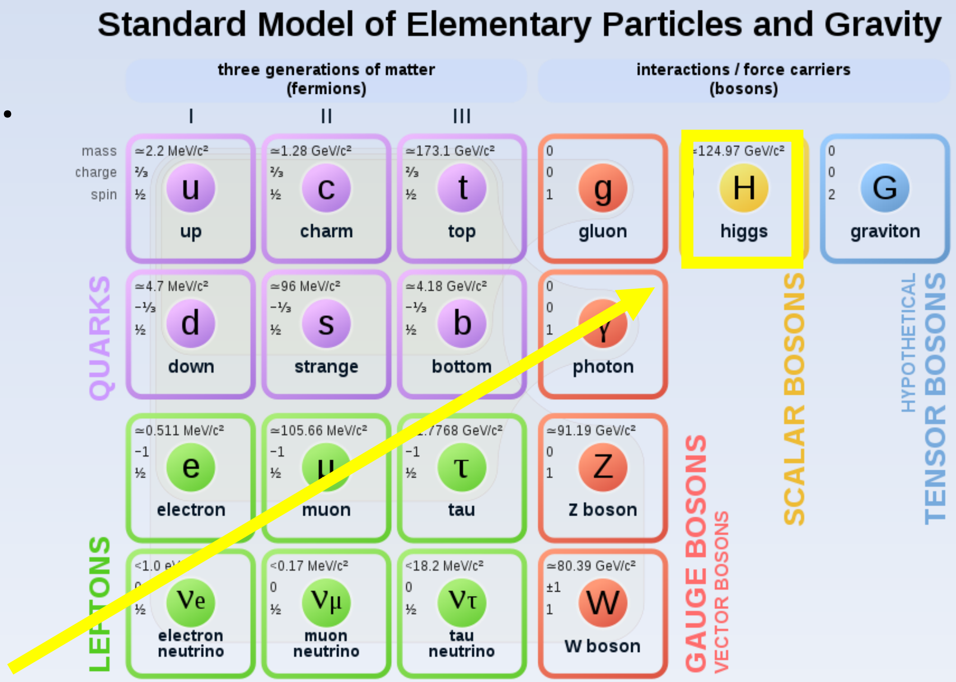
1. Introduction

Altareli and Feruglio impose discrete symmetry(flavor symmetry) among generations.

G. Altarelli and F. Feruglio, Nucl. Phys. B741 (2006), 215-235.

In this study, we choose S_4 symmetry as flavor symmetry.

In addition, we suppose three Higgs doublets model(3HDM).



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We built new flavor model and perform the analysis.

2. S_4 symmetry

S_4 symmetry：Fourth order symmetric group

$(x_1, x_2, x_3, x_4) \rightarrow (x_i, x_j, x_k, x_l)$ $4! = 24$ elements

Representation $1, 1', \underline{2}, 3, 3'$

A_4 symmetry：Fourth order alternating group

Representation $1, 1', 1'', 3_S, 3_A$ 12 elements

Multiplication rule

$$3 \times 3 = 1 + 2 + 3 + 3'$$
$$3' \times 3' = 1 + 2 + 3 + 3'$$
$$3 \times 3' = 1' + 2 + 3 + 3'$$
$$2 \times 2 = 1 + 1' + 2$$
$$2 \times 3 = 3 + 3'$$
$$2 \times 3' = 3 + 3'$$
$$3 \times 1' = 3'$$
$$3' \times 1' = 3$$
$$2 \times 1' = 2$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}_2 \otimes \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}_2 = (\alpha_1\beta_1 + \alpha_2\beta_2)_1 \oplus (-\alpha_1\beta_2 + \alpha_2\beta_1)_{1'} \oplus \begin{pmatrix}\alpha_1\beta_2 + \alpha_2\beta_1\\ \alpha_1\beta_1 - \alpha_2\beta_2\end{pmatrix}_2$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}_3 \otimes \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}_3 = (\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3)_1 \oplus \begin{pmatrix}\frac{1}{\sqrt{2}}(\alpha_2\beta_2 - \alpha_3\beta_3)\\ \frac{1}{\sqrt{6}}(-2\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3)\end{pmatrix}_2$$

$$\oplus \begin{pmatrix}\alpha_3\beta_2 + \alpha_2\beta_3\\ \alpha_1\beta_3 + \alpha_3\beta_1\\ \alpha_2\beta_1 + \alpha_1\beta_2\end{pmatrix}_3 \oplus \begin{pmatrix}\alpha_3\beta_2 - \alpha_2\beta_3\\ \alpha_1\beta_3 - \alpha_3\beta_1\\ \alpha_2\beta_1 - \alpha_1\beta_2\end{pmatrix}_{3'}$$

3. 3HDM

Extend SM Higgs doublet to 3 (12 real scalar fields)

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \phi_3 = \begin{pmatrix} \phi_3^+ \\ \phi_3^0 \end{pmatrix}$$

Higgs Potential in 3HDM under $SU(2)_L \otimes U(1)_Y$

$$V = -\sum_{i,j=1}^{3} m_{ij}^2 (\phi_i^\dagger \phi_j) + \frac{1}{2} \sum_{i,j,k,l=1}^{3} \lambda_{ijkl} (\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_l)$$

Potential minimum conditions

$$\left(\frac{\partial V}{\partial \phi_1}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

$$\left(\frac{\partial V}{\partial \phi_2}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

$$\left(\frac{\partial V}{\partial \phi_3}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

⇩ Spontaneous symmetry breaking

3 degrees of freedom are eaten by W and Z bosons.

⇨ ϕ is represented by the expansion of 9 (=12−3) real scalar fields

$$\phi_i = \begin{pmatrix} \rho_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\chi_i) \end{pmatrix}, i = 1,2,3$$

mass eigenstates ⇨

(ｉ) Three CP-even scalar fields

(ii) Two CP-odd scalar fields

(iii) Four charged scalar fields

4. Flavor model

	$\bar{l}=(\bar{l_e},\bar{l_\mu},\bar{l_\tau})$	$l_R=(e_R,\mu_R)$	τ_R	ν_{eR}	$\nu_R=(\nu_{\mu R}\ ,\nu_{\tau R}\ )$	$\phi=(\phi_1,\phi_2\ ,\phi_3\ )$	X	Θ
$SU(2)_L$	2	1	1	1	1	2	1	1
S_4	3	2	1	1	2	3	2	1
$U(1)_{FN}$	0	+1	0	0	0	0	−1	−1

Lagrangian： $-L_Y = L_l + L_D + L_M + h.c.$

(1) Mass terms of charged leptons： $L_l = \frac{y_{e\mu}}{\Lambda}\bar{l}\phi l_R\Theta + y_\tau\bar{l}\phi\tau_R + \frac{y_l}{\Lambda}\bar{l}\phi l_R X$

