Phenomenological aspecte of Nonlinear-Supersymmetric General Relativity (NLSGR)

- Unification of space, time and matter -

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OUTLINE

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1. Motivation

@ The success of Two SMs, i.e. GR and GWS model,

many unsolved fundamental problems are left, e. g.,

- Unification of two SMs,
- Space-time dimension *four*,
- Three-generations structure of quarks and leptons,
- Tiny Neutrino mass M_ν, proton stability in GUT
- Dark Matter, Dark Enegy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda CDM$, Inflation?

$\Longrightarrow$ SUSY, SUGRA!?,

- Origin of SUSY breaking ?, $\cdots$ etc.

@ GR describes geometry of space-time.

However, unpleasant differences between GR and SUGRA:

- GR $\Longleftrightarrow$ Geometry of Riemann space-time(Physical:[x^μ], GL(4,R))
- SUGRA $\Longleftrightarrow$ Geometry of superspace (Mathematical:[x^μ, θ_α], sPoicaré)

$\Longrightarrow$ New SUSY paradigm on physical space-time!.

@ As for the three-generations structure we found:

Among all SO(N) sP, [$N = 8$ by M. Gell-Mann]

only SO(10) sP is unique which gives **SM with just 3 generations in one irreducible rep.**

- 10 supercharges $Q^I, (I = 1, 2, \cdots .10)$ are decomposed as follows:

$$\underline{\mathbf{10}}_{SO(10)} = \underline{\mathbf{5}}_{SU(5)} + \underline{\mathbf{5}}^*{}_{SU(5)}$$
$$\underline{\mathbf{5}}_{SU(5)} = [\{\underline{\mathbf{3}}^{*c}, \underline{\mathbf{1}}^{ew}, (\tfrac{e}{3}, \tfrac{e}{3}, \tfrac{e}{3}) : Q_a(a = 1, 2, 3)\}, \ \{\underline{\mathbf{1}}^c, \underline{\mathbf{2}}^{ew}, (0, -e) : Q_m(m = 4, 5)\}].$$

$\Leftrightarrow$ **Supercharge** $\underline{5}_{SU(5)}$ has the same quantum numbers as $\underline{5}$ of SU(5) GUT.

- **Massless helicity states of gravity multiplet of SO(10) sP with CPT conjugation** are specified by the helicity $h = (2 - \frac{n}{2})$ and the dimension $\underline{d}_{[n]} = \underline{\frac{10!}{n!(10-n)!}}$:

$$|h >= Q^n Q^{n-1} \cdots Q^2 Q^1 |2 >, \ \ Q^n \ \ (n = a, m, a^*, m^*)\text{: supercharge}$$

$\|h\|$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
$\underline{d}_{[n]}$	$\underline{1}_{[10]}$	$\underline{10}_{[9]}$	$\underline{1}_{[0]}$ $\underline{45}_{[8]}$	$\underline{10}_{[1]}$ $\underline{120}_{[7]}$	$\underline{45}_{[2]}$ $\underline{210}_{[6]}$	$\underline{120}_{[3]}$ $\underline{252}_{[5]}$	$\underline{210}_{[4]}$ $\underline{210}_{[4]}$

@ Dirac particle survivors after *tentative* superHiggs-like mechanism

$SU(3)$	Q_e	$SU(2)\otimes U(1)$
1	0 -1 -2	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$ (N) (E)
3	$5/3$ $2/3$ $-1/3$ $-4/3$	$\begin{pmatrix}u\\ d\end{pmatrix}\begin{pmatrix}c\\ s\end{pmatrix}\begin{pmatrix}t\\ b\end{pmatrix}\begin{pmatrix}h\\ o\end{pmatrix}\begin{pmatrix}a\\ f\end{pmatrix}\begin{pmatrix}g\\ m\end{pmatrix}\begin{pmatrix}r\\ i\\ n\end{pmatrix}$
6	$4/3$ $1/3$ $-2/3$	$\begin{pmatrix}P\\ Q\\ R\end{pmatrix}\begin{pmatrix}X\\ Y\\ Z\end{pmatrix}$
8	0 -1	$\begin{pmatrix}N_1\\ E_1\end{pmatrix}\begin{pmatrix}N_2\\ E_2\end{pmatrix}$

@ SM Higgs-doublet state survives in $h=0$ state.

• $N > 10$ case is unphysical/ugly and excluded.

@ How to construct N=10 SUSY with gravity beyond the No-Go theorem in S-matrix ?

• To circumvent the No-Go theorem degeneracy of space-time is considered.

$$\Downarrow$$

We show in this talk:

• **N=10 SUSY with gravity** is obtained by the geometric description of General Relativity principle on **specific unstable *physical* (Riemann) space-time** whose **tangent space possesses global NLSUSY structure**.

A quick review of NLSUSY:

- Take (flat) space-time specified by x^a for SO(1,3) and ψ_α for SL(2,C).
- Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,

κ is an arbitrary constant with the dimension l^{+2}.

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$

with a global spinor parameter ζ.

- An invariant acction($\sim$ invariant volume) is obtained:

$S = -\frac{1}{2\kappa^2}\int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA}$,

L_{VA} is N=1 Volkov-Akulov model of NLSUSY given by

$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 - t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right]$,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b), \quad t^a{}_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi)$,

which is invariant under N=1 NLSUSY transformation:

$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\zeta}\gamma^a\psi)\partial_a\psi, \quad [\delta_1, \delta_2] = \delta_{\rm P}$.

- ψ is Nambu-Goldstone(NG) fermion for $\frac{superPoincare}{Poincare}$.
- ψ is quantized canonically in compatible with SUSY algebra.

2. Nonlinear-Supersymmetric General Relativity (NLSGR)

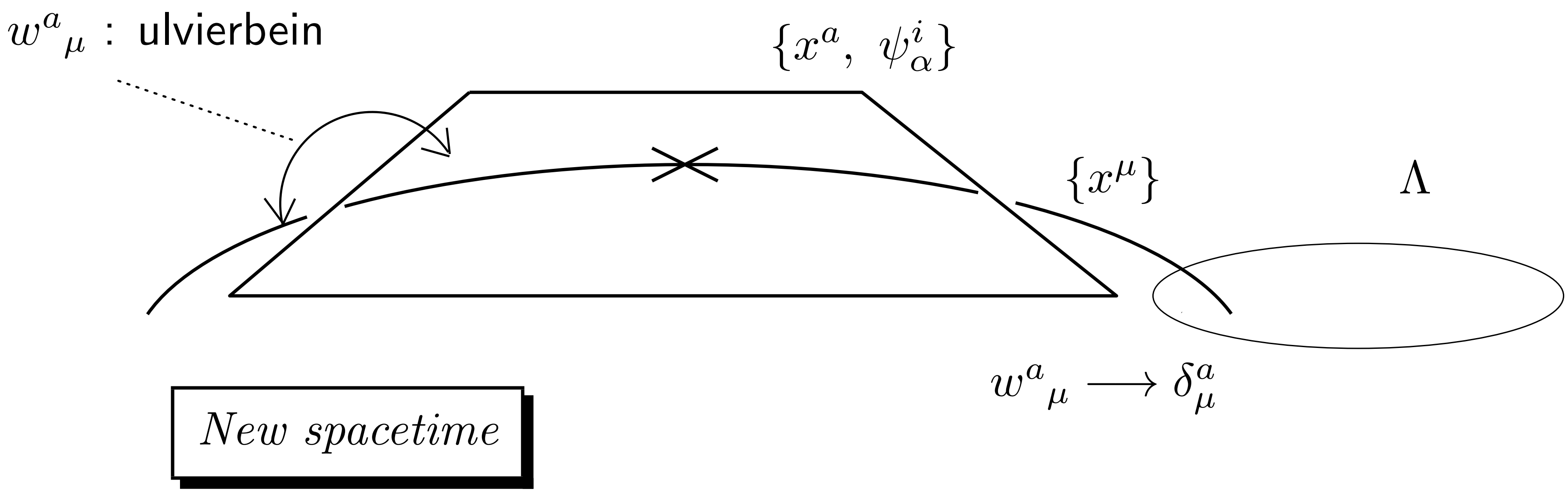
2.1. New Space-time as Ultimate Shape of Nature

We consider new (unstable) *physical* space-time inspired by nonlinear(NL) SUSY:

The tangent space of new space-time is specified by
Grassmann coordinates ψ_α for SL(2,C)
besides the ordinary Minkowski coordinates x^a for SO(1,3),

i.e.,
the coordinate ψ_α of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the NLSUSY NG fermion (called *superon* hereafter) are attached at every curved space-time point besides x^a.

