Lattice calculation of nucleon form factor at physical point

Eigo Shintani（RIKEN-CCS)
K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, T. Yamazaki for PACS Collaboration

8th International Conference on Quarks and Nuclear Physics (QNP2018), Tsukuba, 14 November, 2018

Contents

1. Introduction
2. Set-up
3. Results
4. Summary

Charge radius

Root-mean-square(RMS) radii

$$R_l \equiv \sqrt{\langle r_l^2 \rangle} = -\frac{6}{G_l(0)} \frac{dG_l(q^2)}{dq^2}\Big|_{q^2=0}, \quad l = E, M, A$$

"Proton radius puzzle"

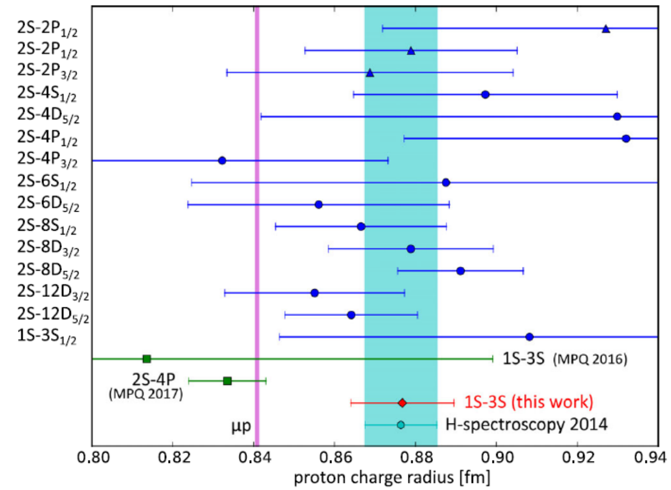
$$R_E = \begin{cases} 0.879(8)\ \text{fm} & \text{e-p scattering} \\ 0.876(8)\ \text{fm} & \text{H-D spectroscopy} \\ 0.8409(4)\ \text{fm} & \mu\ \text{hydrogen} \end{cases}$$

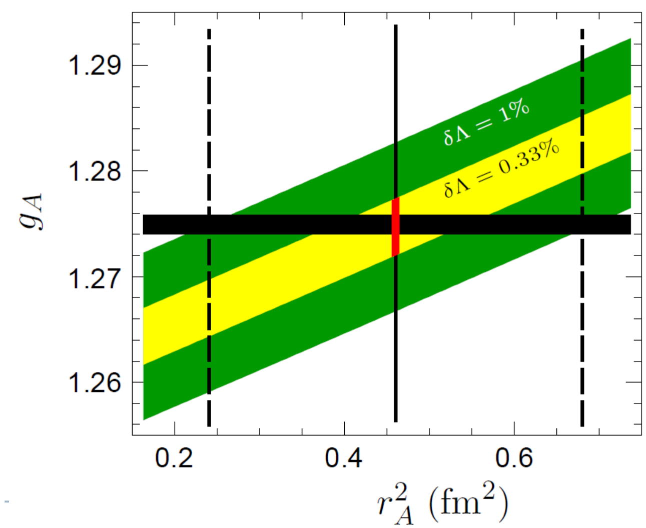
4.7σ deviation between muonic H and H-atom
→ Universality violation of lepton (μ and e) flavor ?

Axial RMS radius

- Input for neutrino physics from N-ν scattering
- Axial RMS from Muon capture
  ⇒ comparable with neutron beta decay

Fleurbaey, et al., PRL120(2018)





Hill et al., 1708.08462

1. Introduction

Role of LQCD for RMS radii

- Ab initio calculation of QCD
  - Theoretical calculation without modeling
  - Comparable with experimental value
    - Possible to obtain high precision by improved algorithm
- Target precision

Proton electric RMS radius : <1%

Possible to determine which experiments, H-atom or μH-atom, are favored in QCD

Axial RMS radius : 1-2%

Comparable with ν-N scattering data

Lattice QCD can determine both radii simultaneously
→ Validity test of LQCD computation

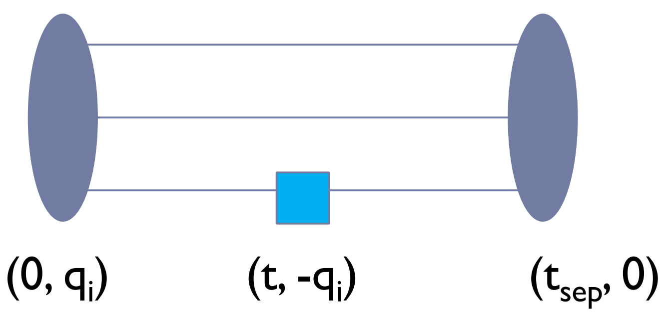
2. Set-up

LQCD with PACS10

- PACS10 parameters

PACS, 1807.06237

  - 2+1 flavor in stout smeared Wilson-clover fermion
  - Iwasaki gauge action, lattice cut-off 2.33 GeV
  - 128^4 lattice size, L=10.8 fm
  - Physical pion (135 MeV pion)
- Three-point function