(2) Mass term of Dirac neutrino： $L_D = y_{De}\bar{l}\tilde{\phi}\nu_{eR} + y_{D\mu\tau}\bar{l}\tilde{\phi}\nu_R$

(3) Mass term of right-handed Majorana neutrino： $L_M = \frac{1}{2}M_{eR}\bar{\nu}^c_{eR}\nu_{eR} + \frac{1}{2}M_{\mu\tau R}\bar{\nu}^c_R\nu_R$

➡ Calculate mass matrices of charged leptons and left-handed Majorana neutrino

Calculation of mass matrices

(1) Mass terms of charged leptons

$$L_l = \boxed{\frac{y_{e\mu}}{\Lambda}\bar{l}\phi l_R \Theta} + y_\tau \bar{l}\phi\tau_R + \frac{y_l}{\Lambda}\bar{l}\phi l_R X$$

$$\frac{y_l}{\Lambda}\underbrace{\begin{pmatrix}\bar{l}_e\\ \bar{l}_\mu\\ \bar{l}_\tau\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3}_{2} \otimes \begin{pmatrix}e_R\\ \mu_R\end{pmatrix}_2 \Theta = \frac{y_{e\mu}}{\Lambda}\begin{pmatrix}\frac{1}{\sqrt{2}}(\bar{l}_\mu\phi_2 - \bar{l}_\tau\phi_3)\\ \frac{1}{\sqrt{6}}(-2\bar{l}_e\phi_1 + \bar{l}_\mu\phi_2 + \bar{l}_\tau\phi_3)\end{pmatrix}_2 \otimes \begin{pmatrix}e_R\\ \mu_R\end{pmatrix}_2 \Theta$$

$$= \frac{y_{e\mu}}{\Lambda}\frac{1}{\sqrt{2}}(\bar{l}_\mu\phi_2 - \bar{l}_\tau\phi_3)e_R\Theta + \frac{y_{e\mu}}{\Lambda}\frac{1}{\sqrt{6}}(-2\bar{l}_e\phi_1 + \bar{l}_\mu\phi_2 + \bar{l}_\tau\phi_3)\mu_R\Theta$$

$$\Downarrow \quad \boxed{< \Theta > = \Theta_0, < \phi >= (v_1, v_2, v_3)}$$

$$= \frac{y_{e\mu}\Theta_0}{\Lambda}\frac{1}{\sqrt{2}}(\bar{\mu}_L v_2 - \bar{\tau}_L v_3)e_R + \frac{y_{e\mu}\Theta_0}{\Lambda}\frac{1}{\sqrt{6}}(-2\bar{e}_L v_1 + \bar{\mu}_L v_2 + \bar{\tau}_L v_3)\mu_R$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}_3 \otimes \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}_3 = \begin{pmatrix}\frac{1}{\sqrt{2}}(\alpha_2\beta_2 - \alpha_3\beta_3)\\ \frac{1}{\sqrt{6}}(-2\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3)\end{pmatrix}_2$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}_2 \otimes \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}_2 = (\alpha_1\beta_1 + \alpha_2\beta_2)_1$$

Calculation of mass matrices

(1) Mass terms of charged leptons

$$L_l = \frac{y_{e\mu}}{\Lambda}\bar{l}\phi l_R \Theta + \boxed{y_\tau \bar{l}\phi\tau_R} + \frac{y_l}{\Lambda}\bar{l}\phi l_R X$$

$$\boxed{\begin{gathered} y_\tau \underbrace{\begin{pmatrix} \bar{l}_e \\ \bar{l}_\mu \\ \bar{l}_\tau \end{pmatrix}_3 \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}_3}_{1} \otimes \tau_R = y_\tau(\bar{l}_e\phi_1 + \bar{l}_\mu\phi_2 + \bar{l}_\tau\phi_3)\tau_R \\ \Big\downarrow \quad < \phi >= (v_1, v_2, v_3) \\ = y_\tau(\bar{e}_L v_1 \tau_R + \bar{\mu}_L v_2 \tau_R + \bar{\tau}_L v_3 \tau_R) \end{gathered}}$$

Mass matrix of charged leptons consisting of one and two terms

$$M_{l1} = \begin{pmatrix} 0 & -\frac{2y_{e\mu}\Theta_0}{\sqrt{6}\Lambda}v_1 & y_\tau v_1 \\ \frac{y_{e\mu}\Theta_0}{\sqrt{2}\Lambda}v_2 & \frac{y_{e\mu}\Theta_0}{\sqrt{6}\Lambda}v_2 & y_\tau v_2 \\ -\frac{y_{e\mu}\Theta_0}{\sqrt{2}\Lambda}v_3 & \frac{y_{e\mu}\Theta_0}{\sqrt{6}\Lambda}v_3 & y_\tau v_3 \end{pmatrix}_{LR}$$

Calculation of mass matrices

(1) Mass terms of charged leptons

$$L_l = \frac{y_{e\mu}}{\Lambda}\bar{l}\phi l_R\Theta + y_\tau\bar{l}\phi\tau_R + \boxed{\frac{y_l}{\Lambda}\bar{l}\phi l_R X}$$

$$\frac{y_l}{\Lambda}\underbrace{\begin{pmatrix}\bar{l}_e\\ \bar{l}_\mu\\ \bar{l}_\tau\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3}_{1,2} \otimes \underbrace{\begin{pmatrix}e_R\\ \mu_R\end{pmatrix}_2 \otimes \begin{pmatrix}X_1\\ X_2\end{pmatrix}_2}_{1,2}$$

$$= \frac{y_{l1}}{\Lambda}\left(\bar{l}_e\phi_1 + \bar{l}_\mu\phi_2 + \bar{l}_\tau\phi_3\right)_1 \otimes \left(e_RX_1 + \mu_RX_2\right)_1 + \frac{y_{l2}}{\Lambda}\begin{pmatrix}\frac{1}{\sqrt{2}}\left(\bar{l}_\mu\phi_2 - \bar{l}_\tau\phi_3\right)\\ \frac{1}{\sqrt{6}}\left(-2\bar{l}_e\phi_1 + \bar{l}_\mu\phi_2 + \bar{l}_\tau\phi_3\right)\end{pmatrix}_2 \otimes \begin{pmatrix}e_RX_2 + \mu_RX_1\\ e_RX_1 - \mu_RX_2\end{pmatrix}_2$$

$$= \frac{y_{l1}}{\Lambda}\left(\bar{l}_e\phi_1e_RX_1 + \bar{l}_e\phi_1\mu_RX_2 + \bar{l}_\mu\phi_2e_RX_1 + \bar{l}_\mu\phi_2\mu_RX_2 + \bar{l}_\tau\phi_3e_RX_1 + \bar{l}_\tau\phi_3\mu_RX_2\right)$$

