Doubly Heavy Baryons Expanded in $1/m_Q$

Takayuki Matsuki

Tokyo Kasei University

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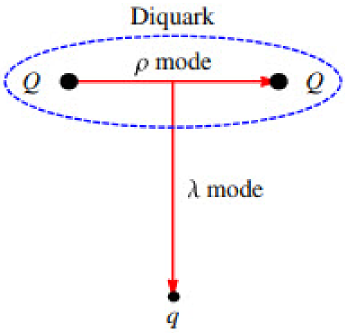
Motivation: discovery of $\Xi_{cc}^{++}(3621)$ by LHCb in 2017

- Discovery of $\Xi_{cc}^{++}(3621)$ by LHCb in 2017
- Discovery of $\Xi_{cc}^{+}(3519)$ by SELEX in 2003
- quark contents: $\Xi(cc\{u,d\}); I = 1/2$ or
  $\Omega(ccs); I = 0$ or
- $\Xi(bb\{u,d\}); I = 1/2$ or $\Omega(bbs); I = 0$
- cc/bb: can be regarded as a diquark; Flavor symmetric, Color antisymmetric $\bar{3}$

Motivation: System

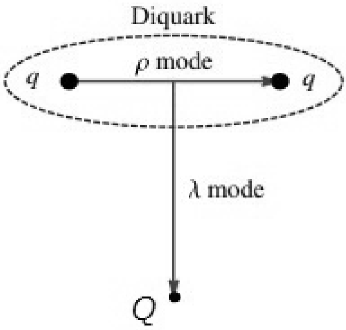
- Study the system



- Study wave funcitons for λ and ρ modes
- Discuss qualitatively narrow widths, mixing angles, and conserved quantities

Motivation: *Qqq* System

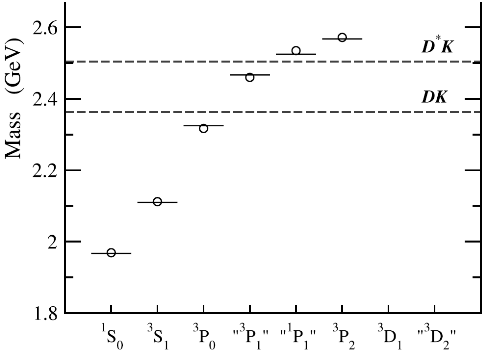
- Compare with the system



- ρ mode wave function becomes complicated

Motivation: Narrow heavy-light mesons

- Discovery of very narrow $D_s(2317, 2460)$ by BaBar & CLEO in 2003



prohibit $D_s(2317) \to D + K$ which leads to $D_s(2317) \to D + \pi$

Motivation: Formulation

- Heavy-light mesons expanded in $1/m_Q$
  MPLA11 (1996) 257; PRD56 (1997) 5646; PTP (2007) 117, 1077
  BS formulation, PRD52 (1995) 5229 by Zeng et al.; PRD 64 (2001) 114004 by DiPierro Eichten

  They are equivalent!

$$(H_0 + H_1 + \cdots)(\psi_0 + \psi_1 + \cdots) = (E_{-1} + E_0 + E_1 \cdots)(\psi_0 + \psi_1 + \cdots)$$

$$\begin{aligned}
H_{-1} &= m_Q \beta_Q, \quad E_{-1} = m_Q \\
H_0 &= \vec{\alpha}_q \cdot \vec{p} + \beta_q m_q - \beta_q \beta_Q S + (1 + \vec{\alpha}_q \cdot \vec{\alpha}_Q) V \\
m_Q H_1 &= -\frac{1}{2}\beta_Q \vec{p}^2 + \beta_q \vec{\alpha}_q \cdot (\vec{p} + \vec{q}/2) S + \frac{1}{2}\vec{\gamma}_Q \cdot \vec{q} V + \cdots
\end{aligned}$$

Motivation: Results of the former formulation

- Reproduce masses of $D_s(2317, 2460)$ below thresholds of $KD^{(*)}$ according to the expansion in $1/m_Q$ (Kinematical, not dynamical like coupled channel)

