Exotic Branes and Superconformal Field Theories

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Contents

1. Exotic branes from F-theory

Consider charged particles in 3D maximal supergravity :

They are D7-brane wrapped on 7-torus and its dualized objects

$$M_{\mathrm{D7}} = \frac{R_1 R_2 \cdots R_7}{g_s \, \ell_s^8} \quad \overset{\mathrm{T}_7}{\nearrow} \quad \underset{\mathrm{S}}{\searrow}$$

$$\mathrm{T}_y \; : \quad R_y \to \frac{\ell_s^2}{R_y} , \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathrm{S} \; : \quad g_s \to \frac{1}{g_s} , \quad \ell_s^2 \to g_s \, \ell_s^2$$

R_y : compact radius of y-direction

g_s : string coupling constant

ℓ_s : string length

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They are D7-brane wrapped on 7-torus and its dualized objects

$$M_{\mathrm{D7}} = \frac{R_1 R_2 \cdots R_7}{g_s \, \ell_s^8} \quad \overset{\mathrm{T}_7}{\nearrow} \quad \frac{R_1 R_2 \cdots R_6}{g_s \, \ell_s^7} = M_{\mathrm{D6}}$$

$$M_{\mathrm{D7}} = \frac{R_1 R_2 \cdots R_7}{g_s \, \ell_s^8} \quad \underset{\mathrm{S}}{\searrow} \quad \frac{R_1 R_2 \cdots R_7}{g_s^3 \, \ell_s^8} \leftarrow \text{exotic!}$$

$$\mathrm{T}_y : \quad R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathrm{S} : \quad g_s \to \frac{1}{g_s}, \quad \ell_s^2 \to g_s \, \ell_s^2$$

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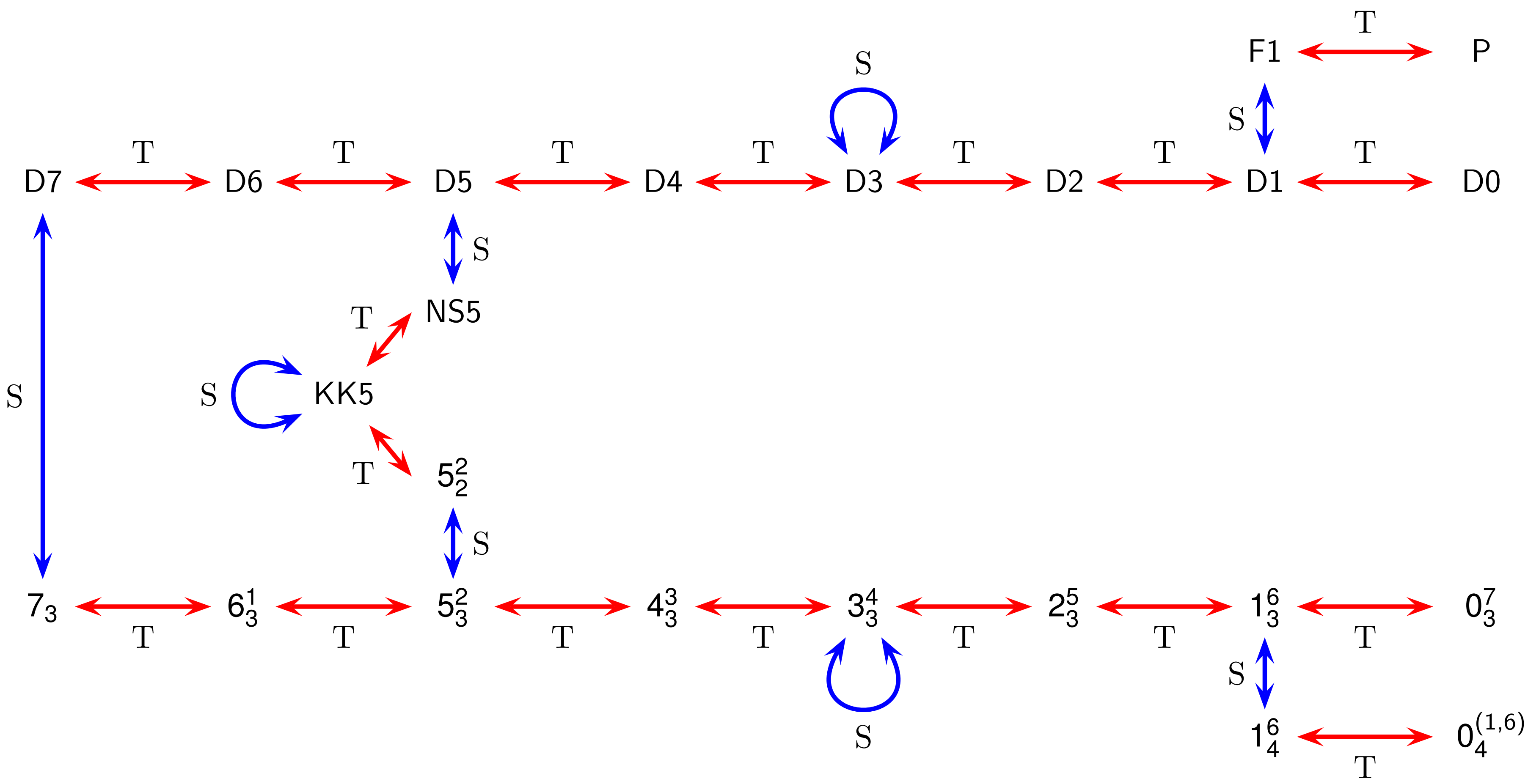

Consider charged particles in 3D maximal supergravity :

They are D7-brane wrapped on 7-torus and its dualized objects

D-dim	IIB				IIA			
3 (240)	D7 (1)	D5 (21)	D3 (35)	D1 (7)	D6 (7)	D4 (35)	D2 (21)	D0 (1)
	F1 (7)	P (7)	NS5 (21)	KK5 (42)	F1 (7)	P (7)	NS5 (21)	KK5 (42)
	1^6_4 (7)	$0_4^{(1,6)}$ (7)	5^2_2 (21)		1^6_4 (7)	$0_4^{(1,6)}$ (7)	5^2_2 (21)	
	7_3 (1)	5^2_3 (21)	3^4_3 (35)	1^6_3 (7)	6^1_3 (7)	4^3_3 (35)	2^5_3 (21)	0^7_3 (1)

# of charged particles $=$ # of $U(1)$ gauge one-form potentials

$=$ # of scalar fields

$$= \dim(E_{8(8)}/SO(16)) = 128 < 240 \, !$$

Consider charged particles in 3D maximal supergravity :

They are D7-brane wrapped on 7-torus and its dualized objects

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	1^6_4 (7)	$0_4^{(1,6)}$ (7)	5^2_2 (21)		1^6_4 (7)	$0_4^{(1,6)}$ (7)	5^2_2 (21)	
	7_3 (1)	5^2_3 (21)	3^4_3 (35)	1^6_3 (7)	6^1_3 (7)	4^3_3 (35)	2^5_3 (21)	0^7_3 (1)

$$b_n^c \text{ has mass (tension)} = \frac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2}{g_s^n \, \ell_s^{b+2c+1}}$$

Consider charged particles in 3D maximal supergravity :

They are D7-brane wrapped on 7-torus and its dualized objects

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eg.) 5^2_2-particle in 3D is uplifted to 5^2_2-brane in 8D($=5+3$) (as codim-2 object).

When exotic 5^2_2-brane in 8D is embedded into 10D,

this does not depend on $2=10-8$ transverse directions. (smeared / KK-reduced)

necessary to keep aspects of codim-2 object

D	U-duality	#	IIB	IIA
10A	1	–	–	–
10B	$SL(2,\mathbb{Z})$	$2 \subset 3$	D7(1) 7_3(1)	–
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	$2 \subset 3$	D7(1) 7_3(1)	D6(1) 6^1_3(1)
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	$6 \subset (8,1)$ $2 \subset (1,3)$	D7(1) 7_3(1) D5(1) 5^2_3(1) NS5(1) 5^2_2(1) KK5(2)	D6(2) 6^1_3(2) KK5(2) NS5(1) 5^2_2(1)
7	$SL(5,\mathbb{Z})$	$20 \subset 24$	D7(1) 7_3(1) D5(3) 5^2_3(3) NS5(3) 5^2_2(3) KK5(6)	D6(3) 6^1_3(3) KK5(6) D4(1) 4^3_3(1) NS5(3) 5^2_2(3)
6	$SO(5,5;\mathbb{Z})$	$40 \subset 45$	D7(1) 7_3(1) D5(6) 5^2_3(6) NS5(6) 5^2_2(6) D3(1) 3^4_3(1) KK5(12)	D6(4) 6^1_3(4) KK5(12) D4(4) 4^3_3(4) NS5(6) 5^2_2(6)
5	$E_{6(6)}(\mathbb{Z})$	$72 \subset 78$	D7(1) 7_3(1) D5(10) 5^2_3(10) NS5(10) 5^2_2(10) D3(5) 3^4_3(5) KK5(20)	D6(5) 6^1_3(5) KK5(20) D4(10) 4^3_3(10) D2(1) 2^5_3(1) NS5(10) 5^2_2(10)
4	$E_{7(7)}(\mathbb{Z})$	$126 \subset 133$	D7(1) 7_3(1) D5(15) 5^2_3(15) NS5(15) 5^2_2(15) D3(15) 3^4_3(15) D1(1) 1^6_3(1) F1(1) 1^6_4(1) KK5(30)	D6(6) 6^1_3(6) KK5(30) D4(20) 4^3_3(20) D2(6) 2^5_3(6) F1(1) 1^6_4(1) NS5(15) 5^2_2(15)
3	$E_{8(8)}(\mathbb{Z})$	$240 \subset 248$	D7(1) 7_3(1) D5(21) 5^2_3(21) NS5(21) 5^2_2(21) D3(35) 3^4_3(35) D1(7) 1^6_3(7) F1(7) 1^6_4(7) P(7) $0^{(1,6)}_4$(7) KK5(42)	D6(7) 6^1_3(7) KK5(42) D4(35) 4^3_3(35) D2(21) 2^5_3(21) F1(7) 1^6_4(7) D0(1) 0^7_3(1) P(7) $0^{(1,6)}_4$(7) NS5(21) 5^2_2(21)

For codim-2, all branes are (un)wrapped on torus along suitable directions.