$$+ \frac{y_{l2}}{\Lambda}\left[\frac{1}{\sqrt{2}}\left(\bar{l}_\mu\phi_2e_RX_2 + \bar{l}_\mu\phi_2\mu_RX_1 - \bar{l}_\tau\phi_3e_RX_2 - \bar{l}_\tau\phi_3\mu_RX_1\right)\right.$$

$$\left. + \frac{1}{\sqrt{6}}\left(-2\bar{l}_e\phi_1e_RX_1 + 2\bar{l}_e\phi_1\mu_RX_2 + \bar{l}_\mu\phi_2e_RX_1 - \bar{l}_\mu\phi_2\mu_RX_2 + \bar{l}_\tau\phi_3e_RX_1 - \bar{l}_\tau\phi_3\mu_RX_2\right)\right]$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}_3 \otimes \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}_3 = \begin{pmatrix}\frac{1}{\sqrt{2}}(\alpha_2\beta_2 - \alpha_3\beta_3)\\ \frac{1}{\sqrt{6}}(-2\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3)\end{pmatrix}_2$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}_2 \otimes \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}_2 = (\alpha_1\beta_1 + \alpha_2\beta_2)_1 \oplus \begin{pmatrix}\alpha_1\beta_2 + \alpha_2\beta_1\\ \alpha_1\beta_1 - \alpha_2\beta_2\end{pmatrix}_2$$

Calculation of mass matrices

(1) Mass terms of charged leptons

$$L_l = \frac{y_{e\mu}}{\Lambda}\bar{l}\phi l_R \Theta + y_\tau \bar{l}\phi\tau_R + \boxed{\frac{y_l}{\Lambda}\bar{l}\phi l_R X}$$

$$< \phi >= (v_1, v_2, v_3), \boxed{< X > = (\mathrm{X}_1, 0)}$$

Mass matrix of charged leptons
consisting of third term

$$M_{l2} = \begin{pmatrix} \left(\frac{y_{l1}}{\Lambda} - \frac{2y_{l2}}{\sqrt{6}\Lambda}\right) v_1 \mathrm{X}_1 & 0 & 0 \\ \left(\frac{y_{l1}}{\Lambda} + \frac{y_{l2}}{\sqrt{6}\Lambda}\right) v_2 \mathrm{X}_1 & \frac{y_{l2}}{\sqrt{2}\Lambda} v_2 \mathrm{X}_1 & 0 \\ \left(\frac{y_{l1}}{\Lambda} - \frac{y_{l2}}{\sqrt{6}\Lambda}\right) v_3 \mathrm{X}_1 & -\frac{y_{l2}}{\sqrt{2}\Lambda} v_3 \mathrm{X}_1 & 0 \end{pmatrix}_{LR}$$

Mass matrix of charged leptons

$$M_l = M_{l1} + M_{l2} = \begin{pmatrix} 0 & -\frac{2y_{e\mu}\Theta_0}{\sqrt{6}\Lambda} v_1 & y_\tau v_1 \\ \frac{y_{e\mu}\Theta_0}{\sqrt{2}\Lambda} v_2 & \frac{y_{e\mu}\Theta_0}{\sqrt{6}\Lambda} v_2 & y_\tau v_2 \\ -\frac{y_{e\mu}\Theta_0}{\sqrt{2}\Lambda} v_3 & \frac{y_{e\mu}\Theta_0}{\sqrt{6}\Lambda} v_3 & y_\tau v_3 \end{pmatrix}_{LR} + \begin{pmatrix} \left(\frac{y_{l1}}{\Lambda} - \frac{2y_{l2}}{\sqrt{6}\Lambda}\right) v_1 \mathrm{X}_1 & 0 & 0 \\ \left(\frac{y_{l1}}{\Lambda} + \frac{y_{l2}}{\sqrt{6}\Lambda}\right) v_2 \mathrm{X}_1 & \frac{y_{l2}}{\sqrt{2}\Lambda} v_2 \mathrm{X}_1 & 0 \\ \left(\frac{y_{l1}}{\Lambda} - \frac{y_{l2}}{\sqrt{6}\Lambda}\right) v_3 \mathrm{X}_1 & -\frac{y_{l2}}{\sqrt{2}\Lambda} v_3 \mathrm{X}_1 & 0 \end{pmatrix}_{LR}$$

Calculation of mass matrices

(2) Mass terms of Dirac neutrino

$L_D = y_{De}\bar{l}\tilde{\phi}\nu_{eR} + y_{D\mu\tau}\bar{l}\tilde{\phi}\nu_R$ Mass matrix of Dirac neutrino $M_D = \begin{pmatrix} y_{De}v_1 & 0 & -2/\sqrt{6}y_{D\mu\tau}v_1 \\ y_{De}v_2 & 1/\sqrt{2}y_{D\mu\tau}v_2 & 1/\sqrt{6}y_{D\mu\tau}v_2 \\ y_{De}v_3 & -1/\sqrt{2}y_{D\mu\tau}v_3 & 1/\sqrt{6}y_{D\mu\tau}v_3 \end{pmatrix}_{LR}$

(3) Mass terms of Majorana neutrino

$L_M = \frac{1}{2}M_{eR}\bar{\nu}_{eR}^c\nu_{eR} + \frac{1}{2}M_{\mu\tau R}\bar{\nu}_R^c\nu_R$ Mass matrix of Majorana neutrino $M_R = \begin{pmatrix} M_{eR} & 0 & 0 \\ 0 & M_{\mu\tau R} & 0 \\ 0 & 0 & M_{\mu\tau R} \end{pmatrix}$

By using type-1 seesaw mechanism, Mass matrix of neutrino $m_\nu = -M_D M_R^{-1} M_D^T$

$$m_\nu = \begin{pmatrix} -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_1^2}{M_{eR}} - \frac{2\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_1^2}{3\mathrm{M}_{\mu\tau R}} & -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_1v_2}{M_{eR}} + \frac{\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_1v_2}{3\mathrm{M}_{\mu\tau R}} & -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_3v_1}{M_{eR}} + \frac{\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_3v_1}{3\mathrm{M}_{\mu\tau R}} \\ -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_1v_2}{M_{eR}} + \frac{\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_1v_2}{3\mathrm{M}_{\mu\tau R}} & -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_2^2}{M_{eR}} - \frac{2\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_2^2}{3\mathrm{M}_{\mu\tau R}} & -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_2v_3}{M_{eR}} + \frac{\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_2v_3}{3\mathrm{M}_{\mu\tau R}} \\ -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_3v_1}{M_{eR}} + \frac{\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_3v_1}{3\mathrm{M}_{\mu\tau R}} & -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_2v_3}{M_{eR}} + \frac{\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_2v_3}{3\mathrm{M}_{\mu\tau R}} & -\frac{e^{2i\phi_{y_{De}}}y_{De}^2v_3^2}{M_{eR}} - \frac{2\mathrm{e}^{2i\phi_{y_{\mu\tau}}}y_{D\mu\tau}^2v_3^2}{3\mathrm{M}_{\mu\tau R}} \end{pmatrix}$$