D_s mass spectra: PTP117 (2007) 1077

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0		$M_{\rm calc}$	$M_{\rm obs}$	χ^2/d.o.f
		p_1/M_0	n_1/M_0			
$^1S_0(0^-)$	1900	0.352×10^{-1}		1967	1969 ± 0.5 [39)]	1.60
		0.270×10^{-1}	0.816×10^{-2}			
$^3S_1(1^-)$		1.102×10^{-1}		2110	2112 ± 0.6 [39)]	1.11
		1.098×10^{-1}	4.076×10^{-4}			
$^3P_0(0^+)$	2095	1.101×10^{-1}		2325	2317 ± 0.6 [39)]	17.78
		1.027×10^{-1}	0.740×10^{-2}			
"$^3P_1$"(1^+)		1.779×10^{-1}		2467	2460 ± 0.9 [39)]	6.05
		1.752×10^{-1}	2.620×10^{-3}			

This happened because we select potentials not appearing as ordinary *LS* and *SS* couplings

Motivation: Mixing angles in heavy quark limit

- Predict mixing angles in HQS limit, e.g., $\tan^{-1}\theta = 1/\sqrt{2}$ between $^1P_1$ and $^3P_1$

$$\begin{pmatrix} j^P = 1^+,\ j_l = 1/2 \\ j^P = 1^+,\ j_l = 3/2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \left|{}^3P_1\right\rangle \\ \left|{}^1P_1\right\rangle \end{pmatrix}$$

In general,

$$\begin{pmatrix} L^P = 1^+,\ j_l = L - 1/2 \\ L^P = 1^+,\ j_l = L + 1/2 \end{pmatrix} = \frac{1}{\sqrt{2L+1}} \begin{pmatrix} \sqrt{L+1} & -\sqrt{L} \\ \sqrt{L} & \sqrt{L+1} \end{pmatrix} \begin{pmatrix} \left|{}^3L_L\right\rangle \\ \left|{}^1L_L\right\rangle \end{pmatrix}$$

$P = (-)^{L+1}$

Preparation: Foldy-Wouthyuisen-Tani transformation

To begin with, we need to introduce FWT transformation to expand the system in $1/m_Q$, which has an important property

- FWT transformation leads to expansion in $1/m_Q$

$$U_{FWT}(\vec{p}) \equiv \exp\left(W\vec{\gamma}\cdot\vec{n}\right), \quad \vec{n} = \frac{\vec{p}}{p}, \quad \tan W = \frac{p}{m_Q + E}$$

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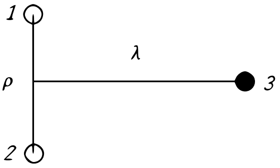
- This operates only on heavy quarks, which makes kinetic terms to free energy

$$U_{FWT}\left(\vec{p}\cdot\vec{\alpha} + m_Q\beta\right) U_{FWT}^{-1} = E\beta$$

Preparation: Jacobi coordinates

To describe QQq system, it is very convinient to use Jacobi coordinates

$$
\begin{aligned}
\vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)\,, \\
\vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) = \frac{1}{\sqrt{6}}\{(\vec{r}_1 - \vec{r}_3) + (\vec{r}_2 - \vec{r}_3)\}
\end{aligned}
$$



Formulation: Scenario

Scenario how to obtain λ and ρ mode wave functions and eigenvalues is as follows.

1. Set up Hamiltonian $H = H_{kin} + H_{int}$
2. Prepare FWT transformation on two heavy quarks
3. Expand H in terms of $1/m_Q$ using FWT transformation
4. Change the coordinates to Jacobi ones
5. Derive lowest order λ and ρ wave functions
6. Try to solve them
7. (Try to numerically solve them togehter with eigenvalues)

Formulation: Hamiltonian

- Kinetic terms

$$H_{kin} = \vec{p}_1 \cdot \vec{\alpha}_1 + m_Q \beta_1 + \vec{p}_2 \cdot \vec{\alpha}_1 + m_Q \beta_2 + \vec{p}_3 \cdot \vec{\alpha}_3 + m_q \beta_3$$