- Fixed source-sink point and moving operator
- Sequential source method
- Source-sink separation t_{sep}/a= 10, 12, 14, 16
- Investigation of excited state contamination

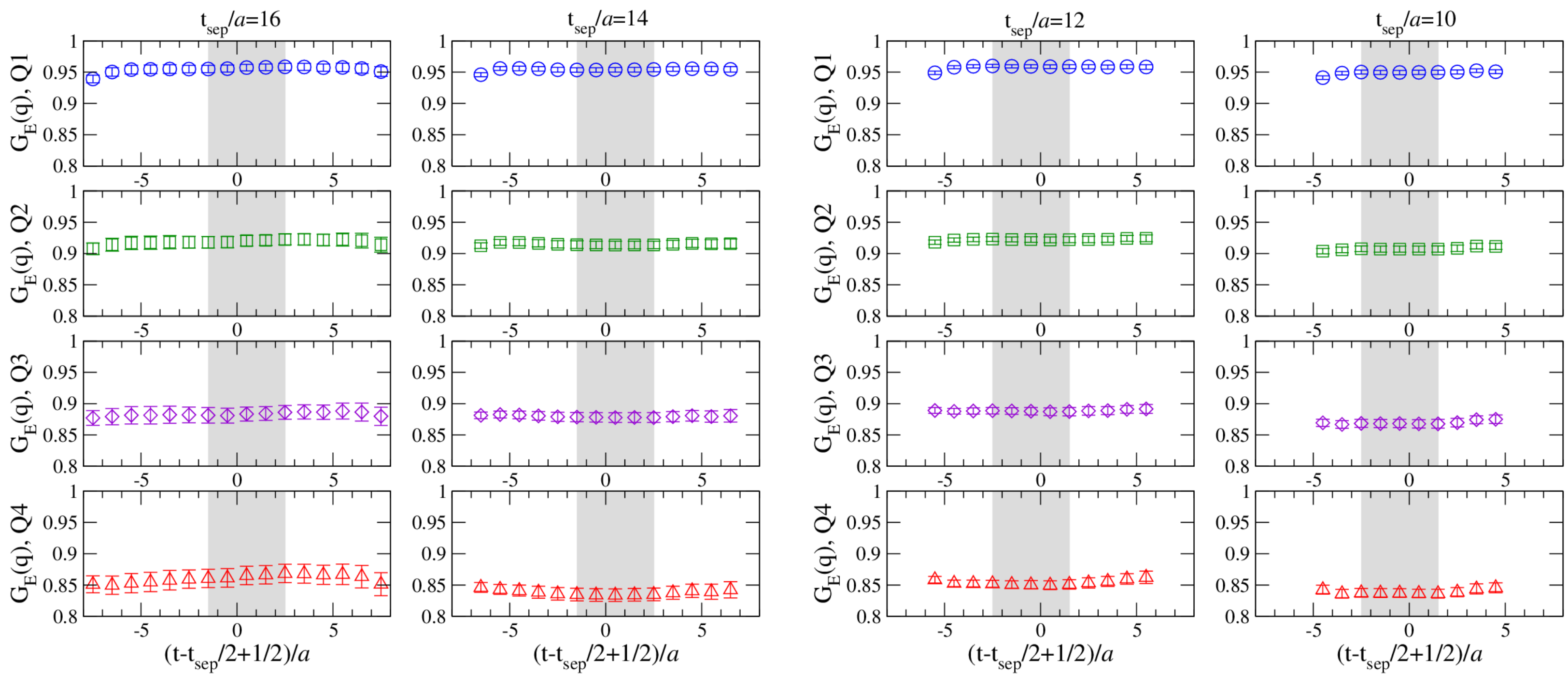
- All-mode-averaging（AMA)
  - Biasless method with combination of high and low precision
  - Deflated SAP + GCR optimization

Mainz, NPB914 (2017)