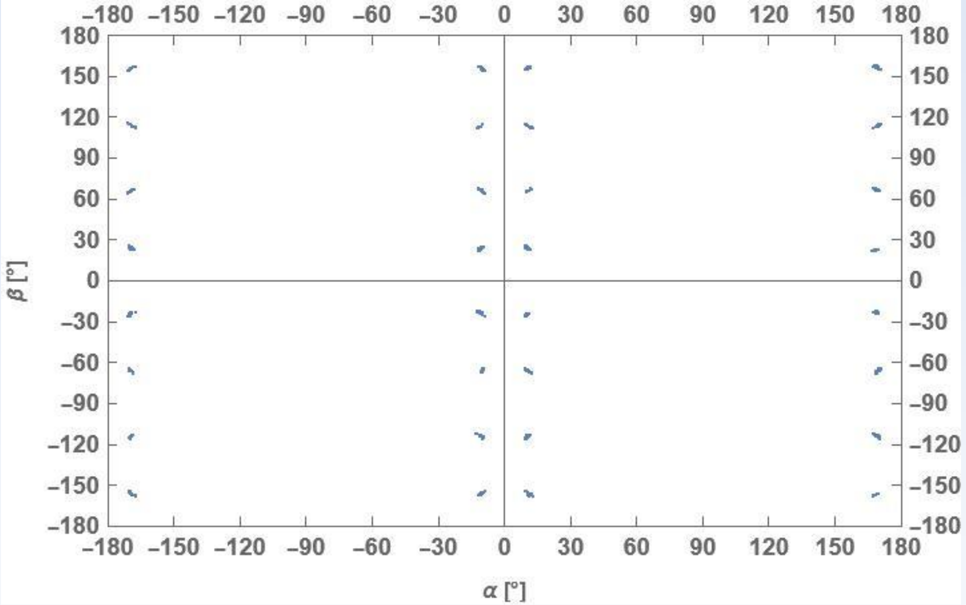
5. Numerical calculation

Satisfy $m_e, m_\mu, m_\tau, \Delta m_{21}^2, \Delta m_{31}^2, \sin^2\theta_{12}, \sin^2\theta_{23}, \sin^2\theta_{13}$

Parameter $\alpha, \beta, y_{e\mu}, y_\tau, y_{l1}, y_{l2}, X_1, m_1, y_{De}, y_{D\mu\tau}, \phi_{y_{De}}$

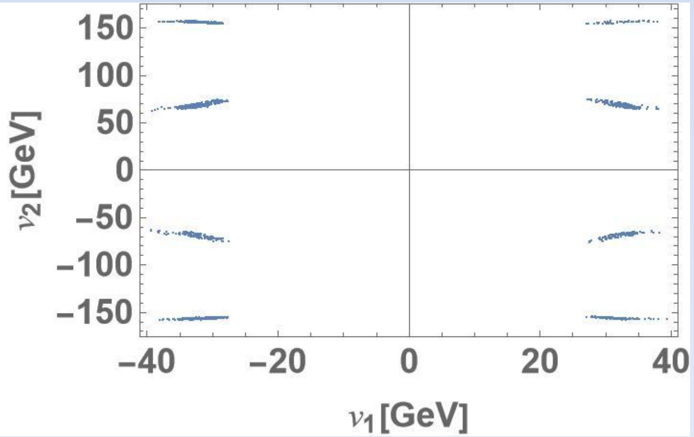
Prediction $\sin^2\theta_{12}, \sin^2\theta_{23}, \sin^2\theta_{13}, \delta_{CP}, m_{light}, m_1 + m_2 + m_3,$
m_{ee}, η_1, η_2

The relation between α and β

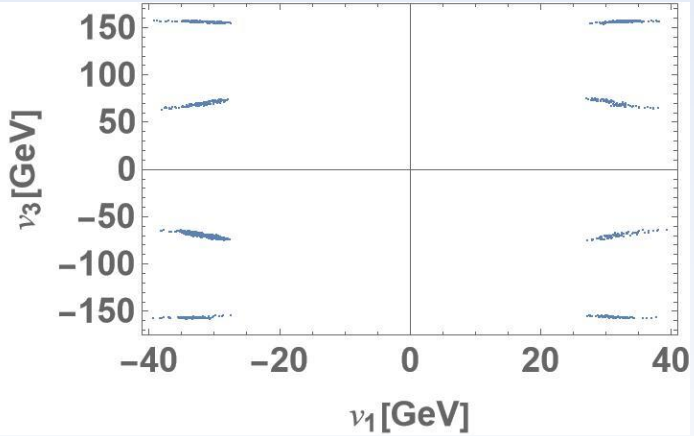


$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v\sin\alpha \\ v\cos\alpha\sin\beta \\ v\cos\alpha\cos\beta \end{pmatrix}$$