$\rightarrow$ Defect branes

Exotic b_n^c-brane :

- charged particle in 3D, codim-2 object in $(b+3)$-dim
- pair with standard b-brane of codim-2 in $(b+3)$-dim
- c smeared transverse directions from 10D viewpoint
- tension proportional to g_s^{-n}

D7-brane (codim-2 object in 10D) has been studied for 20 years : F-theory

Vafa: hep-th/9602022

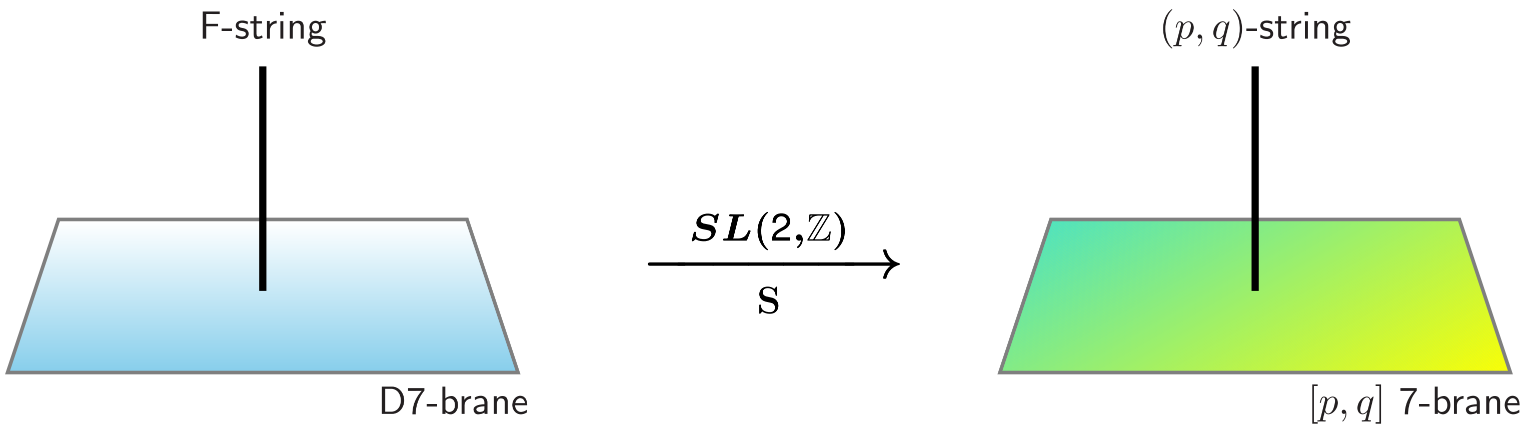
2. **Exotic $SL(2,\mathbb{Z})$ monodromy**

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

D7(1234567) : couple to $\tau(z) = C + \mathrm{i}e^{-\phi}$ $\quad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\vartheta})$

solution : $\tau(z) = \frac{\vartheta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{r}$



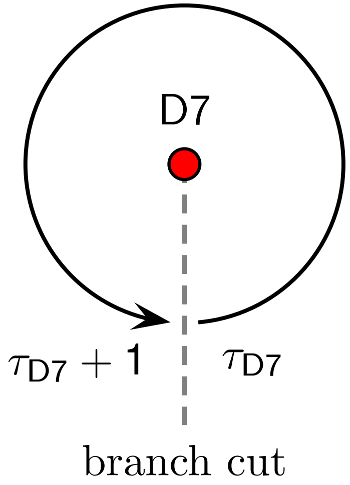
$(1,0)$-string $=$ F1 $\qquad$ $[1,0]$ 7-brane $=$ D7(1234567)

$(0,1)$-string $=$ D1 $\qquad$ $[0,1]$ 7-brane $=$ $7_3(1234567)$

Open D-string is ending on $7_3(1234567)$.

When τ moves around D7-brane counterclockwise,

it receives a magnetic "charge" of D7-brane (monodromy) : $\tau \to \tau + 1$



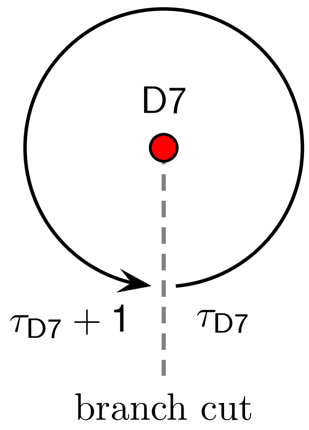
$$\boldsymbol{K}_{[1,0]} \cdot (\tau + 1) = \tau \,, \quad \boldsymbol{K}_{[1,0]} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \in \boldsymbol{SL}(2, \mathbb{Z})$$

D7-brane : (localized in 89-plane)

$$\tau_{\rm D7}(z) \equiv C + {\rm i}\,{\rm e}^{-\phi} = \frac{\vartheta}{2\pi} + \frac{\rm i}{2\pi}\log\frac{\Lambda}{r} \qquad (z = x^8 + {\rm i}x^9 = r\,{\rm e}^{{\rm i}\vartheta})$$

When $\tau_{\rm D7}$ moves around D7-brane counterclockwise $\vartheta \to \vartheta + 2\pi$,

it receives a magnetic “charge” (monodromy) : $\tau_{\rm D7} \to \tau_{\rm D7} + 1$

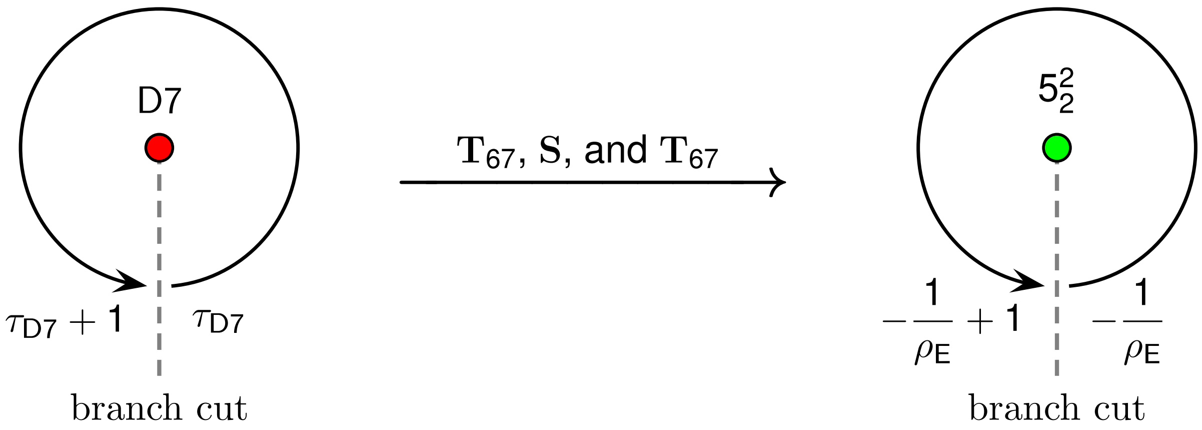


Exotic 5^2_2-brane : (localized in 89-plane, smeared along 67-directions)

$$\rho_{\rm E}(z) \;=\; B_{67} + {\rm i}\sqrt{\det G_{mn}} \;=\; -\Big[\frac{\vartheta}{2\pi} + \frac{\rm i}{2\pi}\log\frac{\Lambda}{r}\Big]^{-1} \qquad (z = x^8 + {\rm i}x^9 = r\,{\rm e}^{{\rm i}\vartheta})$$

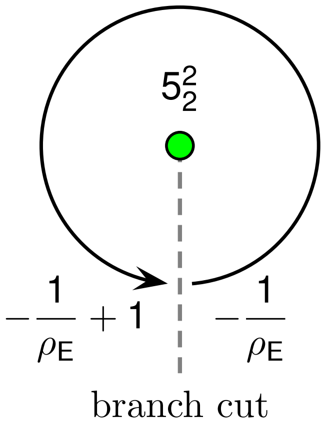
When $\rho_{\rm E}$ moves around 5^2_2-brane counterclockwise $\vartheta \to \vartheta + 2\pi$,

it receives a magnetic "charge" (monodromy) : $-1/\rho_{\rm E} \to -1/\rho_{\rm E} + 1$



$$\mathrm{D7} \xrightarrow{\mathrm{T}_{67}} \mathrm{D5} \xrightarrow{\mathrm{S}} \mathrm{NS5} \xrightarrow{\mathrm{T}_{67}} 5^2_2$$

Exotic 5^2_2-brane : (localized in 89-plane, smeared along 67-directions)



$$\boldsymbol{SL}(2,\mathbb{Z}) : \quad -\frac{1}{\rho_E} \to -\frac{1}{\rho_E} + 1 \qquad \text{where} \quad -\frac{1}{\rho_E} = \rho_{NS5}$$

We cannot remove this shift

by
- B-field gauge transformation
- coordinate transformations

This property comes from

$$\boldsymbol{SL}(2,\mathbb{Z}) \times \boldsymbol{SL}(2,\mathbb{Z}) = \boldsymbol{SO}(2,2;\mathbb{Z})$$

complex structure × complexified Kähler = T_{67}-duality

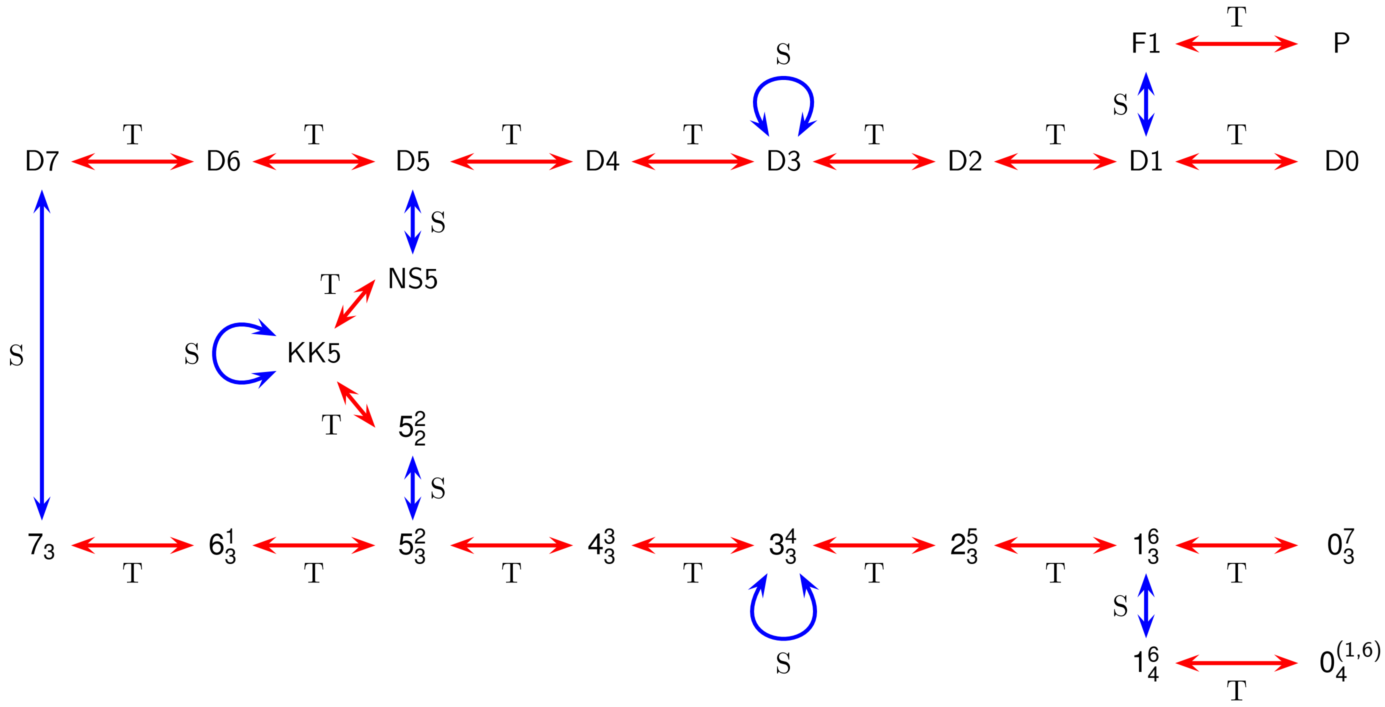
Exotic $SL(2,\mathbb{Z})$ pairs by monodromy matrix $K_{[p,q]}$:

D7 in 10-dim $\xrightarrow{K_{[p,q]}}$ $[p,q]^S_7$-brane

Db in $(b+3)$-dim $\xrightarrow{K_{[p,q]}}$ $[p,q]^E_{db}$-brane

NS5 in 8-dim $\xrightarrow{K_{[p,q]}}$ $[p,q]^T_{s5}$-brane

KK5 in 8-dim $\xrightarrow{K_{[p,q]}}$ $[p,q]^T_{k5}$-brane



TK: arXiv:1602.08606

3. Applications

D	U-duality	IIB	IIA
10B	$SL(2,\mathbb{Z})$	D7 7_3	–
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	D7 7_3	D6 6^1_3
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 NS5 5^2_2
7	$SL(5,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 D4 4^3_3 NS5 5^2_2
6	$SO(5,5;\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 D3 3^4_3 KK5	D6 6^1_3 KK5 D4 4^3_3 NS5 5^2_2
⋮	⋮	⋮	⋮