3. Results

Investigation of excited state

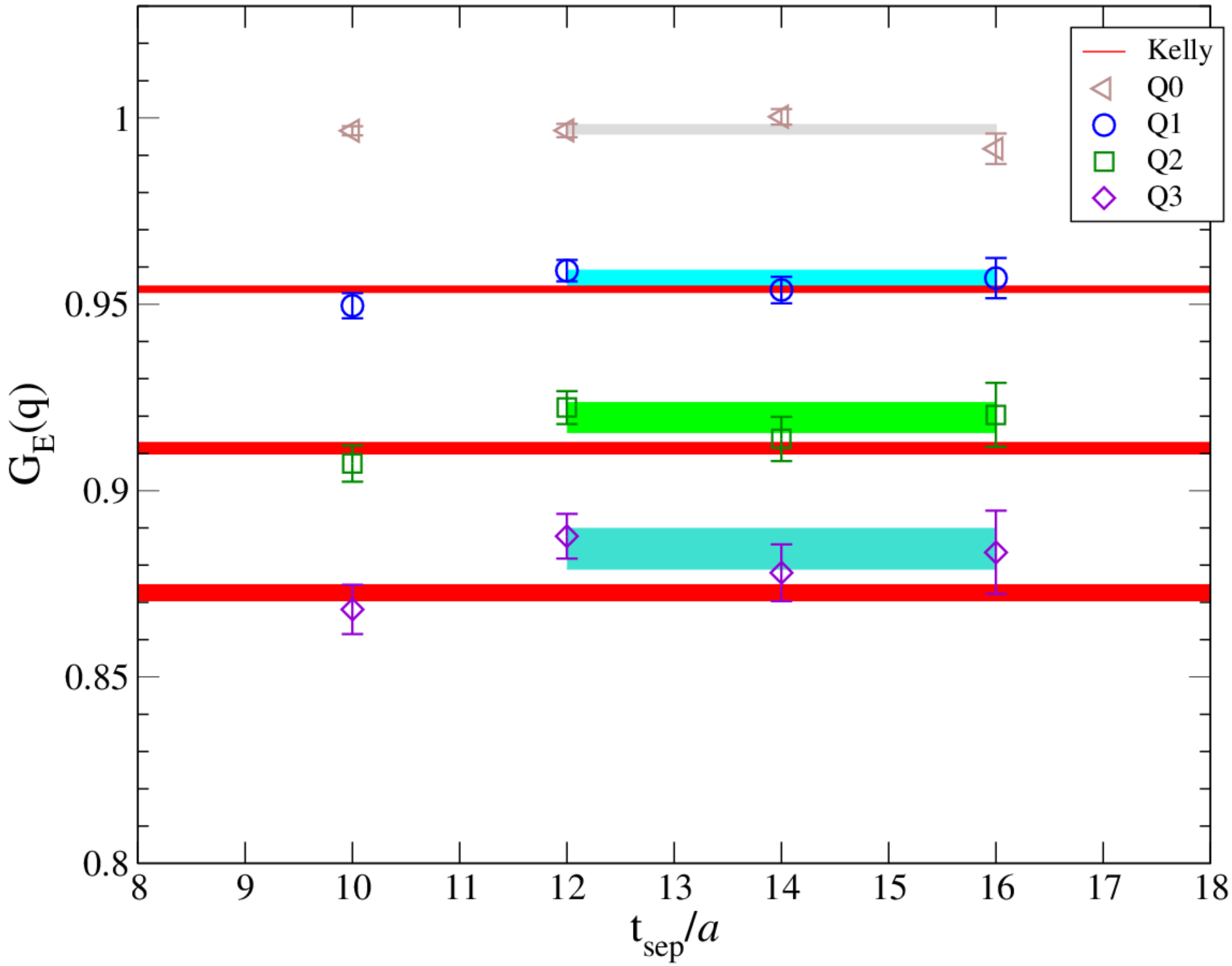
- In the case of G_E



- Using the exponential smeared source
- Clear plateau signal appears at each q^2 ⇒ small excited state contamination
- Fitting range: symmetric at $t_{sep}/2$

