Tribaryon configurations and three nucleon repulsions at short distance

(The 8th International Conference on Quarks and Nuclear Physics)

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Hyperon puzzle in neutron stars

Massive ($2M_{\odot}$) neutron stars vs softening of EOS by hyperon mixing

→ Hyperon puzzle

Hyperon puzzle in neutron stars

Massive ($2M_\odot$) neutron stars vs softening of EOS by hyperon mixing

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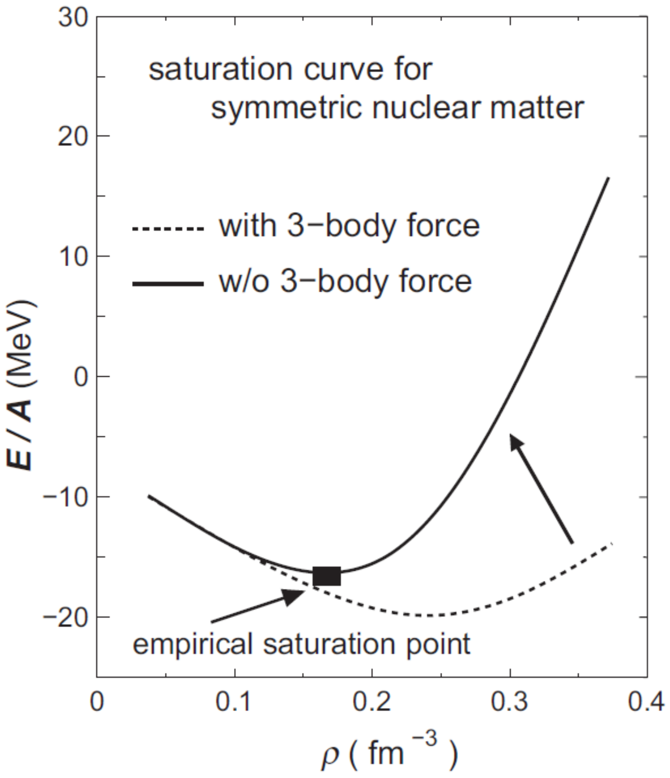
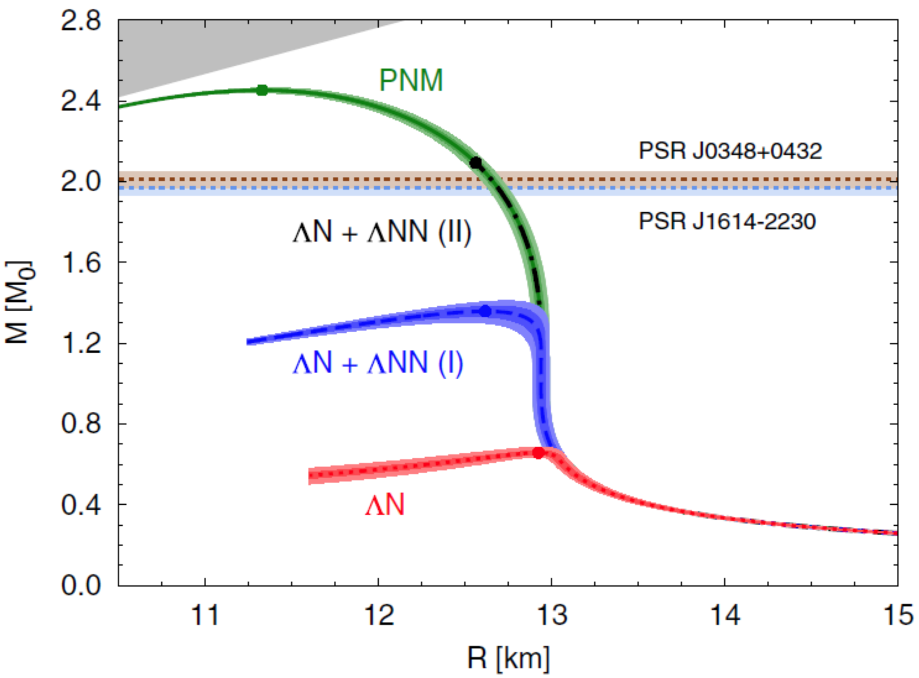
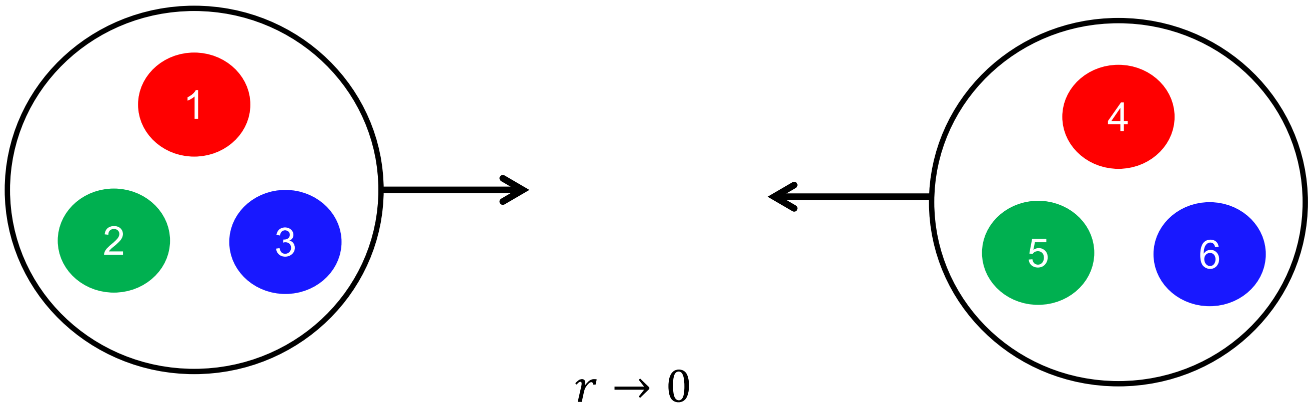


Y. Sakuragi, PTEP 2016 (2016) 06A106

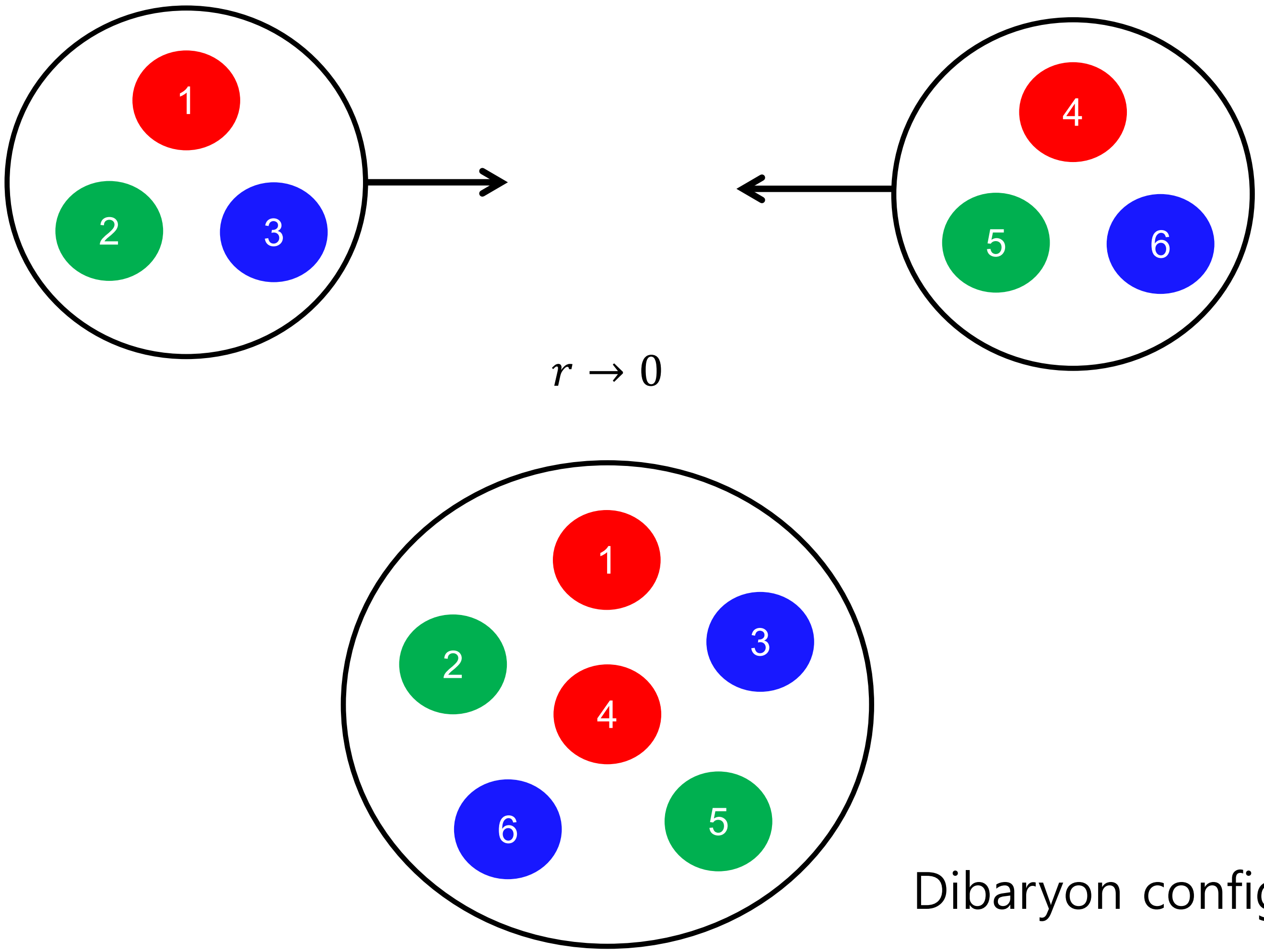


D. Lonardoni et al, PRL 114, 092301 (2015)

Baryon-baryon interaction



Baryon-baryon interaction



Dibaryon configuration

Baryon-baryon interaction

baryon ⊗ baryon → dibaryon

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$
$$8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35$$
$$10 \otimes 10 = \overline{10} \oplus 27 \oplus 28 \oplus 35.$$

TABLE II. Energies relative to threshold for dibaryon systems with an exact $SU(3)_F$ symmetry. The entries are the energy E, the threshold energy E_T and the relative energy $\Delta E = E - E_T$ for the various dibaryons. The same value $a(6q) = a(3q)$ has been used throughout.

