Effects of Attractive KK^{bar} and Repulsive KK Interactions in KKK^{bar} three-body resonance

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Hadron physics

Hadrons as composite systems of hadrons
(Hadron molecules) have been discussed intensively
Physically,hadron number is not a good quantum number
So, the concept of 'compositeness' is important to characterize hadrons

Few-body physics

Resonances in coupled-channel systems
Three-body force
Relativistic effects
Efimov physics

KKK^{bar}-$K\pi\pi$-$K\pi\eta$ three-body system

This system is very interesting

- Three-body boson system
- Coupled-channel problem
- Strongly interacting : Attractive KK^{bar} and Repulsive KK
- Relativistic vs non-relativistic

K(1460) (0^-, I=1/2)

- Near to (just below) the threshold of KKK^{bar}(1685 MeV)
- The same quantum numbers with those of ground state of KKK^{bar}

Experiments
- G.W. Brandenburg, et al., PRL36(1976)1239
- C. Daum, et al., NPB187(1981)1

Theories as a KKK^{bar} resonance
- A.M. Torres, D. Jido, and Y. Kanada-Enyo, PRC83, 065205, 2011

Experiment on K(1460) at SLAC

G.W. Brandenburg, et al., PRL36(1976)1239

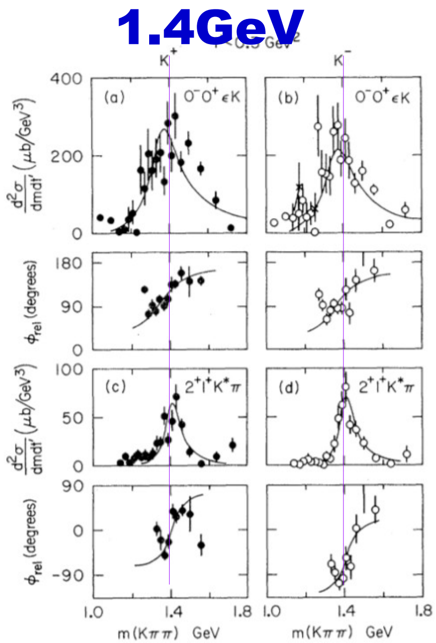


FIG. 1. The mass dependence of the 0^-0^+ ϵK and the 2^+1^+ $K^*\pi$ waves. Crosses correspond to ambiguous solutions, and $\varphi_{\rm rel}$ is measured with respect to the 1^+0^+ ρK wave. The curves correspond to simple Breit-Wigner parametrizations.

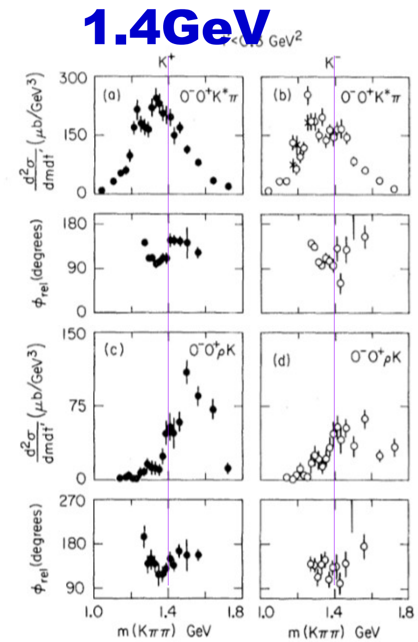


FIG. 2. The mass dependence of the 0^-0^+ $K^*\pi$ and the 0^-0^+ ρK waves. Crosses correspond to ambiguous solutions, and $\varphi_{\rm rel}$ is measured with respect to the 1^+0^+ ρK wave.

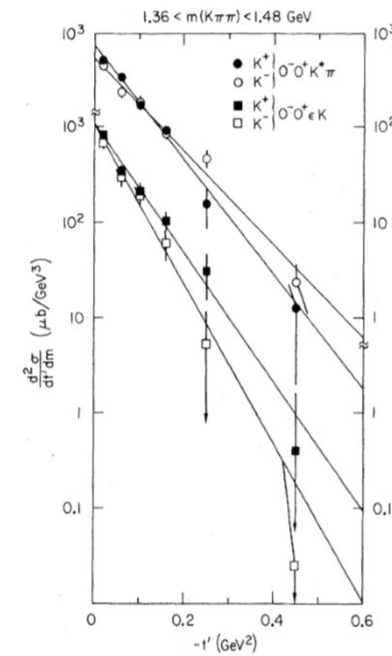


FIG. 3. The momentum transfer dependence of the 0^-0^+ $K^*\pi$ and 0^-0^+ ϵK waves. The lines are exponentials with the parameter values given in Table I.