- **New unstable space(U space:**



(Non-compact groups SO(1,3) and SL(2,C) for space-time symmetry are analogous to compact groups SO(3) and SU(2) for gauge symmetry of 't Hooft-Polyakov monopole.)

- We will see that $SO(1,3) \cong SL(2,C)$ **is crucial** for NLSGR scenario.

4 dimensional space-time is singled out.

2.2. Nonlinear-Supersymmetric General Relativity (NLSGR)

The geometrical arguments of Einstein general relativity(GR) can be extended to new (unstable) space-time.

- **Unified vierbein $w^a{}_\mu(x)$ of new space-time:**

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi) \equiv w^a{}_b e^b{}_\mu$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a \equiv e^\mu{}_b w^b{}_a,$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, .., N)$$

(Note that Grassmann d.o.f. induces **the imaginary part of $w^a{}_\mu(x)$** .)

- **N-extended NLSGR action of Eienstein-Hilbert(EH)-type for new space-time.** $\Longrightarrow$

N-extended NLSGR action:

$$L_{\rm NLSGR}(w) = -\frac{c^4}{16\pi G}|w|\{\Omega(w) + \Lambda\}, \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \tag{2}$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I\gamma^a\partial_\mu\psi^I - \partial_\mu\bar{\psi}^I\gamma^a\psi^I), (I = 1, 2, .., N) \tag{3}$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: **the unified vierbein of new space-time(*ulvierbein*)**
- $e^a{}_\mu(x)$: **the ordinary vierbein for the local SO(1,3) d.o.f.of GR,**
- $t^a{}_\mu(\psi(x))$: **the mimic vierbein for the local SL(2,C) d.o.f. composed of the stress-energy-momentum of NG fermion $\psi(x)^I$(called *superons*),**
- $\Omega(w)$: **Ricci scalar curvature of new space-time computed in terms of $w^a{}_\mu$,**
- $s_{\mu\nu} \equiv w^a{}_\mu\eta_{ab}w^b{}_\nu$, $s^{\mu\nu}(x) \equiv w^\mu{}_a(x)\eta^{ab}w^\nu{}_a(x)$: **metric tensors of new space-time.**
- G : **the Newton gravitational constant.**
- $\Lambda > 0$: **cosmological constant required by the correct NLSUSY structure of tangent space.**

• **NLSGR scenario fixes the arbitrary constatnt κ^2 to**

$$\kappa^2 = \left(\frac{c^4\Lambda}{8\pi G}\right)^{-1},$$

with the dimension $(length)^4 \sim (enegy)^{-4}$.

• $\Lambda > 0$ **in the action $L_{\rm NLSGR}$ allows**
negative dark energy density interpretation of $\frac{\Lambda}{G}$
in the Einstein equation. $\rightarrow$ **Sec.4.**

• **No-go theorem for $\underline{N > 8}$ <u>SUSY</u> has been circumvented**
by the global NLSUSY, i.e. by the vacuum(flat space) degeneracy.

• **Note that $SO(1, D-1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2 - 1)$ holds**
for only $\underline{D = 4, d = 2}$.

NLSGR scenario predicts 4 dimensional space-time.

2.3. Symmetries of NLSGR(N-extended action)

- **Space-time symmetries** $(\sim sP)$**:**

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \tag{4}$$

- **Internal symmetries for N-extended NLSUSY GR (N-superons** ψ^I $(I = 1, 2, ..N)$**):**

$$[\text{global SO(N)}] \otimes [\text{local U}(1)^{\text{N}}] \otimes [\text{chiral}]. \tag{5}$$

For example:

- **Invariance under the new NLSUSY transformation**;

$$\delta_\zeta \psi^I = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_\rho\psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_{[\mu}e^a{}_{\rho]}. \tag{6}$$

induces GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{7}$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and $\xi^\rho = -i\kappa\bar{\zeta}^I\gamma^\rho\psi^I$.

- **Commutators of two new NLSUSY transformations (??) on ψ^I and $e^a{}_\mu$ close to GL(4,R),**

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu\partial_\mu\psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \tag{8}$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2 - \xi_1^\rho\xi_2^\sigma e_a{}^\mu\partial_{[\rho}e^a{}_{\sigma]}$.
q.e.d.

- **New NLSUSY (??) is the square-root of GL(4,R);**

$$[\delta_1, \delta_2] = \delta_{\mathrm{GL(4,R)}}, \quad i.e. \quad \delta \sim \sqrt{\delta_{\mathrm{GL(4,R)}}}.$$

c.f. SUGRA (LSUSY)

$$[\delta_1, \delta_2] = \delta_P + \delta_L + \delta_g$$

- **The ordinary local GL(4,R) invariance is manifest by the construction.**

- **Invariance under new local Lorentz transformation**;

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I\gamma_5\gamma_d\psi^I(\partial_\mu\epsilon_{bc}) \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

(??) induce the familiar local Lorentz transformation on $w^a{}_\mu$**:**

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{10}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,
e.g., the new form on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ **is given by** $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$.
q.e.d.

3. Evolution of NLSGR and linearization of NLSUSY:

3.1. Big Collapse of new space-time toward true vacuum

@ $\Lambda > 0$ allows $L_{\rm NLSGR}(w)$ breaks down spontaneously(Big Collapse) to ordinary Riemann space-time(graviton) and NG fermion(superon): $L_{\rm SGM}(e,\psi)$.

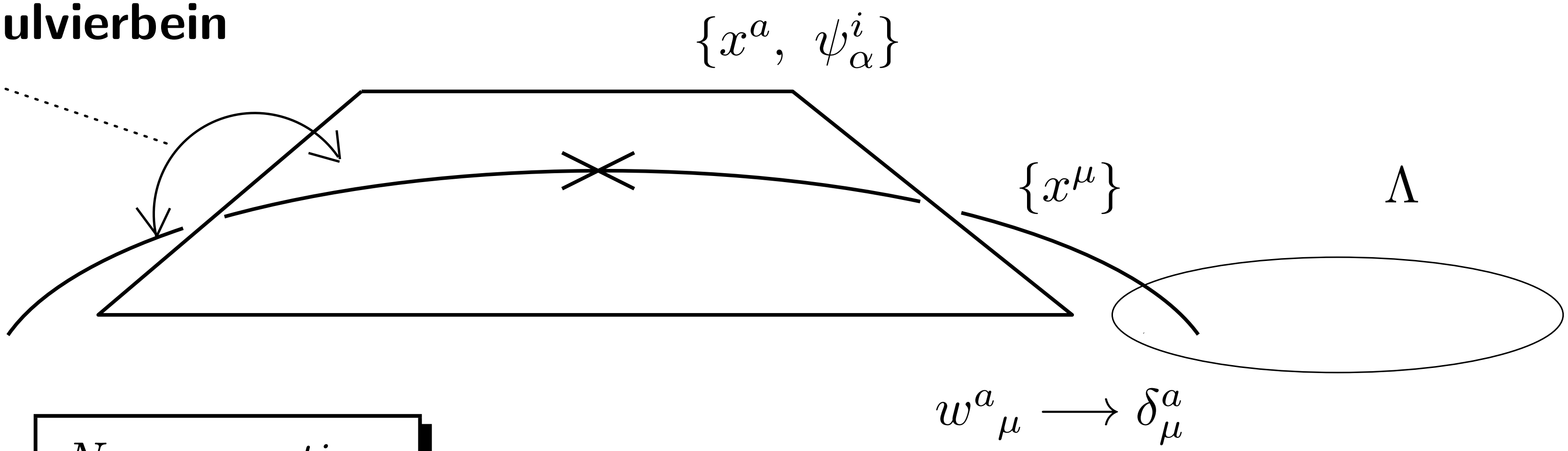
$$L_{\rm NLSGR}(w) = L_{\rm SGM}(e,\psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + |w(\psi^I)|\Lambda + \tilde{T}(e,\psi^I)\}. \qquad (12)$$

- $R(e)$: the Ricci scalar curvature of ordinary Riemann space-time
- Λ : the cosmological constant
- $|w(\psi^I)| = \det w^a{}_b = \det\{\delta^a{}_b + t^a{}_b(\psi^I)\}$: NLSUSY action for superon
- $\tilde{T}(e,\psi^I)$: the gravitational interaction of superon

@ Big Collapse may induce the rapid spacial expansion of space-time.