Defect branes (codim-2 branes) in diverse dimensions

Bergshoeff, Ortín, Riccioni: arXiv:1109.4484

D	U-duality	IIB	IIA
10B	$SL(2,\mathbb{Z})$	D7 7_3	–
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	D7 7_3	D6 6^1_3
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 NS5 5^2_2
7	$SL(5,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 D4 4^3_3 NS5 5^2_2
6	$SO(5,5;\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 D3 3^4_3 KK5	D6 6^1_3 KK5 D4 4^3_3 NS5 5^2_2
⋮	⋮	⋮	⋮

F-theory

Greene, Shapere, Vafa, Yau: NPB337 (1990) 1
Vafa: hep-th/9602022
and many works

D	U-duality	IIB	IIA
10B	$SL(2,\mathbb{Z})$	D7 7_3	–
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	D7 7_3	D6 6^1_3
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 NS5 5^2_2
7	$SL(5,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 D4 4^3_3 NS5 5^2_2
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⋮	⋮	⋮	⋮

3D T_3 theory and its mirror dual

Benini, Tachikawa, Xie: arXiv:1007.0992

D	U-duality	IIB	IIA
10B	$SL(2,\mathbb{Z})$	D7 7_3	–
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	D7 7_3	D6 6^1_3
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 NS5 5^2_2
7	$SL(5,\mathbb{Z})$	D7 7_3 D5 5^2_3 NS5 5^2_2 KK5	D6 6^1_3 KK5 D4 4^3_3 NS5 5^2_2
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⋮	⋮	⋮	⋮

Defect (p,q) 5-branes for 6D $\mathcal{N}=(2,0)$

TK: arXiv:1410.8403

D	U-duality	IIB	IIA
10B	$SL(2,\mathbb{Z})$	D7 7_3	–
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	D7 7_3	D6 6_3^1
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	D7 7_3 D5 5_3^2 NS5 5_2^2 KK5	D6 6_3^1 KK5 NS5 5_2^2
7	$SL(5,\mathbb{Z})$	D7 7_3 D5 5_3^2 NS5 5_2^2 KK5	D6 6_3^1 KK5 D4 4_3^3 NS5 5_2^2
6	$SO(5,5;\mathbb{Z})$	D7 7_3 D5 5_3^2 NS5 5_2^2 D3 3_3^4 KK5	D6 6_3^1 KK5 D4 4_3^3 NS5 5_2^2
⋮	⋮	⋮	⋮

G-theory

Martucci, Morales, Ricci Pacifici: arXiv:1207.6120
Braun, Fucito, Morales: arXiv:1308.0553
Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785, 1411.4786
Font, García-Etxebarria, Lüst, Massai, Mayrhofer: arXiv:1603.09361

Towards 6D $\mathcal{N} = (2, 0)$

via defect (p, q) 5-branes collapsing

Single defect (p,q) 5-brane is a solution in type IIA SUGRA : (non-vanishing p,q case)

$$
\begin{aligned}
ds^2 &= ds^2_{012345} + \frac{H}{q^2 K}\Big[(dx^6)^2 + (dx^7)^2\Big] + H\Big[(dr)^2 + r^2 (d\vartheta)^2\Big] \\
B_{67} &= -\frac{p}{q} - \frac{V}{q^2 K}, \qquad e^{2\phi} = \frac{H}{q^2 K}, \qquad \rho = B_{67} + i\sqrt{\det G_{mn}} \\
H &= \frac{1}{2\pi}\log\frac{\Lambda}{r}, \qquad V = \frac{\vartheta}{2\pi}, \qquad K = H^2 + V^2
\end{aligned}
$$

TK: arXiv:1410.8403

Special cases (1, 0) : defect NS5-brane ($[1,0]^T_{s5}$), (0, 1) : exotic 5^2_2-brane ($[0,1]^T_{s5}$)

Complex scalar ρ has $SL(2,\mathbb{Z})_\rho \subset SO(2,2;\mathbb{Z})$ monodromy matrix

$$
\begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}
$$

Completely parallel to the discussion of $[p,q]$ 7-barne, except for the stringy meaning of $SL(2,\mathbb{Z})$

Now, we assign various defect (p, q) 5-branes as follows :

$\mathbf{A}$-brane : $(1, 0)$ 5-brane (= defect NS5)

$\mathbf{B}$-brane : $(1, -1)$ 5-brane

$\mathbf{C}$-brane : $(1, +1)$ 5-brane

We can consider collapsible defect 5-branes by $\mathbf{A}, \mathbf{B}, \mathbf{C}$-branes as in F-theory

branes	symmetry	fixed point ρ_*
$\mathbf{A}^7\mathbf{B}\mathbf{C}^2$	E_8	$\mathrm{e}^{\mathrm{i}\pi/3}$
$\mathbf{A}^6\mathbf{B}\mathbf{C}^2$	E_7	i
$\mathbf{A}^5\mathbf{B}\mathbf{C}^2$	E_6	$\mathrm{e}^{\mathrm{i}\pi/3}$
$\mathbf{A}^4\mathbf{B}\mathbf{C}$	D_4	$^\forall\rho$
$\mathbf{A}\mathbf{C}$	H_0	$\mathrm{e}^{\mathrm{i}\pi/3}$
$\mathbf{A}^2\mathbf{C}$	H_1	i
$\mathbf{A}^3\mathbf{C}$	H_2	$\mathrm{e}^{\mathrm{i}\pi/3}$
$\mathbf{A}^n$	A_n	$\mathrm{i}\infty$
$\mathbf{A}^{n+4}\mathbf{B}\mathbf{C}$	D_{n+4}	$\mathrm{i}\infty$

Since each brane is of NS-type in IIA,
6D $\mathcal{N} = (2, 0)$ theory with symmetry should be realized.

Orientifold 5-plane ($\mathrm{ONS5}^-_{\mathrm{A}}$) can be constructed
by collapsing $\mathbf{B}$ and $\mathbf{C}$.

$\rho_2 = \sqrt{\det G_{mn}} = \mathrm{e}^{+2\phi}$ is not small.

G-theory

auxiliary K3 fibration over $\mathbb{C}\boldsymbol{P}^1$ in "14D" theory

(a review)

Martucci, Morales, Ricci Pacifici: arXiv:1207.6120

Braun, Fucito, Morales: arXiv:1308.0553

☞ Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785, 1411.4786

Font, García-Etxebarria, Lüst, Massai, Mayrhofer: arXiv:1603.09361

Begin with a warped CY 3-fold in type IIB theory

$$\mathrm{d}s^2_{\text{string}} = \mathrm{e}^{2A} \sum_{\mu=0}^{3} \mathrm{d}x^\mu \mathrm{d}x_\mu + \underbrace{\overbrace{\sum_{m,n=1}^{4} g_{mn}\, \mathrm{d}y^m \mathrm{d}y^n}^{\text{compact } \mathcal{M}_4} + 2\mathrm{e}^{2D}|h(z)|^2\, \mathrm{d}z\mathrm{d}\overline{z}}_{\text{CY}_3}$$

with

$$\Omega_3 = h\mathrm{d}z \wedge \big[(\mathrm{d}y^4 - \tau \mathrm{d}y^1) \wedge (\mathrm{d}y^3 - \sigma \mathrm{d}y^2) - \beta(\mathrm{d}y^1 \wedge \mathrm{d}y^4 - \mathrm{d}y^2 \wedge \mathrm{d}y^3) - \beta^2 \mathrm{d}y^1 \wedge \mathrm{d}y^2\big]$$

$$J = \mathrm{d}y^1 \wedge \mathrm{d}y^4 + \mathrm{d}y^2 \wedge \mathrm{d}y^3 + \frac{\mathrm{i}}{2}\mathrm{e}^{2D}|h|^2 \mathrm{d}z \wedge \mathrm{d}\overline{z}$$

$$\text{with} \quad \mathrm{d}\Omega_3 = 0 = \mathrm{d}J$$

Assume that A, D, h and τ, σ, β vary only over z-plane.

Rescaling by e^{-2A}, we obtain $\mathcal{N} = 2$ theory in 4D Minkowski spacetime.

Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785

Perform string dualities :

$$\text{CY}_3 \xrightarrow{\text{T}_{12}} \text{sol. B} \xrightarrow{\text{S}} \text{sol. C} \xrightarrow{\text{T}_{14}} \text{sol. A}$$

$$\mathrm{d}\Omega_3 = 0 = \mathrm{d}J \,, \qquad \mathrm{e}^{2D} = \sigma_2 \tau_2 - \beta_2^2$$

$$g_{mn} = \mathrm{e}^{-2D} \begin{pmatrix} \sigma_2 & \beta_2 & \beta_2 \sigma_1 - \beta_1 \sigma_2 & -\beta_1 \beta_2 + \sigma_2 \tau_1 \\ \beta_2 & \tau_2 & -\beta_1 \beta_2 + \sigma_1 \tau_2 & \beta_2 \tau_1 - \beta_1 \tau_2 \\ \beta_2 \sigma_1 - \beta_1 \sigma_2 & -\beta_1 \beta_2 + \sigma_1 \tau_2 & \mathrm{Im}(\overline{\beta}^2 \sigma) + |\sigma|^2 \tau_2 & |\beta|^2 \beta_2 + \mathrm{Im}(\beta \overline{\sigma \tau}) \\ -\beta_1 \beta_2 + \sigma_2 \tau_1 & \beta_2 \tau_1 - \beta_1 \tau_2 & |\beta|^2 \beta_2 + \mathrm{Im}(\beta \overline{\sigma \tau}) & \mathrm{Im}(\overline{\beta}^2 \tau) + |\tau|^2 \sigma_2 \end{pmatrix}$$

Special case ($\beta = 0$) :

$$\mathrm{d}s^2_{\text{CY}} = \frac{1}{\tau_2} \big| \mathrm{d}y^1 + \tau \mathrm{d}y^4 \big|^2 + \frac{1}{\sigma_2} \big| \mathrm{d}y^2 + \sigma \mathrm{d}y^3 \big|^2 + 2 \sigma_2 \tau_2 |h(z)|^2 \, \mathrm{d}z \mathrm{d}\overline{z}$$

Perform string dualities :

$$\mathrm{CY}_3 \xrightarrow{\mathrm{T}_{12}} \text{sol. B} \xrightarrow{\mathrm{S}} \text{sol. C} \xrightarrow{\mathrm{T}_{14}} \text{sol. A}$$

$$\mathrm{e}^{2D} = \mathrm{e}^{2\phi} = \sigma_2 \tau_2 - \beta_2^2 \,, \qquad A = 0$$

$$B_{(2)} = \tau_1 \, \mathrm{d}y^1 \wedge \mathrm{d}y^4 + \sigma_1 \, \mathrm{d}y^2 \wedge \mathrm{d}y^3 - \beta_1 (\mathrm{d}y^2 \wedge \mathrm{d}y^4 + \mathrm{d}y^1 \wedge \mathrm{d}y^3)$$

$$g_{mn} = \begin{pmatrix} \tau_2 & -\beta_2 & 0 & 0 \\ -\beta_2 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_2 & -\beta_2 \\ 0 & 0 & -\beta_2 & \tau_2 \end{pmatrix}$$

general systems of intersecting NS5-branes, and their U-duals

All branes are codim-2 in 6D

Perform string dualities :

$$\mathrm{CY}_3 \xrightarrow{\mathrm{T}_{12}} \text{sol. B} \xrightarrow{\mathrm{S}} \text{sol. C} \xrightarrow{\mathrm{T}_{14}} \text{sol. A}$$

$$\mathrm{e}^{2D} = \mathrm{e}^{-2A} = \mathrm{e}^{-\phi} = \sqrt{\sigma_2 \tau_2 - \beta_2^2}$$

$$C_{(2)} = \tau_1 \, \mathrm{d}y^1 \wedge \mathrm{d}y^4 + \sigma_1 \, \mathrm{d}y^2 \wedge \mathrm{d}y^3 - \beta_1 (\mathrm{d}y^2 \wedge \mathrm{d}y^4 + \mathrm{d}y^1 \wedge \mathrm{d}y^3)$$