The relation between v_1 and v_2

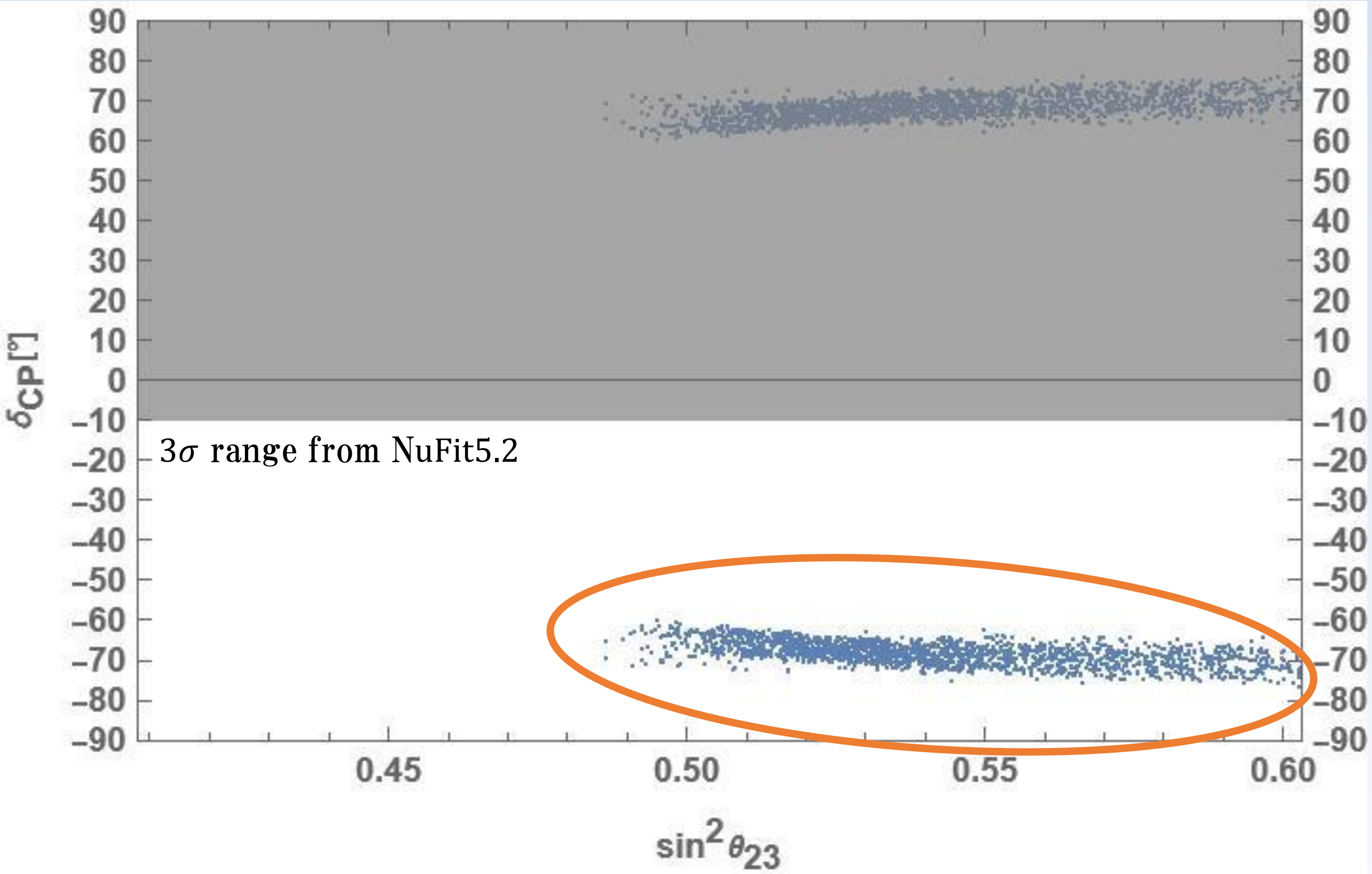


The relation between v_1 and v_3



Numerical calculation

Prediction of δ_{CP} and $\sin^2\theta_{23}$



NuFIT 5.2
$0.408 \leq \sin^2\theta_{23} \leq 0.603$

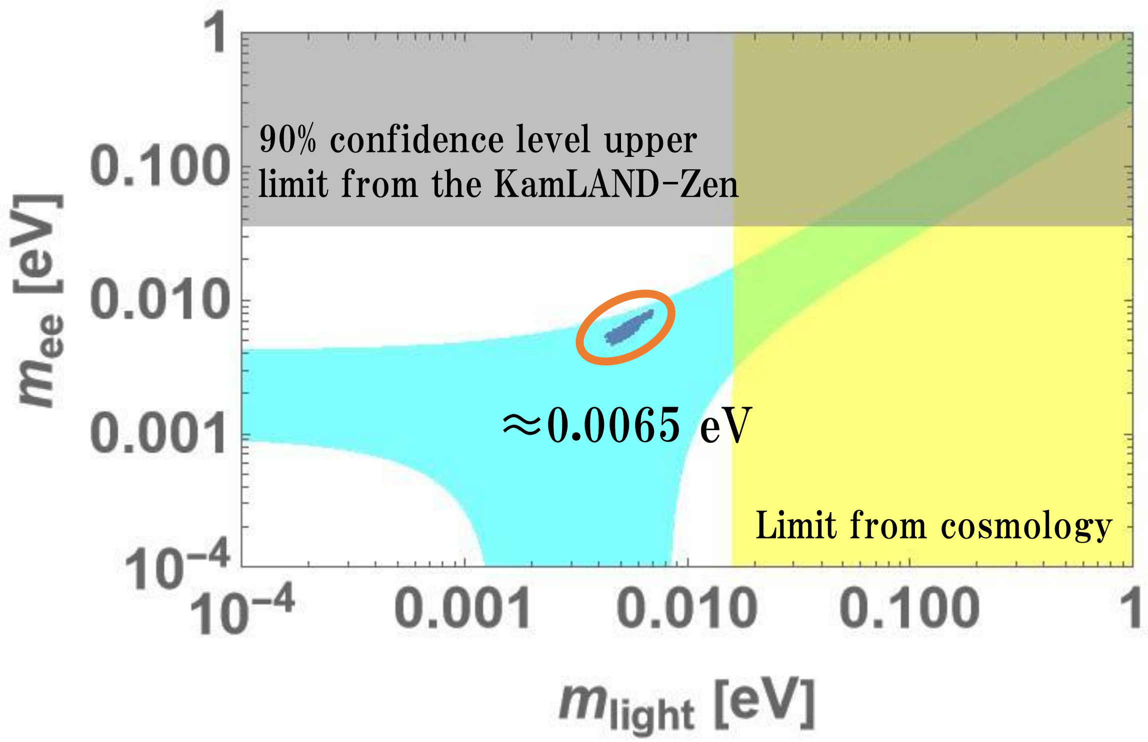
$0.488 \leq \sin^2\theta_{23} \leq 0.603$

$\delta_{CP} \approx -67.7°$

Strong prediction of δ_{CP}

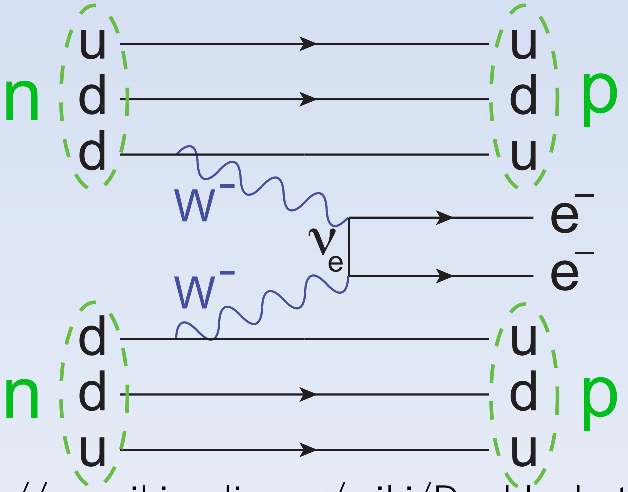
Numerical calculation

Prediction of the effective neutrino mass m_{ee} in the $0\nu\beta\beta$ decay experiment and the lightest neutrino mass m_{light}



Our model can be confirmed in the near future.