@ $L_{\rm SGM}(e,\psi^I)$ evolves toward the true vacuum by constituting gravitational composite (massless) eigenstates of broken LSUSY SO(N) sP, which is the ignition of the Big Bang SMs scenario. $\Longrightarrow$

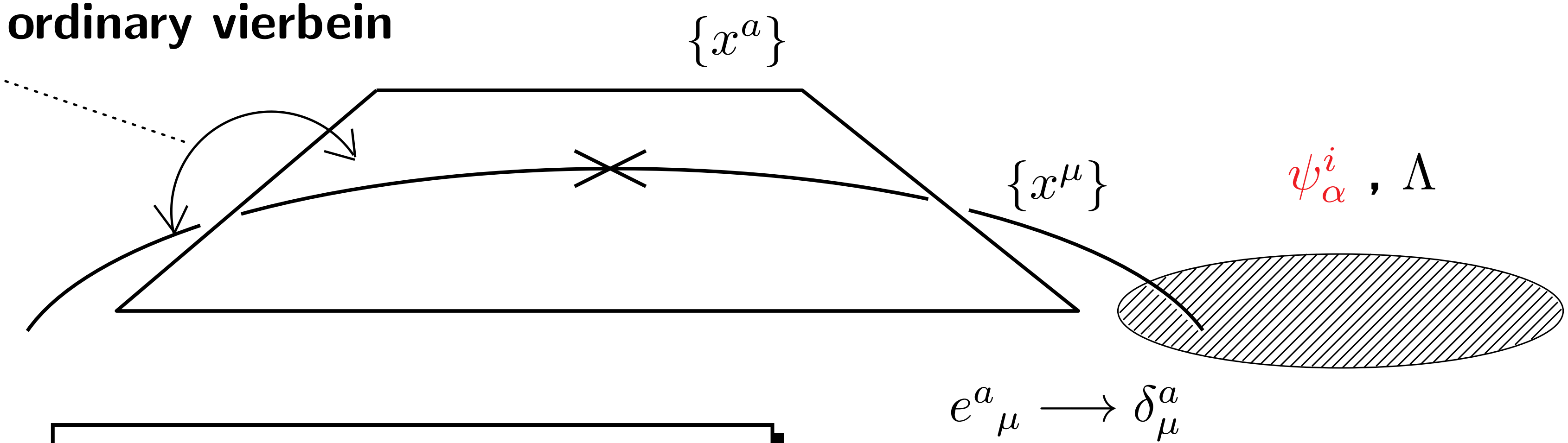
$w^a{}_\mu$: **ulvierbein**



New spacetime

$\Downarrow$ **Big Collapse**

$e^a{}_\mu$: **ordinary vierbein**



Riemann spacetime $\oplus$ **matter**

Ignition of the Big Bang

• **The Noether's theorem gives the conserved supercurrent:**

$$S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}{e_a}^{\mu}\gamma^a\psi^I + \cdots . \tag{13}$$

• **The supercurrent couples the graviton and the superon(NG fermion) to the vacuum**

$$< {e_b}^{\nu}{\psi_\beta}^J|{S_\alpha}^{I\mu}|0 >= i\sqrt{\frac{c^4\Lambda}{8\pi G}}\delta^{\mu\nu}\delta^{IJ}(\gamma_b)_{\alpha\beta} \tag{14}$$

with the strength $g_{sv} = \sqrt{\frac{c^4\Lambda}{8\pi G}} = \kappa^{-1}$.

3.2. Linearization of NLSUSY and vacuum of $L_{\rm SGM}(e, \psi^I)$

The graviton is the universal attractive force and creates all possible gravitational composites of superons, which are helicity eigenstates of LSUSY sP algebra of asymptotic space-time symmetry

• By linearizing NLSUSY we anticipate that we will obtain the (SUGRA-analogue) broken global LSUSY theory expressed in terms of the composite of superon and ${e^a}_\mu$:

$$L_{\rm LSUSY}({e^a}_\mu, \psi_\nu(e,\psi), M(e,\psi), N(e,\psi), \cdots)$$

which satisfies:

NL/L SUSY relation (*equivalence*):

$$L_{\rm NLSGR}(w) = L_{\rm SGM}(e,\psi) = L_{\rm LSUSY}({e^a}_\mu, \psi_\nu, M, N, \cdots).$$

- **LSUSY of higher-spin massless supermultiplet of SGM:**

For the massless supermultiplet with the highest-helicity ($\pm 3, \pm \frac{5}{2}$) of SGM, we take the symmetric tensor(-spinor) gauge field multiplet ($\varphi_{\mu\nu\rho}, \lambda_{\mu\nu}$).

The equations of motion and the local gauge transformations are given by :

$$
\begin{aligned}
&\Gamma_\sigma{}^\sigma{}_{,\mu_1\mu_2\mu_3} = \partial^\rho\partial_\rho\varphi_{\mu_1\mu_2\mu_3} - \sum_{\mu(1)}\partial_{\mu_1}\partial_\rho\varphi^\rho{}_{\mu_2\mu_3} + \sum_{\mu(2)}\partial_{\mu_1}\partial_{\mu_2}\varphi^\rho{}_{\rho\mu_3} = 0,\\
&\delta_\xi\varphi_{\mu_1\mu_2\mu_3} = \sum_{\mu(1)}\partial_{\mu_1}\xi_{\mu_2\mu_3} \equiv \partial_{\mu_1}\xi_{\mu_2\mu_3} + \partial_{\mu_2}\xi_{\mu_1\mu_3} + \partial_{\mu_3}\xi_{\mu_1\mu_2},\\
&\gamma^\rho\Gamma^{(1)}{}_{\rho,\mu_1\mu_2} = \gamma^\rho\partial_\rho\lambda_{\mu_1\mu_2} - \gamma^\rho\partial_{\mu_1}\lambda_{\rho\mu_2} - \gamma^\rho\partial_{\mu_2}\lambda_{\mu_1\rho} = 0,\\
&\delta_\epsilon\lambda_{\mu_1\mu_2} = \partial_{\mu_1}\epsilon_{\mu_2} + \partial_{\mu_2}\epsilon_{\mu_1},\\
&\xi_\rho{}^\rho = 0, \quad \gamma^\rho\epsilon_\rho = 0
\end{aligned}
\tag{15}
$$

(The arguments are systematic by using the generalized Christoffel symbol $\Gamma_{\rho\sigma,\mu_1\mu_2\mu_3}$.)

The naive extension of the familiar global LSUY for the on-shell gauge vector supermultiplet:

$$\delta_\zeta \varphi_{abc} = -i\bar{\zeta}\gamma_{\dot{a}}\lambda_{\dot{b}\dot{c}},$$
$$\delta_\zeta \lambda_{ab} = \sigma^{cd}\partial_c\varphi_{d\dot{a}\dot{b}}\zeta, \quad (16)$$

produce on-shell for φ_{abc} the familiar SUSY algebra:

$$[\delta_2, \delta_1]\varphi_{abc} = \delta_P + \delta_G,$$
$$\delta_P = (i\bar{\zeta}_1\gamma^d\zeta_2)\partial_d\varphi_{abc}, \quad \delta_G = \partial_{\dot{a}}\{(i\bar{\zeta}_1\gamma^d\zeta_2)\varphi_{d\dot{b}\dot{c}}\}, \quad (17)$$

while for λ_{ab} similar analogues result with the extra term δ_Σ containing $i\bar{\zeta}_1\sigma^{de}\zeta_2$:

$$[\delta_2, \delta_1]\lambda_{ab} = \delta_P + \delta_G + \delta_{\tilde{G}} + \delta_\Sigma,$$
$$\delta_P = -24(i\bar{\zeta}_1\gamma^d\zeta_2)\partial_d\lambda_{ab}, \quad \delta_G = \partial_{\dot{a}}\{(i\bar{\zeta}_1\gamma^d\zeta_2)(8\lambda_{db} - 4\gamma_b\gamma^c\lambda_{dc} + \dots)\},$$
$$\delta_{\tilde{G}} = \partial_{\dot{a}}\{(i\bar{\zeta}_1\sigma^{cd}\zeta_2)(\gamma_c\lambda_{db} + \dots)\}, \quad \delta_\Sigma = 2(i\bar{\zeta}_1\sigma^{cd}\zeta_2)(\gamma_{\dot{a}}\partial_c\lambda_{d\dot{b}} + \gamma_c\partial_d\lambda_{\dot{a}\dot{b}}), \quad (18)$$