$$g_{mn} = \mathrm{e}^{2A} \begin{pmatrix} \tau_2 & -\beta_2 & 0 & 0 \\ -\beta_2 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_2 & -\beta_2 \\ 0 & 0 & -\beta_2 & \tau_2 \end{pmatrix}$$

general systems of intersecting D5-branes, and their U-duals

All branes are codim-2 in 6D

Perform string dualities :

$$\mathrm{CY}_3 \xrightarrow{\mathrm{T}_{12}} \text{sol. B} \xrightarrow{\mathrm{S}} \text{sol. C} \xrightarrow{\mathrm{T}_{14}} \text{sol. A}$$

$$g_{mn} = \mathrm{e}^{\phi-2A}\delta_{mn}\,, \quad \mathrm{e}^{2D} = \mathrm{e}^{-2A} = \sqrt{\sigma_2\tau_2 - \beta_2^2}\,, \quad \mathrm{e}^{-\phi} = \tau_2$$

$$C = \tau_1$$

$$C_{(4)} = \Big(-\sigma_1 + \frac{2\beta_1\beta_2}{\tau_2} - \frac{\tau_1\beta_2^2}{\tau_2^2}\Big)\,\mathrm{d}y^1\wedge\mathrm{d}y^2\wedge\mathrm{d}y^3\wedge\mathrm{d}y^4$$

$$B_{(2)} = -\frac{\beta_2}{\tau_2}\,(\mathrm{d}y^1\wedge\mathrm{d}y^2 - \mathrm{d}y^3\wedge\mathrm{d}y^4)$$

$$C_{(2)} = \Big(\beta_1 - \frac{\tau_1\beta_2}{\tau_2}\Big)\,(\mathrm{d}y^1\wedge\mathrm{d}y^2 - \mathrm{d}y^3\wedge\mathrm{d}y^4)$$

general systems of parallel D3- and D7-branes, and their U-duals

All branes are codim-2 in 6D

In each solution, $\tau(z)$, $\sigma(z)$, $\beta(z)$ can be interpreted as

complex structure deformations of auxiliary K3 surface fibred over z-plane

$$\mathbb{R}^{1,3} \times \underbrace{\mathcal{M}_4 \times \mathbb{C} \times \text{auxiliary K3}}_{\mathcal{M}_{10}} \quad \longleftarrow \ \text{14D}$$

with $\tau, \sigma, \beta \in \dfrac{SO(2,3)}{SO(2) \times SO(3)}$: K3 with Picard number $20 - 3$

U-duality $SO(2,3)$ is generated by

$$\begin{aligned}
S&: \quad \tau \to -\frac{1}{\tau}\,, \qquad \sigma \to -\frac{1}{2\tau}\beta^2\,, \qquad \beta \to \frac{1}{\tau}\beta \\
T&: \quad \tau \to \tau + 1 \\
W&: \quad \beta \to \beta + 1 \\
R&: \quad \sigma \leftrightarrow \tau
\end{aligned}$$

τ, σ, β can be regarded as parameters of genus-2 surface with monodromy $SO(2,3) \simeq Sp(4)$

auxiliary K3 is genus-2 Riemann surface fibred over $\mathbb{C}P^1$

For U-duality invariance and global compactness, we choose

$$\mathrm{e}^{2D-2A}|h(z)|^2\,\mathrm{d}z\mathrm{d}\overline{z} \;=\; (\sigma_2\tau_2 - \beta_2^2)\,|h(z)|^2\,\mathrm{d}z\mathrm{d}\overline{z}$$

$$\text{with} \quad h(z) \;=\; \left(\frac{\chi_{12}(\Pi)}{\prod_{i=1}^{24}(z-z_i)}\right)^{\frac{1}{12}}$$

$\chi_{12}(\Pi)$: cusp form of weight 12 of genus-2 Riemann surface with $\Pi = \begin{pmatrix} \sigma & \beta \\ \beta & \tau \end{pmatrix}$

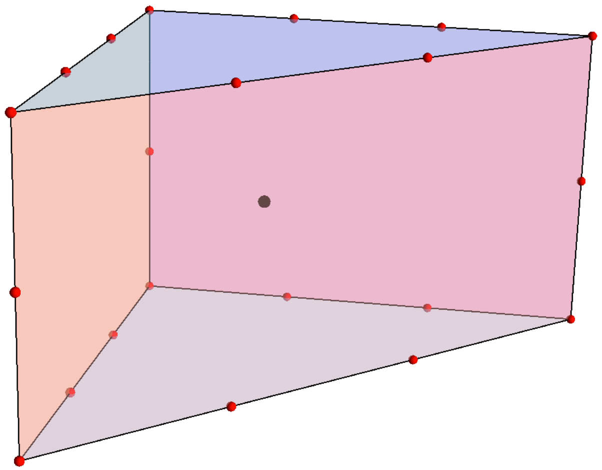
Malmendier, Morrison: arXiv:1406.4873

In a special case $\beta = 0 \;\to\; \tau, \sigma \in \left(\frac{SL(2)}{U(1)}\right)^2$

$$h(z) \;\to\; \frac{\eta(\sigma)^2\eta(\tau)^2}{\prod_{i=1}^{24}(z-z_i)^{\frac{1}{12}}}$$

In both cases, the number of branes introduced is 24.

An example of auxiliary K3 with Picard number $20 - 2$ (2 complex deformations)



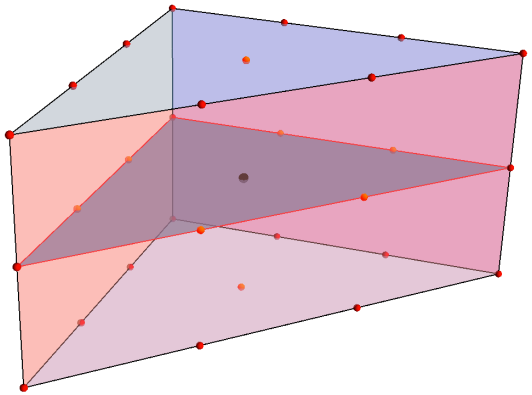
This K3 is fibred over $\mathbb{C}\boldsymbol{P}^1$.

At degeneration point of K3 (or its subspace), the symmetry is enhanced.

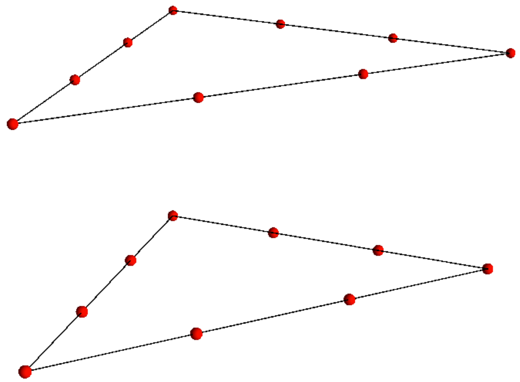
This symmetry ehancement can be seen as the edges in the above polyhedron.

(Any details are skipped, sorry!)

Figures from Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785

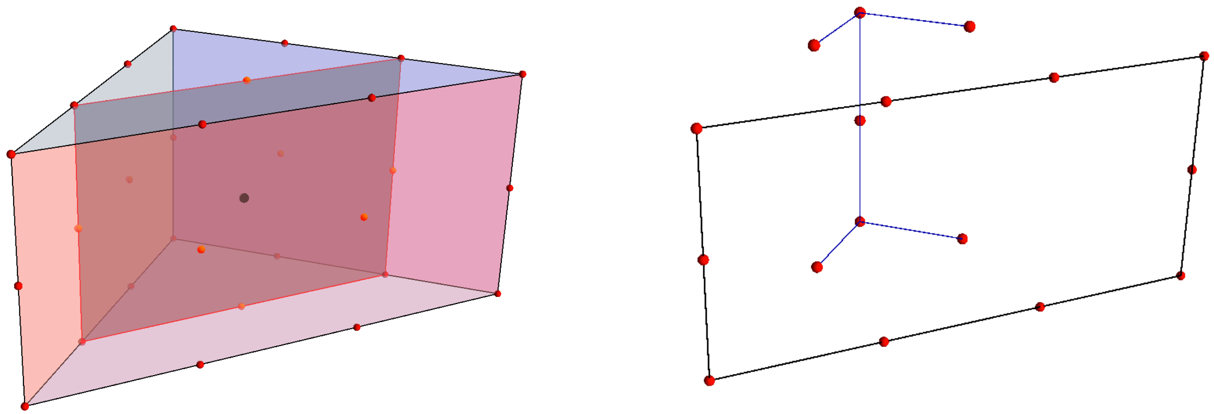


A sub-polyhedron describing elliptic curve in K3

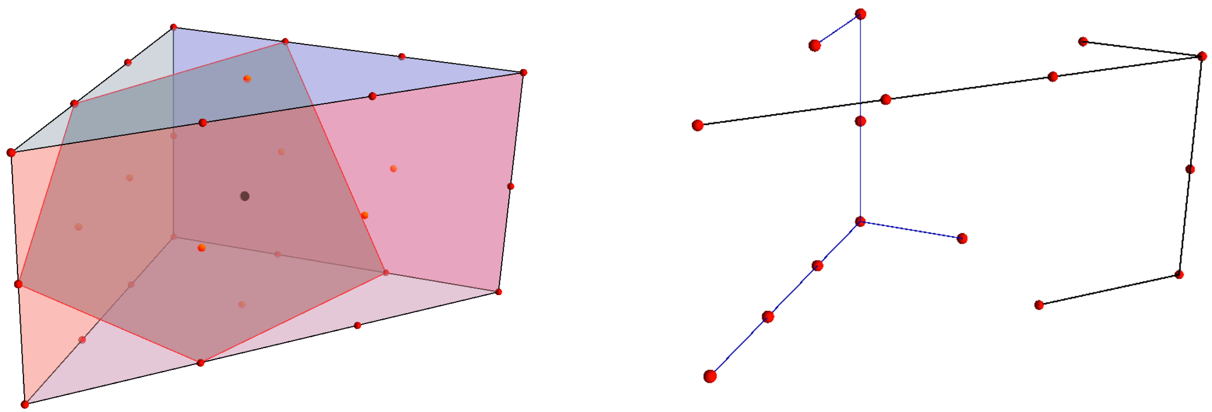


Affine Dynkin diagram for $SU(9) \times SU(9)$
Each symmetry appears at each degeneration point.