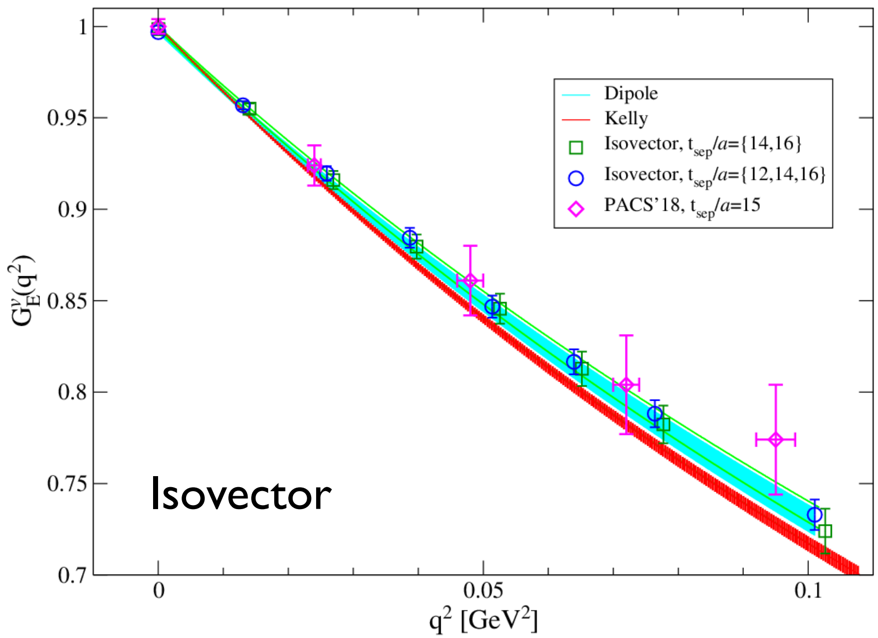
3. Results

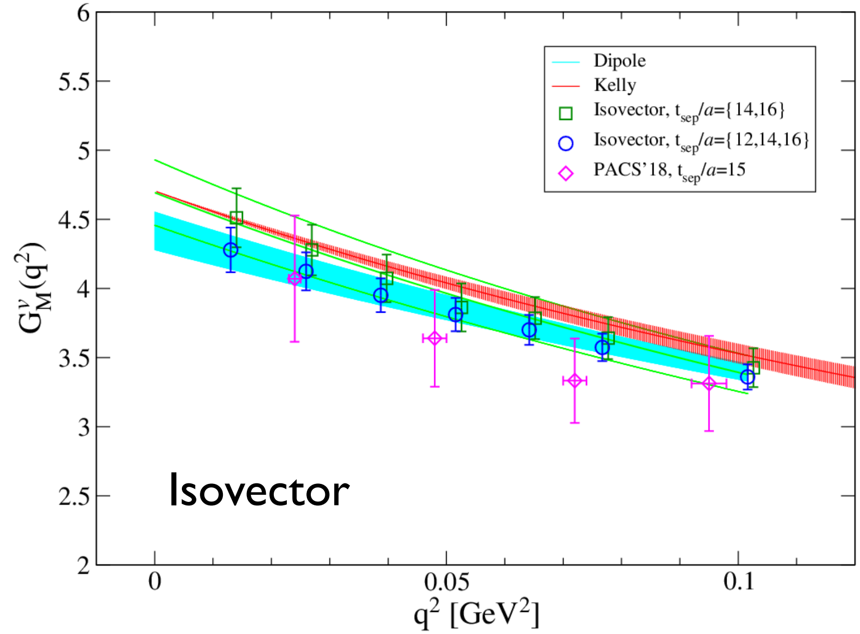
t_{sep} dependence



3. Results

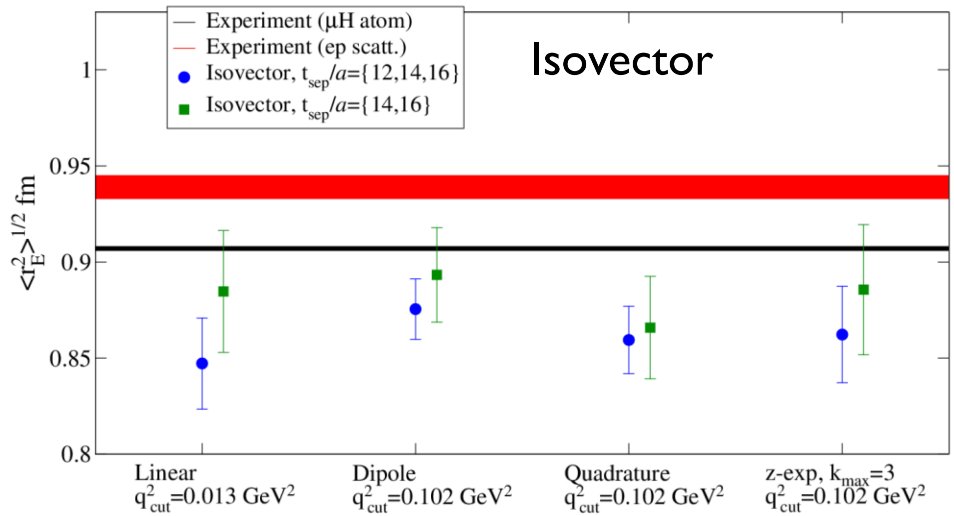
q^2 dependence for EM form factor

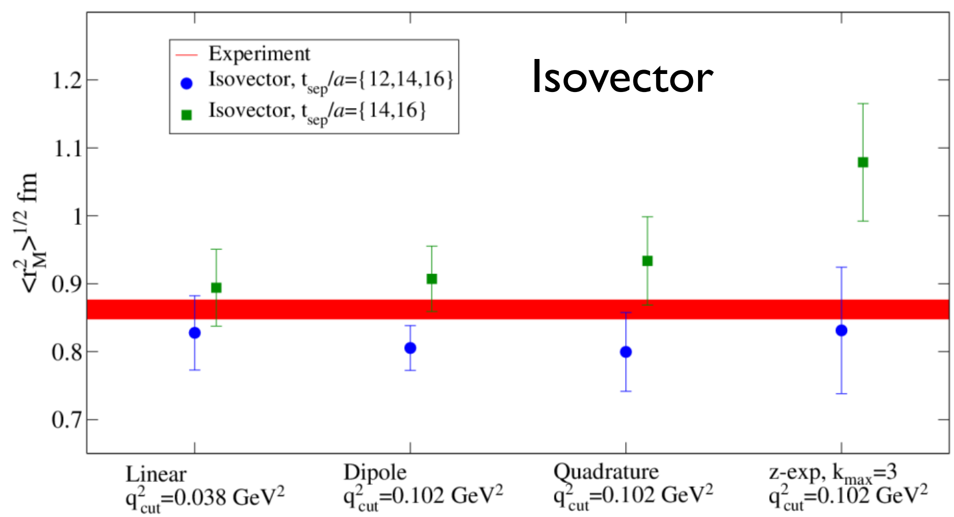




- Compared to the previous study (PACS 1807.03974), the statistical accuracy is much improved.
- The lowest q^2 points is increased, which is crucial to reduce systematic error in RMS radius.