The generalization of (16) and the off-shell extention of the supermultiplet will close the algebra., e.g., the simlplest minimal off-shell supermultiplet $(40_F, 20_B, 10_B, 10_B)$**:**

$$\delta_\zeta \lambda_{ab} = \sigma^{cd}\partial_c \phi_{d\dot{a}\dot{b}}\zeta + \cdots,$$
$$\delta_\zeta \phi_{abc} = -i\bar{\zeta}\gamma_{\dot{a}}\lambda_{\dot{b}\dot{c}} + \cdots,,$$
$$\delta_\zeta D_{ab} = -i\bar{\zeta}\gamma^5 \partial\!\!\!/ \lambda_{ab} + \cdots,.$$
$$\delta_\zeta E_{ab} = -i\bar{\zeta}\partial\!\!\!/ \lambda_{ab} + \cdots,. \qquad (19)$$

3.3. NL/L SUSY relation (equivalence)

We demonstrate NL/L SUSY relation for N=2 SUSY in *flat space*:

$$L_{\rm NLSGR}(w) = L_{\rm SGM}(e,\psi) \to \ L_{\rm NLSUSY}(\psi) = L_{\rm LSUSY}(v^a(\psi), \phi(\psi), \cdots)$$

($N=2$ SGM reduces to $N=2$ NLSUSY of Λ term of NLSGR)

@ N=2, d=2 NLSUSY model:

$$L_{NLSUSY} = -\frac{1}{2\kappa^2}|w_{NLSUSY}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \qquad (20)$$

$|w_{NLSUSY}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$, $t^a{}_b = -i\kappa^2(\bar\psi_j \gamma^a \partial_b \psi^j - \bar\psi_j \gamma^a \partial_b \psi^j)$, $(j = 1, 2)$,

which is invariant under N=2 NLSUSY transformation,

$\delta_\zeta \psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar\zeta_k \gamma^a \psi^k - \bar\zeta_k \gamma^a \psi^k)\partial_a \psi^j, (j = 1, 2)$.

N=2, d=2 LSUSY Theory (SUSY QED):

- **Helicity states of N=2 vector supermultiplet:**

$$\begin{pmatrix} +1 \\ +\frac{1}{2},\ +\frac{1}{2} \\ 0 \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell minimal vector supermultiplet: $(v^a, \lambda^i, A, \phi, D;$ i=1,2$)$ in *WZ gauge*. (A and ϕ are two singlets, 0^+ and 0^-, scalar fields.)

- **Helicity states of N=2 scalar supermultiplet:**

$$\begin{pmatrix} +\frac{1}{2} \\ 0,\ 0 \\ -\frac{1}{2} \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets: (χ, B^i, ν, F^i; $i = 1, 2$), B^i and F^i are complex.

• **The most genaral $N=2, d=2$ SUSYQED action ($m=0$ case) :**

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf}, \tag{21}$$

$$\begin{aligned}
L_{V0} &= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D, \\
L'_{\Phi 0} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}|F^i|^2, \\
L_e &= e\left\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5 \nu \right. \\
&\quad \left. + B^i(\bar{\lambda}^i \chi - \epsilon^{ij}\bar{\lambda}^j \nu) - \frac{1}{2}|B^i|^2 D \right\} + \{h.c.\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)|B^i|^2, \\
L_{Vf} &= f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}
\end{aligned} \tag{22}$$

• **Note that**
$J=0$ **states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.**

$L_{\mathrm{N=2SUSYQED}}$ is invariant under $N=2$ LSUSY transformation:

- **For the minimal vector off-shell supermultiplet:**

$$\delta_\zeta v^a = -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j,$$

$$\delta_\zeta \lambda^i = (D - i\partial\!\!\!/ A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\partial\!\!\!/\phi\zeta^j,$$

$$\delta_\zeta A = \bar{\zeta}^i\lambda^i,$$

$$\delta_\zeta \phi = -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j,$$

$$\delta_\zeta D = -i\bar{\zeta}^i\partial\!\!\!/\lambda^i. \tag{23}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{24}$$

where $\zeta^i, i = 1,2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation for only v^a with $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2$, $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\, v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- **For the two scalar off-shell supermultiplets:**

$$
\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial\!\!\!/ B^i)\zeta^i - e\epsilon^{ij} V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij} \bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij} (F^i + i\partial\!\!\!/ B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial\!\!\!/ \chi - i\epsilon^{ij} \bar{\zeta}^j \partial\!\!\!/ \nu \\
&\quad -e\{\epsilon^{ij} \bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i) B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\chi = \Xi^a \partial_a \chi - e\theta\nu, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]B^i = \Xi^a \partial_a B^i - e\epsilon^{ij} \theta B^j, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\nu = \Xi^a \partial_a \nu + e\theta\chi, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]F^i = \Xi^a \partial_a F^i + e\epsilon^{ij} \theta F^j,
\end{aligned}
\tag{25}
$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ **and the U(1) gauge parameter** θ**.**

$N = 2$ NL/L SUSY relation(*equivalence*):

$$L_{\rm N=2SUSYQED} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} = L_{\rm N=2NLSUSY} + [\text{surface terms}], \quad (26)$$

is realized by
(I) commutator-based (heuristic aspect) linearization
or
(II) superfield-based (systematic aspect) linearization.
Z.B.
(I) Commutator-based linearization:

@The product of Lorentz tensors composed of ψ^i and $|w|$ play a basic role.

$Q^n \sim (\psi + \bar\psi\gamma \cdot \partial\psi\psi + \cdots)^n \sim (\psi)^n$, $(\psi)^n \equiv 0, n > 4$,

- **For such Lorentz tensors (of current) of ψ^i miltiplied by $|w|$,**

$$b^i{}_A{}^{jk}{}_B{}^{l\cdots m}{}_C{}^n \left((\psi^i)^{2(n-1)}|w|\right) = \kappa^{2n-3}\bar\psi^i\gamma_A\psi^j\bar\psi^k\gamma_B\psi^l\cdots\bar\psi^m\gamma_C\psi^n|w|, \quad (27)$$

$$f^{ij}{}_A{}^{kl}{}_B{}^{m\cdots n}{}_C{}^p \left((\psi^i)^{2n-1}|w|\right) = \kappa^{2(n-1)}\psi^i\bar\psi^j\gamma_A\psi^k\bar\psi^l\gamma_B\psi^m\cdots\bar\psi^n\gamma_C\psi^p|w|, \quad (28)$$

the variations under the NLSUSY transformations become

$$\delta_\zeta b^i{}_A{}^{jk}{}_B{}^{l\cdots m}{}_C{}^n = \kappa^{2(n-1)} \left[\left\{ \left(\bar{\zeta}^i \gamma_A \psi^j + \bar{\psi}^i \gamma_A \zeta^j \right) \bar{\psi}^k \gamma_B \psi^l \cdots \bar{\psi}^m \gamma_C \psi^n + \cdots \right\} |w| + \kappa \partial_a \left(\xi^a \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \cdots \bar{\psi}^m \gamma_C \psi^n |w| \right) \right], \quad (29)$$

$$\delta_\zeta f^{ij}{}_A{}^{kl}{}_B{}^{ml\cdots n}{}_C{}^p = \kappa^{2n-1} \left[\left\{ \zeta^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \cdots \bar{\psi}^n \gamma_C \psi^p + \psi^i \left(\bar{\zeta}^j \gamma_A \psi^k + \bar{\psi}^j \gamma_A \zeta^k \right) \bar{\psi}^l \gamma_B \psi^m \cdots \bar{\psi}^n \gamma_C \psi^p + \cdots \right\} |w| + \kappa \partial_a \left(\xi^a \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \cdots \bar{\psi}^n \gamma_C \psi^p |w| \right) \right], \quad (30)$$

where $\xi^a = i\kappa \bar{\zeta}^i \gamma^a \psi^i$.

- **These show that the tables of products of Lorentz tensors (of currents) of ψ^i multiplied by $|w|$ give a <u>finite</u> representation of NLSUSY algebra.**
- **The self-contained part of Lorentz-tensor table corresponds to smaller LSUSY multiplet.**
- **They satisfy the basic commutator under NLSGR tr.**

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v),$$

where $\delta_P(v)$ is a translation with a parameter $v^a = 2i(\bar{\zeta}^i_{1L} \gamma^a \zeta^i_{2L} - \bar{\zeta}^i_{1R} \gamma^a \zeta^i_{2R})$

- These results show that

the commutator-based linearization closes on the all possible Lorentz tensors composed of ψ^i and gives a finite dimensional representation of sP algebra.