Figures from Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785

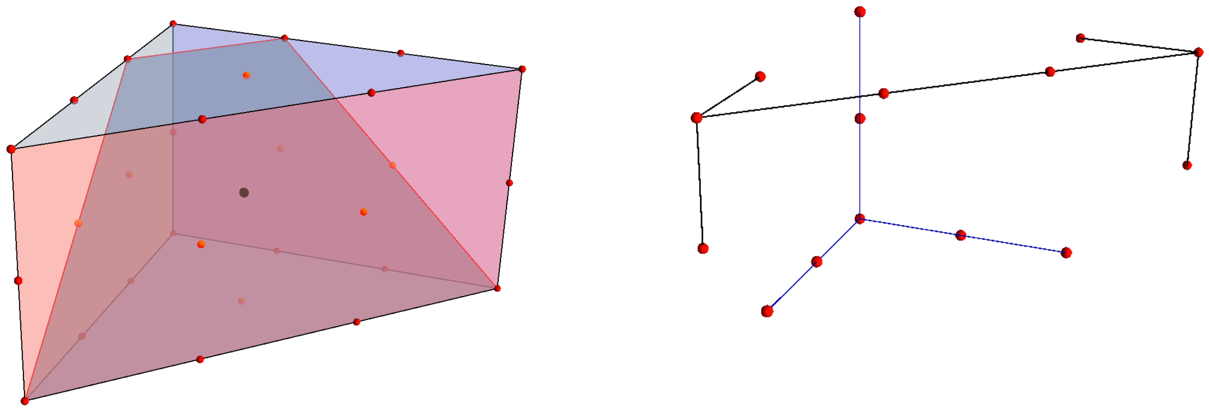


$SO(12) \times SU(10)$

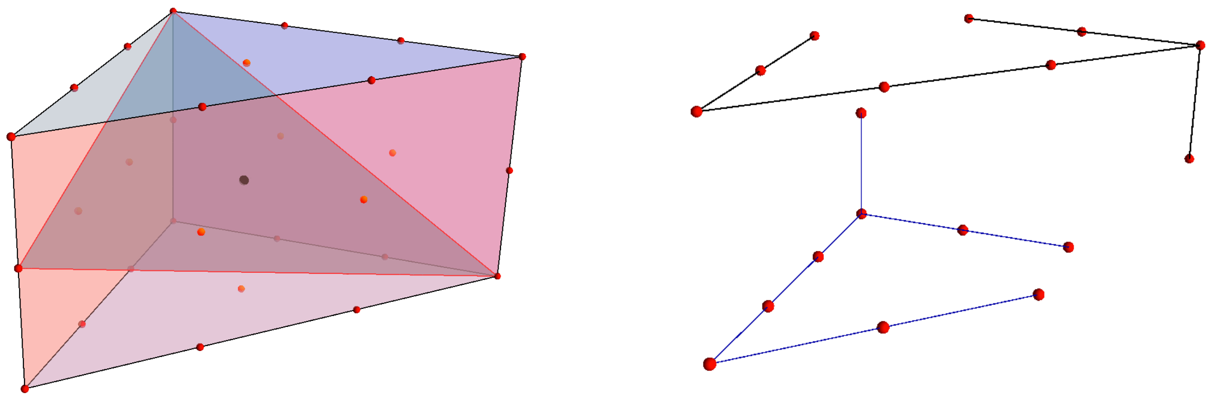


$E_7 \times E_7$

Figures from Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785



$SO(14) \times E_6$



$E_8 \times E_8$

Figures from Candelas, Constantin, Damian, Larfors, Morales: arXiv:1411.4785

In any case, the space $\mathcal{M}_{10} = \mathcal{M}_4 \times$ auxiliary K3 $\ltimes \mathbb{C}\boldsymbol{P}^1$ is compact.

In order to balance the energy in $\mathcal{M}_{10}$, we should introduce orientifold-planes.

The O-planes should also be given by codim-2 branes, in such a way that

$$\text{O7}^- = \text{colliding } \mathbf{B}\text{- and } \mathbf{C}\text{-branes} \quad \text{in F-theory.}$$

Sen: hep-th/9605150

	$\mathbf{B}$	$\mathbf{C}$
sol.A	$[1,-1]^{\boldsymbol{E}}_{\boldsymbol{d}3}$, $[1,-1]^{\boldsymbol{S}}_{7}$	$[1,1]^{\boldsymbol{E}}_{\boldsymbol{d}3}$, $[1,1]^{\boldsymbol{S}}_{7}$
sol.B	$[1,-1]^{\boldsymbol{T}}_{\boldsymbol{s}5}$	$[1,+1]^{\boldsymbol{T}}_{\boldsymbol{s}5}$
sol.C	$[1,-1]^{\boldsymbol{E}}_{\boldsymbol{d}5}$	$[1,+1]^{\boldsymbol{E}}_{\boldsymbol{d}5}$

Each is an exotic $\boldsymbol{SL}(2,\mathbb{Z})$ pair (a subgroup of U-duality)

$[1,0]^{\boldsymbol{E}}_{\boldsymbol{dp}}$: defect D$\boldsymbol{p}$ $[0,1]^{\boldsymbol{E}}_{\boldsymbol{dp}}$: exotic $\boldsymbol{p}^{7-\boldsymbol{p}}_3$ $[1,0]^{\boldsymbol{T}}_{\boldsymbol{s}5}$: defect NS5 $[0,1]^{\boldsymbol{T}}_{\boldsymbol{s}5}$: exotic 5^2_2 $[\boldsymbol{p},\boldsymbol{q}]^{\boldsymbol{S}}_7$: $[\boldsymbol{p},\boldsymbol{q}]$ 7-brane

4. Summary and discussions

- Exotic branes are codim-2 objects in D-dim string theory.
- Exotic $SL(2,\mathbb{Z})$ monodromy and its extension.
- Towards 6D $\mathcal{N} = (2,0)$ and 4D $\mathcal{N} = 2$ via generating orientifold planes.

- Genuinely exotic configurations? (by exceptional field theory)
- Quantitative predictions?

Thanks

Appendix

Consider charged particles in $D \geq 4$ maximal supergravity :

# of charged particles $=$ # of $U(1)$ gauge one-form potentials

They can be regarded as standard branes (Dp, NS5, KK5, F1, P)
(un)wrapped on torus T^{10-D} in type II string theory

D	#	IIB	IIA
10A	1	–	D0$_{(1)}$
10B	0	–	–
9	3	D1$_{(1)}$ F1$_{(1)}$ P$_{(1)}$	F1$_{(1)}$ D0$_{(1)}$ P$_{(1)}$
8	3×2	D1$_{(2)}$ F1$_{(2)}$ P$_{(2)}$	D2$_{(1)}$ F1$_{(2)}$ D0$_{(1)}$ P$_{(2)}$
7	10	D3$_{(1)}$ D1$_{(3)}$ F1$_{(3)}$ P$_{(3)}$	D2$_{(3)}$ F1$_{(3)}$ D0$_{(1)}$ P$_{(3)}$
6	16	D3$_{(4)}$ D1$_{(4)}$ F1$_{(4)}$ P$_{(4)}$	D4$_{(1)}$ D2$_{(6)}$ F1$_{(4)}$ D0$_{(1)}$ P$_{(4)}$
5	27	D5$_{(1)}$ NS5$_{(1)}$ D3$_{(10)}$ D1$_{(5)}$ F1$_{(5)}$ P$_{(5)}$	NS5$_{(1)}$ D4$_{(5)}$ D2$_{(10)}$ F1$_{(5)}$ D0$_{(1)}$ P$_{(5)}$
4	28×2	D5$_{(6)}$ NS5$_{(6)}$ KK5$_{(6)}$ D3$_{(20)}$ D1$_{(6)}$ F1$_{(6)}$ P$_{(6)}$	D6$_{(1)}$ NS5$_{(6)}$ KK5$_{(6)}$ D4$_{(15)}$ D2$_{(15)}$ F1$_{(6)}$ D0$_{(1)}$ P$_{(6)}$

Transformations via string duality :

$$\mathrm{T}_y \; : \quad R_y \to \frac{\ell_s^2}{R_y}\,, \quad g_s \to \frac{\ell_s}{R_y}\, g_s$$

$$\mathrm{S} \; : \quad g_s \to \frac{1}{g_s}\,, \quad \ell_s^2 \to g_s\, \ell_s^2$$

R_y : compact radius of y-direction

g_s : string coupling constant

ℓ_s : string length

$$\text{tension of } b_n^c \; = \; \frac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2}{g_s^n \, \ell_s^{b+2c+1}}$$

$$\text{tension of } b_n^{(d,c)} \; = \; \frac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2 \, (R_{b+c+1} \cdots R_{b+c+d})^3}{g_s^n \, \ell_s^{b+2c+3d+1}}$$

# of defect branes of codim-2 in D-dim $(\equiv n_{\mathrm{D}})$

$$= \text{\# of supersymmetric Wess-Zumino couplings}$$

$$= \dim G - (\mathrm{rank}\, T + 1)$$

$$= \dim G - \mathrm{rank}\, G$$

$$= 2 \times \underbrace{\text{\# of } (D-2)\text{-form central charges in maximal SUSY algebra}}_{= \dim H}$$

D	n_{D}	G	H	T
10A	0	$\mathbb{R}^+$	1	1
10B	2	$SL(2,\mathbb{R})$	$SO(2)$	1
9	2	$SL(2,\mathbb{R}) \times \mathbb{R}^+$	$SO(2) \times \mathbb{R}^+$	$SO(1,1)$
8	$6+2$	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	$SO(2,2)$
7	20	$SL(5,\mathbb{R})$	$SO(5)$	$SO(3,3)$
6	40	$SO(5,5)$	$SO(5) \times SO(5)$	$SO(4,4)$
5	72	$E_{6(6)}$	$USp(8)$	$SO(5,5)$
4	126	$E_{7(7)}$	$SU(8)$	$SO(6,6)$
3	240	$E_{8(8)}$	$SO(16)$	$SO(7,7)$

Bergshoeff, Ortín, Riccioni: arXiv:1109.4484

IIB action in Einstein frame :

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G_{\mathrm{E}}}\,\Big\{R_{\mathrm{E}} - \frac{\partial_M\bar{\tau}\,\partial^M\tau}{2(\tau_2)^2} - \frac{1}{2}F^i_{(3)}\cdot\mathcal{M}_{ij}\,F^j_{(3)} - \frac{1}{4}|\widetilde{F}_{(5)}|^2\Big\}$$
$$-\frac{\epsilon_{ij}}{8\kappa^2}\int C_{(4)}\wedge F^i_{(3)}\wedge F^j_{(3)}$$

$$\tau \equiv C + \mathrm{i}e^{-\phi} \equiv \tau_1 + \mathrm{i}\tau_2\,, \qquad \mathcal{M}_{ij} \equiv \frac{1}{\tau_2}\begin{pmatrix} 1 & -\tau_1 \\ -\tau_1 & |\tau|^2 \end{pmatrix}$$

$$F^i_{(3)} \equiv \begin{pmatrix} dC_{(2)} \\ dB_{(2)} \end{pmatrix}, \qquad \widetilde{F}_{(5)} \equiv dC_{(4)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}$$

$SL(2,\mathbb{Z})$ S-duality

$$\tau \to \frac{a\tau+b}{c\tau+d}, \qquad \Lambda^i{}_j = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$F^i_{(3)} \to \Lambda^i{}_j\,F^j_{(3)}\,, \qquad \widetilde{F}_{(5)} \to \widetilde{F}_{(5)}\,, \qquad G^{\mathrm{E}}_{MN} \to G^{\mathrm{E}}_{MN}$$

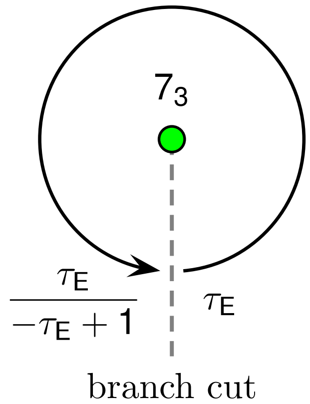
$$\mathcal{M} \to \Lambda^{-\mathrm{T}}\mathcal{M}\Lambda^{-1}$$

Monodromy matrix $K_{[1,0]}$ is transformed to $K_{[p,q]}$ for $[p,q]$ 7-brane :

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to g \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} , \quad g = \begin{pmatrix} p & r \\ q & s \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$K_{[p,q]} = g \, K_{[1,0]} \, g^{-1} = \begin{pmatrix} 1 + pq & -p^2 \\ q^2 & 1 - pq \end{pmatrix}$$

ex) monodromy $K_{[0,1]}$ for 7_3-brane : $\quad K_{[0,1]} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