	(F,S)									
	$(\overline{10},3)$	(27,2)	(8,2)	(35,1)	(10,1)	$(\overline{10},1)$	(8,1)	(28,0)	(27,0)	(1,0)
E	16	16	−4	$\frac{80}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	$-\frac{28}{3}$	48	8	−24
E_T	16	0	0	0	−16	−16	−16	16	−16	−16
ΔE	0	16	−4	$\frac{80}{3}$	$\frac{56}{3}$	$\frac{56}{3}$	$\frac{20}{3}$	32	24	−8

B. Silvestre-Brac and J. Leandri, Phys. Rev. D 45, 4221 (1992)

Transformation coefficients

TABLE 11

Transformation coefficients between the physical basis states (with CC denoting a hidden-color state) shown on the left of each block, and the symmetry basis states whose structure is identified above each block by the orbital symmetry [f] and isopsin-spin symmetry {f''}; each block is for the designated (TS) pairs

$T=1 \quad S=0$ $T=0 \quad S=1$	[6]{33}	[42]{33}	[42]{51}
N^2	$\sqrt{\frac{1}{9}}$	$\sqrt{\frac{4}{9}}$	$-\sqrt{\frac{4}{9}}$
Δ^2	$\sqrt{\frac{4}{45}}$	$\sqrt{\frac{16}{45}}$	$\sqrt{\frac{25}{45}}$
CC	$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$	0

S-wave orbital

M. Harvey, Nucl. Phys. A 352, 301 (1981)

Transformation coefficients

Baryon ⊗ baryon

Dibaryon

(F,S)	$8 \otimes 8$	$8 \otimes 10$	$10 \otimes 10$
(1,0)	1		
(8,1)	$\frac{4}{9}$	$\frac{5}{9}$	
(8,2)		1	
(10,1)	$\frac{1}{9}$	$\frac{8}{9}$	
$(\overline{10},1)$	$\frac{5}{9}$		$\frac{4}{9}$
$(\overline{10},3)$			1
(27,0)	$\frac{5}{9}$		$\frac{4}{9}$
(27,2)		$\frac{4}{9}$	$\frac{5}{9}$
(28,0)			1
(35,1)		$\frac{4}{9}$	$\frac{5}{9}$

$$\langle F_1 \rangle = \langle F_8 \otimes F_8 \rangle = \frac{1}{8}\Lambda\Lambda + \frac{3}{8}\Sigma\Sigma + \frac{1}{2}N\Xi$$

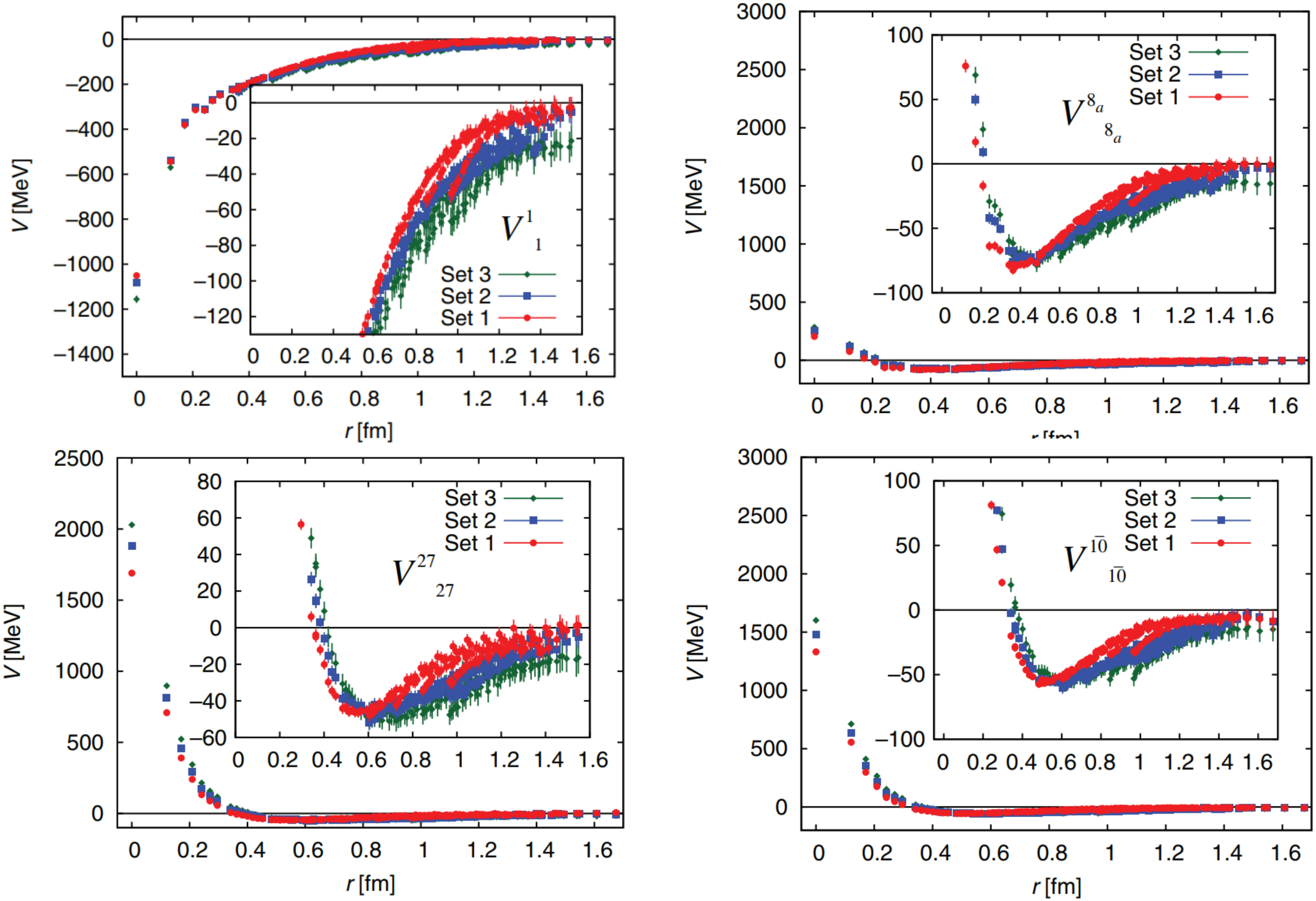
$$\begin{aligned}\langle F_{27} \rangle &= \frac{5}{9}\langle F_8 \otimes F_8 \rangle + \frac{4}{9}\langle F_{10} \otimes F_{10} \rangle \\ &= \frac{5}{9}(\frac{27}{40}\Lambda\Lambda + \frac{1}{40}\Sigma\Sigma + \frac{3}{10}N\Xi) + \frac{4}{9}\Sigma^*\Sigma^*\end{aligned}$$

$$\begin{aligned}\langle F_{10} \rangle &= \frac{1}{9}\langle F_8 \otimes F_8 \rangle + \frac{8}{9}\langle F_8 \otimes F_{10} \rangle \\ &= \frac{1}{9}(\frac{1}{2}\Sigma\Lambda + \frac{1}{6}\Sigma\Sigma + \frac{1}{3}N\Xi) + \frac{8}{9}(\frac{1}{3}\Sigma\Sigma^* + \frac{1}{3}N\Xi^* + \frac{1}{3}\Xi\Delta)\end{aligned}$$

$$\begin{aligned}\langle F_{\overline{10}} \rangle &= \frac{5}{9}\langle F_8 \otimes F_8 \rangle + \frac{4}{9}\langle F_{10} \otimes F_{10} \rangle \\ &= \frac{5}{9}(\frac{1}{2}\Sigma\Lambda + \frac{1}{6}\Sigma\Sigma + \frac{1}{3}N\Xi) + \frac{4}{9}(\frac{1}{3}\Sigma^*\Sigma^* + \frac{2}{3}\Delta\Xi^*)\end{aligned}$$

$$\begin{aligned}\langle F_8 \rangle &= \frac{4}{9}\langle F_8 \otimes F_8 \rangle + \frac{5}{9}\langle F_8 \otimes F_{10} \rangle \\ &= \frac{4}{9}N\Xi + \frac{5}{9}(\frac{3}{5}\Sigma\Sigma^* + \frac{2}{5}N\Xi^*)\end{aligned}$$