$K^\pm\, p \rightarrow K^\pm\, \pi^+ \pi^-\, p$

… We have presented evidence for the existence of a $K\pi\pi$ pseudoscalar resonance, the K' of mass ~1400 MeV and width ~250 MeV. The predominant decay mode is εK with weaker couplings to $K^*\pi$ and, possibly, ρK. ...

Experiment on K(1460) at SLAC

C. Daum, et al., NPB187(1981)1

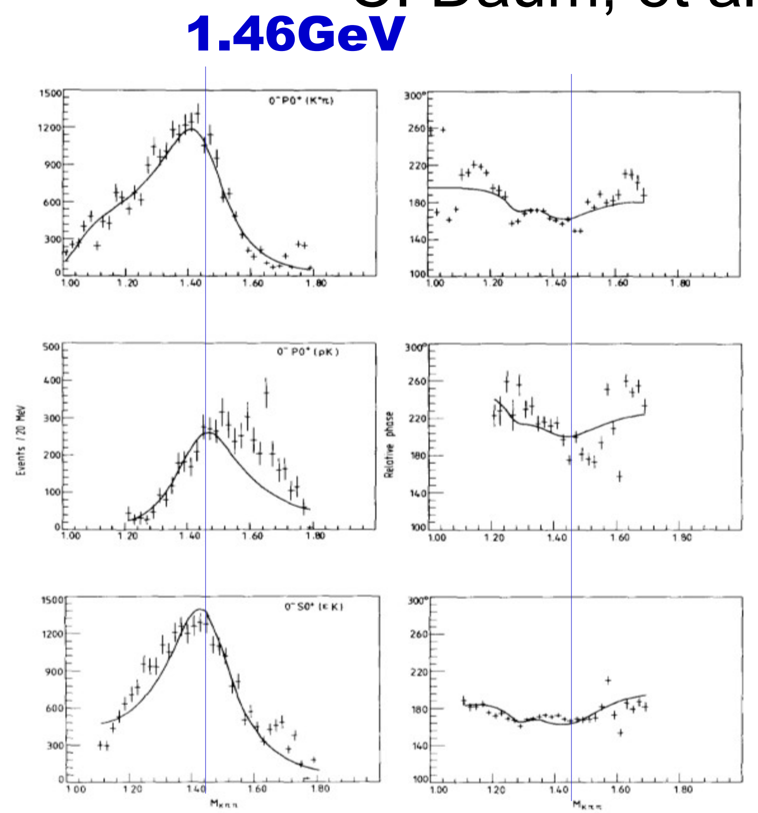


Fig. 18. Intensities and phases of 0^- waves for $1.0 \leq M_{K\pi\pi} \leq 1.8$ GeV, $0 \leq |t'| \leq 0.7$ GeV^2. The phases are measured relative to $1^+S0^+(K^*\pi)$. The curves show the result of fitting with a single resonance and a simple background in all three channels. The fitted phases have been corrected for the variation of the reference phase.

$K^- p \rightarrow K^- \pi^+ \pi^- p$

… The fits represent well the qualitative behaviour of the 0^- amplitudes and these data constitute strong evidence for the existence of a diffractively produced 0^- meson with a mass ~1.46 GeV and width ~260 MeV. The fitted partial widths are

$\Gamma_{K^*\pi}$=109 MeV, $\Gamma_{\rho K}$=34 MeV, $\Gamma_{\varepsilon K}$=117 MeV.

….

Theoretical works on K(1460)

