

From hadrons at unphysical quark masses to coupled-channel reaction dynamics in the laboratory

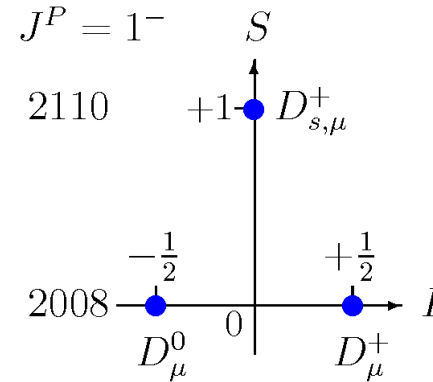
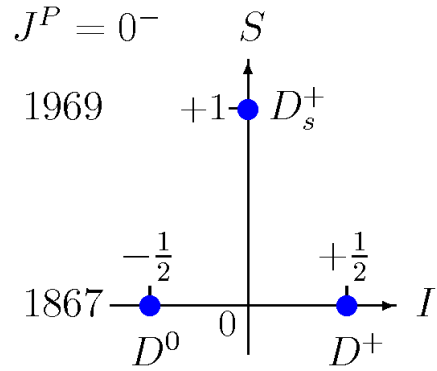
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- ✓ Chiral extrapolation for charmed meson masses
- ✓ Coupled-channel dynamics for charmed mesons
- ✓ Summary and outlook

Chiral Lagrangian for charmed mesons

✓ **Heavy-light mesons** ($c\bar{q}$) SU(3) anti-triplet $[\bar{3}]$



$$\begin{aligned} \mathcal{L} = & (\partial_\mu D)(\partial^\mu \bar{D}) - M^2 D \bar{D} - (\partial_\mu D^{\mu\alpha})(\partial^\nu \bar{D}_{\nu\alpha}) + \frac{1}{2} \tilde{M}^2 D^{\mu\alpha} \bar{D}_{\mu\alpha} \\ & + 2g_P \left\{ D_{\mu\nu} U^\mu (\partial^\nu \bar{D}) - (\partial^\nu D) U^\mu \bar{D}_{\mu\nu} \right\} \\ & - \frac{i}{2} \tilde{g}_P \epsilon^{\mu\nu\alpha\beta} \left\{ D_{\mu\nu} U_\alpha (\partial^\tau \bar{D}_{\tau\beta}) + (\partial^\tau D_{\tau\beta}) U_\alpha \bar{D}_{\mu\nu} \right\} \end{aligned}$$

covariant derivative $\partial_\mu \rightarrow \partial_\mu + \frac{1}{2} e^{-i \frac{\Phi}{2f}} \partial_\mu e^{+i \frac{\Phi}{2f}} + \frac{1}{2} e^{+i \frac{\Phi}{2f}} \partial_\mu e^{-i \frac{\Phi}{2f}}$

- chiral symmetry : $f \sim 90$ MeV chiral SU(3) limit value of f_π
- hadronic decay of $D^* \rightarrow D\pi$ implies $|g_P| = 0.57 \pm 0.07$
- heavy-quark spin symmetry : $\tilde{g}_P = g_P$ and $M = \tilde{M}$ as $m_c \rightarrow \infty$

Chiral SU(3) for heavy-light meson resonance

✓ Coupled-channel interaction from chiral SU(3) symmetry

S \ I	0	1/2	1	3/2
+2		$(D_s K)$		
+1	$(DK, D_s \eta)$		$(D_s \pi, DK)$	
0		$(D\pi, D\eta, D_s K)$		$(D\pi)$
-1	$(\bar{K}D)$		$(\bar{K}D)$	