- Compared to experimental value, LQCD result is close to Kelly's parameterization.

Electric and magnetic RMS radii





4 kinds of fitting function

- Linear
- Dipole
- Quadrature
- z-expansion

Fit range:

$q^2 < 0.03$GeV^2 for linear function

$q^2 < 0.1$ GeV^2 for others

t_{sep} range:

t_{sep}/a = {12, 14, 16} ⇒ 1.02 fm – 1.35 fm

{14,16} ⇒ 1.18 fm – 1.35 fm

Electric RMS radius

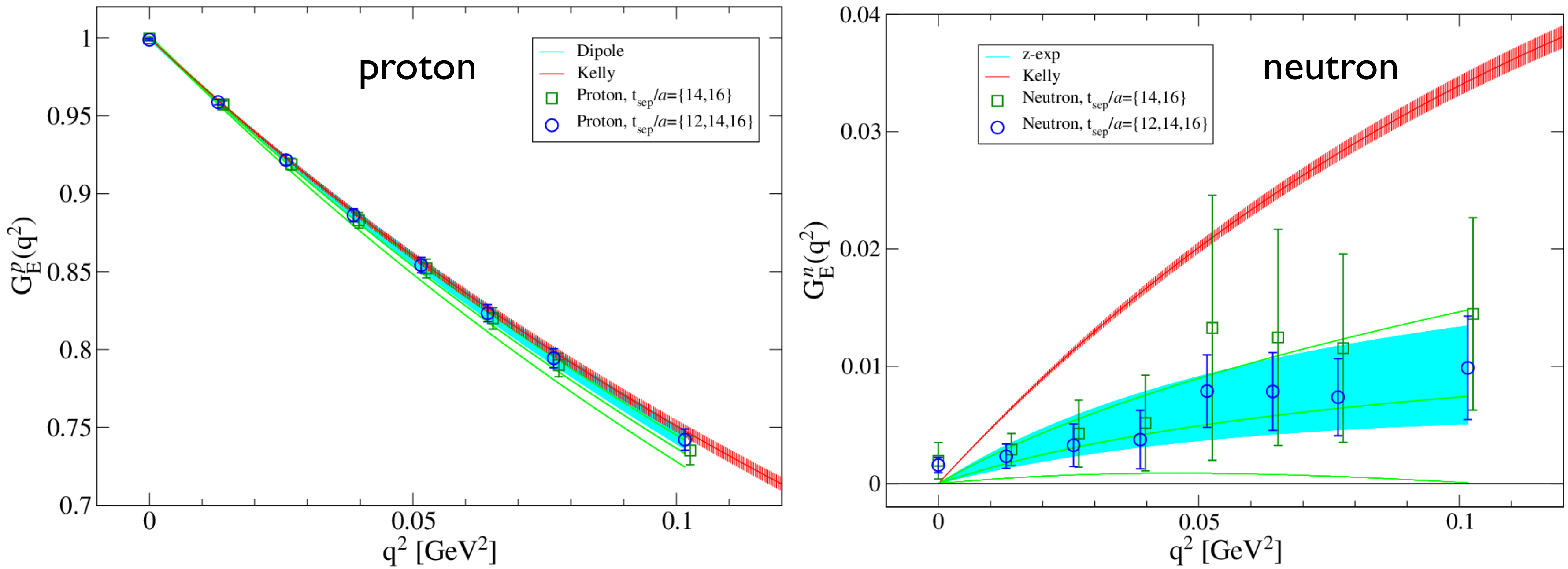
- Good agreement within 1σ error
- Close to the μH-atom experiment