A.M. Torres, D. Jido, and Y. Kanada-Enyo, PRC83, 065205, 2011

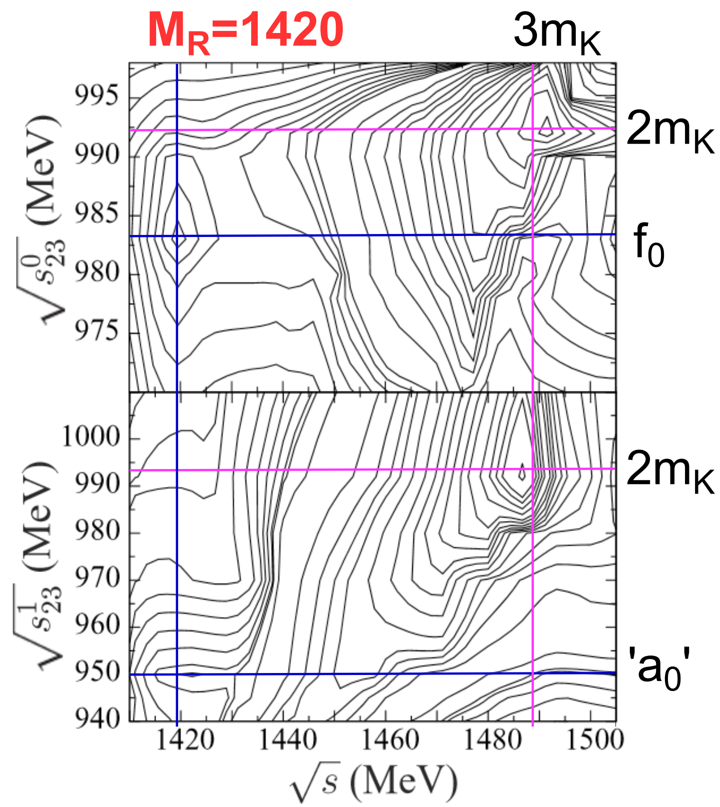


FIG. 2. Contour plots of the three-body squared amplitudes $|T_R^{(1/2,0)}|^2$ and $|T_R^{(1/2,1)}|^2$ for the $KK\bar{K} \to KK\bar{K}$ transition with total $I = 1/2$ as functions of the total three-body energy, $\sqrt{s}$, and the invariant mass of the $K\bar{K}$ subsystem with $I_{23} = 0$ (top) or the invariant mass of the $K\bar{K}$ subsystem with $I_{23} = 1$ (bottom).

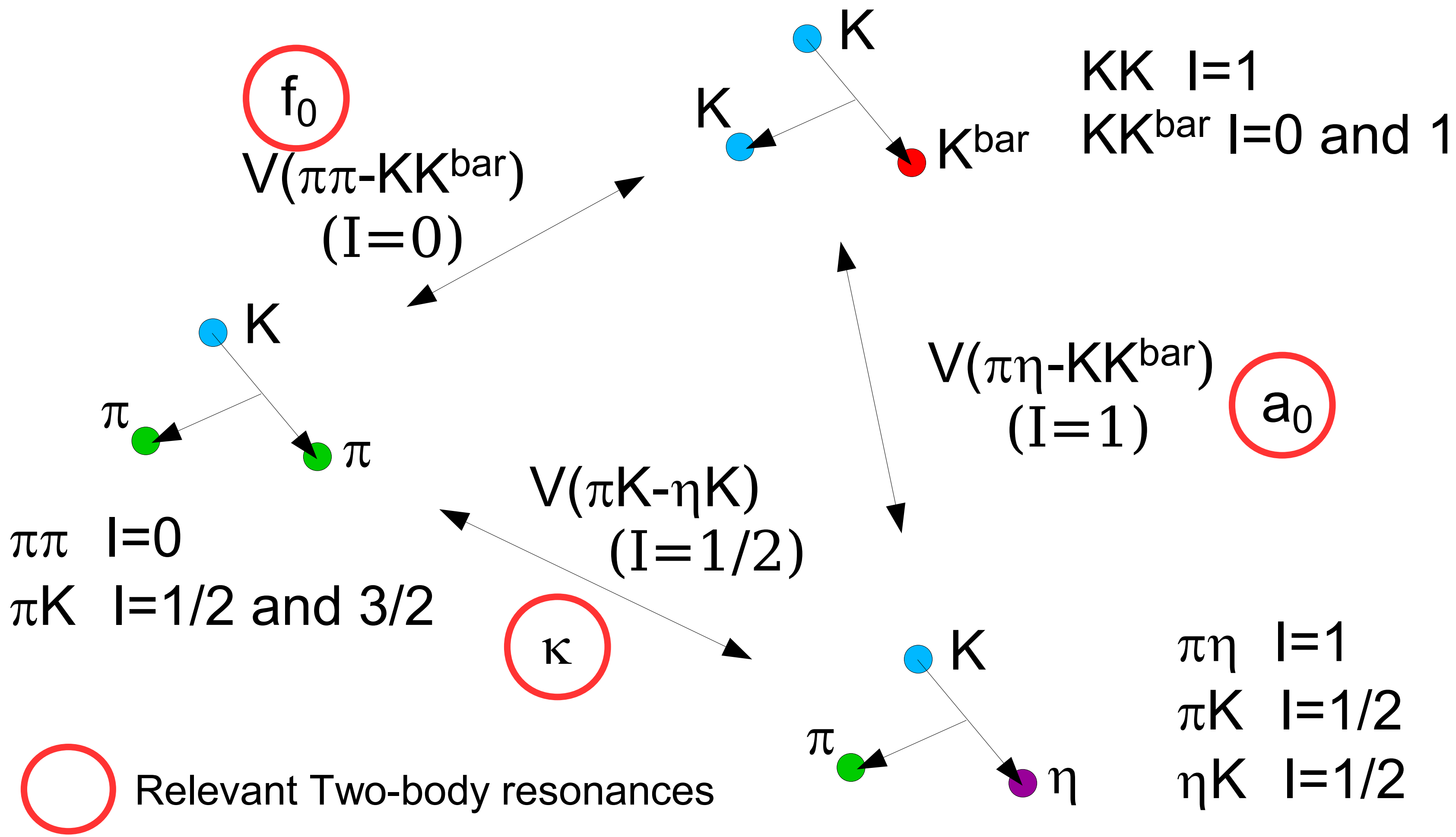
TABLE I. Comparison of the results of the Faddeev calculation and the potential model. The spatial structure of the $KK\bar{K}$ quasi-bound state obtained with the potential model is also shown in the table.