- Assign each composite Lorentz tensor to the component field of the LSUSY supermultiplet including the auxiliary field (susy compositenenss),

which reproduces the familiar LSUSY transformation among the supermultiplet under the NLSUSY transformations of constituents ψ^i.

- Substituting SUSY compositeness into $L_{\mathrm{N=2LSUSYQED}}$,

we obtain $L_{\mathrm{N=2NLSUSY}}$, i .e. the NL/L SUSY relation(equivalence).

• **SUSY compositeness for the vector off-shell minimal supermultiplet:**

$$
\begin{aligned}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\
\lambda^i &= \xi\psi^i|w|, \\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\
D &= \frac{\xi}{\kappa}|w|,
\end{aligned} \tag{31}
$$

where ξ is a VEV factor of the auxiliary field D.

• **Note that ψ^i is the low energy leading term of the supercharge Q^i.**

• **SUSY compositeness for scalar off-shell minimal supermultiplets:**

$$
\begin{aligned}
&\chi = \xi^i \left[\psi^i|w| + \frac{i}{2}\kappa^2\partial_a\{\gamma^a\psi^i\bar{\psi}^j\psi^j|w|\}\right] \\
&B^i = -\kappa\left(\frac{1}{2}\xi^i\bar{\psi}^j\psi^j - \xi^j\bar{\psi}^i\psi^j\right)|w|, \\
&\nu = \xi^i\epsilon^{ij}\left[\psi^j|w| + \frac{i}{2}\kappa^2\partial_a\{\gamma^a\psi^j\bar{\psi}^k\psi^k|w|\}\right], \\
&F^i = \frac{1}{\kappa}\xi^i\left\{|w| + \frac{1}{8}\kappa^3\partial_a\partial^a(\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w|)\right\} - i\kappa\xi^j\partial_a(\bar{\psi}^i\gamma^a\psi^j|w|) \\
&\underline{-\frac{1}{4}e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w|}. \qquad (32)
\end{aligned}
$$

• **The quartic fermion self-interaction term in F^i realizes the local $U(1)$ gauge symmetry of LSUSY.**

• **ξ^i is the VEV factor of the auxiliary field F^i.**

• **SUSY compositeness produces under NLSUSY transformation**

a new off-shell commutator algebra which closes on only a translation:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{33}$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}^i_{1L}\gamma^a\zeta^i_{2L} - \bar{\zeta}^i_{1R}\gamma^a\zeta^i_{2R}) \tag{34}$$

• **Note that the commutator does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.**

• **Substituting SUSY copositeness into $L_{\mathrm{N=2LSUSYQED}}$,
we find NL/L SUSY relation for the <u>minimal</u> supermultiplet:**

$$L_{\mathrm{N=2LSUSYQED}} \;=f(\xi,\xi^{i})L_{\mathrm{N=2NLSUSY}}+[\text{suface terms}], \tag{35}$$

$$f(\xi,\xi^{i})=\xi^{2}-(\xi^{i})^{2}=1. \tag{36}$$

$\Rightarrow$ LSUSY may be regarded as composite eigenstates of space-time symmetries.

• **NL/L SUSY relation bridges naturally
the cosmology and the low energy particle physics in NLSGR. ($\Rightarrow$ Sec. 4).**

• **The direct linearization of highly nonlinear SGM action (??)
in curved space remains to be carried out.**
$\rightarrow$

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the cosmological term Λ of SGM and materializes the true vacuum of SGM
(as gravitational composites of NG fermion)

SM can be a low energy effective theory of SGM/NLSGR.

(II) Linearization of NLSUSY by the superfield formulation($d = 2$)

- **General superfields** are given for the $N = 2$ vector supermultiplet by

$$\mathcal{V}(x, \theta^i) = C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2}\bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2}\bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i \gamma_5 \theta^j \phi(x)$$
$$- \frac{i}{4}\epsilon^{ij}\bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2}\bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8}\bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \quad (37)$$

and for the $N = 2$ scalar supermultiplet by

$$\Phi^i(x, \theta^i) = B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij}\bar{\theta}^j \nu(x) - \frac{1}{2}\bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i\bar{\theta}^i \partial\!\!\!/ B^j(x)\theta^j$$
$$+ \frac{i}{2}\bar{\theta}^j \theta^j (\bar{\theta}^i \partial\!\!\!/ \chi(x) - \epsilon^{ik}\bar{\theta}^k \partial\!\!\!/ \nu(x)) + \frac{1}{8}\bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \quad (38)$$

- **Extend the superspace to the following superspace** (x'^a, θ') **with** $-\kappa\psi(x)$**,**

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{39}$$

and denotes the extended superfields on (x'^a, θ'^i) **and their** θ**-expansions as**

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \tag{40}$$

- **Extended global SUSY transformations** $\tilde{\delta} = \delta^L(x.\theta) + \delta^{NL}(\psi)$ **on** (x'^a, θ'^i) **give:**

$$\tilde{\delta}\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \tilde{\delta}\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{41}$$

- **Therefore, the conditions(SUSY invariant constraints):**

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \xi^I_{\mathcal{V}}(\text{constant}) \quad \tilde{\varphi}^I_{\Phi}(x) = \xi^I_{\Phi}(\text{constant}), \tag{42}$$

make extended superfields invariant under the extended SUSY transformations. which provide SUSY compositeness.

- **Putting constants in the most general case as follows:**

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \tag{43}$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \tag{44}$$

- **We obtain straightforwardly SUSY compositeness $\varphi^I_{\mathcal{V}} = \varphi^I_{\mathcal{V}}(\psi)$ for the vector supermultiplet**

$$\begin{aligned}
C &= \xi_c + \kappa \bar{\psi}^i \xi_\Lambda^i + \frac{1}{2}\kappa^2 (\xi_M^{ij} \bar{\psi}^i \psi^j - \xi_M^{ii} \bar{\psi}^j \psi^j) + \frac{1}{4}\xi_\phi \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - \frac{i}{4}\xi_v^a \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \\
&\quad -\frac{1}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \xi_\lambda^j - \frac{1}{8}\xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j, \\
\Lambda^i &= \xi_\Lambda^i + \kappa (\xi_M^{ij} \psi^j - \xi_M^{jj} \psi^i) + \frac{1}{2}\xi_\phi \kappa \epsilon^{ij} \gamma_5 \psi^j - \frac{i}{2}\xi_v^a \kappa \epsilon^{ij} \gamma_a \psi^j \\
&\quad -\frac{1}{2}\xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2}\kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi_\lambda^j) \\
&\quad -\frac{1}{2}\xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i\kappa \partial\!\!\!/ C(\psi) \psi^i,
\end{aligned}$$

$$
\begin{aligned}
M^{ij} &= \xi_M^{ij} + \kappa\bar{\psi}^{(i}\xi_\lambda^{j)} + \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j + i\kappa\epsilon^{(i|k|}\epsilon^{j)l}\bar{\psi}^k \partial\!\!\!/\Lambda^l(\psi) - \frac{1}{2}\kappa^2\epsilon^{ik}\epsilon^{jl}\bar{\psi}^k\psi^l\partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\xi_\lambda^j - \frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\partial\!\!\!/\Lambda^j(\psi) + \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2 C(\psi), \\
v^a &= \xi_v^a - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\xi_\lambda^j - \frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j - \kappa\epsilon^{ij}\bar{\psi}^i\partial\!\!\!/\gamma^a\Lambda^j(\psi) + \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j\partial^2 C(\psi) \\
&\quad -i\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^b\psi^j\partial^a\partial_b C(\psi), \\
\lambda^i &= \xi_\lambda^i + \xi\psi^i - i\kappa\partial\!\!\!/ M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa\epsilon^{ab}\epsilon^{ij}\gamma_a\psi^j\partial_b\phi(\psi) \\
&\quad -\frac{1}{2}\kappa\epsilon^{ij}\left\{\psi^j\partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab}\gamma_5\psi^j F_{ab}(\psi)\right\} \\
&\quad -\frac{1}{2}\kappa^2\{\partial^2\Lambda^i(\psi)\bar{\psi}^j\psi^j - \partial^2\Lambda^j(\psi)\bar{\psi}^i\psi^j - \gamma_5\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma_5\psi^j \\
&\quad -\gamma_a\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j + 2\partial\!\!\!/\partial_a\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j\} - \frac{i}{2}\kappa^3\partial\!\!\!/\partial^2 C(\psi)\psi^i\bar{\psi}^j\psi^j, \\
D &= \frac{\xi}{\kappa} - i\kappa\bar{\psi}^i\partial\!\!\!/\lambda^i(\psi)
\end{aligned}
$$