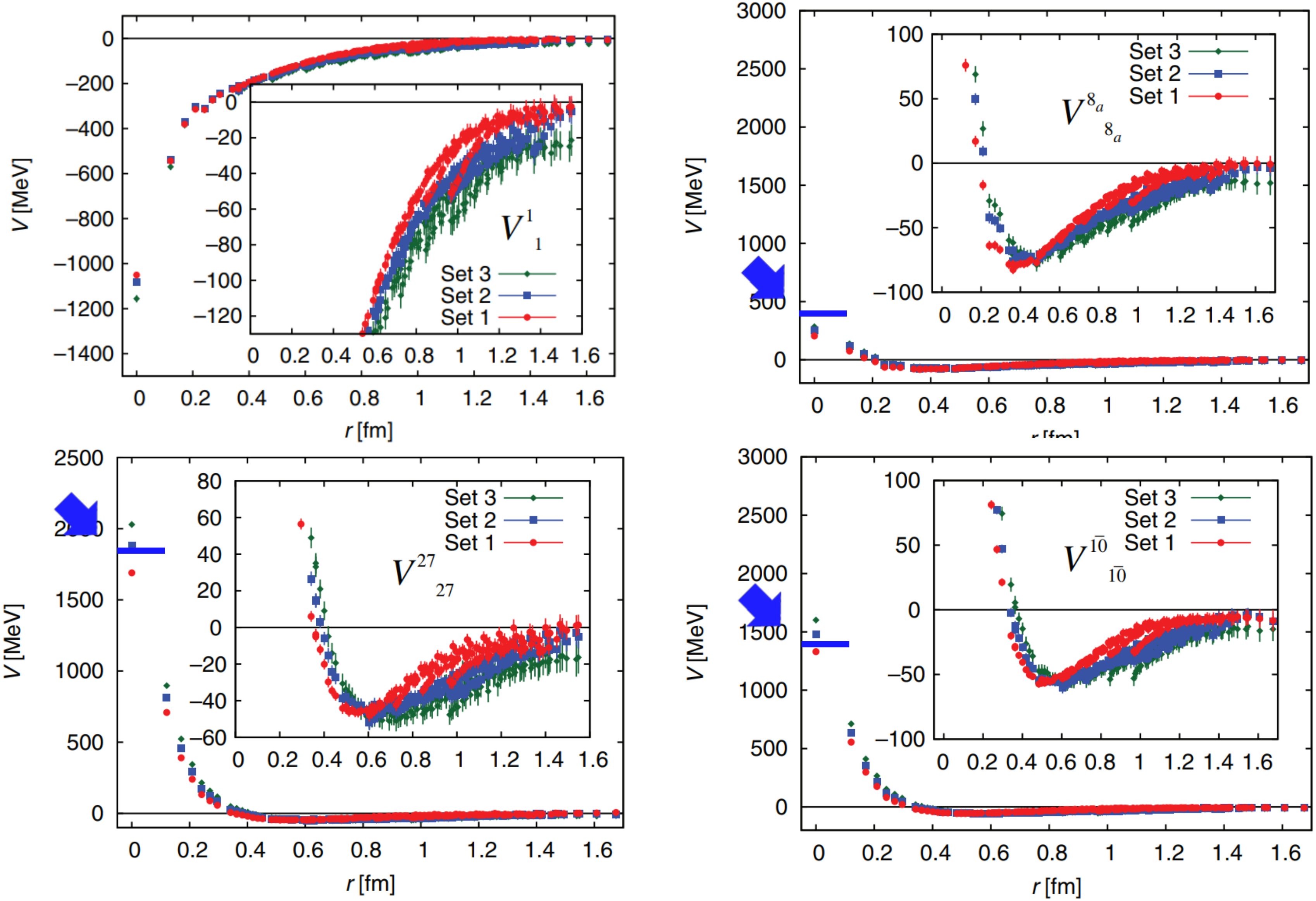
Lattice QCD & Quark Model



QNP2018

K. Sasaki *et al.*(HAL QCD Collaboration), PTEP 2015, 113B01

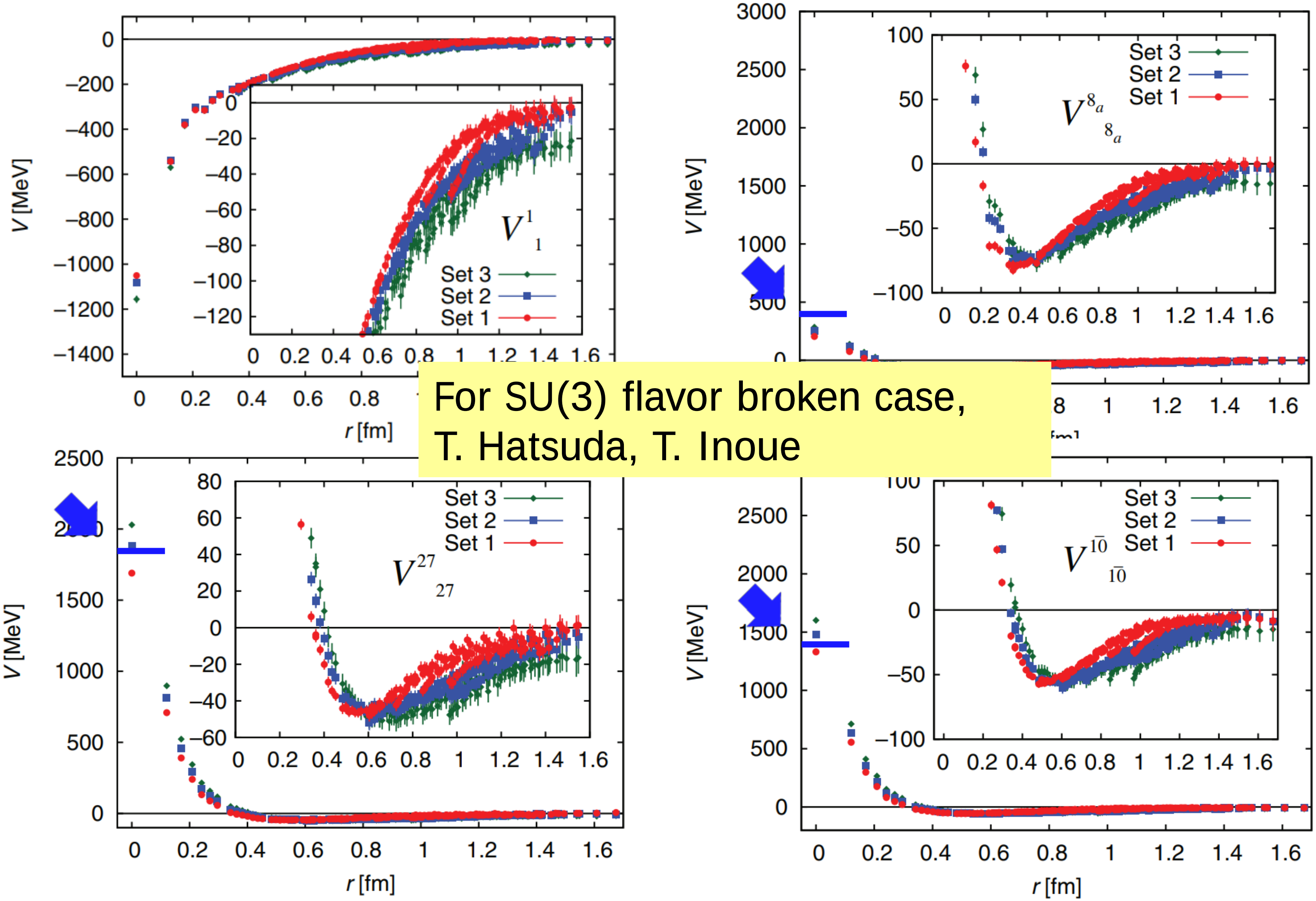
Lattice QCD & Quark Model



QNP2018

K. Sasaki *et al.*(HAL QCD Collaboration), PTEP 2015, 113B01

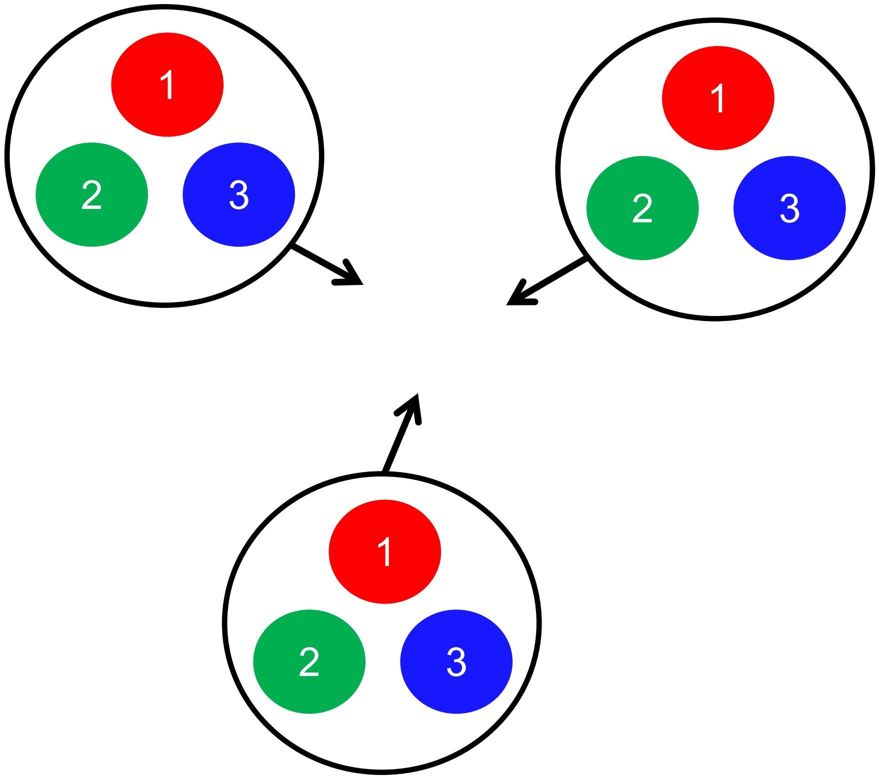
Lattice QCD & Quark Model



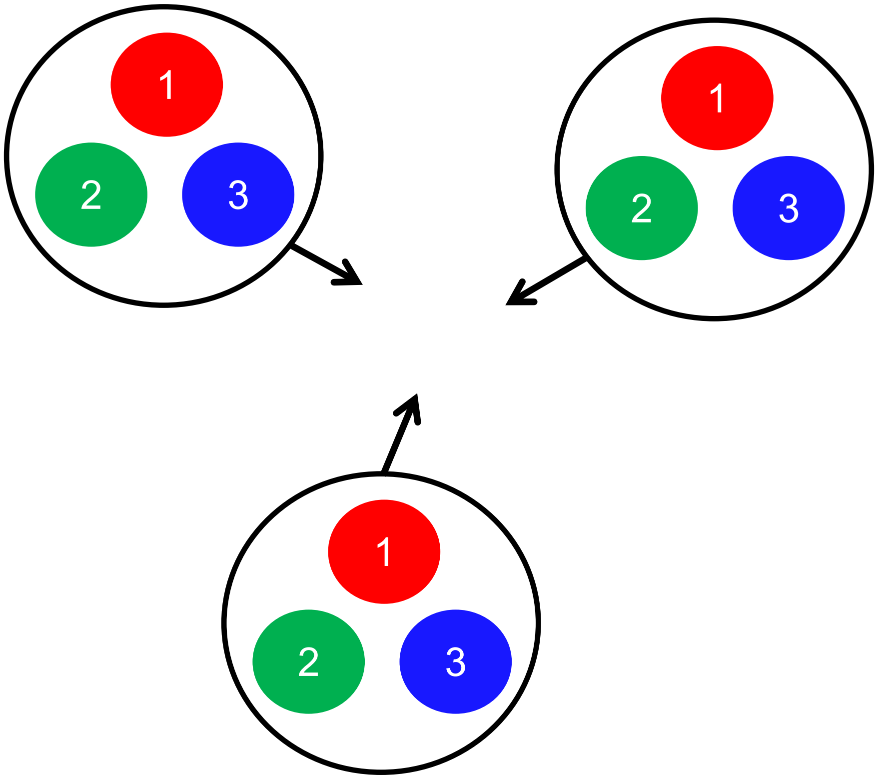
For SU(3) flavor broken case,
T. Hatsuda, T. Inoue

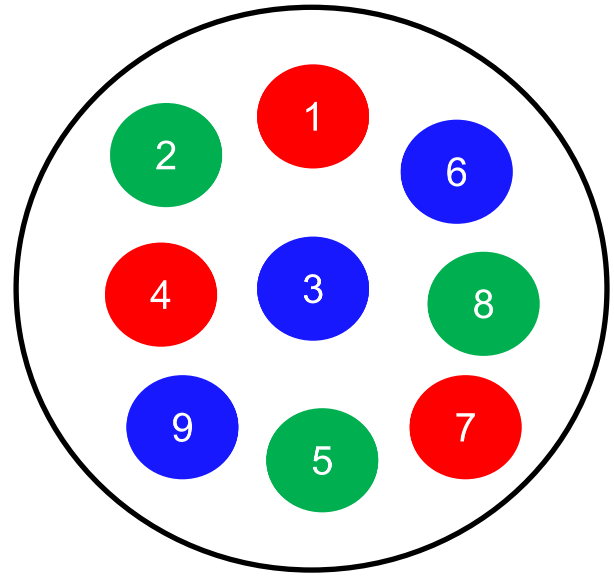
K. Sasaki *et al.*(HAL QCD Collaboration), PTEP 2015, 113B01