Model	Faddeev calculation	Potential Model
Mass (MeV)	~1420	1467
Width (MeV)	~50	110
Root mean squared radius (fm)		1.6
K-K distance (fm)		2.8
(KK)-$\bar{K}$ distance (fm)		1.7
K_2-$\bar{K}_3$ distance (fm)[a]		1.6
K_1-$(K_2\bar{K}_3)$ distance (fm)[a]		2.6

[a]The values of the K_2-$\bar{K}_3$ and K_1-$(K_2\bar{K}_3)$ distances are obtained before making the symmetrization of K_1K_2.

Faddeev calc.
M~1420MeV, Γ~50MeV
Variational calc.
M=1467MeV, Γ=110MeV

Our model of Coupled-Channel KKK^{bar}-K$\pi\pi$-K$\pi\eta$ system



Total : J=0,I=1/2, L=0(S-wave only)

Semi-Relativistic complex-scaled Coupled-Channel Three-Body Equation

A=1:KKK^{bar}, A=2:Kππ, A=3:Kπη

$$\hat{H}^{AA'} = \delta_{AA'} \sum_{i=1}^{3} \sqrt{m_i^{A2} + \boldsymbol{p}_i^2} + \sum_{(i,j)} V_{ij}^{AA'}(r_{ij}), \quad \sum_i \boldsymbol{p}_i = 0$$

$$\boldsymbol{r}_{ij} \to \boldsymbol{r}_{ij} e^{i\theta}, \qquad \boldsymbol{p}_i \to \boldsymbol{p}_i e^{-i\theta}$$: complex rotation

$$\hat{H}_\theta^{AA'} = \delta_{AA'} \sum_{i=1}^{3} \sqrt{m_i^{A2} + \boldsymbol{p}_i^2 e^{-2i\theta}} + \sum_{(i,j)} V_{ij}^{AA'}(r_{ij} e^{i\theta}), \quad \sum_i \boldsymbol{p}_i = 0$$

$$\sum_{A'} H_\theta^{AA'} \Psi^{A'} |A'\rangle = E_\theta \Psi^A |A\rangle$$: coupled-channel equation

$$\Psi^A = \Phi_1^A(\boldsymbol{\rho}_1^A, \boldsymbol{R}_1^A) + \Phi_2^A(\boldsymbol{\rho}_2^A, \boldsymbol{R}_2^A) + \Phi_3^A(\boldsymbol{\rho}_3^A, \boldsymbol{R}_3^A)$$

$\boldsymbol{\rho}_i^A$, $\boldsymbol{R}_i^A$: Jacobi coordinates in channel A

Three-Body Wave Function in the Gaussian Expansion Method

$$\Psi^{A}=\Phi_{1}^{A}(\boldsymbol{\rho}_{1,}^{A}\boldsymbol{R}_{1}^{A})+\Phi_{2}^{A}(\boldsymbol{\rho}_{2,}^{A}\boldsymbol{R}_{2}^{A})+\Phi_{3}^{A}(\boldsymbol{\rho}_{3,}^{A}\boldsymbol{R}_{3}^{A})$$

$$\Phi_{i}^{A}(\boldsymbol{\rho}_{i}^{A},\boldsymbol{R}_{i}^{A})=\sum_{\alpha\beta}C_{(i\alpha\beta)}^{A}N_{\alpha}N_{\beta}\exp(-\rho_{i}^{A2}/\rho_{\alpha}^{2})\exp(-R_{i}^{A2}/R_{\beta}^{2})$$

↓

Generalized Eigenvalue Problem

$$\sum_{B(j\alpha'\beta')}H_{\theta(i\alpha\beta),(j\alpha'\beta')}^{AB}C_{(j\alpha'\beta')}^{B}=E_{\theta}\sum_{(j\alpha'\beta')}N_{(i\alpha\beta),(j\alpha'\beta')}^{A}C_{(j\alpha'\beta')}^{A}$$

$H_{\theta(i\alpha\beta),(j\alpha'\beta')}^{AB}$: complex symmetric matrix (not Hermitian)