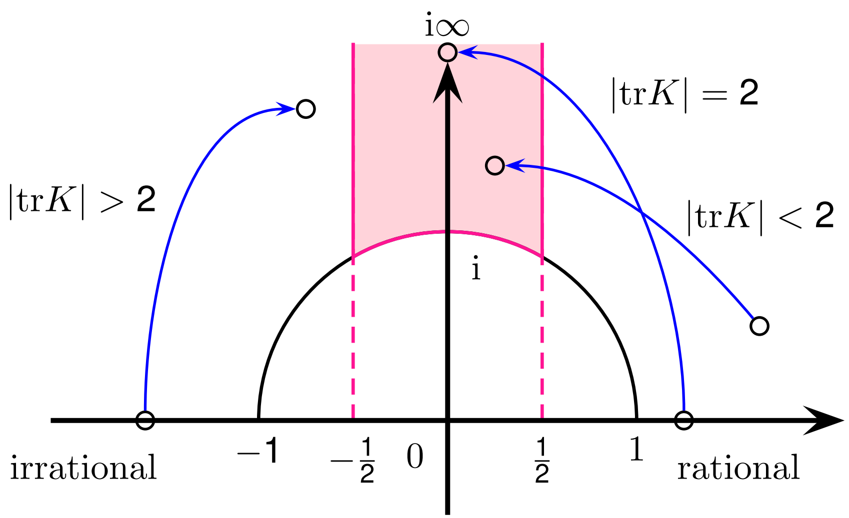


$$\tau_E = -\frac{1}{\tau_{D7}}$$

$|\mathrm{tr}K|$ is a good character to classify defect branes :

$$K \cdot \tau_* = \frac{a\tau_* + b}{c\tau_* + d} = \tau_* \,, \qquad K = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$\therefore \quad \tau_* = \frac{1}{2c}\Big\{(a-d) \pm \sqrt{(\mathrm{tr}K)^2 - 4}\Big\}$$



$|\mathrm{tr}K| = 2$: parabolic (collapsible)

$|\mathrm{tr}K| < 2$: elliptic (collapsible)

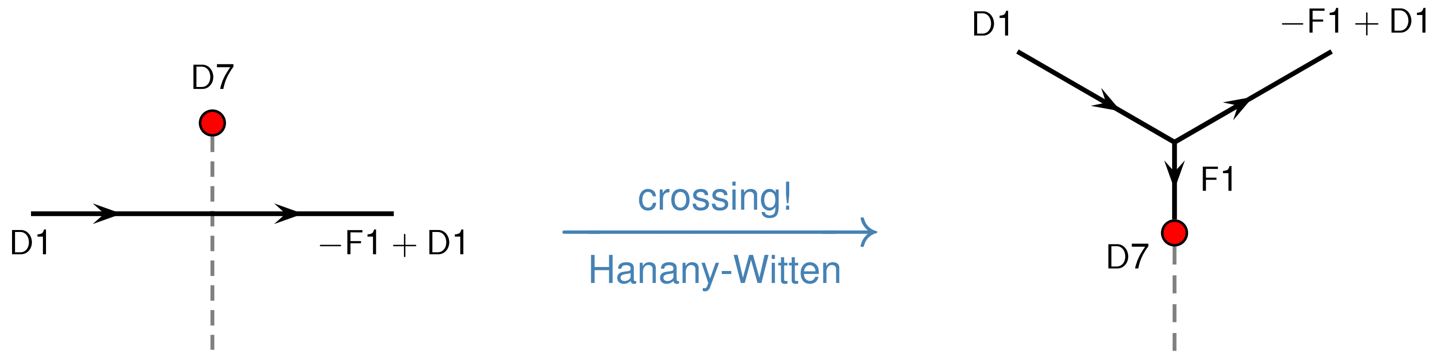
$|\mathrm{tr}K| > 2$: hyperbolic (non-collapsible)

tr$K$	monodromy	branes	collapsible?	symmetry	type
+2	$T^{-n} = K_A^n$	A^n	yes	A_{n-1} $(n \geq 1)$	I_n
	$\mathbb{1} = T^0 = K_C K_B K_C K_B K_A^8$	$\widehat{\mathrm{E}}_9 \equiv \mathrm{A}^8\mathrm{BCBC}$	yes	$\widehat{E}_9$	I_0
	$T^{\lvert n\rvert} = K_C K_B K_C K_B K_A^{8-\lvert n\rvert}$	$\mathrm{A}^{8-\lvert n\rvert}\mathrm{BCBC}$	no	$\widehat{E}_{9-\lvert n\rvert}$ $(n \leq -1)$	
+1	$ST \sim K_C K_A$	$\mathrm{H}_0 \equiv \mathrm{AC}$	yes	H_0	II
	$(ST)^{-1} \sim K_C^2 K_B K_A^7$	$\mathrm{E}_8 \equiv \mathrm{A}^7\mathrm{BC}^2$	yes	E_8	II^*
0	$S \sim K_C K_A^2$	$\mathrm{H}_1 \equiv \mathrm{A}^2\mathrm{C}$	yes	H_1	III
	$-S \sim K_C^2 K_B K_A^6$	$\mathrm{E}_7 \equiv \mathrm{A}^6\mathrm{BC}^2$	yes	E_7	III^*
−1	$-(ST)^{-1} \sim K_C K_A^3$	$\mathrm{H}_2 \equiv \mathrm{A}^3\mathrm{C}$	yes	H_2	IV
	$-ST \sim K_C^2 K_B K_A^5$	$\mathrm{E}_6 \equiv \mathrm{A}^5\mathrm{BC}^2$	yes	E_6	IV^*
−2	$-T^{-n} = K_C K_B K_A^{n+4}$	$\mathrm{D}_{n+4} \equiv \mathrm{A}^{n+4}\mathrm{BC}$	yes	D_{n+4} $(n \geq 1)$	I_n^*
	$-\mathbb{1} = -T^0 = K_C K_B K_A^4$	$\mathrm{D}_4 \equiv \mathrm{A}^4\mathrm{BC}$	yes	D_4	I_0^*
	$-T = K_C K_B K_A^3$	$\mathrm{A}^3\mathrm{BC}$	no	D_3	
	$-T^2 = K_C K_B K_A^2$	$\mathrm{A}^2\mathrm{BC}$	no	D_2	
	$-T^3 = K_C K_B K_A$	ABC	no	D_1	
	$-T^4 = K_C K_B$	BC	no	–	

$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\mathrm{A} = [1, 0]$-brane $\mathrm{B} = [1, -1]$-brane $\mathrm{C} = [1, +1]$-brane

Consider a D-string crossing the branch cut of D7-brane from the left.

A new string and a junction appear by Hanany-Witten transition.



Note: D7-brane is stretched in 1234567-directions.

This is a string junction in F-theory.

Gaberdiel and Zwiebach: hep-th/9709013
DeWolfe and Zwiebach: hep-th/9804210
etc..

[Generalization]

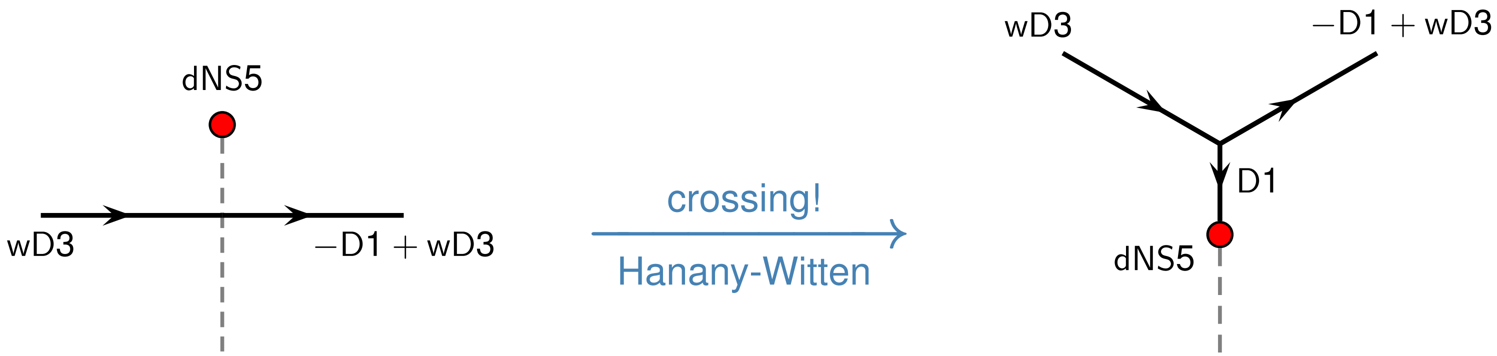
$(\boldsymbol{r}, \boldsymbol{s})$-string crossing the branch cut of $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane :

w/ charge conservation law : $\boldsymbol{qr - ps} = \pm 1$

Consider a D3-brane wrapped on two-torus (wD3) and defect NS5-brane (dNS5).

If dNS5 goes across wD3,

a new D-string and a junction appear by Hanany-Witten transition.



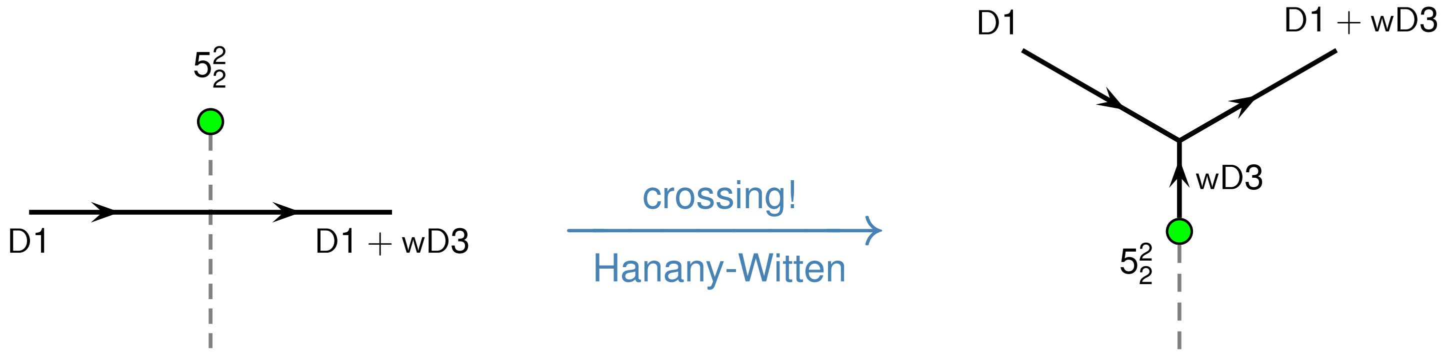
defect b-brane : b-brane of codim-2 in $(b+3)$-dim

Consider a D-string and 5_2^2-brane.

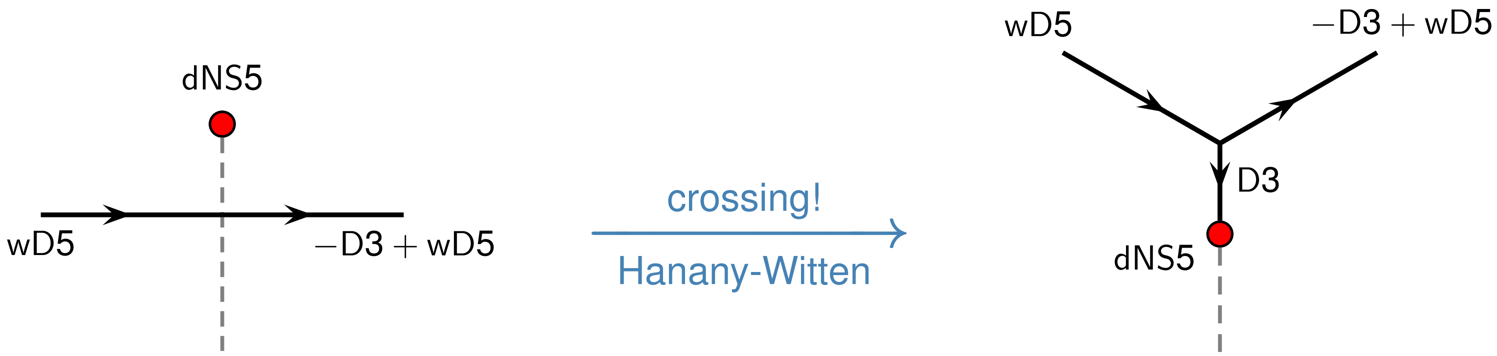
D-string charge is jumped by monodromy.