$$8 \otimes \bar{3} = \bar{15} \oplus 6 \oplus \bar{3}$$

- Weinberg- Tomozawa interaction

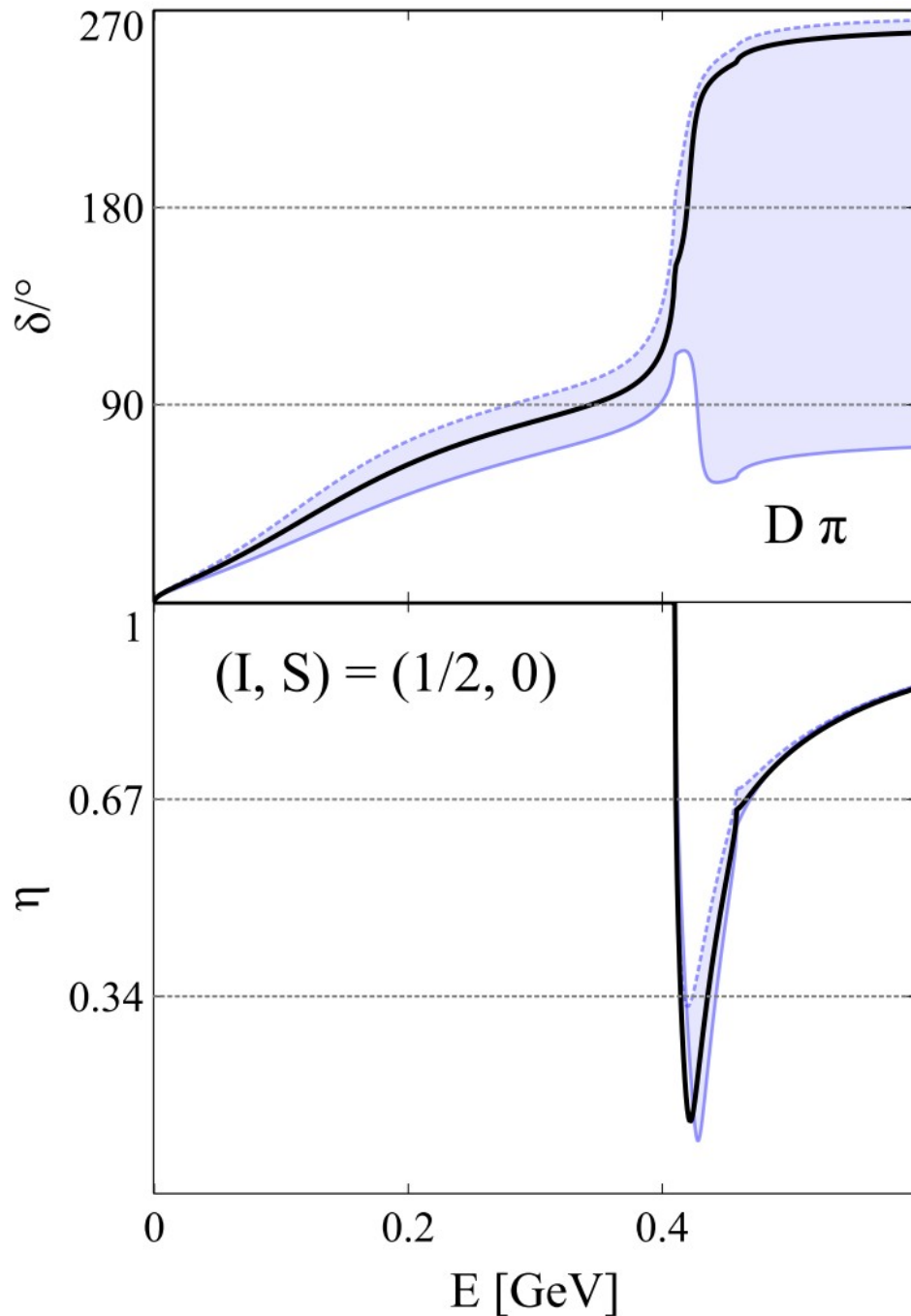
$$\mathcal{L}_{WT} = \frac{1}{8f^2} D[\Phi, (\partial_\mu \Phi)](\partial^\mu \bar{D}) + \dots$$

$$V_{WT}^{[\bar{15}]} : V_{WT}^{[6]} : V_{WT}^{[3]} = -1 : 1 : 3$$

- strong attraction in anti-triplet and sextet
- dynamically generated $J^P = 0^+$ and $J^P = 1^+$ meson resonances in particular $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$

✓ Are there exotic flavour sextet resonances?