Formulation: Hamiltonian

- Kinetic terms

$$H_{kin} = \vec{p}_1 \cdot \vec{\alpha}_1 + m_Q\beta_1 + \vec{p}_2 \cdot \vec{\alpha}_1 + m_Q\beta_2 + \vec{p}_3 \cdot \vec{\alpha}_3 + m_q\beta_3$$

- Interation terms

$$H_{int} = V_{13} + V_{23} + V_{12} + \beta_1\beta_3 S_{13} + \beta_2\beta_3 S_{23} + \beta_1\beta_2 S_{12},$$

with

$$V_{ij} = (1 - \vec{\alpha}_i \cdot \vec{\alpha}_j)\, V(r_{ij}), \quad V(r_{ij}) = -\frac{2\alpha_s}{3}\frac{1}{r_{ij}}, \quad S_{ij} = \frac{r_{ij}}{a^2} + b,$$

Formulation: FWT transformation on QQ & expansion

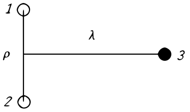
$$
\begin{aligned}
H_{-1} &= (\beta_1 + \beta_2)\, m_Q, \\
H_0 &= \vec{p}_3 \cdot \vec{\alpha}_3 + \beta_3 m_q + \beta_1\beta_2 S_{12}(r_{12}) + \beta_1\beta_3 S_{13}(r_{13}) \\
&\quad + \beta_2\beta_3 S_{23}(r_{23}) + V(r_{12}) + V(r_{13}) + V(r_{23}), \\
2m_Q H_1 &= \left(\beta_1 p_1^2 + \beta_2 p_2^2\right) + (\vec{q}_1 \cdot \vec{\gamma}_1 + \vec{q}_2 \cdot \vec{\gamma}_2)\,(\vec{p}_3 \cdot \vec{\alpha}_3 + m_Q \beta_3) \\
&\quad - [\beta_2\,(2\vec{p}_1 + \vec{q}_1) \cdot \vec{\alpha}_1 + \beta_1\,(2\vec{p}_2 + \vec{q}_2) \cdot \vec{\alpha}_2]\; S_{12}(r_{12}) \\
&\quad - \beta_3\,(2\vec{p}_1 + \vec{q}_1) \cdot \vec{\alpha}_1\; S_{13}(r_{13}) - \beta_3\,(2\vec{p}_2 + \vec{q}_2) \cdot \vec{\alpha}_2\; S_{23}(r_{23}) \\
&\quad - \Big[\beta_1 \vec{\alpha}_2 \cdot (2\vec{p}_1 + \vec{q}_1) + \beta_2 \vec{\alpha}_1 \cdot (2\vec{p}_2 + \vec{q}_2) + i\vec{\alpha}_1 \cdot \beta_2\; \vec{q}_2 \times \vec{\Sigma}_2 \\
&\quad + i\; \vec{\alpha}_2 \cdot \beta_1\; \vec{q}_1 \times \vec{\Sigma}_1\Big]\; V(r_{12}) \\
&\quad - \Big[\beta_1 \vec{\alpha}_3 \cdot (2\vec{p}_1 + \vec{q}_1) + i\vec{\alpha}_3 \cdot \beta_1\; \vec{q}_1 \times \vec{\Sigma}_1\Big]\; V(r_{13}) \\
&\quad - \Big[\beta_2 \vec{\alpha}_3 \cdot (2\vec{p}_2 + \vec{q}_2) + i\vec{\alpha}_3 \cdot \beta_2\; \vec{q}_2 \times \vec{\Sigma}_2\Big]\; V(r_{23}),
\end{aligned}
$$

Formulation: Lowest order λ mode equation

Lowest order λ mode Schrödinger equation:

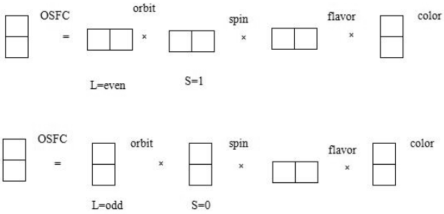
$$\left[-\vec{p}_{\lambda'} \cdot \vec{\alpha}_3 + m_q\beta_3 + 2V(\lambda') + 2\beta_3 S(\lambda')\right]\psi_0 = E_0\psi_0$$

where $\lambda' = \lambda/\sqrt{6}$, $2V = -4\alpha_s/(3r)$, and $2S = 2r/a^2 + 2b$.
Twice of confining potential and the same one-gluon exchange potential as $\bar{3}$
That is, there are two color sources for confining potential, $2S$, but $3 \times 3 = 6 + \bar{3}$ for one-gluon exchange potential, $2V$



Formulation: Lowest order ρ mode equation

Lowest order ρ mode Schrödinger equation must be antisymmetric in flavor, color, spin, and orbital angular momentum as shown in the figure



$$(s_\rho = 0, L_\rho = 1), \quad \text{or} \quad (s_\rho = 1, L_\rho = 0)$$

That is, for a diquark, either ${}^3S_1$ or ${}^1P_1$. So the ground state of QQq is given by
${}^{2s_\rho+1}L_{\rho_{j_\rho}} \; {}^{2s_\lambda+1}L_{\lambda_{j_\lambda}} = \; {}^3S_1 \; {}^1S_{1/2}, \; J^P = 1/2^+, 3/2^+ \; (P = (-)^{L_\rho+L_\lambda})$

Formulation: Lowest order ρ mode equation

Lowest order ρ mode Schrödinger equation must satisfy

$$H_\rho \psi_\rho = E_\rho \psi_\rho, \quad H_\rho = \frac{\vec{p}_\rho^{\,2}}{2m_Q} + S\left(\sqrt{2}\rho\right) + V\left(\sqrt{2}\rho\right)$$

This is one component of ψ_ρ. There are other components.
Total energy of doubly heavy baryon is given by

$$E_{tot} = E_0 + E_\rho + \left\langle \frac{p_\lambda^2}{2m_Q} \right\rangle$$

$$
\begin{aligned}
y_{jm}^{k}(\Omega) &= \frac{1}{\sqrt{2(j+1)}} \begin{pmatrix} \sqrt{j+1-m}\, Y_{j+1/2}^{m-1/2} \\ -\sqrt{j+1+m}\, Y_{j+1/2}^{m+1/2} \end{pmatrix}, \\
y_{jm}^{-k}(\Omega) &= \frac{1}{\sqrt{2j}} \begin{pmatrix} \sqrt{j+1-m}\, Y_{j+1/2}^{m-1/2} \\ -\sqrt{j+1+m}\, Y_{j+1/2}^{m+1/2} \end{pmatrix} = (\vec{\sigma}\cdot\vec{n})\; y_{jm}^{k}(\Omega),
\end{aligned}
$$

with Y_j^m spherical hamonics and $k = j_\lambda + 1/2$. A general solution is

$$
\psi_{jm}^{(\lambda)k} = \frac{1}{r} \begin{pmatrix} u_k(r) \\ -iv_k(r)\,(\vec{\sigma}\cdot\vec{n}) \end{pmatrix} y_{jm}^{k}(\Omega),
$$

$$
\begin{pmatrix} m_q + 2S + 2V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - 2S + 2V \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = E_0^k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}
$$

Results: Conserved quantum numbers

We find the quantum number similar to one for heavy-light mesons

$$K = -\beta_q \left(\vec{\Sigma}_q \cdot \vec{L}_\lambda + 1 \right), \quad K\psi_\ell = k\psi_\ell,$$
$$k = \pm \left(j_\lambda + \frac{1}{2} \right),$$

where k appears in Schrödinger eq. for λ mode wf.