$N_{(i\alpha\beta),(j\alpha'\beta')}^{A}$: real symmetric matrix

→ complex eigenvalues

We need meson-meson potentials:

$V_{KK\text{-}KK}$ (I=1)

$V_{KKbar\text{-}KKbar}$, $V_{KKbar\text{-}\pi\pi}$, $V_{\pi\pi\text{-}\pi\pi}$ (I=0)

$V_{KKbar\text{-}KKbar}$, $V_{KKbar\text{-}\pi\eta}$, $V_{\pi\eta\text{-}\pi\eta}$(I=1)

$V_{\pi K\text{-}\pi K}$,$V_{\pi K\text{-}\eta K}$, $V_{\eta K\text{-}\eta K}$(I=1/2)

$V_{\pi K\text{-}\pi K}$(I=3/2)

↓

Coupled-Channel One-range Gaussian potentials

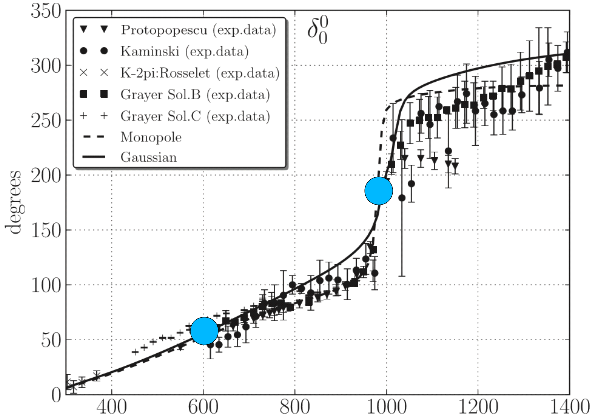
$$V_{mm\text{-}mm}(r) = V_{mm\text{-}mm} \exp(-(r/r_G)^2)$$

($V_{\pi\eta\text{-}\pi\eta}=V_{\pi K\text{-}\eta K}=V_{\eta K\text{-}\eta K}=0$ is assumed in present calculation)

We try various ranges : r_G=0.3, 0.4, 0.5, 0.6, 0.7 fm

Determination of KK^{bar}-ππ(I=0), KK^{bar}-πη (I=1)potentials

I=0



f_0: M=980MeV, Γ=70MeV

$$\delta_{L=0}^{I=0}(\pi\pi)\, at\ 600\,\mathrm{MeV} = 55\ \mathrm{degree}$$

Strongly attractive

r_G(fm)	$V_{KKbar\text{-}KKbar}$	$V_{KKbar\text{-}\pi\pi}$	$V_{\pi\pi\text{-}\pi\pi}$
0.3	-2006	-417	-1877
0.4	-1327	-383	-1164
0.5	-969	-350	-800
0.6	-763	-322	-590
0.7	-635	-305	-456

I=1

r_G(fm)	$V_{KKbar\text{-}KKbar}$	$V_{KKbar\text{-}\pi\eta}$	$V_{\pi\eta\text{-}\pi\eta}$
0.3	-1335	-1099	0
0.4	-928	-686	0
0.5	-695	-480	0
0.6	-550	-362	0
0.7	-456	-288	0

a_0: M=980MeV, Γ=70MeV

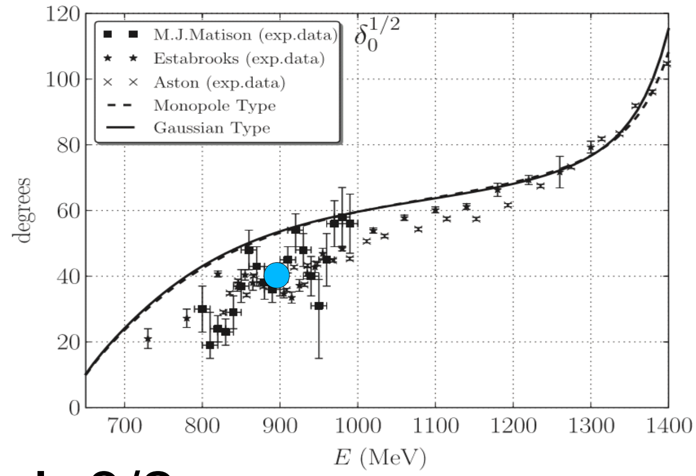
KK^{bar} :strongly attractive

We assume Vπη-πη=0 (tentatively)