Magnetic RMS radius

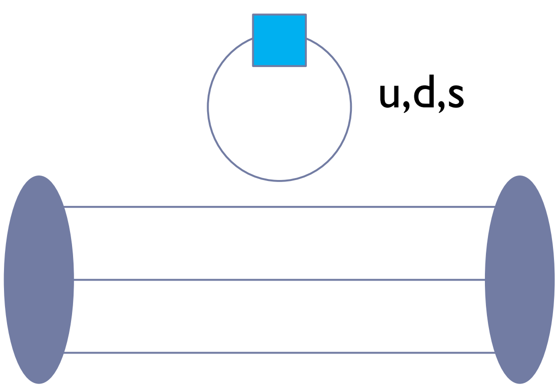
- Relatively large t_{sep} dependence
- Good consistency with experiment

3. Results

Proton's and neutron's RMS radius



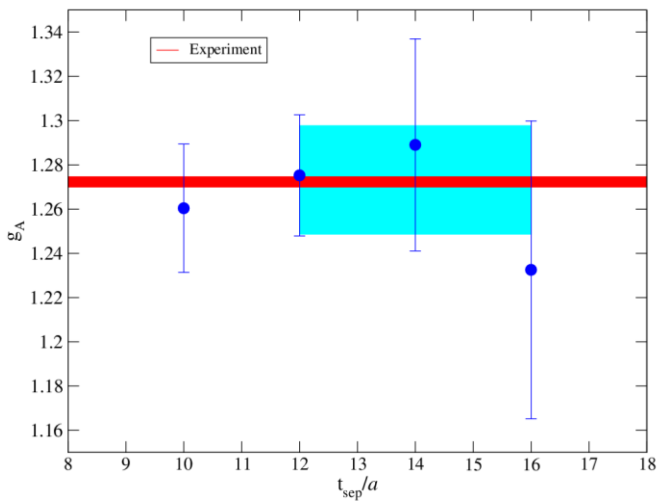
- Missing disconnected diagram
  ⇒ O(m_s-m_{ud}) effect
  ⇒ slightly positive contribution to $G_E(q^2)$.
- Not so large discrepancy from the experimental value, but visible effect.