Three-body interaction



Three-body interaction





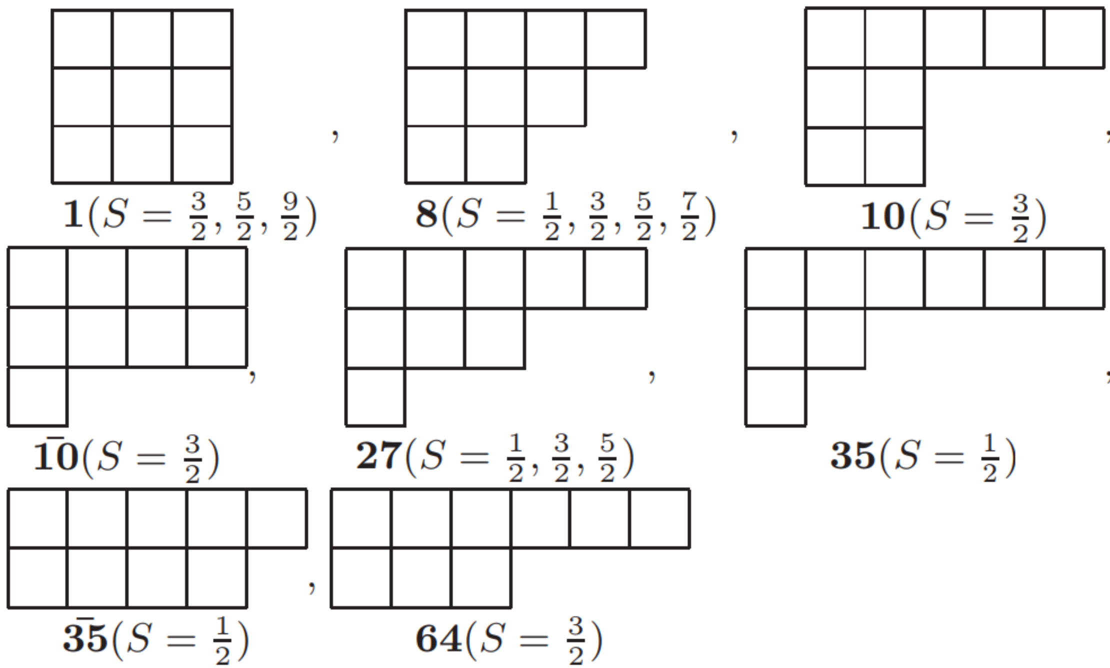
Tribaryon configuration

Flavor state of the tribaryon

Wave function = Orbital ⊗ Color ⊗ Flavor ⊗ Spin

$$[333]_{FS} = [63]_F \otimes [63]_S + [54]_F \otimes [54]_S + [621]_F \otimes [54]_S + [531]_F \otimes [72]_S + [531]_F \otimes [63]_S + [531]_F \otimes [54]_S + [522]_F \otimes [63]_S + [441]_F \otimes [63]_S + [432]_F \otimes [81]_S + [432]_F \otimes [72]_S + [432]_F \otimes [63]_S + [432]_F \otimes [54]_S + [333]_F \otimes [9]_S + [333]_F \otimes [72]_S + [333]_F \otimes [63]_S.$$

Flavor and spin states of tribaryon :



$\mathbf{1}(S = \frac{3}{2}, \frac{5}{2}, \frac{9}{2})$, $\mathbf{8}(S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2})$, $\mathbf{10}(S = \frac{3}{2})$, $\overline{\mathbf{10}}(S = \frac{3}{2})$, $\mathbf{27}(S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2})$, $\mathbf{35}(S = \frac{1}{2})$, $\overline{\mathbf{35}}(S = \frac{1}{2})$, $\mathbf{64}(S = \frac{3}{2})$

Hyperfine potential

$$K = -\sum_{i<j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$$
$$= N(N-10) + \frac{4}{3} S(S+1) + 4C_F + 2C_C$$

SU(3) flavor symmetric limit

$K_{tribaryon}$ (rows 1 – 64)

$K_{B1} + K_{B2} + K_{B3}$ → V

Flavor	$-\sum_{i<j} \lambda_i \lambda_j \sigma_i \cdot \sigma_j$				
	$S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$	$S=\frac{7}{2}$	$S=\frac{9}{2}$
1		-4	$\frac{8}{3}$		24
8	4	8	$\frac{44}{3}$	24	
10		20			
$\overline{\mathbf{10}}$		20			
27	24	28	$\frac{104}{3}$		
35	40				
$\overline{\mathbf{35}}$	40				
64		56			
V	-24	-24	-8	8	24

TABLE I: The expectation value of color-spin interaction of the tribaryon for each flavor in SU(3) flavor symmetry. Empty boxes represent states that are not allowed by Pauli principle. V is for the lowest threshold three baryon states.

$$E_B = H_{\text{tribaryon}} - H_{\text{baryon1}} - H_{\text{baryon2}} - H_{\text{baryon3}}$$

$$\delta = 1 - \frac{m_u}{m_s}$$

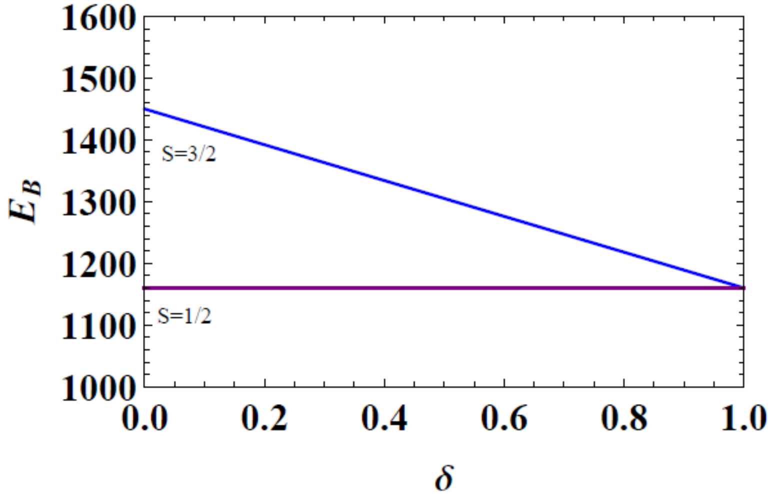


FIG. 1: E_B of q^8s with $I = 2$(unit: MeV).

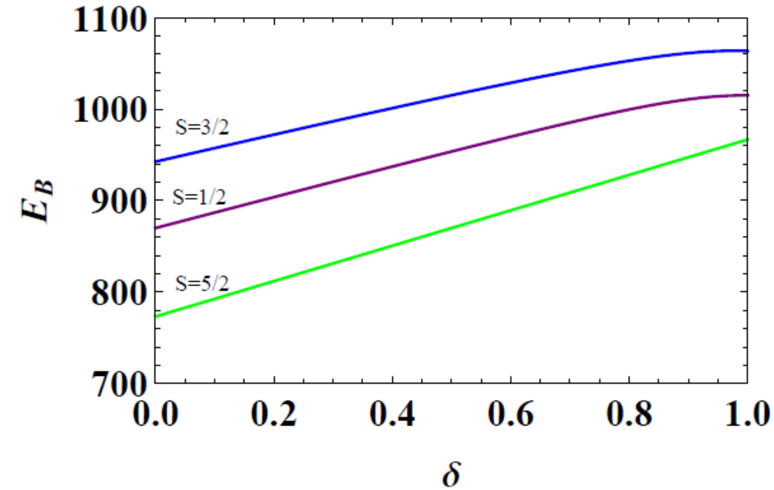


FIG. 2: E_B of q^8s with $I = 1$(unit: MeV).

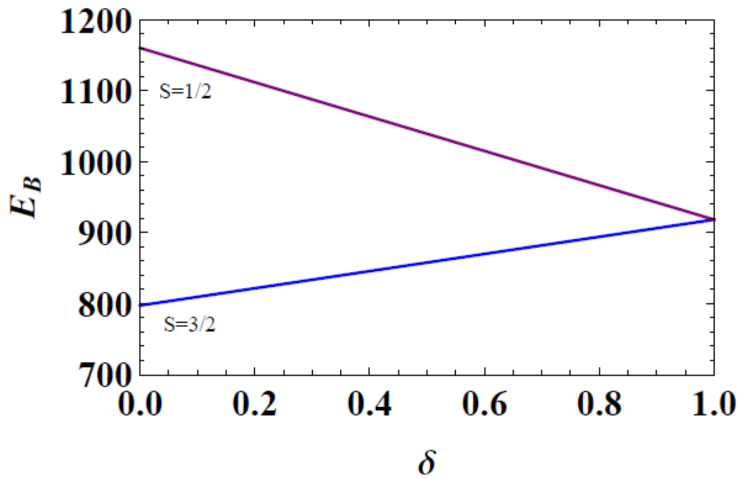


FIG. 3: E_B of q^8s with $I = 0$(unit: MeV).

Strangeness = -1

ΛNN

ΣNN

$\vdots$

$$E_B = H_{\text{tribaryon}} - H_{\text{baryon1}} - H_{\text{baryon2}} - H_{\text{baryon3}}$$

$$\delta = 1 - \frac{m_u}{m_s}$$

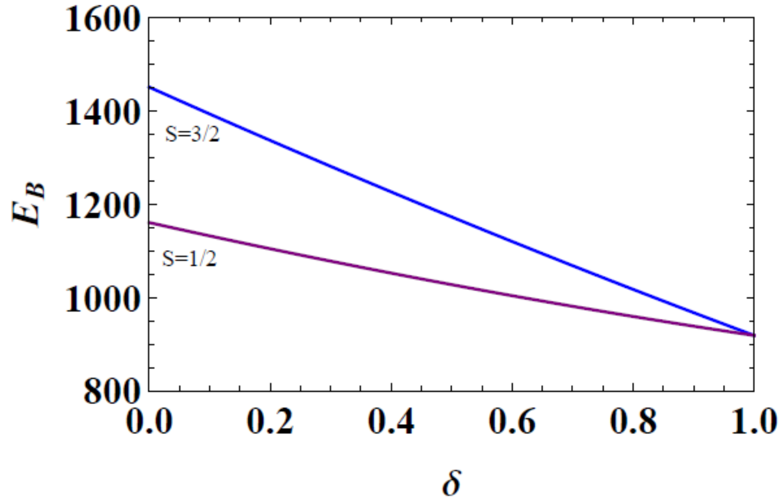


FIG. 4: E_B of q^7s^2 with $I = \frac{5}{2}$(unit: MeV).

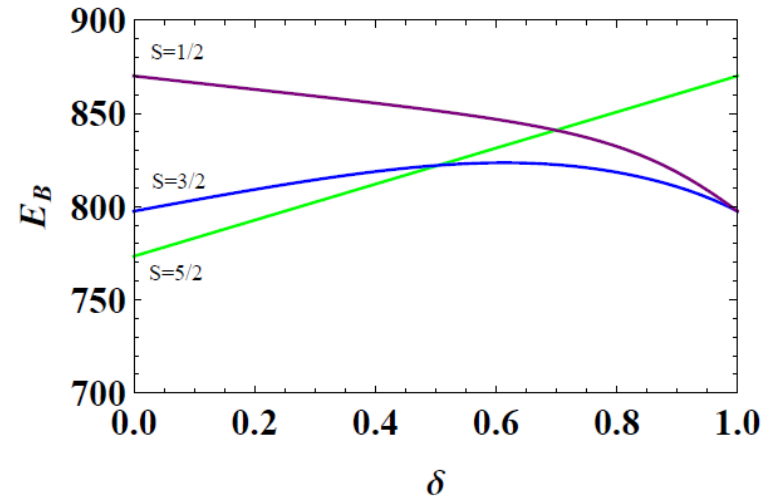


FIG. 5: E_B of q^7s^2 with $I = \frac{3}{2}$(unit: MeV).