$$+\frac{1}{2}\kappa^2\left\{\bar{\psi}^i\psi^j\partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2\phi(\psi)\right.$$

$$\left.+\frac{i}{2}\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial^2 v^a(\psi) - i\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial_a\partial_b v^b(\psi)\right\}$$

$$-\frac{i}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\partial\!\!\!/\partial^2\Lambda^j(\psi) + \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j\partial^4 C(\psi), \quad (45)$$

and SUSY compositeness for the scalar multiplet $\varphi_\Phi^I = \varphi_\Phi^I(\psi)$:

$$B^i = \xi_B^i + \kappa(\bar{\psi}^i\xi_\chi - \epsilon^{ij}\bar{\psi}^j\xi_\nu) - \frac{1}{2}\kappa^2\{\bar{\psi}^j\psi^j F^i(\psi) - 2\bar{\psi}^i\psi^j F^j(\psi) + 2i\bar{\psi}^i\partial\!\!\!/ B^j(\psi)\psi^j\}$$

$$-i\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) - \epsilon^{ik}\bar{\psi}^k\partial\!\!\!/\nu(\psi)\} + \frac{3}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 B^i(\psi),$$

$$\chi = \xi_\chi + \kappa\{\psi^i F^i(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^i\}$$

$$-\frac{i}{2}\kappa^2[\partial\!\!\!/\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}]$$

$$+\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\partial\!\!\!/ F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j,$$

$$
\begin{aligned}
\nu &= \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^j\} \\
&-\frac{i}{2}\kappa^2[\partial\!\!\!/\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}] \\
&+\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial\!\!\!/ F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
F^i &= \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\partial\!\!\!/\nu(\psi)\} \\
&-\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar{\psi}^i\partial\!\!\!/ F^j(\psi)\psi^j \\
&+\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 F^i(\psi). \qquad (46)
\end{aligned}
$$

• Choosing the following Lorentz invariant and SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{47}$$

give previous SUSY compositeness for the minimal supermultiplet.

Therefore,

• under SUSY invariant constraints,

the $N = 2$ NLSUSY action $S_{N=2\text{NLSUSY}}$ is related to $N = 2$ SUSY QED action:

$$L_{N=2\text{SUSYQED}} \equiv L_{\mathcal{V}\text{free}} + L_{\mathcal{V}f} + L_{\text{gauge}} = f(\xi, \xi^i) S_{N=2\text{NLSUSY}} \tag{48}$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

• NL/L SUSY relation bridges the cosmology and the low energy particle physics in NLSGR scenario $\Longrightarrow$ Sec. 4.

@ SGM scenario predicts the magnitude of the bare gauge coupling constant.

For more general SUSY invariant constraints(vev of 0^+ auxiliary field):

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \tag{49}$$

NL/L SUSY relation gives

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \ e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \tag{50}$$

where e is the bare gauge coupling constant.

• This mechanism is natural and favorable for SGM scenario as a theory of everything.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

3.3. $N = 3$ NL/L SUSY relation and SUSY Yang-MIlls theory

- **Physical helicity states of $N = 3$ LSUSY vector supermultiplet:**

$$\left[\ \underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right)\ \right] + [\text{CPT conjugate}], \tag{51}$$

where $\underline{n}(\lambda)$ means the dimension $\underline{n}$ and the helicity λ, are accomodated in $N = 3$ off-shell vector supermultiplet($d = 2$):

- $N = 3$ **superYang-Mills(SUSYYM) minimal off-shell gauge multiplet,**

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), {\chi_\alpha}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I = 1, 2, \cdots, dim.G) \tag{52}$$

Each component field belongs to the adjoint representation of the YM gauge group G:
$[T^I, T^J] = if^{IJK}T^K$ **and denoted as $\varphi^i = \varphi^{iI}T^I$, etc..**

- $N=3$ **(pure) SUSYYM action:**

$$S_{\rm SYM} = \int d^2x \ {\rm tr}\Big\{-\frac{1}{4}(F_{ab})^2+\frac{i}{2}\bar{\lambda}^i \not{D}\lambda^i+\frac{1}{2}(D_aA^i)^2+\frac{i}{2}\bar{\chi}\not{D}\chi+\frac{1}{2}(D_a\phi)^2+\frac{1}{2}(D^i)^2$$
$$-ig\{\epsilon^{ijk}A^i\bar{\lambda}^j\lambda^k-[A^i,\bar{\lambda}^i]\chi+\phi(\bar{\lambda}^i\gamma_5\lambda^i+\bar{\chi}\gamma_5\chi)\}$$
$$+\frac{1}{4}g^2([A^i,A^j]^2+2[A^i,\phi]^2)\Big\}, \quad (53)$$

where g is the gauge coupling constant, D_a and F_{ab} are the covariant derivative and the YM gauge field strength defined as

$$D_a\varphi = \partial_a\varphi - ig[v_a,\varphi],$$
$$F_{ab} = \partial_a v_b - \partial_b v_a - ig[v_a,v_b]. \quad (54)$$

• **SUSYYM action is invariant under $N=3$ LSUSY transformations:**

$$
\begin{aligned}
\delta_\zeta v^a &= i\bar{\zeta}^i \gamma^a \lambda^i, \\
\delta_\zeta \lambda^i &= \epsilon^{ijk}(D^j - i\not{\partial} A^j)\zeta^k + \frac{1}{2}\epsilon^{ab}F_{ab}\gamma_5\zeta^i - i\gamma_5\not{\partial}\phi\zeta^i \\
&\quad + ig([A^i, A^j]\zeta^j + \epsilon^{ijk}[A^j, \phi]\gamma_5\zeta^k), \\
\delta_\zeta A^i &= \epsilon^{ijk}\bar{\zeta}^j\lambda^k - \bar{\zeta}^i\chi, \\
\delta_\zeta \chi &= (D^i + i\not{\partial} A^i)\zeta^i + ig(\epsilon^{ijk}A^iA^j\zeta^k - [A^i, \phi]\gamma_5\zeta^i), \\
\delta_\zeta \phi &= \bar{\zeta}^i\gamma_5\lambda^i, \\
\delta_\zeta D^i &= -i\epsilon^{ijk}\bar{\zeta}^j\not{\partial}\lambda^k - i\bar{\zeta}^i\not{\partial}\chi + ig(\bar{\zeta}^i[\lambda^j, A^j] + \bar{\zeta}^j[\lambda^i, A^j] - \bar{\zeta}^j[\lambda^j, A^i] \\
&\quad - \epsilon^{ijk}\bar{\zeta}^j[\chi, A^k] + \epsilon^{ijk}\bar{\zeta}^j\gamma_5[\lambda^k, \phi] + \bar{\zeta}^i\gamma_5[\chi, \phi]),
\end{aligned} \tag{55}
$$

$$
[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{56}
$$

where $\delta_G(\theta)$ **means** $\delta_G(\theta)\varphi = ig[\theta, \varphi]$ **and** $\delta_g(\theta)$ **is the** $U(1)$ **gauge transformation only for** v^a **with** $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ijk}\bar{\zeta}_1^i\zeta_2^j A^k - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$**.**

- **SUSY invariant(composite) relations for $N = 3$ YM off-shell gauge supermultiplet**

$$
\begin{aligned}
v^{aI} &= -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
\lambda^{iI} &= \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) \\
&\quad +\frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4), \\
A^{iI} &= \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5), \\
\chi^I &= \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\partial\!\!\!/\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4), \\
\phi^I &= -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
D^{iI} &= \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k)\} \\
&\quad -\frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5), \qquad (57)
\end{aligned}
$$

- **Arbitrary real constants ξ^{iI} of auxirially fields D^{iI} bridge $N=3$ SUSY and the YM gauge group G.**

- **Substituting (??) into the SYM action (??), we can show the NL/L SUSY relation for $N=3$ SUSY:**

$$L_{\rm SUSYYM}(\psi) = -(\xi^{iI})^2 L_{\rm NLSUSY} + [\text{surface terms}]. \tag{58}$$

3.4. Linearization of $L_{SGM}(e,\psi)$ in curved space-time

@ Commutator-based linearization:

- **Show**

NL/L SUSY relation (*equivalence*):

$$L_{\rm NLSGR}(w) = L_{\rm SGM}(e,\psi) = L_{\rm LSUSY}(e^a{}_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \cdots).$$

- **NLSUSY transformations**

$$\delta_\zeta \psi^i = \frac{1}{\kappa}\zeta^i + \xi^\mu \partial_\mu \psi^i, \quad \delta_\zeta e^a{}_\mu = 2i\kappa \bar{\zeta}^i \gamma^\nu \psi^i \partial_{[\mu} e^a_{\nu]}, \quad \xi^\mu = i\kappa \bar{\psi}^j \gamma^\mu \zeta^j.$$

- **Based commutator algebra, LSUSY as well,**

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{GL(4,R)}(\Xi^\mu), \ \Xi^\mu = 2(i\bar{\zeta}_1^i \gamma^\mu \zeta_2^i - \xi^\nu{}_1 \xi^\kappa{}_2 e^\mu{}_a \partial_{[\mu} e^a{}_{\nu]})$$

- **In curved space-time, Lorentz- and Riemann-tensor functionals with γ-matrices**

$$F_M^{IA} = F_M^{IA}(\psi^i, e^a_\mu, \partial\psi^i, \partial_{[\nu} e^a{}_{\kappa]})$$

satisfies the commutator algebra, favourable to the SUGRA-like LSUSY structure.