$0\nu\beta\beta$ decay



https://en.wikipedia.org/wiki/Double_beta_decay

Decay rate

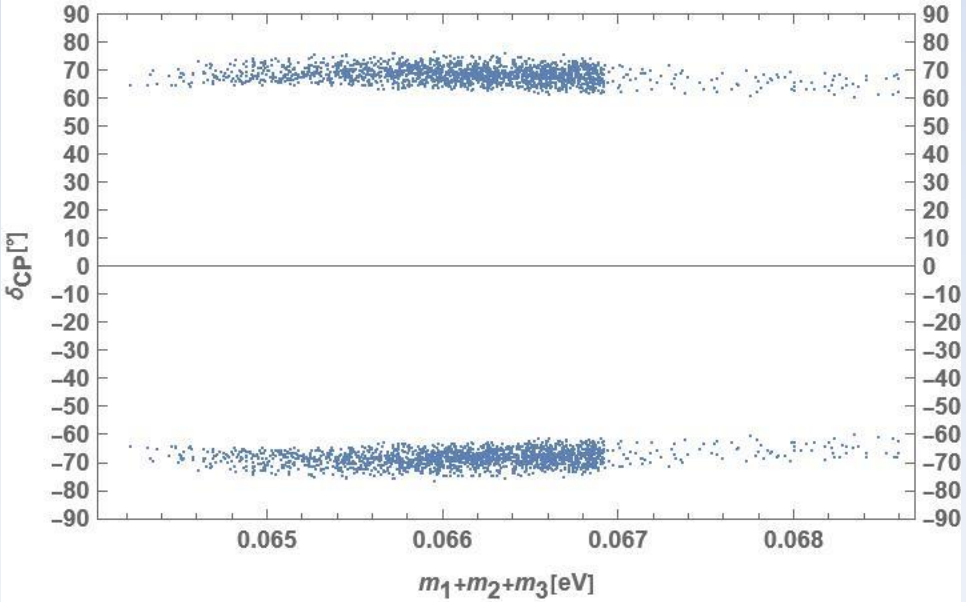
$$\Gamma \propto m_{ee}^2$$

Effective mass of electron neutrino

$$m_{ee} = \left|m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2\right|$$

Numerical calculation

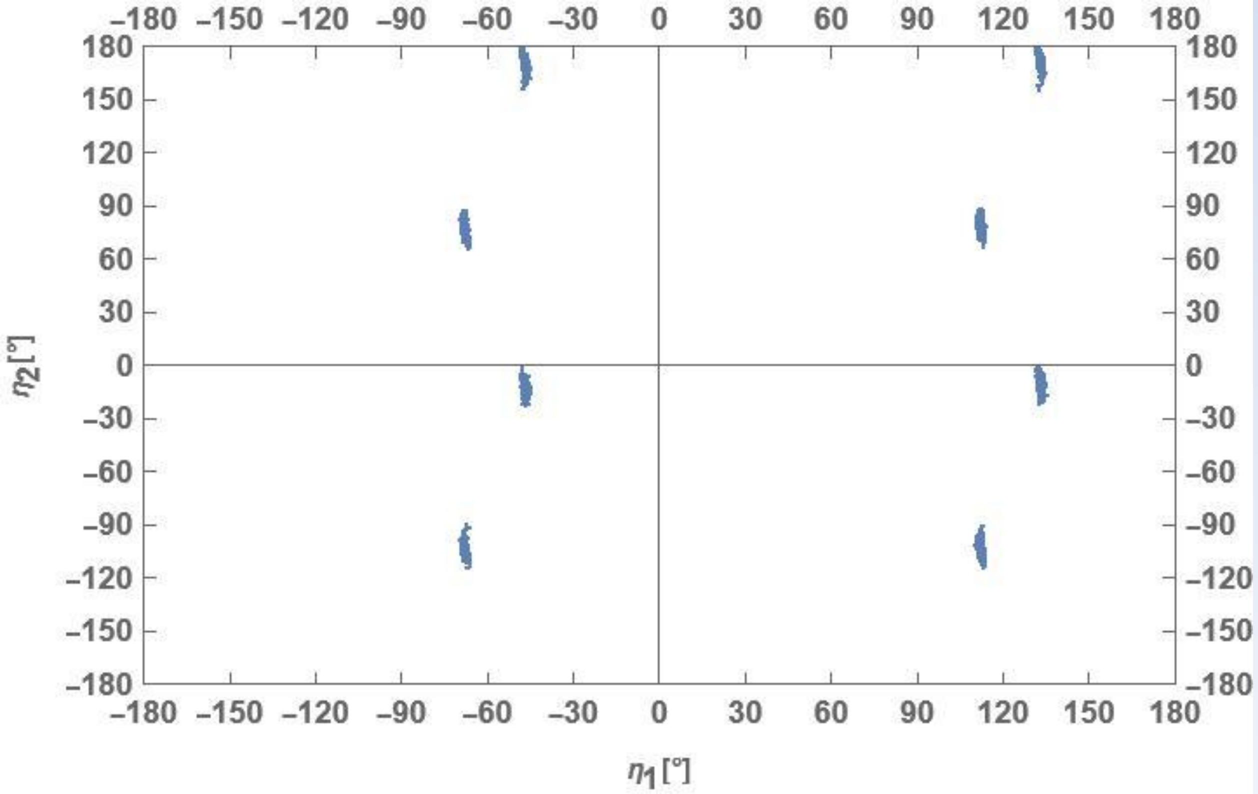
Prediction of sum of neutrino mass
$m_1 + m_2 + m_3$ and δ_{CP}



Limit from cosmology
(arXiv:1807.06209)
$m_1 + m_2 + m_3 \leq 0.12$eV

Numerical calculation

Prediction of Majorana phases η_1, η_2



$$\eta_1 = \arg\left[\frac{U_{e1}U_{e3}^*}{\cos\theta_{12}\cos\theta_{13}\sin\theta_{13}e^{i\delta_{CP}}}\right], \quad \eta_2 = \arg\left[\frac{U_{e2}U_{e3}^*}{\sin\theta_{12}\cos\theta_{13}\sin\theta_{13}e^{i\delta_{CP}}}\right]$$