If 5_2^2-brane goes across D-string,

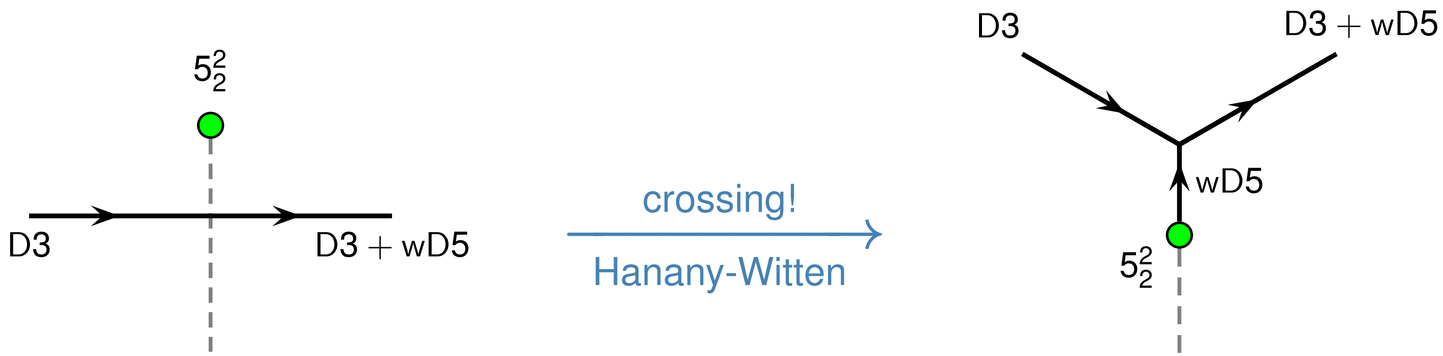
a new wD3 and a junction appear (Hanany-Witten transition).



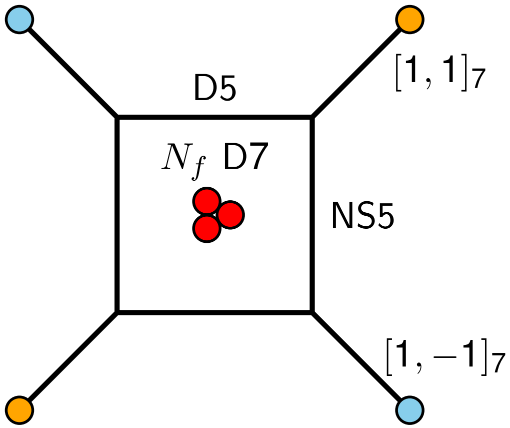
- defect NS5-brane and D5-brane wrapped on two-torus :



- 5^2_2-brane and D3-brane :



Example of brane configurations with 7-branes :



5D $\mathcal{N} = 1$ $SU(2)$ gauge theory on 5-brane web

with N_f D7-branes

$\rightarrow$ SCFT with E_{N_f+1} symmetry

Symmetry is purely determined by the singularity of fibers :

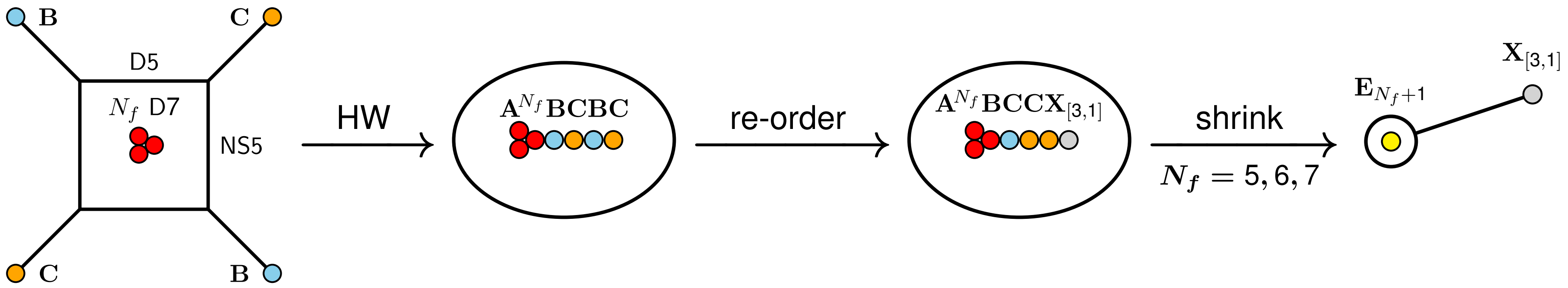
Kodaira classification (ADE-type singularity on K3 surface)

Seiberg: hep-th/9608111

Aharony, Hanany: hep-th/9704170

DeWolfe, Hanany, Iqbal, Katz: hep-th/9902179

5D $\mathcal{N} = 1$ SCFT from $SU(2)$ gauge theory with N_f flavors :



$\mathbf{A}$: D7-brane, $\mathbf{B}$: $[1,-1]_7$-brane, $\mathbf{C}$: $[1,1]_7$-brane

SCFT :

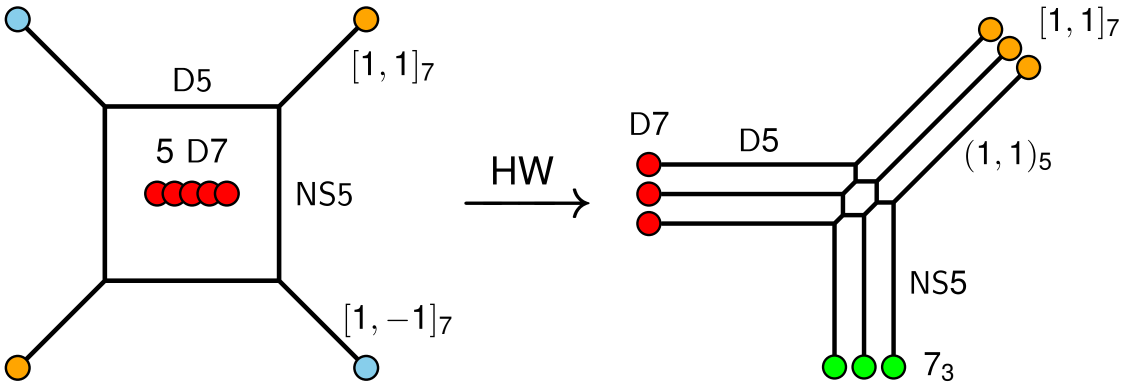
N_f	0	1	2	3	4	5	6	7
symmetry	$E_1 = A_1$	$E_2 = A_1 \times U(1)$	$E_3 = A_2 \times A_1$	$E_4 = A_4$	$E_5 = D_5$	E_6	E_7	E_8

They are purely determined by the singularity of fibers :

Kodaira classification (ADE-type singularity on K3 surface)

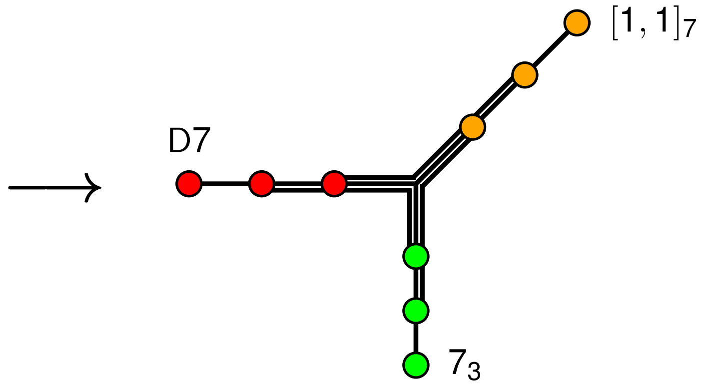
DeWolfe, Hanany, Iqbal, Katz: hep-th/9902179

5D theory on 5-branes with $N_f = 5$ D7-branes :



IIB	0	1	2	3	4	5	6	7	8	9
5 D7	–	–	–	–	–	–	–	–		
D5	–	–	–	–	–				–	
NS5	–	–	–	–	–					–
$(1,1)_5$	–	–	–	–	–				angle	

—— 5D theory ——

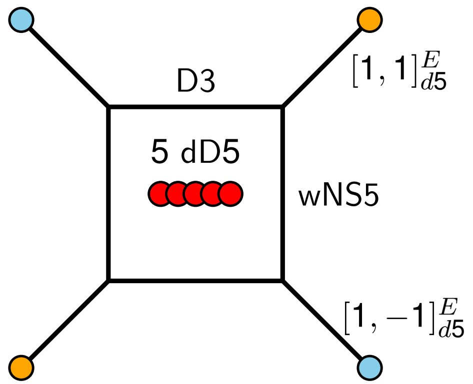


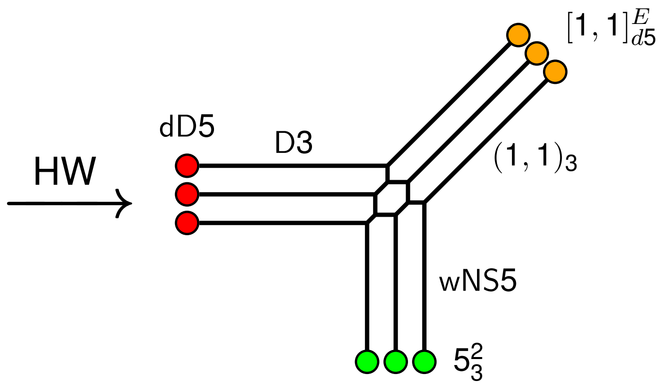
5D T_3 theory with E_6 symmetry

Gaiotto, Witten: arXiv:0804.2902, 0807.3720
Benini, Benvenuti, Tachikawa: arXiv:0906.0359

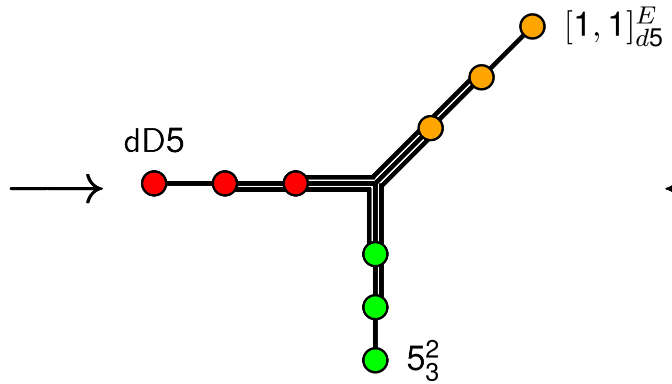
D7-branes and 7_3-branes are involved.

3D theory on 3-branes with 5 defect D5-branes :

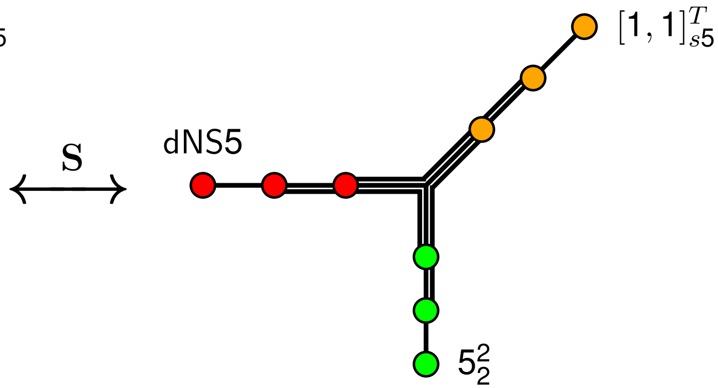




IIB	0	1	2	③	④	5	6	7	8	9
5 dD5	—	—	—			—	—	—		
D3	—	—	—						—	
wNS5	—	—	—	—	—					—
$(1,1)_3$	—	—	—						angle	
	– 3D theory –			(smeared)						

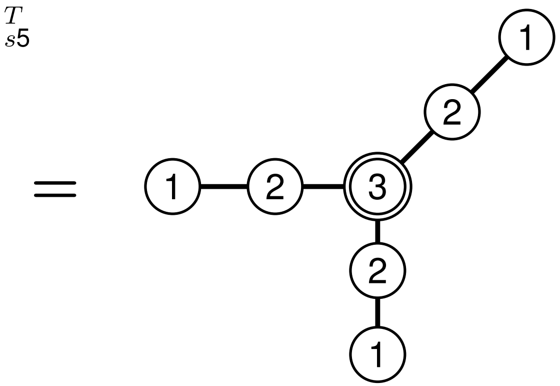


3D T_3 theory



mirror of 3D T_3 theory

=



star-shaped quiver

(dD5, 5^2_3) and (dNS5, 5^2_2) are involved.