*Theoretically,Vπη-πη may be weak

Determination of πK(I=1/2), πK(I=3/2) potentials

I=1/2

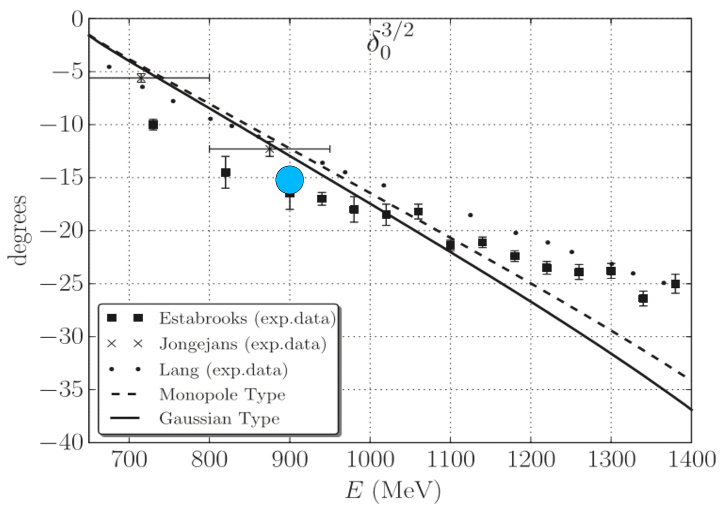


$$\delta_{L=0}^{I=1/2}(\sqrt{s}=900\,\text{MeV})=40\,\text{degree}$$

rG(fm)	V_{KK0}(MeV)
0.3	-1488.7
0.4	-895.0
0.5	-607.1
0.6	-405.1
0.7	-356.8

attractive

I=3/2

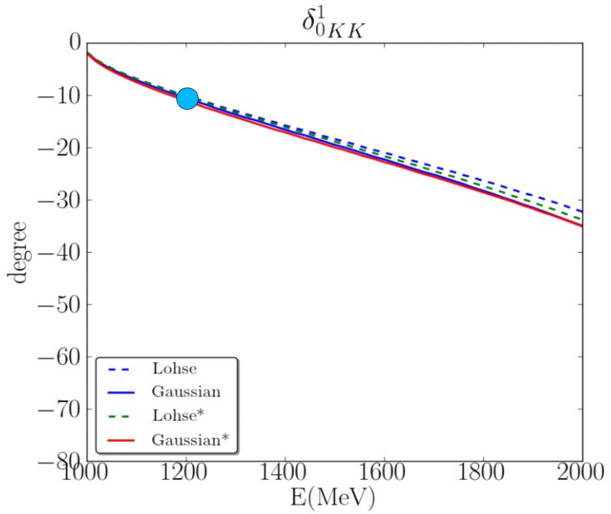


$$\delta_{L=0}^{I=3/2}(\sqrt{s}=900\,\text{MeV})=-15\,\text{degree}$$

rG(fm)	V_{KK0}(MeV)
0.3	3694.0
0.4	1174.0
0.5	555.6
0.6	323.8
0.7	215.2

repulsive

Determination of KK-KK(I=1) potentials



No experimental phase shifts

Theoretical predictions :

Repulsive interaction

$$\delta^{I=1}_{L=0}(\sqrt{s} = 1200\,\mathrm{MeV}) = -10\,\mathrm{degree}$$

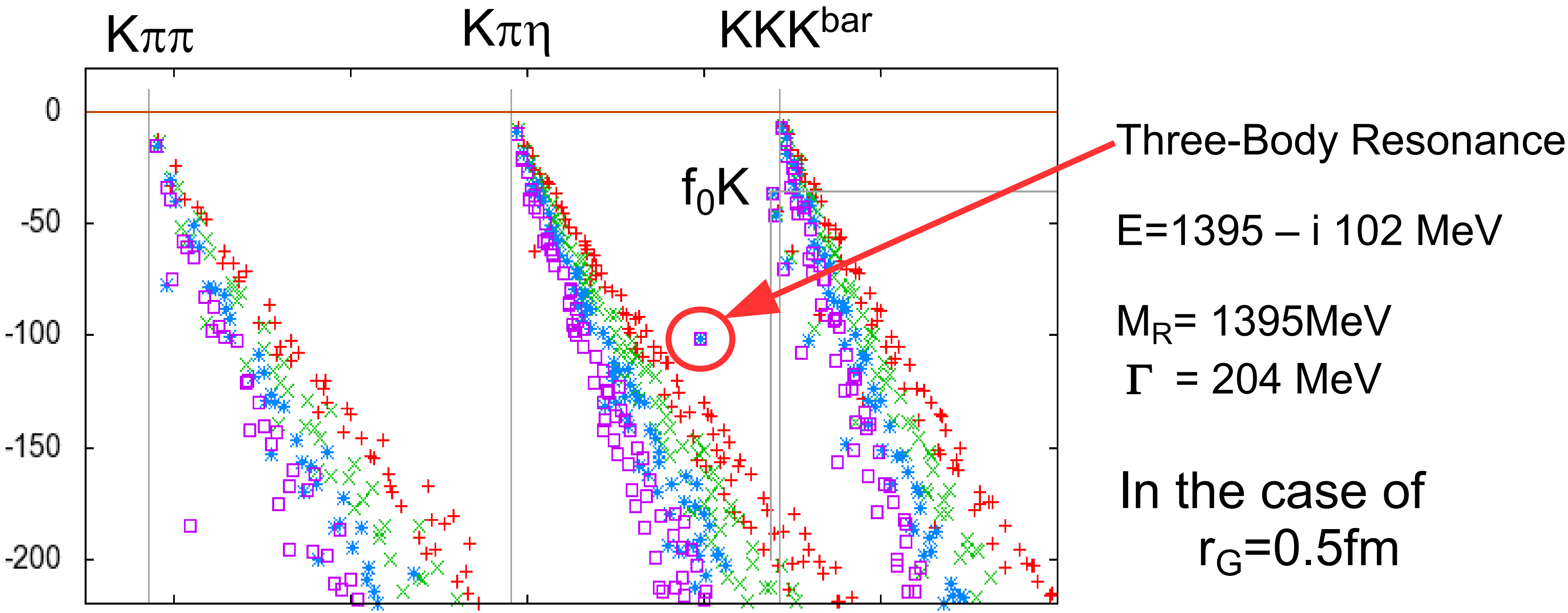
rG(fm)	V_{KK0}(MeV)
0.3	903.0
0.4	366.3
0.5	196.4
0.6	124.5
0.7	88.3