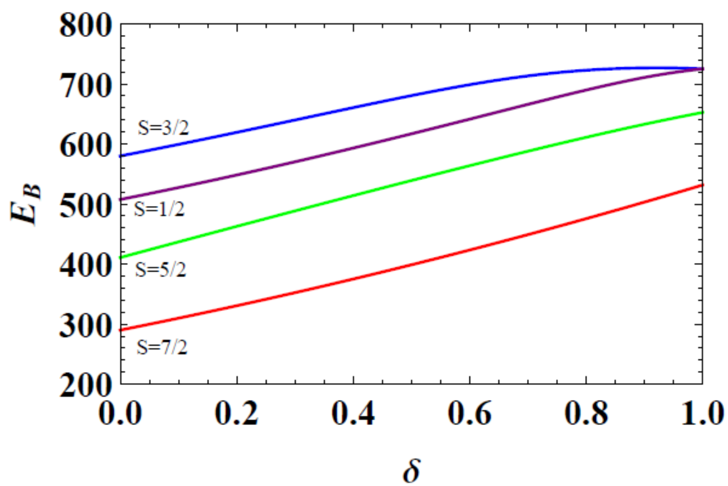


FIG. 6: E_B of q^7s^2 with $I = \frac{1}{2}$(unit: MeV).

Strangeness = −2

$\Lambda\Lambda N$

$\Lambda\Sigma N$

$\Sigma\Sigma N$

$\Xi N N$

⋮

$$E_B = H_{\text{tribaryon}} - H_{\text{baryon1}} - H_{\text{baryon2}} - H_{\text{baryon3}}$$

$$\delta = 1 - \frac{m_u}{m_s}$$

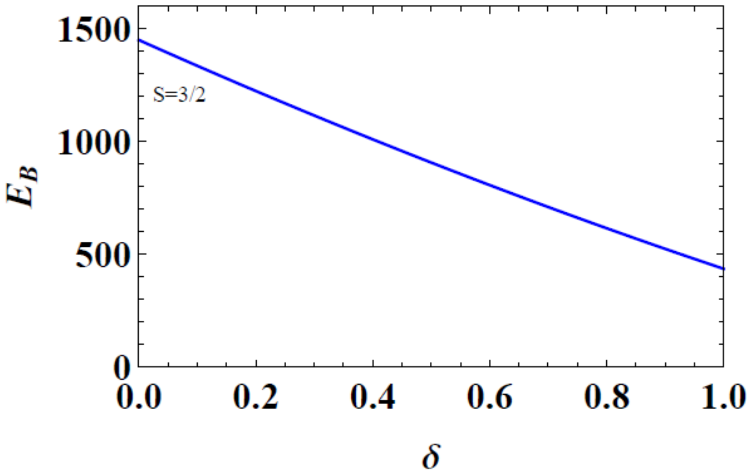


FIG. 7: E_B of $q^6 s^3$ with $I = 3$(unit: MeV).

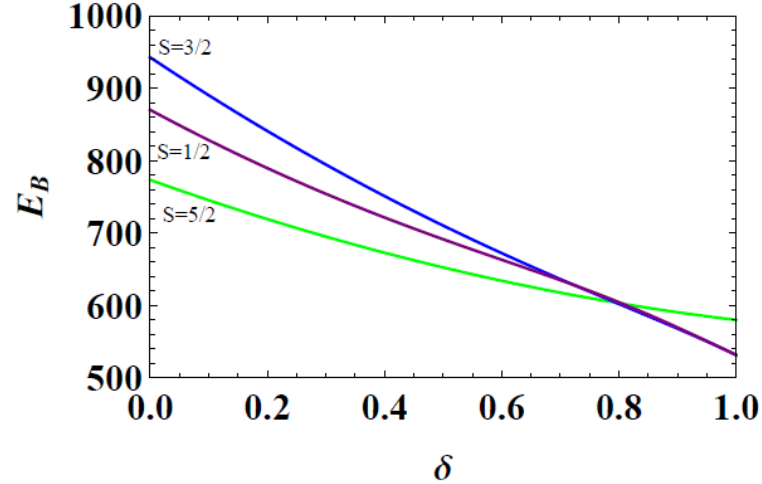


FIG. 8: E_B of $q^6 s^3$ with $I = 2$(unit: MeV).

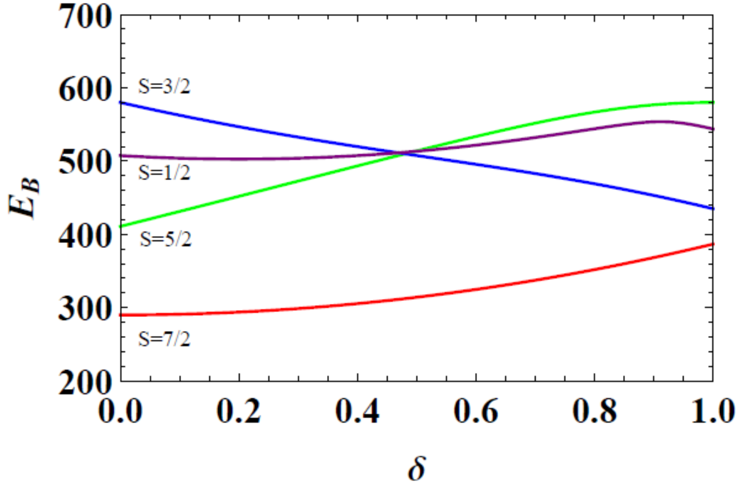


FIG. 9: E_B of $q^6 s^3$ with $I = 1$(unit: MeV).

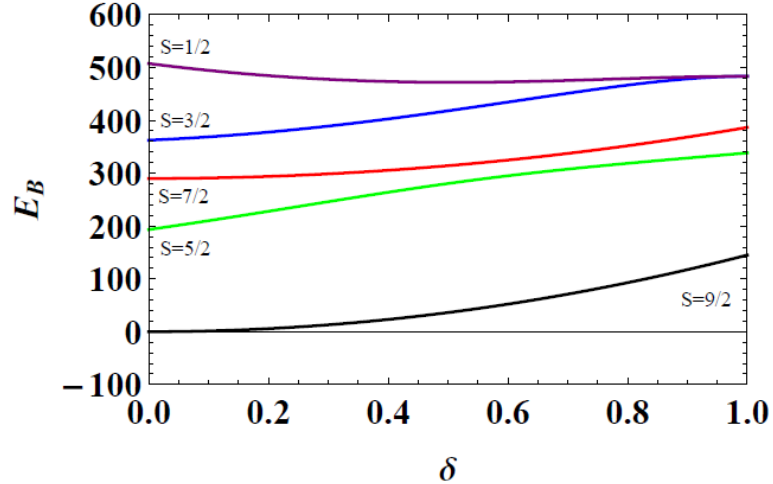


FIG. 10: E_B of $q^6 s^3$ with $I = 0$(unit: MeV).

$S = -3$

$\Lambda\Lambda\Lambda$

$\Lambda\Lambda\Sigma$

$\Lambda\Sigma\Sigma$

$\Xi\Lambda N$

$\Xi\Sigma N$

$\vdots$

$$E_B = H_{\text{tribaryon}} - H_{\text{baryon1}} - H_{\text{baryon2}} - H_{\text{baryon3}}$$

$$\delta = 1 - \frac{m_u}{m_s}$$

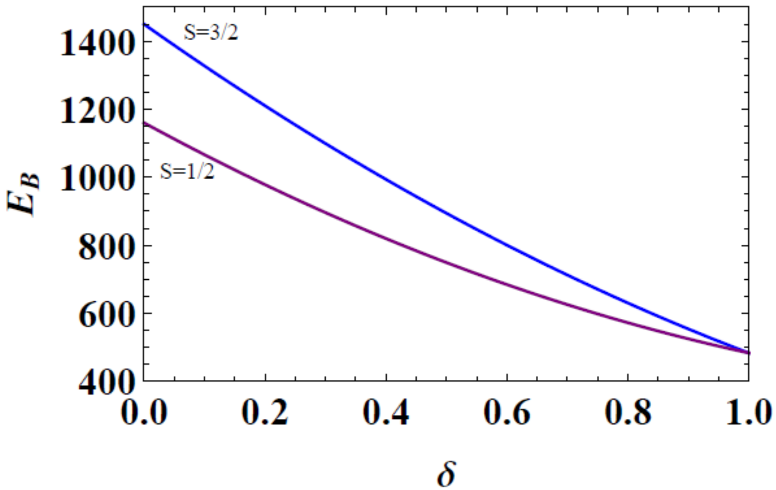


FIG. 11: E_B of q^5s^4 with $I = \frac{5}{2}$(unit: MeV).

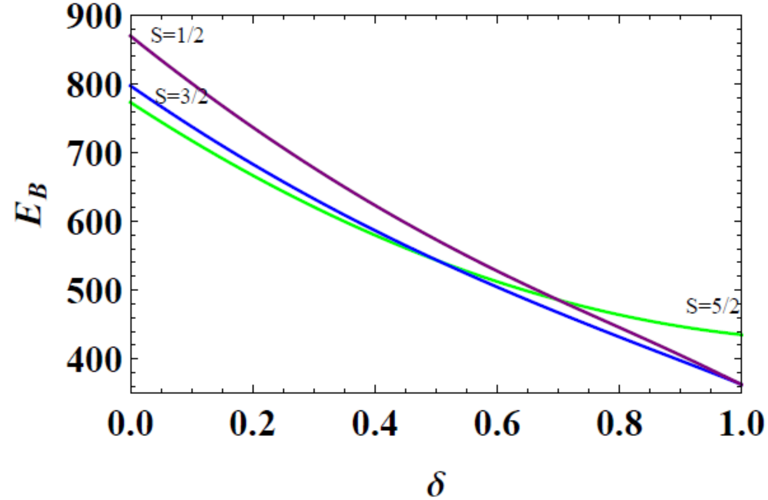


FIG. 12: E_B of q^5s^4 with $I = \frac{3}{2}$(unit: MeV).

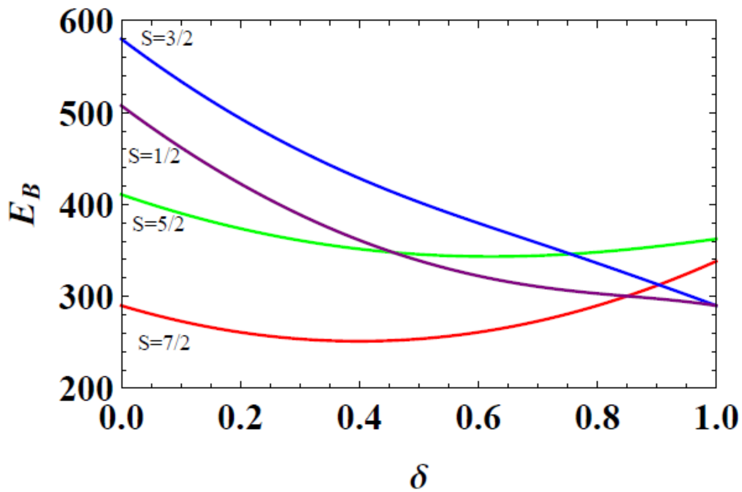


FIG. 13: E_B of q^5s^4 with $I = \frac{1}{2}$(unit: MeV).