Conclusions and summary

- $1/m_Q$ expansion works well for doubly heavy baryon systems

Conclusions and summary

- $1/m_Q$ expansion works well for doubly heavy baryon systems
- solutions to λ mode wave functions are obtained

$$\psi_{jm}^{(\lambda)k} = \frac{1}{r}\left(\begin{array}{c} u_k(r) \\ -iv_k(r)\,(\vec{\sigma}\cdot\vec{n}) \end{array}\right) y_{jm}^k(\Omega),$$

$$\left(\begin{array}{cc} m_q + 2S + 2V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - 2S + 2V \end{array}\right)\left(\begin{array}{c} u_k(r) \\ v_k(r) \end{array}\right) = E_0^k \left(\begin{array}{c} u_k(r) \\ v_k(r) \end{array}\right)$$

Conclusions and summary

- We expect that ρ mode excitation energies are smaller than λ mode ones, $E_\rho < E_\lambda$ So the first excitation is $n_\rho L_\rho n_\lambda L_\lambda = 1P1S$ which is next of $1S1S$. $E(1P1S) < E(1S1P)$. Below is taken from Qi-Fang Lü et al. PRD96 (2017) 114006.

PHYSICAL REVIEW D **96,** 114006 (2017)

TABLE III. Predicted mass spectra of Ξ_{cc}, Ω_{cc}, Ξ_{bb}, and Ω_{bb}. The units are in MeV.

$(N_d L_d n_q l_q)J^P$	Ξ_{cc}	Ω_{cc}	Ξ_{bb}	Ω_{bb}
$(1S1s)1/2^+$	3606	3715	10 138	10 230
$(1S1s)3/2^+$	3675	3772	10 169	10 258
$(1S1p)1/2^-{}_{S_T=1/2}$	3998	4087	10 525	10 605
$(1S1p)3/2^-{}_{S_T=1/2}$	4014	4107	10 526	10 610
$(1S1p)1/2^-{}_{S_T=3/2}$	3985	4081	10 504	10 591
$(1S1p)3/2^-{}_{S_T=3/2}$	4025	4114	10 528	10 611
$(1S1p)5/2^-$	4050	4134	10 547	10 625
$(1S2s)1/2^+$	4172	4270	10 662	10 751
$(1S2s)3/2^+$	4193	4288	10 675	10 763
$(1P1s)1/2^-$	3873	3986	10 364	10 464
$(1P1s)3/2^-$	3916	4020	10 387	10 482

Conclusions and summary

- Narrow decay widths are expected for the following processes, when $M(\Omega_{cc}) < M(\Xi_{cc}) + M(K)$
  For exmaple, in this case, the process $\Omega_{cc} \to \Xi_{cc} + K$ is prohibited. Hence the **strong decay width of Ω_{cc} becomes very narrow**. There is only one channel allowed, $\Omega_{cc} \to \Xi_{cc} + \pi$ through 10^{-2} times suppresed due to $\eta - \pi^0$ mixing. Both Ω_{cc} and Ξ_{cc} have the same J but have the opposite P.

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Conclusions and summary

Mixing angles:

- Heavy quark symmetric mixing occurs for heavy-light mesons between $^1P_1$ and $^3P_1$. This is not the case for doubly heavy baryons because there so many states with the same J^P.

$(N_d L_d n_q l_q) J^P$	Ξ_{cc}	Ω_{cc}	Ξ_{bb}	Ω_{bb}
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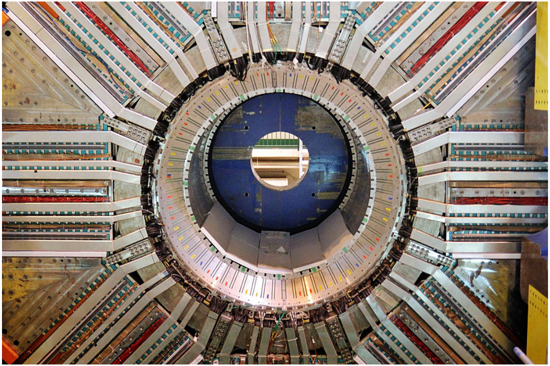
$(2S1s)1/2^+$	4004	4118	10 464	10 566

Thanks for your attention!



Belle II