We introduce an artificial factor *f*

to discuss the effect of repulsive KK interaction

$$V_{KK\text{-}KK} = f \times V_{KK0} \exp(-(r/r_G)^2)$$

An example of Eigenvalues in Coupled-Channel KKK^{bar}-Kππ-Kπη system



E=1395 – i 102 MeV

M_R= 1395MeV

Γ = 204 MeV

In the case of r_G=0.5fm

θ=0.35(+), 0.40(x), 0.45(※), 0.50(□)

KKK^{bar}-$K\pi\pi$-$K\pi\eta$ Three-Body Resonance Poles

$3m_K$=1485 MeV

	Two-channel* KKK^{bar}-$K\pi\pi$		Three-channel KKK^{bar}-$K\pi\pi$-$K\pi\eta$	
Range:r_G (fm)	Re(E) (MeV)	Im(E) (MeV)	Re(E) (MeV)	Im(E) (MeV)
0.3	-- (1432.7)	-- (-42.8)	1436.0	-57.1
0.4	1464.7 (1406.5)	-48.7 (-59.0)	1406.7	-74.4
0.5	1457.7 (1400.8)	-72.0 (-86.1)	**1395.2**	**-101.9**
0.6	1465.3 (1405.6)	-87.3 (-110.2)	**1396.1**	**-133.0**
0.7	-- (1440.2)	-- (-138.8)	--	--