4. Low energy particle physics and Cosmology of NLSGR

4.1. Low Energy Particle Physics of NLSGR :

@ As we have seen that

$N=2$ SGM is essentially N=2 NLSUSY action in tangent(flat)) space-time, we focus on N=2 NLSUSY action for extracting physical implications of SGM.

- **The low energy theorem for NLSUSY gives the following superon(massless NG fermion)-vacuum coupling**

$$< \psi^j{}_\alpha(x)|J^{k\mu}{}_\beta|0 >= i\sqrt{\frac{c^4\Lambda}{8\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{jk} + \cdots, \tag{59}$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\gamma^\mu\psi^k + \cdots$ is the conserved supercurrent.

$\sqrt{\frac{c^4\Lambda}{8\pi G}} = \frac{1}{\sqrt{2}\kappa}$ is the coupling constant ($\equiv g_{sv}$) of superon with the vacuum.

$\Longrightarrow {m_\nu}^4 \sim \rho_{DE},$,

- **How to extract the vacuum configuration of SGM.**

In Riemann-flat space-time, NL/L SUSY relation(equivalence) gives:

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\text{suface terms}] = L_{N=2\mathrm{SUSYQED}}. \quad (60)$$

- **We study vacuum structures of $N=2$ LSUSY QED action** **in stead of $N=2$ SGM.**

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\mathrm{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \quad (61)$$

• **Substituting the solution of the equation of motion for the auxiliary field D we obtain**

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa}\right\}^2 + \frac{1}{2}e^2(A^2+\phi^2)|B^i|^2 \geq 0. \tag{62}$$

• **Two different types of vacua $V=0$ exist in (A,ϕ,B^i)-space:**

$$(I)\quad A=\phi=0,\quad |\tilde{B}^i|^2 = -k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i,\quad k^2 = \frac{\xi}{f\kappa}\right) \tag{63}$$

and

$$(II)\quad |\tilde{B}^i|^2 = 0,\quad A^2-\phi^2 = k^2. \tag{64}$$

• **Expansions of $A,\phi,\tilde{B}^i$ around vacuum values give**

low energy particles $\hat{A},\hat{\phi},\hat{B}^i$ in the true vacuum.

- **For the type (I) vacuum with $SO(2)$ symmetry for $(\tilde{B}^1, \tilde{B}^2)$,** $\quad e\xi < 0$**,**

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a\rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a\hat{A})^2 + (\partial_a\hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \cdots,
\end{aligned}
\tag{65}
$$

and following mass spectra

$$m_\rho^2 = m_{\hat{A}}^2 = m_{\hat{\phi}}^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa},$$
$$m_{\lambda^i} = m_\chi = m_\nu = 0. \tag{66}$$

- **The vacuum breaks both SUSY and the local $U(1)(O(2))$ spontaneously and Higgs-Kibble mechanism works.**
- **All bosons have the same mass and all fermions remain massless.**
- λ^i **are NG fermions.**

- **For the type (II) vacuum with $SO(1,1)$ symmetry for (A,ϕ), e.g. $f\xi > 0$,**

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2k^2\hat{A}^2\} \\
& + \frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2k^2(|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& + \frac{1}{2}(\partial_a \hat{\phi})^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/\chi + \bar{\nu}\partial\!\!\!/\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots . \qquad (67)
\end{aligned}
$$

and following mass spectra:

$$
\begin{aligned}
m_{\hat{A}}^2 &= m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
m_{\hat{B}^1}^2 &= m_{\hat{B}^2}^2 = m_{\chi}^2 = m_{\nu}^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
m_{v_a} &= m_{\hat{\phi}} = 0,
\end{aligned} \tag{68}
$$

which produces mass hierarchy by the factor $\frac{e}{f}$ independent of κ. $(\kappa^{-2} = \frac{c^4\Lambda}{16\pi G})$

• The vacuum breaks both **SUSY** and $SO(1,1)$ for (A, ϕ) and restores(maintains) $SO(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which produces NG-Boson $\hat{\phi}$ and massless photon v_a and gives soft masses $< A >$ to λ^i.

- **We have shown explicitly that**

N=2 LSUSY QED, i.e. the matter sector(Λ term) of $N=2$ SGM (in flat-space), possesses a true vacuum type (II).

- **The resulting model describes qualitatively**

lepton-Higgs-U(1) sector analogue of SM:

one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\hat{B}^1 + i\hat{B}^2$),
one neutral complex scalar ($\hat{A} + i\hat{\phi}$),

which are composites of superons.

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the NLSUSY cosmological term of SGM
as composites of NG fermion in the true vacuum.

4. Cosmological implications of SGM scenario

The variation of SGM action $L_{\mathrm{SGM}}(e,\psi)$ with respect to $e^a{}_\mu$ yields
Einstein equation equipping with matter and cosmological term:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e,\psi) - g_{\mu\nu}\frac{c^4\Lambda}{8\pi G}\}. \tag{69}$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

• **Note that the cosmological term $-\frac{c^4\Lambda}{8\pi G}$ can be interpreted as**
the negative energy density of space-time,
i.e. the dark energy density ρ_D.

• Big collapse may induce 3 dimensional expansion of space-time by Pauli principle:

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \Phi_{\mu\nu}(e,\psi)\}dx^\mu dx^\nu.$$
$$\{\psi(x), \bar{\psi}(y)\} = 0 \ \Rightarrow \ \{\psi(x), \bar{\psi}(y)\} = \delta^{(3)}(\mathbf{x} - \mathbf{y})$$

• Big Collapse produces composite (massless) eigenstates of SO(N) sP angebra due to the universal gravitational force,

which is the ignition of the Big Bang(BB) SM scenario.

• As shown in the toy model, the vacuum of the composite SGM scenario may explain naturally observed mysterious (numerical) relations:

$$dark\ energy\ density\ \rho_D \sim O(\kappa^{-2}) \sim {m_\nu}^4 \sim (10^{-12} GeV)^4 \sim {g_{sv}}^2,$$

provided ${\lambda_D}^0$ is identified with neutrino and $f\xi \sim O(1)$.

5. Rarita-Schwinger NLSGR

- **New SUSY algebra containing spinor-vector generators Q^{μ}_{α}:**

$$\{Q^{\mu}_{\alpha}, Q^{\nu}_{\beta}\} = \varepsilon^{\mu\nu\lambda\rho} P_{\lambda} (\gamma_{\rho}\gamma_{5}C)_{\alpha\beta}, \quad (70)$$

$$[Q^{\mu}_{\alpha}, P^{\nu}] = 0, \quad (71)$$

$$[Q^{\mu}_{\alpha}, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho}Q^{\mu})_{\alpha} + i\eta^{\lambda\mu}Q^{\rho}_{\alpha} - i\eta^{\rho\mu}Q^{\lambda}_{\alpha}, \quad (72)$$

where Q^{μ}_{α} are vector-spinor generators satisfying Majorana condition $Q^{\mu}_{\alpha} = C_{\alpha\beta}\overline{Q}^{\mu}_{\alpha}$.

- **Consider the following global (3/2 super)translations:**

$$\psi^{a}_{\alpha} \longrightarrow \psi^{a}_{\alpha} + \zeta^{a}_{\alpha}. \quad (73)$$

$$x_{a} \longrightarrow x_{a} + i\kappa\varepsilon_{abcd}\bar{\psi}^{b}\gamma^{c}\gamma_{5}\zeta^{d}, \quad (74)$$

where ζ^{a}_{α} is a constant Majorana tensor-spinor parameter.