6. Potential analysis

Check the Higgs VEV $< \phi >= (v_1, v_2, v_3)$

3HDM + S_4symmetry

Consider ϕ as S_4 triplet $\phi = (\phi_1, \phi_2, \phi_3)$

Calculate Higgs potential $V = -\mu^2 \underline{\phi^\dagger \phi} + \lambda \underline{\left(\phi^\dagger \phi\right)^2} + c \underline{\phi^\dagger \phi X^\dagger X} + k \underline{\phi^\dagger \phi \Theta^\dagger \Theta} + \left(g \underline{\phi^\dagger \phi X^\dagger \Theta} + h.c.\right)$

$$\boxed{\phi^\dagger \phi} = \begin{pmatrix} \phi_1^\dagger \\ \phi_2^\dagger \\ \phi_3^\dagger \end{pmatrix}_3 \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}_3 = \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3\right)_1 = \boxed{|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2}$$

Potential analysis

Calculate Higgs potential $V = -\mu^2\underline{\phi^\dagger\phi} + \lambda\underline{(\phi^\dagger\phi)^2} + c\underline{\phi^\dagger\phi X^\dagger X} + k\underline{\phi^\dagger\phi\Theta^\dagger\Theta} + (g\underline{\phi^\dagger\phi\Theta^\dagger X} + h.c.)$

$$\boxed{(\phi^\dagger\phi)^2} = \underbrace{\underline{\begin{pmatrix}\phi_1^\dagger\\ \phi_2^\dagger\\ \phi_3^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3}}_{1,2,3,3'} \otimes \underbrace{\underline{\begin{pmatrix}\phi_1^\dagger\\ \phi_2^\dagger\\ \phi_3^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3}}_{1,2,3,3'} = (|\phi_1|^2+|\phi_2|^2+|\phi_3|^2)^2 \oplus \underline{\begin{pmatrix}\frac{1}{\sqrt{2}}(\phi_2^\dagger\phi_2-\phi_3^\dagger\phi_3)\\ \frac{1}{\sqrt{6}}(-2\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3)\end{pmatrix}_2} \otimes \underline{\begin{pmatrix}\frac{1}{\sqrt{2}}(\phi_2^\dagger\phi_2-\phi_3^\dagger\phi_3)\\ \frac{1}{\sqrt{6}}(-2\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3)\end{pmatrix}_2}$$

$$\oplus \underline{\begin{pmatrix}\phi_3^\dagger\phi_2+\phi_2^\dagger\phi_3\\ \phi_1^\dagger\phi_3+\phi_3^\dagger\phi_1\\ \phi_2^\dagger\phi_1+\phi_1^\dagger\phi_2\end{pmatrix}_3} \otimes \underline{\begin{pmatrix}\phi_3^\dagger\phi_2+\phi_2^\dagger\phi_3\\ \phi_1^\dagger\phi_3+\phi_3^\dagger\phi_1\\ \phi_2^\dagger\phi_1+\phi_1^\dagger\phi_2\end{pmatrix}_3} \oplus \underline{\begin{pmatrix}\phi_3^\dagger\phi_2-\phi_2^\dagger\phi_3\\ \phi_1^\dagger\phi_3-\phi_3^\dagger\phi_1\\ \phi_2^\dagger\phi_1-\phi_1^\dagger\phi_2\end{pmatrix}_{3'}} \otimes \underline{\begin{pmatrix}\phi_3^\dagger\phi_2-\phi_2^\dagger\phi_3\\ \phi_1^\dagger\phi_3-\phi_3^\dagger\phi_1\\ \phi_2^\dagger\phi_1-\phi_1^\dagger\phi_2\end{pmatrix}_{3'}}$$

$$= \boxed{\begin{array}{l} (|\phi_1|^2+|\phi_2|^2+|\phi_3|^2)^2 \\ \oplus\frac{2}{3}(|\phi_1|^4+|\phi_2|^4+|\phi_3|^4-|\phi_1|^2|\phi_2|^2-|\phi_2|^2|\phi_3|^2-|\phi_3|^2|\phi_1|^2) \\ \oplus\left[(\phi_1^\dagger\phi_2)^2+(\phi_2^\dagger\phi_3)^2+(\phi_3^\dagger\phi_1)^2+|\phi_1|^2|\phi_2|^2+|\phi_2|^2|\phi_3|^2+|\phi_3|^2|\phi_1|^2+h.c.\right] \\ \oplus\left[(\phi_1^\dagger\phi_2)^2+(\phi_2^\dagger\phi_3)^2+(\phi_3^\dagger\phi_1)^2-|\phi_1|^2|\phi_2|^2-|\phi_2|^2|\phi_3|^2-|\phi_3|^2|\phi_1|^2+h.c.\right] \end{array}}$$

Potential analysis

Calculate Higgs potential $V = -\mu^2\underline{\phi^\dagger\phi} + \lambda\underline{\left(\phi^\dagger\phi\right)^2} + c\underline{\phi^\dagger\phi X^\dagger X} + k\underline{\phi^\dagger\phi\Theta^\dagger\Theta} + \left(g\underline{\phi^\dagger\phi\Theta^\dagger X} + h.c.\right)$

$$\boxed{\phi^\dagger\phi X^\dagger X} = \underbrace{\begin{pmatrix}\phi_1^\dagger\\ \phi_2^\dagger\\ \phi_3^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3}_{1,2} \otimes \underbrace{\begin{pmatrix}X_1^\dagger\\ X_2^\dagger\end{pmatrix}_2 \otimes \begin{pmatrix}X_1\\ X_2\end{pmatrix}_2}_{1,2} = \left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right)_1 \otimes \left(X_1^\dagger X_1 + X_2^\dagger X_2\right)_1 + \begin{pmatrix}\frac{1}{\sqrt{2}}\left(\phi_2^\dagger\phi_2 - \phi_3^\dagger\phi_3\right)\\ \frac{1}{\sqrt{6}}\left(-2\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right)\end{pmatrix}_2 \otimes \begin{pmatrix}X_1^\dagger X_2 + X_2^\dagger X_1\\ X_1^\dagger X_1 - X_2^\dagger X_2\end{pmatrix}_2$$

$$< X > = (\mathrm{X}_1, 0)$$

$$= \boxed{|\mathrm{X}_1|^2\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2\right) + \frac{|\mathrm{X}_1|^2}{\sqrt{6}}\left(-2|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2\right)}$$

$$\boxed{\phi^\dagger\phi\Theta^\dagger\Theta} = \begin{pmatrix}\phi_1^\dagger\\ \phi_2^\dagger\\ \phi_3^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3 \Theta^\dagger\Theta = \boxed{|\Theta_0|^2\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2\right)}$$