Strangeness = −4

$\Xi\Lambda\Lambda$
$\Xi\Lambda\Sigma$
$\Xi\Sigma\Sigma$
$\Xi\Xi N$
⋮

$$E_B = H_{\text{tribaryon}} - H_{\text{baryon1}} - H_{\text{baryon2}} - H_{\text{baryon3}}$$

$$\delta = 1 - \frac{m_u}{m_s}$$

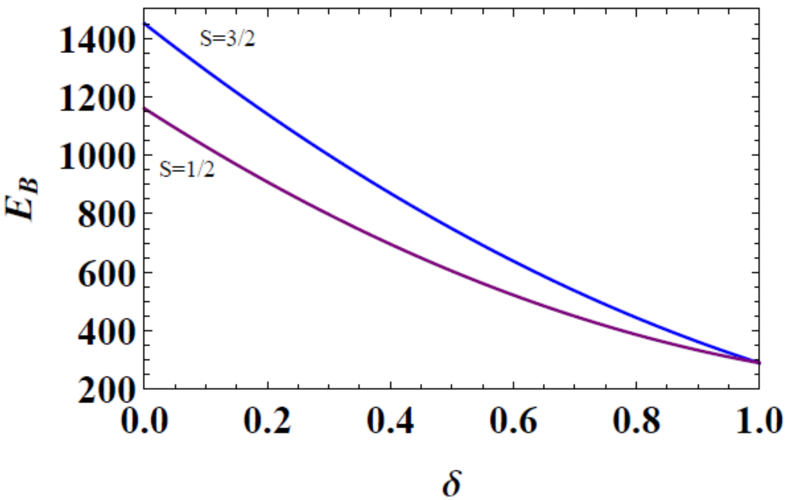


FIG. 14: E_B of q^4s^5 with $I = 2$(unit: MeV).

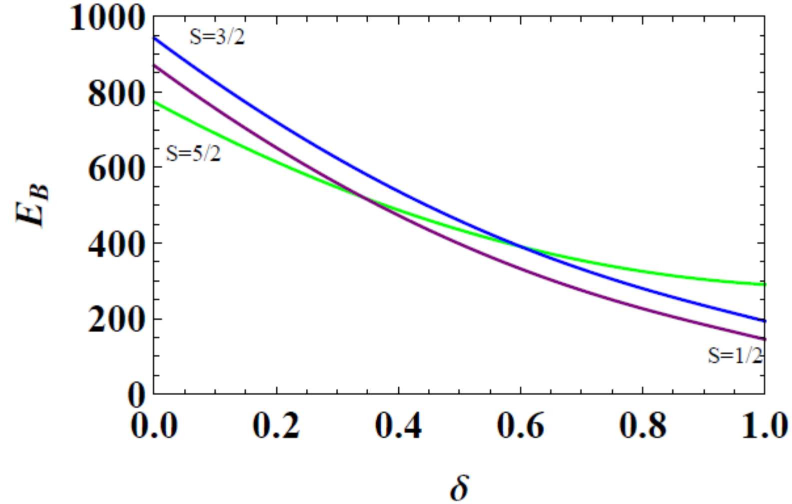


FIG. 15: E_B of q^4s^5 with $I = 1$(unit: MeV).

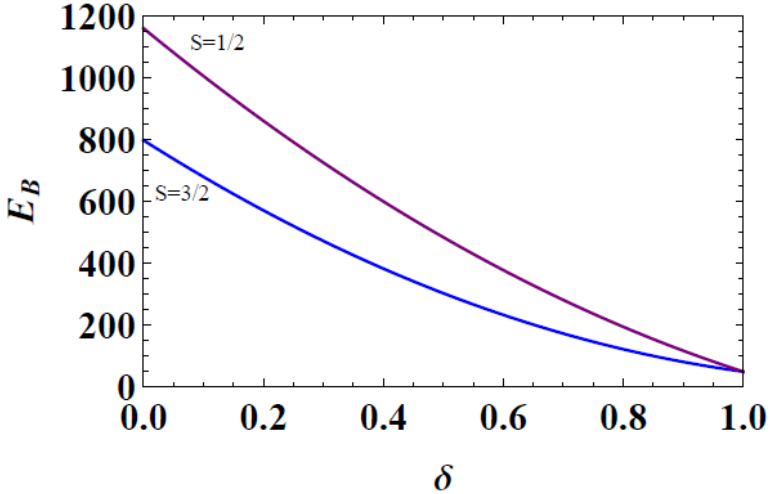


FIG. 16: E_B of q^4s^5 with $I = 0$(unit: MeV).

Strangeness $= -5$

$\Xi\Xi\Lambda$

$\Xi\Xi\Sigma$

$\Omega\Lambda\Lambda$

$\Omega\Xi N$

$\vdots$

$$E_B = H_{\text{tribaryon}} - H_{\text{baryon1}} - H_{\text{baryon2}} - H_{\text{baryon3}}$$

$$\delta = 1 - \frac{m_u}{m_s}$$

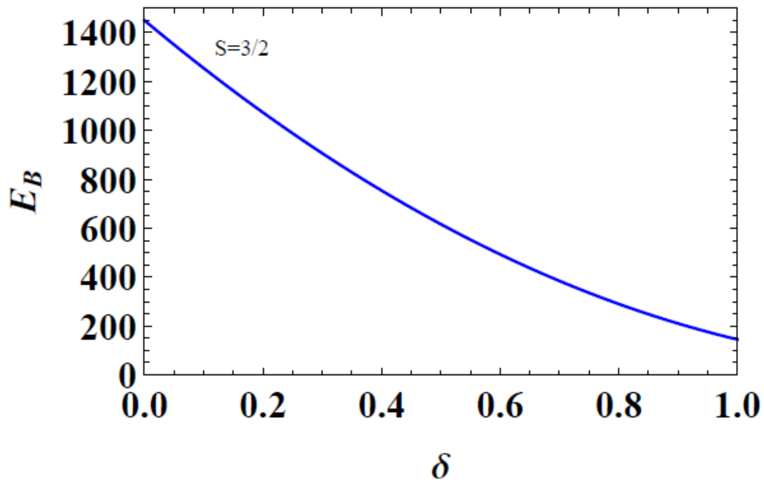


FIG. 17: E_B of $q^3 s^6$ with $I = \frac{3}{2}$(unit: MeV).

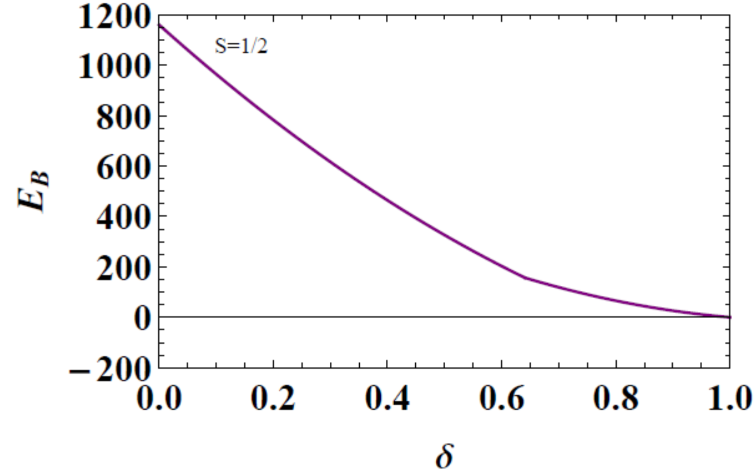


FIG. 18: E_B of $q^3 s^6$ with $I = \frac{1}{2}$(unit: MeV).