- **The invariant differential forms become:**

$$\omega_{a} = dx_{a} + i\kappa\varepsilon_{abcd}\bar{\psi}^{b}\gamma^{c}\gamma_{5}d\psi^{d}. \quad (75)$$

• **Invariant action of nonlinear representation of vector-spinor SUSY:**

$$S = \frac{1}{\kappa}\int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa}\int \det w_{ab} d^4x, \tag{76}$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i\kappa\varepsilon_{acde}\bar{\psi}^c\gamma^d\gamma_5\partial_b\psi^e, \tag{77}$$

• **By similar geometrical arguments to SGM**
we obtain vector-spinor NLSUSY GR:

$$L_{vsNLSUSYGR} = -\frac{c^3}{16\pi G}|w|\{\Omega(w^a{}_\mu) + \Lambda\}, \tag{78}$$

$$|w| = detw^a{}_\mu = det(e^a{}_\mu + t^a{}_\mu), \tag{79}$$

• **Unified vierbeins are:**

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = i\kappa\varepsilon^{abcd}\bar{\psi}_b\gamma_c\gamma_5\partial_\mu\psi_d, \tag{80}$$

• $L_{vsNLSUSYGR}$ **possesses similar symmetry properties as SGM.**

6. Summary

NLSGR(SGM) for unity of nature:

• Ultimate entity; Unstable $d = 4$ space-time:$[x^a, \psi_\alpha{}^N; x^\mu]$ described by $[L_{\rm NLSGR}(w^a{}_\mu)]$: NLSGR on New space-time with $\Lambda > 0$

$\Longrightarrow$ Big Collapse (BC)

• The creation of Riemann space-time [graviton $e^a{}_\mu$] and massless fermionic matter [superon $\psi_\alpha{}^{\rm N}$]
$[L_{\rm SGM} = L_{\rm EH}(e) - \Lambda + T(\psi.e)]$: Einstein GR with $\Lambda > 0$ and N superon

• The universal attractive force graviton dictates the phase transition by forming gravitational composite LSUSY supermultiplet corresponding to (massless) eigenstates of space-time symmetry SO(10) sP.

$\Longrightarrow$ Ignition of Big Bang Scenario toward ΛCDMSM scenario

• In flat space-time, broken N-LSUSY theory emerges from the N-NLSUSY cosmological term of $L_{\rm SGM}(e, \psi)$ [NL/L SUSY relation]. $\longleftrightarrow$ BCS vs GL

The cosmological term is the origin of everything!

Predictions and Speculations

@SO(10) sP algebra with $\underline{10} = \underline{5}_{\rm SU(5)} + \underline{5}^*_{\rm SU(5)}$:
superon-quintet model(SQM) of matter

- **Two new $1^{\rm C}$ particles besides SM particles:**

  One double-charge spin 1/2 fermion $E^{2\pm}$: $\epsilon^{abc}Q_aQ_bQ_c\epsilon^{mn}Q^*{}_mQ^*{}_n$,

  One neutral spin 1/2 fermion N: $\epsilon^{abc}Q_aQ_bQ_c\epsilon^{mn}Q_mQ_n$,

  One neutral vector boson S: $\delta^{ab}Q_aQ^*{}_b$

- **Proton decay diagrams of SU(5) GUT in composite SQM view are forbidden by superon slection rule. $\Rightarrow$ stable proton**

@Field theory via Linearization:

- **NLSGR(SGM) scenario predicts 4 dimensional space-time.**
- **The bare gauge coupling constant is determined.**
- **N-L/NL SUSY relation $\Longleftrightarrow$ superon-quintet hypothesis**

cosmological term $\leftrightarrow$ dark energy density $\leftrightarrow$ SUSY Br. $\rightarrow m_\nu$

Many Open Questions ! e.g.,

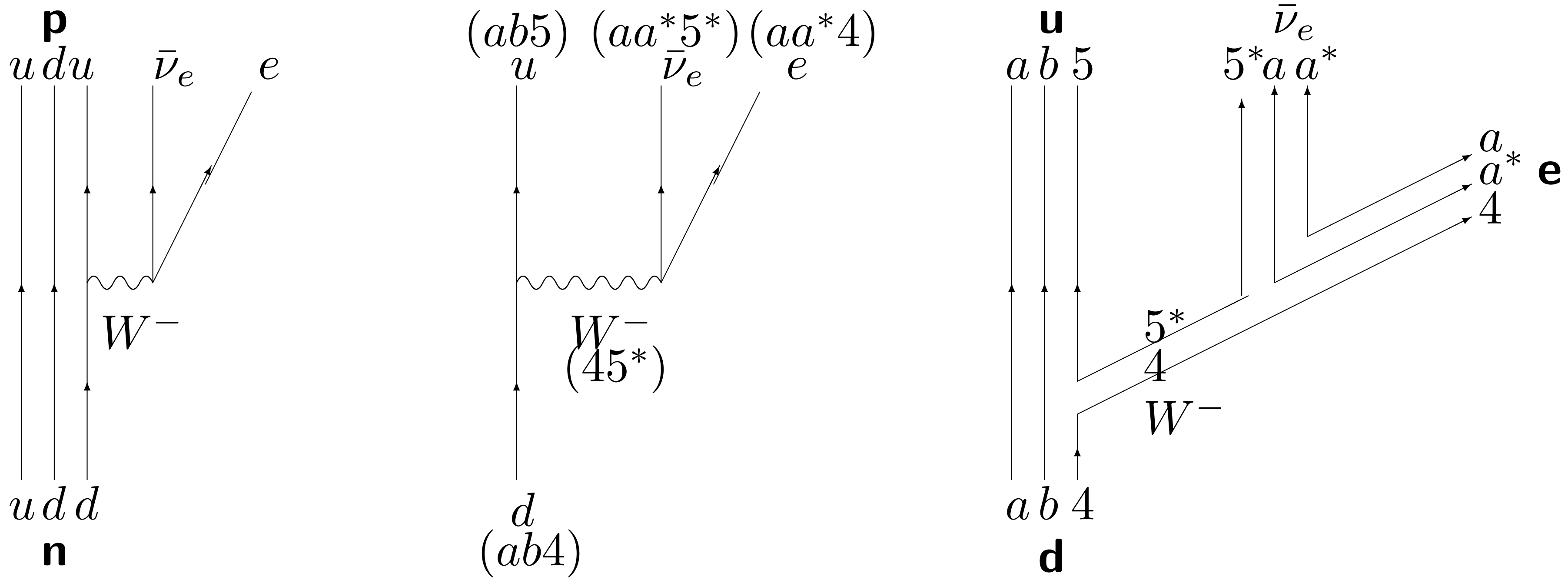
- **Direct linearization of SGM action,**

i.e. NL/L SUSY relation in curved space-time, LSUSY of high spin states .

- **Revisit SM and GUT from $N = 10$ SQM composite diagram viewpoints:**

(e, ν_e): $\delta^{ab} Q_a {Q^*}_b Q_m$, (μ, ν_μ): $\delta^{ab} Q_a {Q^*}_b \epsilon^{lm} Q_l Q_m Q^*_n$, (τ, ν_τ): $\epsilon^{abc} Q_b Q_c \epsilon^{ade} {Q^*}_d Q^*_e Q_m$
(u, d): $\epsilon^{abc} Q_b Q_c Q_m$, (c, s): $\epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c {Q^*}_n$, (t, b): $\epsilon^{abc} Q_a Q_b Q_c {Q^*}_d Q_m$,
$HiggsBoson$: $\delta^{ab} Q_a {Q^*}_b Q_l {Q^*}_m$, $GaugeBoson$: $Q_a {Q^*}_b$, $\cdots$.

e.g. SQM diagram for β-decay.



- **SQM diagram interpretation of dangerous Feynman diagrams of SM and GUT (e.g., proton decay, FCNC, $\cdots$).**

- **The thirmodynamics of the universe in NLSGR/SGM.**

- **Physical consequences of spin $\frac{3}{2}$ NLSGR.**

[Ref.] K. Shima, Invited talk at *Conference on Cosmology, Gravitational Waves and Particles*, **6-10, January, 2017, NTU, Singapore(Uploaded at YouTube by IAS). Proceeding of** *CCGWP*, **ed. Harald Fritzsch, (World Scientific, Singapo re, 2017), 301.**
K. Shima Invited talk at *14th Rencontres du Vietnam on Windows on the Universe, 25th Anniversary*, **8/5-11, 2018, Quy Nhon, Vietnam. Proceedings of Windows on the Universe, ed. Jacques Dumarchez. at press.**