( ):$V_{KKbar\text{-}KKbar}(I=1)=V_{KKbar\text{-}KKbar}(I=0)$ is assumed

[exp K(1460)] 1400 -125

1460 -130

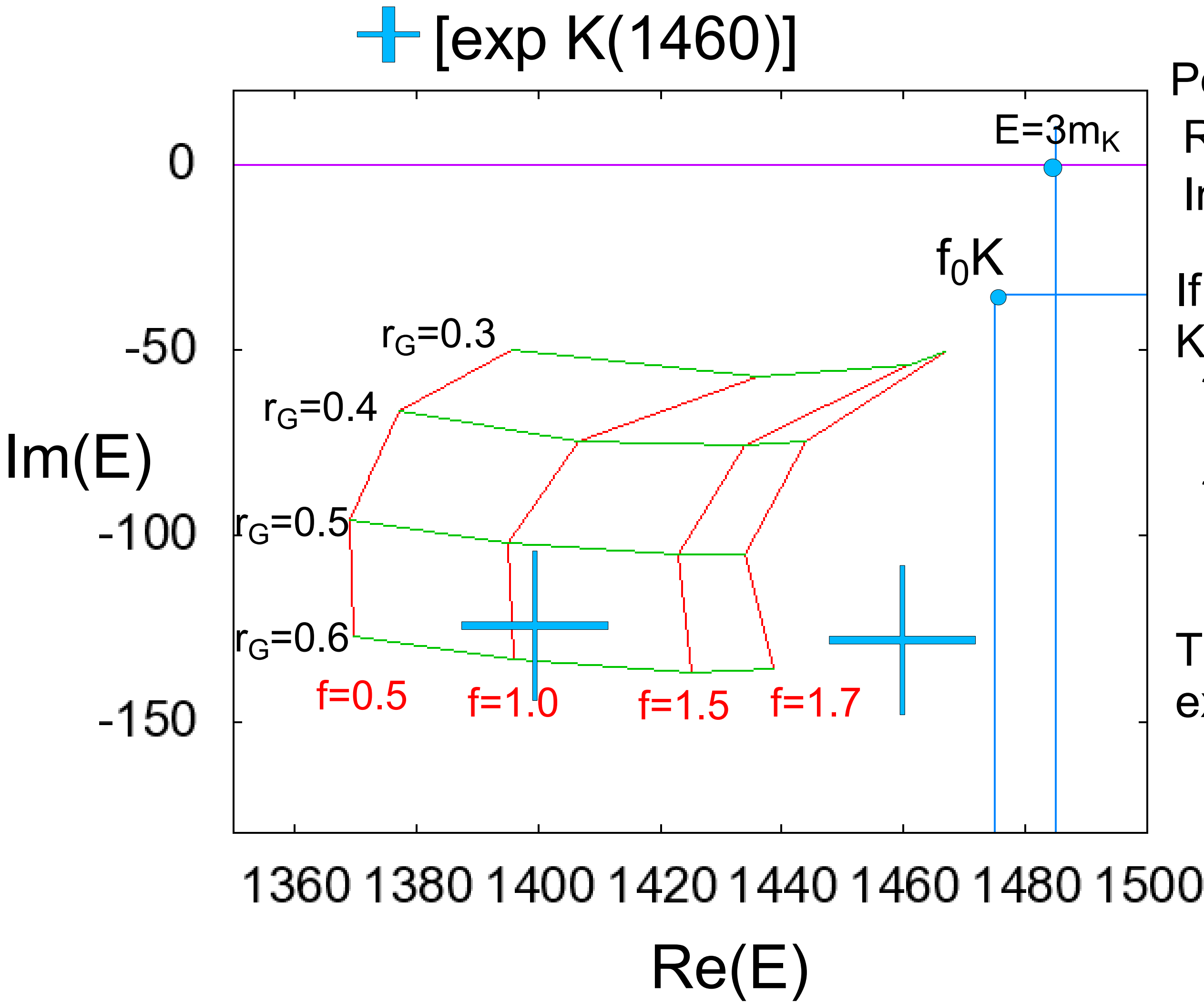
For r_G=0.7fm no pole

Pole position, especially Imaginary part, depends strongly on the potnetial range r_G

A good fit to K(1460) for r_G=0.5-0.6fm

$K\pi\eta$ channel is important to explain the width

KKK^{bar}-$K\pi\pi$-$K\pi\eta$ resonance pole positions
Dependence on KK interaction

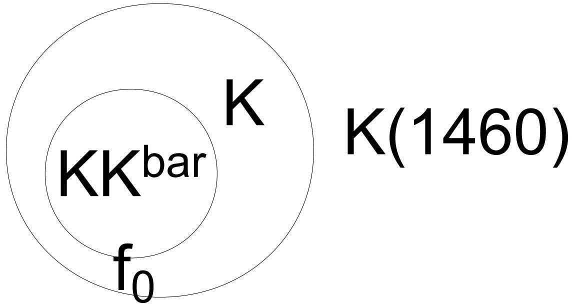


$$V_{KK\text{-}KK} = f \times V_{KK0} \exp(-(r/r_G)^2)$$

Pole position E_p
- $\mathrm{Re}(E_p)$ depends strongly on f
- $\mathrm{Im}(E_p)$ depends strongly on r_G

If this pole can be interpreted as K(1460),
- *KK Interaction must be repulsive.
- *KK Interaction range: r_G~0.5-0.6fm is supported.

The three-body resonance is expected to have a f_0K structure.



Summary

We solved the KKK^{bar}-$K\pi\pi$-$K\pi\eta$ three-meson problem, using the complex scaling method and found a three-body resonance pole.

Semi-relativistic Hamiltonian

Gaussian Expansion method

The resonance position depends strongly on the range of two-body potentials even if the potentials are adjusted to reproduce the same two-body properties.

This resonance can be interpreted as K(1460) discovered in two SLAC experiments. For this interpretation, repulsive KK interaction is essential and the potential range r_G=0.5-0.6fm is supported.

Future plans:

To use more realistic two-body interactions

Two- or Three-range gaussian potentials

Inculsion of $V_{\pi\eta-\pi\eta}$,$V_{\pi K\text{-}V\eta K}$, $V_{\eta K\text{-}\eta K}$

To intoroduce the P-wave components

To calculate the partial decay widths : εK, $K^*\pi$, ρK modes

Thank you for your attention