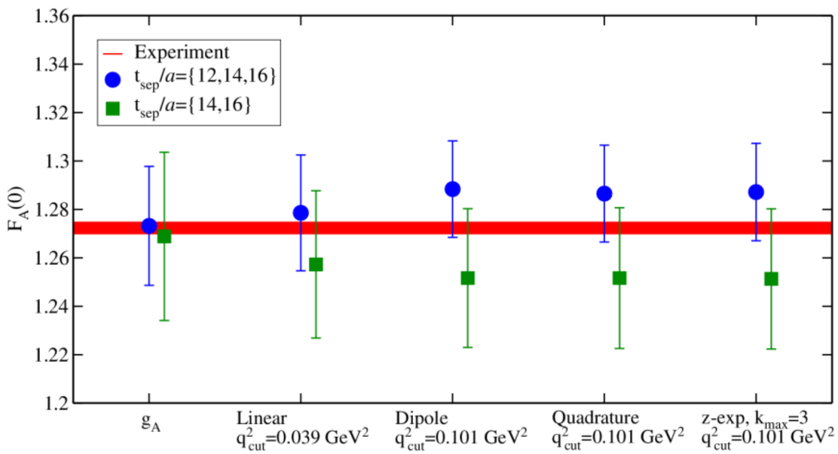


3. Results

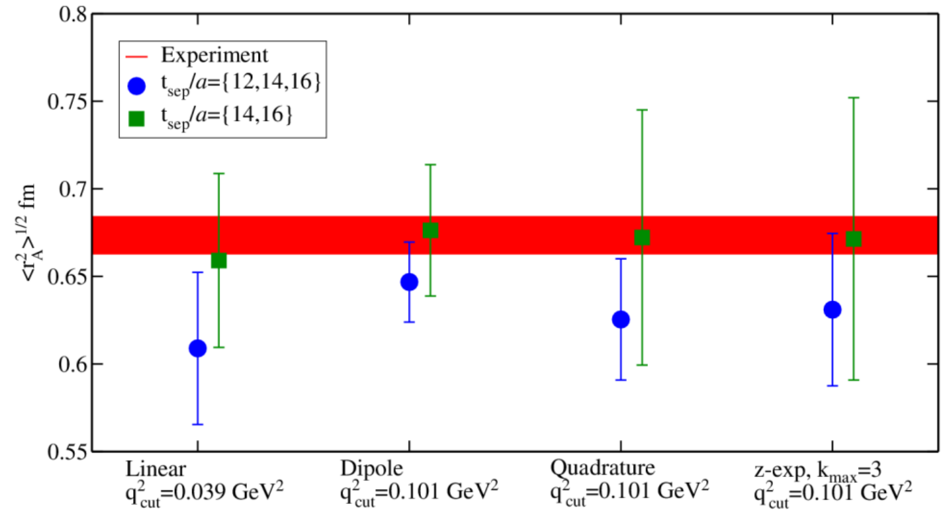
Axial charge and RMS radius

➢ Axial charge





➢ Axial RMS radius

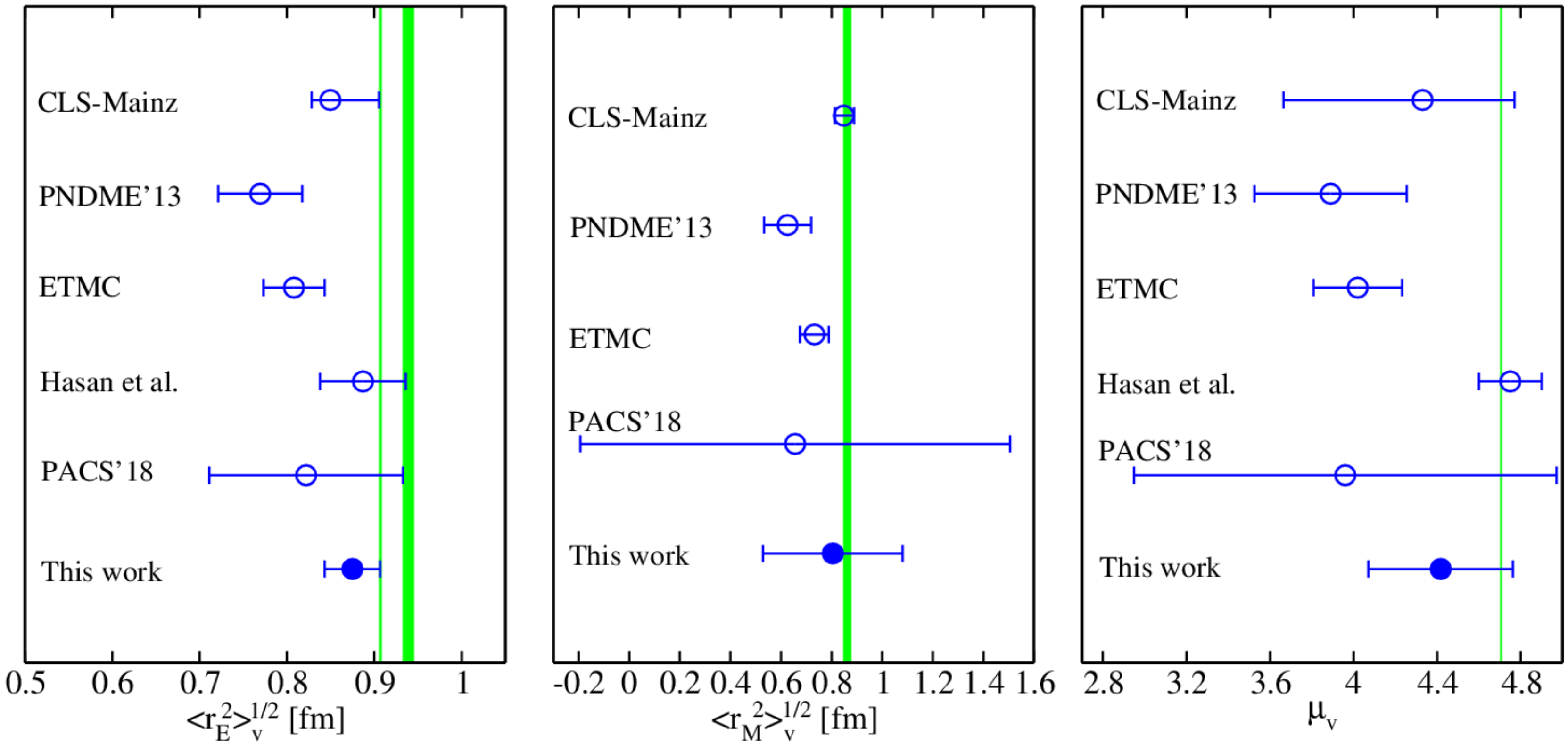


- Not large excited state effect.
- good consistency between different fitting functions.
- Both g_A and RMS radius are in good agreement with experimental value.