Strangeness = −6

$\Xi\Xi\Xi$

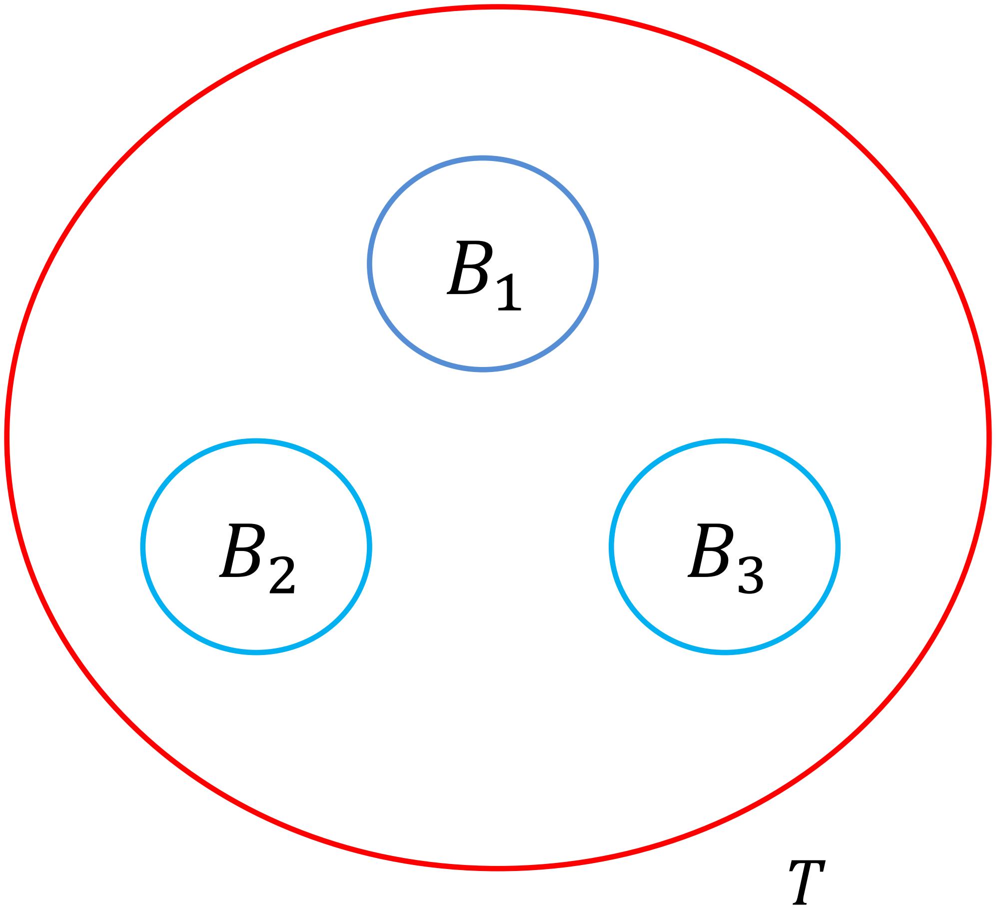
$\Omega\Xi\Lambda$

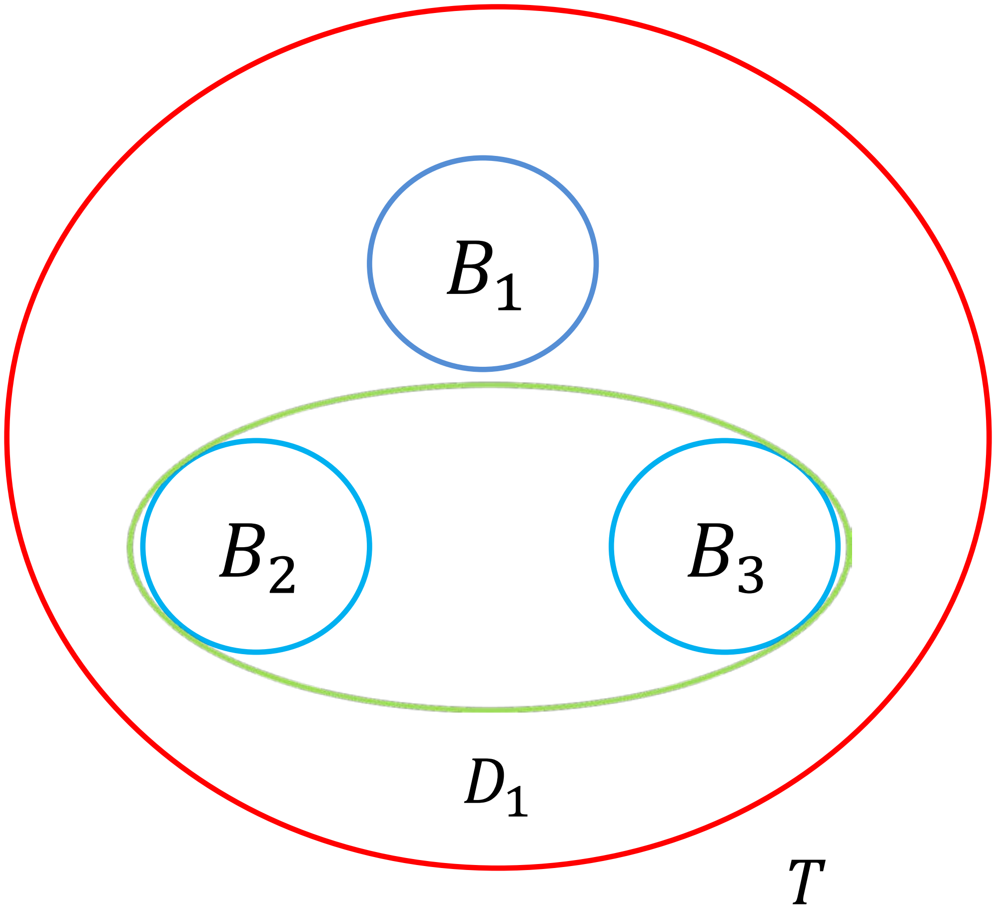
$\Omega\Xi\Sigma$

$\Omega\Omega N$

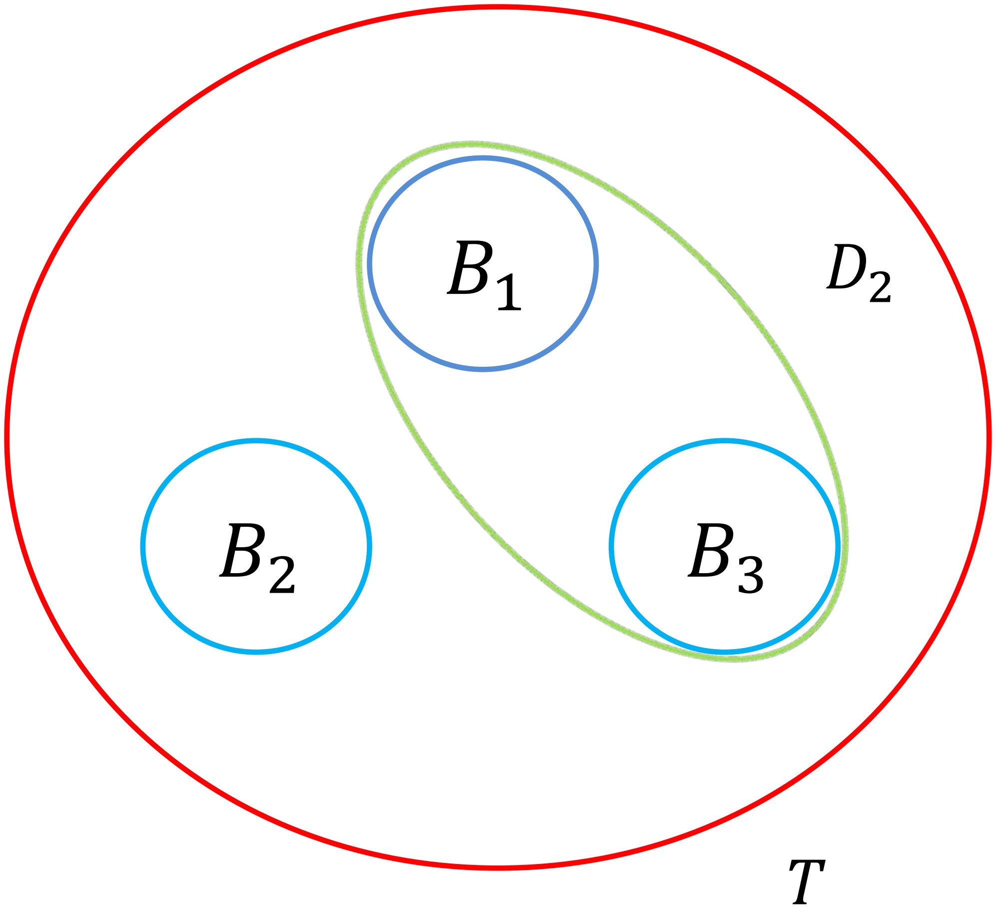
⋮

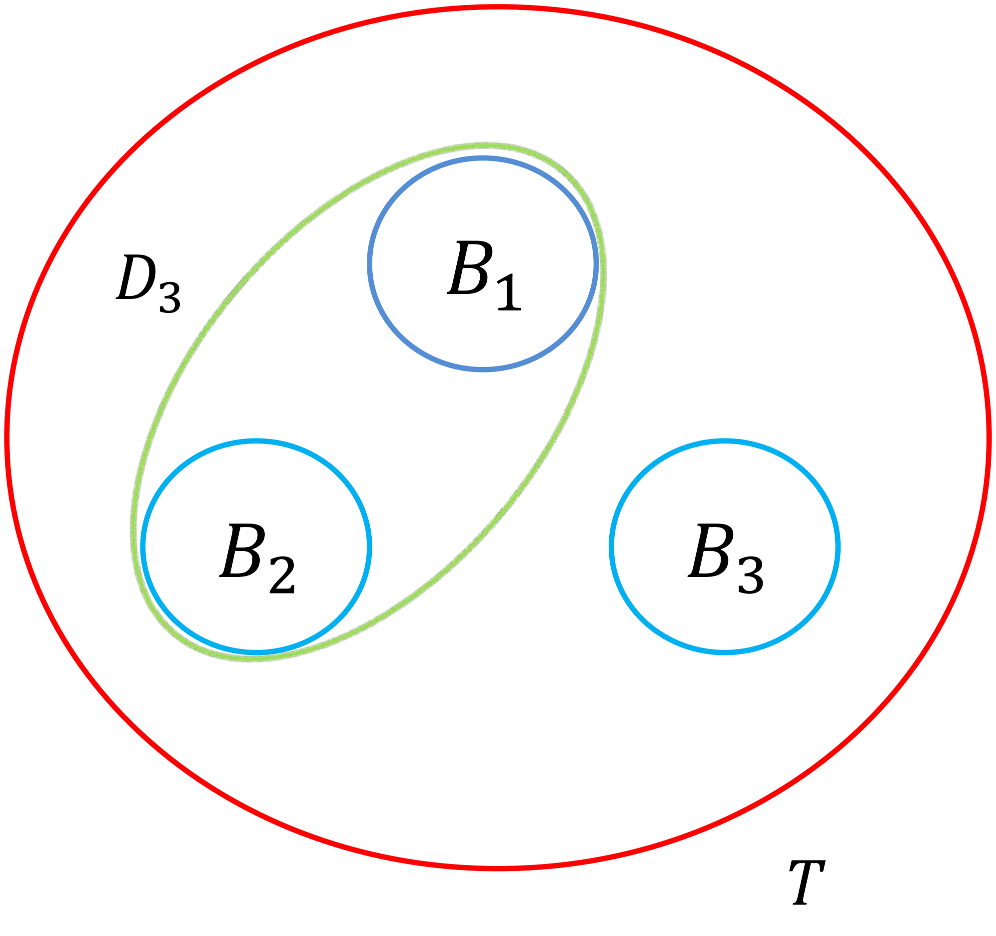
QNP2018

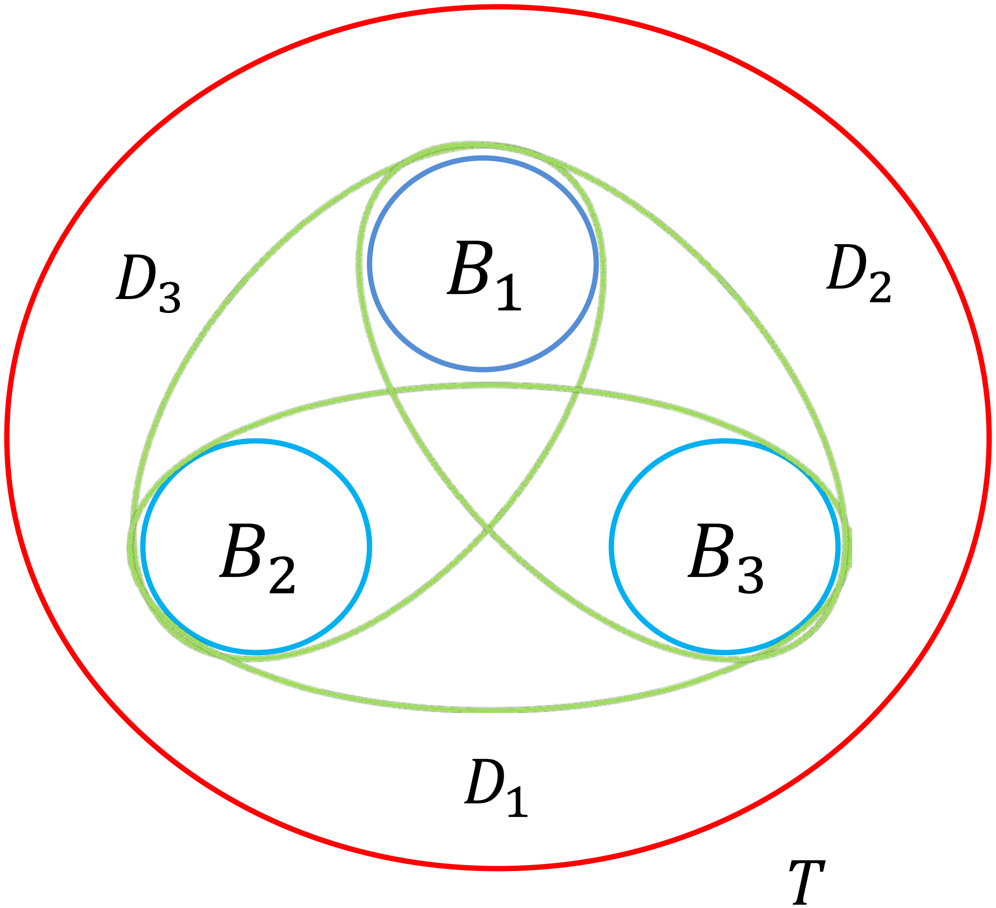

B_1
B_2
B_3
T



B_1
B_2
B_3
D_1
T


B_1
D_2
B_2
B_3
T


D_3
B_1
B_2
B_3
T



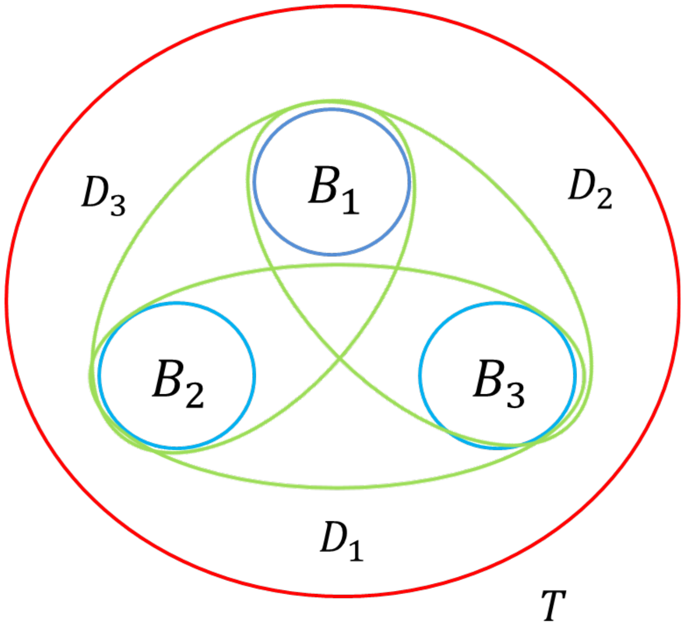
D_3
B_1
D_2
B_2
B_3
D_1
T

Pure three-body force

$$E_{2-\text{body},i} = M_{D,i} - M_{B_{i,1}} - M_{B_{i,2}}$$

$$M_{\text{tribaryon}} = \sum_{i=1}^{3} E_{2-\text{body},i} + E_{3-\text{body}} + \sum_{i=1}^{3} M_{B_i}$$

$$E_{3-\text{body}} = M_{\text{tribaryon}} - \sum_{i=1}^{3} E_{2-\text{body},i} - \sum_{i=1}^{3} M_{B_i}$$
$$= M_{\text{tribaryon}} - \sum_{i=1}^{3} M_{\text{dibaryon},i} + \sum_{i=1}^{3} M_{B_i}$$



Transformation coefficients

Baryon ⊗ baryon

Dibaryon

(F,S)	$8 \otimes 8$	$8 \otimes 10$	$10 \otimes 10$
(1,0)	1		
(8,1)	$\frac{4}{9}$	$\frac{5}{9}$	
(8,2)		1	
(10,1)	$\frac{1}{9}$	$\frac{8}{9}$	
($\bar{10}$,1)	$\frac{5}{9}$		$\frac{4}{9}$
($\bar{10}$,3)			1
(27,0)	$\frac{5}{9}$		$\frac{4}{9}$
(27,2)		$\frac{4}{9}$	$\frac{5}{9}$
(28,0)			1
(35,1)		$\frac{4}{9}$	$\frac{5}{9}$

$$\langle F_1 \rangle = \langle F_8 \otimes F_8 \rangle = \frac{1}{8}\Lambda\Lambda + \frac{3}{8}\Sigma\Sigma + \frac{1}{2}N\Xi$$

$$\langle F_{27} \rangle = \frac{5}{9}\langle F_8 \otimes F_8 \rangle + \frac{4}{9}\langle F_{10} \otimes F_{10} \rangle$$
$$= \frac{5}{9}(\frac{27}{40}\Lambda\Lambda + \frac{1}{40}\Sigma\Sigma + \frac{3}{10}N\Xi) + \frac{4}{9}\Sigma^*\Sigma^*$$

$$\langle F_{10} \rangle = \frac{1}{9}\langle F_8 \otimes F_8 \rangle + \frac{8}{9}\langle F_8 \otimes F_{10} \rangle$$
$$= \frac{1}{9}(\frac{1}{2}\Sigma\Lambda + \frac{1}{6}\Sigma\Sigma + \frac{1}{3}N\Xi) + \frac{8}{9}(\frac{1}{3}\Sigma\Sigma^* + \frac{1}{3}N\Xi^* + \frac{1}{3}\Xi\Delta)$$

$$\langle F_{\bar{10}} \rangle = \frac{5}{9}\langle F_8 \otimes F_8 \rangle + \frac{4}{9}\langle F_{10} \otimes F_{10} \rangle$$
$$= \frac{5}{9}(\frac{1}{2}\Sigma\Lambda + \frac{1}{6}\Sigma\Sigma + \frac{1}{3}N\Xi) + \frac{4}{9}(\frac{1}{3}\Sigma^*\Sigma^* + \frac{2}{3}\Delta\Xi^*)$$

$$\langle F_8 \rangle = \frac{4}{9}\langle F_8 \otimes F_8 \rangle + \frac{5}{9}\langle F_8 \otimes F_{10} \rangle$$
$$= \frac{4}{9}N\Xi + \frac{5}{9}(\frac{3}{5}\Sigma\Sigma^* + \frac{2}{5}N\Xi^*)$$

Transformation coefficients

Tribaryon → Baryon $\otimes$ Dibaryon

Transformation coefficients

Tribaryon → Baryon ⊗ Dibaryon

Tribaryon (columns); Baryon ⊗ Dibaryon (rows)

(F,S)	$(1,\frac{3}{2})$	$(1,\frac{5}{2})$	$(1,\frac{9}{2})$	$(8,\frac{1}{2})$	$(8,\frac{3}{2})$	$(8,\frac{5}{2})$	$(8,\frac{7}{2})$	$(10,\frac{3}{2})$	$(\bar{10},\frac{3}{2})$	$(27,\frac{1}{2})$	$(27,\frac{3}{2})$	$(27,\frac{5}{2})$	$(35,\frac{1}{2})$	$(\bar{35},\frac{1}{2})$	$(64,\frac{3}{2})$
$(8,\frac{1}{2})\otimes(1,0)$				$\frac{1}{16}$											