$$< \Theta > = \Theta_0$$

$$\boxed{\phi^\dagger\phi\Theta^\dagger X} = \underbrace{\begin{pmatrix}\phi_1^\dagger\\ \phi_2^\dagger\\ \phi_3^\dagger\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}_3}_{2} \otimes \begin{pmatrix}X_1\\ X_2\end{pmatrix}_2 \Theta^\dagger = \begin{pmatrix}\frac{1}{\sqrt{2}}\left(\phi_2^\dagger\phi_2 - \phi_3^\dagger\phi_3\right)\\ \frac{1}{\sqrt{6}}\left(-2\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right)\end{pmatrix}_2 \otimes \begin{pmatrix}X_1\\ X_2\end{pmatrix}_2 \Theta^\dagger = \boxed{\frac{\Theta_0^*\mathrm{X}_1}{\sqrt{2}}\left(|\phi_2|^2 - |\phi_3|^2\right)}$$

$$< X > = (\mathrm{X}_1, 0), < \Theta > = \Theta_0$$

Vacuum structure

Potential

$$V = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2 + c\phi^\dagger\phi X^\dagger X + k\phi^\dagger\phi\Theta^\dagger\Theta + \left(g\phi^\dagger\phi\Theta^\dagger X + h.c.\right)$$
$$= \frac{\Lambda_1}{2}(|\phi_1|^4 + |\phi_2|^4 + |\phi_3|^4) + \Lambda_2(|\phi_1|^2|\phi_2|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_3|^2|\phi_1|^2)$$
$$+ \frac{\Lambda_3}{2}\left[\left(\phi_1^\dagger\phi_2\right)^2 + \left(\phi_2^\dagger\phi_3\right)^2 + \left(\phi_3^\dagger\phi_1\right)^2 + h.c.\right]$$
$$+ \left(-\mu^2 + c_1\mathrm{X}_1^2 + \frac{2c_2\mathrm{X}_1^2}{\sqrt{6}}\right)|\phi_1|^2 + \left(-\mu^2 + c_1\mathrm{X}_1^2 + \frac{c_2\mathrm{X}_1^2}{\sqrt{6}} + \frac{g\mathrm{X}_1\Theta_0}{\sqrt{2}}\right)|\phi_2|^2 + \left(-\mu^2 + c_1\mathrm{X}_1^2 + \frac{c_2\mathrm{X}_1^2}{\sqrt{6}} - \frac{g\mathrm{X}_1\Theta_0}{\sqrt{2}}\right)|\phi_3|^2$$

Potential minimum conditions

$$\left(\frac{\partial V}{\partial\phi_i}\right)_{\phi_1=v_1,\phi_2=v_2,\phi_3=v_3} = 0, \quad i = 1,2,3$$

vacuum expectation values

$$v_1 = \pm\sqrt{\frac{\mu'^2}{\Lambda_1 + 2\Lambda'} - \frac{2\Lambda_1 c_2{}'}{(\Lambda_1 + 2\Lambda')(\Lambda_1 - \Lambda')}}$$
$$v_2 = \pm\sqrt{\frac{\mu'^2}{\Lambda_1 + 2\Lambda'} - \frac{\Lambda_1 - 2\Lambda'}{(\Lambda_1 + 2\Lambda')(\Lambda_1 - \Lambda')}c'_2 - \frac{g'}{\Lambda_1 - \Lambda'}}$$
$$v_3 = \pm\sqrt{\frac{\mu'^2}{\Lambda_1 + 2\Lambda'} - \frac{\Lambda_1 - 2\Lambda'}{(\Lambda_1 + 2\Lambda')(\Lambda_1 - \Lambda')}c'_2 + \frac{g'}{\Lambda_1 - \Lambda'}}$$

Rewrite VEV with
v and α, β

$$\langle\phi\rangle = \begin{pmatrix} v\sin\alpha \\ v\cos\alpha\sin\beta \\ v\cos\alpha\cos\beta \end{pmatrix}$$

v : Higgs VEV
α, β : free parameter

7. Summary

We consider S_4 symmetry as flavor symmetry.
We consider Higgs field ϕ as S_4 triplet.

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We build new flavor model by using 3HDM and S_4 symmetry.
We calculate mass matrices of charged leptons and neutrinos under new flavor model.

Mass matrix of charged leptons

$$M_l = M_{l1} + M_{l2}$$

$$= \begin{pmatrix} 0 & -\frac{2y_{e\mu}\Theta_0}{\sqrt{6}\Lambda}v_1 & y_\tau v_1 \\ \frac{y_{e\mu}\Theta_0}{\sqrt{2}\Lambda}v_2 & \frac{y_{e\mu}\Theta_0}{\sqrt{6}\Lambda}v_2 & y_\tau v_2 \\ -\frac{y_{e\mu}\Theta_0}{\sqrt{2}\Lambda}v_3 & \frac{y_{e\mu}\Theta_0}{\sqrt{6}\Lambda}v_3 & y_\tau v_3 \end{pmatrix}_{LR} + \begin{pmatrix} \left(\frac{y_{l1}}{\Lambda} - \frac{2y_{l2}}{\sqrt{6}\Lambda}\right)v_1\mathrm{X}_1 & 0 & 0 \\ \left(\frac{y_{l1}}{\Lambda} + \frac{y_{l2}}{\sqrt{6}\Lambda}\right)v_2\mathrm{X}_1 & \frac{y_{l2}}{\sqrt{2}\Lambda}v_2\mathrm{X}_1 & 0 \\ \left(\frac{y_{l1}}{\Lambda} - \frac{y_{l2}}{\sqrt{6}\Lambda}\right)v_3\mathrm{X}_1 & -\frac{y_{l2}}{\sqrt{2}\Lambda}v_3\mathrm{X}_1 & 0 \end{pmatrix}_{LR}$$

Mass matrix of left-handed Majorana neutrinos

$$m_\nu = -M_D M_R^{-1} M_D^\dagger$$

$$M_D = \begin{pmatrix} y_{De}v_1 & 0 & -2/\sqrt{6}y_{D\mu\tau}v_1 \\ y_{De}v_2 & 1/\sqrt{2}y_{D\mu\tau}v_2 & 1/\sqrt{6}y_{D\mu\tau}v_2 \\ y_{De}v_3 & -1/\sqrt{2}y_{D\mu\tau}v_3 & 1/\sqrt{6}y_{D\mu\tau}v_3 \end{pmatrix}_{LR} \quad M_R = \begin{pmatrix} M_{eR} & 0 & 0 \\ 0 & M_{\mu\tau R} & 0 \\ 0 & 0 & M_{\mu\tau R} \end{pmatrix}$$

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We perform numerical analysis and calculate δ_{CP} , effective mass m_{ee} and Majorana phases η_1, η_2.
We obtain strong predictions of δ_{CP} and m_{ee} ($m_{ee} \approx 0.0065[\mathrm{eV}]$).
→This flavor model can be confirmed by neutrino experiments in the near future.
Finally, we analysis the Higgs potential and we obtain the Higgs VEV $< \phi > = (v_1, v_2, v_3)$.