Intriligator, Seiberg: hep-th/9609207
Benini, Tachikawa, Xie: arXiv:1007.0992

- D7(1234567) :

$$z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\vartheta}$$

$$\begin{aligned} \mathrm{d}s^2 &= \frac{1}{(\tau_2)^{1/2}}\,\mathrm{d}x^2_{01234567} + (\tau_2)^{1/2}|f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\ \mathrm{e}^{2\phi} &= (\tau_2)^{-2} \\ C_{(0)} &= \tau_1 \\ C_{(8)} &= -\frac{1}{\tau_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^7 \\ \tau(z) &= \tau_1 + \mathrm{i}\tau_2 \;=\; C_{(0)} + \mathrm{i}\mathrm{e}^{-\phi} \;=\; \frac{\vartheta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\frac{\Lambda}{r} \end{aligned}$$

Sakatani: arXiv:1412.8769

- $7_3(1234567)$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{(\tau_2')^{1/2}}\,\mathrm{d}x^2_{01234567} + (\tau_2')^{1/2}|f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= (\rho_2')^{-2} \\
C'_{(0)} &= \tau_1'\,, \qquad C'_{(8)} \;=\; -\frac{1}{\tau_2'}\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^7 \\
\tau'(z) &= \tau_1' + \mathrm{i}\tau_2' \;=\; C'_{(0)} + \mathrm{i}\mathrm{e}^{-\phi'} \;=\; -\frac{1}{\tau_{\mathrm{D7}}} \\
\tau_1' &= -\frac{\tau_1}{|\tau|^2}\,, \qquad \tau_2' \;=\; \frac{\tau_2}{|\tau|^2} \\
\tau_2'|f'|^2 &= \tau_2|f|^2\,, \qquad |f'| \;=\; |\tau||f|
\end{aligned}
$$

- defect NS5(12345) smeared along 67-directions : $z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\vartheta}$

$$
\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}x^2_{012345} + \rho_2\,\mathrm{d}x^2_{67} + \rho_2|f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \rho_2 \\
B_{(2)} &= \rho_1\,\mathrm{d}x^6 \wedge \mathrm{d}x^7\,, \qquad B_{(6)} = \frac{1}{\rho_2}\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho(z) &= \rho_1 + \mathrm{i}\rho_2 = B^{(2)}_{67} + \mathrm{i}\mathrm{e}^{2\phi} = B^{(2)}_{67} + \mathrm{i}\sqrt{\det G_{mn}} = \frac{\vartheta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\frac{\Lambda}{r} \\
\tau &= (\text{complex structure of } T^2_{67}) = \mathrm{i} \\
m, n &= 6, 7
\end{aligned}
$$

- 5^2_2(12345,67) :

$$
\begin{aligned}
\mathrm{d}s'^2 &= \mathrm{d}x^2_{012345} + \rho'_2\, \mathrm{d}x^2_{67} + \rho'_2 |f'|^2\, \mathrm{d}z \mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \rho'_2 \\
B'_{(2)} &= \rho'_1\, \mathrm{d}x^6 \wedge \mathrm{d}x^7 \,, \qquad B'_{(6)} = \frac{1}{\rho'_2} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho'(z) &= \rho'_1 + \mathrm{i}\rho'_2 = B'^{(2)}_{67} + \mathrm{i}\mathrm{e}^{2\phi'} = B'^{(2)}_{67} + \mathrm{i}\sqrt{\det G'_{mn}} = -\frac{1}{\rho_{\mathrm{NS5}}} \\
\rho'_1 &= -\frac{\rho_1}{|\rho|^2} \,, \qquad \rho'_2 = \frac{\rho_2}{|\rho|^2} \\
\tau' &= (\text{complex structure of } \widetilde{T}^2_{67}) = \mathrm{i} = -\frac{1}{\tau_{\mathrm{NS5}}} \\
\rho'_2 |f'|^2 &= \rho_2 |f|^2 \,, \qquad m, n = 6, 7
\end{aligned}
$$

- defect KK5(12345,7) smeared along 67-directions :

$$
\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}x^2_{012345} + \tau_2\, \mathrm{d}x^2_6 + \frac{1}{\tau_2}\big(\mathrm{d}x^7 - \tau_1\, \mathrm{d}x^6\big)^2 + \tau_2 |f|^2\, \mathrm{d}z \mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi} &= 1\,, \qquad B_{(2)} \;=\; 0\,, \qquad B_{(6)} \;=\; 0 \\
\rho &= \rho_1 + \mathrm{i}\rho_2 \;=\; B^{(2)}_{67} + \mathrm{i}\sqrt{\det G_{mn}} \;=\; \mathrm{i} \\
\tau(z) &= (\text{complex structure of } T^2_{67}) \;=\; \tau_1 + \mathrm{i}\tau_2 \;=\; \frac{\vartheta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{r} \\
m, n &= 6, 7
\end{aligned}
$$

- defect D3(123) smeared along 4567-directions : $z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\vartheta}$

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{(\tau_2)^{1/2}}\,\mathrm{d}x^2_{0123} + (\tau_2)^{1/2}\,\mathrm{d}x^2_{4567} + (\tau_2)^{1/2}|f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= 1 \\
\widetilde{C}_{(4)} &= \tau_1\,\mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \wedge \mathrm{d}x^7 \\
C_{(4)} &= -\frac{1}{\tau_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \\
\tau(z) &= \tau_1 + \mathrm{i}\tau_2 \;=\; \widetilde{C}^{(4)}_{4567} + \mathrm{i}\sqrt{\det G_{mn}} \\
m, n &= 4, 5, 6, 7
\end{aligned}
$$

- $3^4_3(123, 4567)$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{(\tau_2')^{1/2}}\,\mathrm{d}x^2_{0123} + (\tau_2')^{1/2}\,\mathrm{d}x^2_{4567} + (\tau_2')^{1/2}|f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= 1 \\
\widetilde{C}'_{(4)} &= \tau_1'\,\mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \wedge \mathrm{d}x^7\,, \qquad C'_{(4)} \;=\; -\frac{1}{\tau_2'}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \\
\tau'(z) &= \tau_1' + \mathrm{i}\tau_2' \;=\; \widetilde{C}'^{(4)}_{4567} + \mathrm{i}\sqrt{\det G'_{mn}} \;=\; -\frac{1}{\tau_{\mathrm{D3}}} \\
\tau_1' &= -\frac{\tau_1}{|\tau|^2}\,, \qquad \tau_2' \;=\; \frac{\tau_2}{|\tau|^2} \\
\tau_2'|f'|^2 &= \tau_2|f|^2\,, \qquad |f'| \;=\; |\tau||f|\,, \qquad m,n = 4,5,6,7
\end{aligned}
$$

- defect D$p(12\cdots p)$ smeared along $a_1\cdots a_{7-p}$-directions : $z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\vartheta}$

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{(\tau_2)^{1/2}}\,\mathrm{d}x^2_{012\cdots p} + (\tau_2)^{1/2}\,\mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\tau_2)^{1/2}|f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= (\tau_2)^{\frac{3-p}{2}} \\
C_{(7-p)} &= \tau_1\,\mathrm{d}x^{a_1}\wedge\cdots\wedge\mathrm{d}x^{a_{7-p}} \\
C_{(p+1)} &= -\frac{1}{\tau_2}\,\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^p \\
\tau(z) &= \tau_1 + \mathrm{i}\tau_2 \;=\; C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi} \;=\; C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{-\phi}\sqrt{\det G_{mn}} \\
m, n &= a_1, \ldots, a_{7-p}
\end{aligned}
$$

- $p_3^{7-p}(12\cdots p, a_1\cdots a_{7-p})$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{(\tau_2')^{1/2}}\,\mathrm{d}x^2_{012\cdots p} + (\tau_2')^{1/2}\,\mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\tau_2')^{1/2}|f'|^2\,\mathrm{d}z\mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi'} &= (\tau_2')^{\frac{3-p}{2}} \\
C'_{(7-p)} &= \tau_1'\,\mathrm{d}x^{a_1}\wedge\cdots\wedge\mathrm{d}x^{a_{7-p}}\,, \qquad C'_{(p+1)} \;=\; -\frac{1}{\tau_2'}\,\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^p \\
\tau'(z) &= \tau_1' + \mathrm{i}\tau_2' \;=\; C'^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi'} \;=\; -\frac{1}{\tau_{\mathrm{D}p}} \\
\tau_1' &= -\frac{\tau_1}{|\tau|^2}\,, \qquad \tau_2' \;=\; \frac{\tau_2}{|\tau|^2} \\
\tau_2'|f'|^2 &= \tau_2|f|^2\,, \qquad |f'| \;=\; |\tau||f|\,, \qquad m,n = a_1,\ldots,a_{7-p}
\end{aligned}
$$

- defect F1(1) smeared along 234567-directions :

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{\rho_2}\,\mathrm{d}x_{01}^2 + \mathrm{d}x_{234567}^2 + |f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \frac{1}{\rho_2} \\
B_{(6)} &= \rho_1\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B_{(2)} \;=\; -\frac{1}{\rho_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \\
\rho(z) &= \rho_1 + \mathrm{i}\rho_2 \;=\; B^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi}
\end{aligned}$$

- $1^6_4(1{,}234567)$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{\rho'_2}\,\mathrm{d}x^2_{01} + \mathrm{d}x^2_{234567} + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \frac{1}{\rho'_2} \\
B'_{(6)} &= \rho'_1\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B'_{(2)} \;=\; -\frac{1}{\rho'_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \\
\rho'(z) &= \rho'_1 + \mathrm{i}\rho'_2 \;=\; B'^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi'} \;=\; -\frac{1}{\rho_{\mathrm{F1}}} \\
\rho'_2|f'|^2 &= \rho_2|f|^2\,, \qquad |f'| \;=\; |\rho||f|
\end{aligned}
$$

- defect P smeared along 1234567-directions :

$$
\begin{aligned}
ds^2 &= -2dx^0 dx^1 + \rho_2\, dx_1^2 + dx_{234567}^2 + |f|^2\, dz d\bar{z} \\
e^{2\phi} &= \frac{1}{\rho_2}\,, \qquad B_{(2)} = 0\,, \qquad B_{(6)} = 0 \\
\rho(z) &= \rho_1 + i\rho_2 = ie^{-2\phi}
\end{aligned}
$$

- $0_4^{(1,6)}(,234567,1)$:

$$
\begin{aligned}
ds^2 &= -2dx^0 dx^1 + \rho_2'\, dx_1^2 + dx_{234567}^2 + |f'|^2\, dz d\bar{z} \\
e^{2\phi'} &= \frac{1}{\rho_2'} = \frac{|\rho|^2}{\rho_2}\,, \qquad B'_{(2)} = 0\,, \qquad B'_{(6)} = 0 \\
\rho'(z) &= \rho_1' + i\rho_2' = ie^{-2\phi'} = -\frac{1}{\rho_{\rm P}}\,, \qquad \rho_2'|f'|^2 = \rho_2|f|^2\,, \qquad |f'| = |\rho||f|
\end{aligned}
$$