$(8,\frac{1}{2})\otimes(8,1)$	$\frac{7}{16}$			$\frac{1}{6}$	$\frac{1}{8}$			$\frac{41}{240}$	$\frac{5}{48}$	$\frac{10}{81}$	$\frac{7}{1296}$				
$(8,\frac{1}{2})\otimes(8,2)$	$\frac{7}{48}$	$\frac{4}{9}$			$\frac{43}{200}$	$\frac{7}{50}$		$\frac{17}{80}$	$\frac{1}{80}$		$\frac{21}{400}$	$\frac{2}{675}$			
$(8,\frac{1}{2})\otimes(10,1)$				$\frac{5}{24}$	$\frac{1}{8}$			$\frac{1}{30}$		$\frac{5}{162}$	$\frac{7}{81}$		$\frac{1}{6}$		
$(8,\frac{1}{2})\otimes(\bar{10},1)$				$\frac{1}{24}$	$\frac{1}{40}$				$\frac{1}{6}$	$\frac{1}{162}$	$\frac{7}{405}$			$\frac{1}{6}$	
$(8,\frac{1}{2})\otimes(\bar{10},3)$						$\frac{1}{90}$	$\frac{1}{6}$					$\frac{14}{135}$			
$(8,\frac{1}{2})\otimes(27,0)$				$\frac{5}{48}$						$\frac{1}{9}$			$\frac{1}{150}$	$\frac{1}{6}$	
$(8,\frac{1}{2})\otimes(27,2)$					$\frac{1}{100}$	$\frac{21}{100}$		$\frac{1}{30}$	$\frac{2}{15}$		$\frac{31}{135}$	$\frac{26}{225}$			$\frac{1}{12}$
$(8,\frac{1}{2})\otimes(28,0)$													$\frac{4}{25}$		
$(8,\frac{1}{2})\otimes(35,1)$								$\frac{7}{30}$		$\frac{14}{81}$	$\frac{5}{81}$		$\frac{2}{15}$		$\frac{1}{12}$
$(10,\frac{3}{2})\otimes(1,0)$								$\frac{1}{40}$							
$(10,\frac{3}{2})\otimes(8,1)$				$\frac{5}{48}$	$\frac{1}{40}$	$\frac{7}{80}$				$\frac{5}{81}$	$\frac{7}{405}$	$\frac{4}{135}$	$\frac{1}{15}$		
$(10,\frac{3}{2})\otimes(8,2)$				$\frac{1}{80}$	$\frac{1}{600}$	$\frac{361}{3600}$	$\frac{2}{15}$			$\frac{1}{15}$	$\frac{7}{75}$	$\frac{28}{675}$	$\frac{1}{5}$		
$(10,\frac{3}{2})\otimes(10,1)$									$\frac{1}{6}$	$\frac{8}{81}$	$\frac{7}{162}$	$\frac{2}{27}$			
$(10,\frac{3}{2})\otimes(\bar{10},1)$	$\frac{7}{20}$	$\frac{2}{5}$		$\frac{1}{6}$	$\frac{9}{100}$	$\frac{7}{50}$				$\frac{7}{81}$	$\frac{529}{8100}$	$\frac{7}{675}$			$\frac{1}{30}$
$(10,\frac{3}{2})\otimes(\bar{10},3)$	$\frac{1}{15}$	$\frac{7}{45}$	1		$\frac{7}{75}$	$\frac{16}{225}$	$\frac{1}{4}$				$\frac{7}{75}$	$\frac{98}{675}$			$\frac{7}{40}$
$(10,\frac{3}{2})\otimes(27,0)$					$\frac{1}{4}$			$\frac{7}{120}$	$\frac{1}{12}$		$\frac{1}{9}$				$\frac{1}{30}$
$(10,\frac{3}{2})\otimes(27,2)$				$\frac{2}{15}$	$\frac{1}{25}$	$\frac{6}{25}$	$\frac{9}{20}$	$\frac{7}{30}$	$\frac{1}{3}$	$\frac{7}{45}$	$\frac{4}{135}$	$\frac{49}{225}$	$\frac{2}{15}$	$\frac{1}{3}$	$\frac{5}{24}$
$(10,\frac{3}{2})\otimes(28,0)$															$\frac{7}{40}$
$(10,\frac{3}{2})\otimes(35,1)$										$\frac{7}{81}$	$\frac{16}{81}$	$\frac{7}{27}$	$\frac{2}{15}$	$\frac{1}{3}$	$\frac{5}{24}$

Summary

1. We have identified compact tribaryon configurations in terms of SU(3) flavor and spin quantum numbers that are allowed within the Pauli principle.

2. While compact configurations are possible for certain quantum numbers, we found that the color-spin interaction for all the allowed configurations are highly repulsive with respect to three baryon channel.

3. This is the microscopic proof that the three body nuclear force should be repulsive in all channels, which are consistent with the recently established neutron star mass limit.

Thank you