3. Results

Comparison with other LQCD results

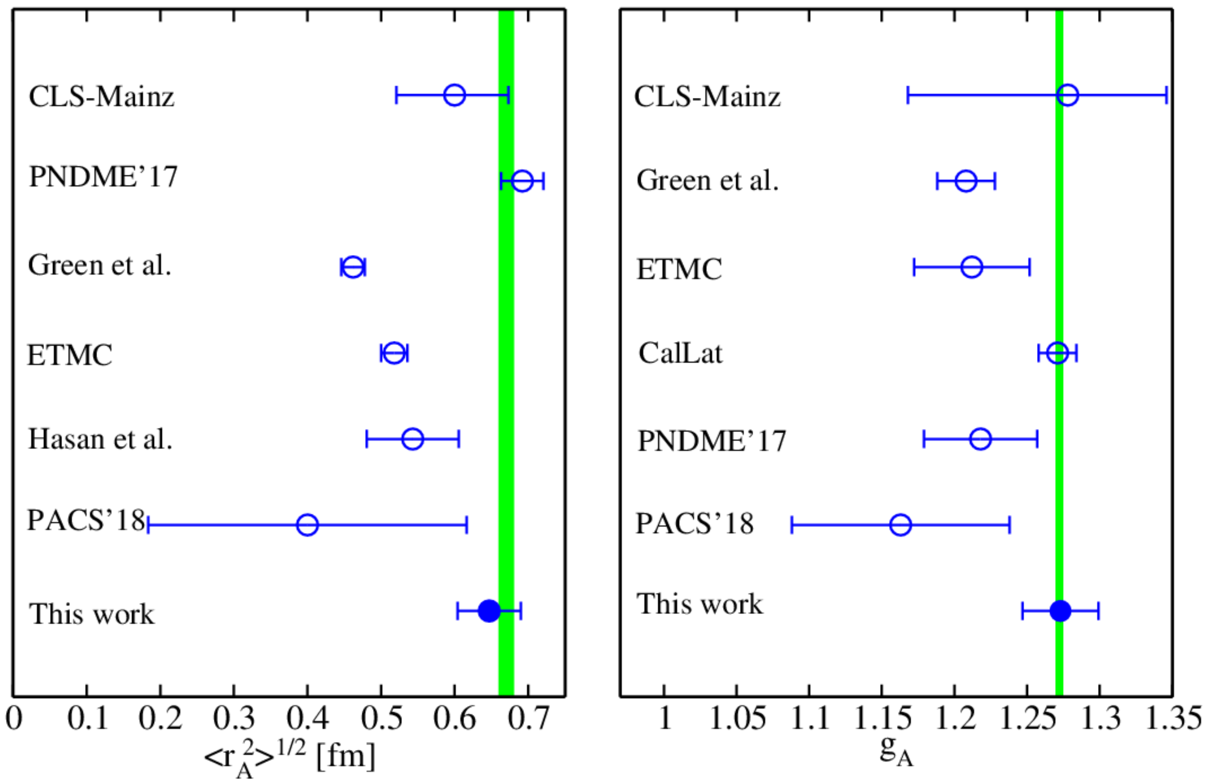
- Electric and magnetic RMS radii and moment



Precision: electric RMS radius ~ 4%, magnetic RMS radius ~ 34%

Comparison with other LQCD results

- Axial charge and RMS radius



Precision: axial RMS radius ~ 7%, axial charge ~ 2%

Summary and outlook

- LQCD computation of nucleon form factor on L=10.8 fm box.
- High precision calculation at the physical point.
- Suppress the systematic uncertainty of excited state and finite size correction.
- Coincide with experimental values.
  Electric charge RMS radius is close to μH atom experiment
- In the future
  - More high precision comparable with experimental accuracy.
  - Study of lattice cut-off effect.

Backup

Nucleon form factor

- Matrix element

$$\langle N(P')|j_\alpha^{\rm em}|N(P)\rangle = \bar{u}_N(P')\Big(\gamma_\alpha F_1^N(q^2) + \sigma_{\alpha\beta}\frac{q_\beta}{2M_N}F_2^N(q^2)\Big)u_N(P)$$

$$\langle p(P')|A_\alpha^+|n(P)\rangle = \bar{u}_p(P')\Big(\gamma_\alpha\gamma_5 F_A(q^2) + iq_\alpha\gamma_5 F_P(q^2)\Big)u_n(P)$$

$$\langle p(P')|P_\alpha^+|n(P)\rangle = \bar{u}_p(P')\gamma_5 G_P(q^2)u_n(P)$$

Electric and magnetic form factor

$$G_E^N(q^2) = F_1^N - \frac{q^2}{4M_N^2}F_2^N, \quad G_M^N(q^2) = F_1^N + F_2^N$$

$$G_E^v = G_E^p - G_E^n, \quad G_M^v = G_M^p - G_M^n$$ (isovector形)

$$G_E^v(0) = 1, \quad G_M^v(0) = \mu_m^p - \mu_m^n = 4.70589$$

Axial form factor

$$F_A(q^2), \quad F_A(0) = g_A$$ (軸正電荷)

Pseudo scalar form factor

$$G_P(q^2) \qquad F_P(q^2)$$ （induced 擬スカラー)