Leading order prediction for πD phase shift in $I = 1/2$



✓ Two resonances in phase shift

- a broad state from the anti-triplet
- a narrow state from the sextet
- a variation of the matching scale
- s-wave phase shift required in unitarity studies of CKM matrix from $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$ decays (LHCb and Belle)

✓ How stable is the prediction?

- the broad state from the anti-triplet may move further into the complex plane
- the narrow state from the sextet always at $E \sim 0.4$ GeV
- are higher order counter terms important?

Chiral SU(3) correction terms for charmed mesons

$$\begin{aligned}
 \mathcal{L}^{(2)} = & - (4c_0 - 2c_1) D \bar{D} \text{tr} \chi_+ - 2c_1 D \chi_+ \bar{D} \\
 & - 4(2c_2 + c_3) D \bar{D} \text{tr} (U_\mu U^\mu) + 4c_3 D U_\mu U^\mu \bar{D} \\
 & - \frac{1}{M^2} (4c_4 + 2c_5) (\partial_\mu D)(\partial_\nu \bar{D}) \text{tr} [U^\mu, U^\nu]_+ + \frac{1}{M^2} 2c_5 (\partial_\mu D) [U^\mu, U^\nu]_+ (\partial_\nu \bar{D}) \\
 & - i c_6 \epsilon^{\mu\nu\rho\sigma} \left(D [U_\mu, U_\nu]_- \bar{D}_{\rho\sigma} - D_{\rho\sigma} [U_\nu, U_\mu]_- \bar{D} \right)
 \end{aligned}$$

covariant derivative $\partial_\mu \rightarrow \partial_\mu + \frac{1}{2} e^{-i \frac{\Phi}{2f}} \partial_\mu e^{+i \frac{\Phi}{2f}} + \frac{1}{2} e^{+i \frac{\Phi}{2f}} \partial_\mu e^{-i \frac{\Phi}{2f}},$

✓ How to determine the low-energy constants?

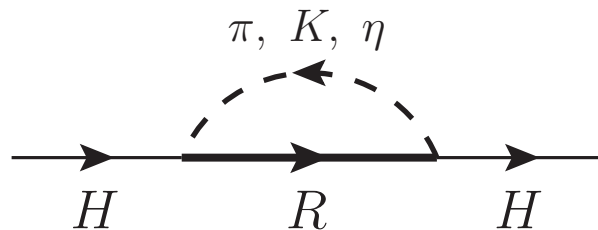
- additional 6 terms parameterized by \tilde{c}_n involving two $J^P = 1^-$ fields ($D_{\alpha\beta} \dots \bar{D}_{\mu\nu}$)
- from heavy-quark spin symmetry $\tilde{c}_n = c_n$ at $m_c \rightarrow \infty$
- from large- N_c $c_0 \simeq \frac{c_1}{2}, \quad c_2 \simeq -\frac{c_3}{2}, \quad c_4 \simeq -\frac{c_5}{2},$
- use QCD lattice data on the quark-mass dependence of the D meson masses

Quark-mass dependence of hadron masses

✓ A challenge for chiral SU(3) symmetry

- the conventional formulation of three flavour χ PT does not converge
- conventional χ PT inconsistent with three-flavor QCD lattice simulations

✓ Loops depend sensitively on internal masses



- develop a chiral expansion in terms of on-shell hadron masses
- reorganize conventional χ PT keeping its model independence
- keep renormalization scale and reparametrization invariance

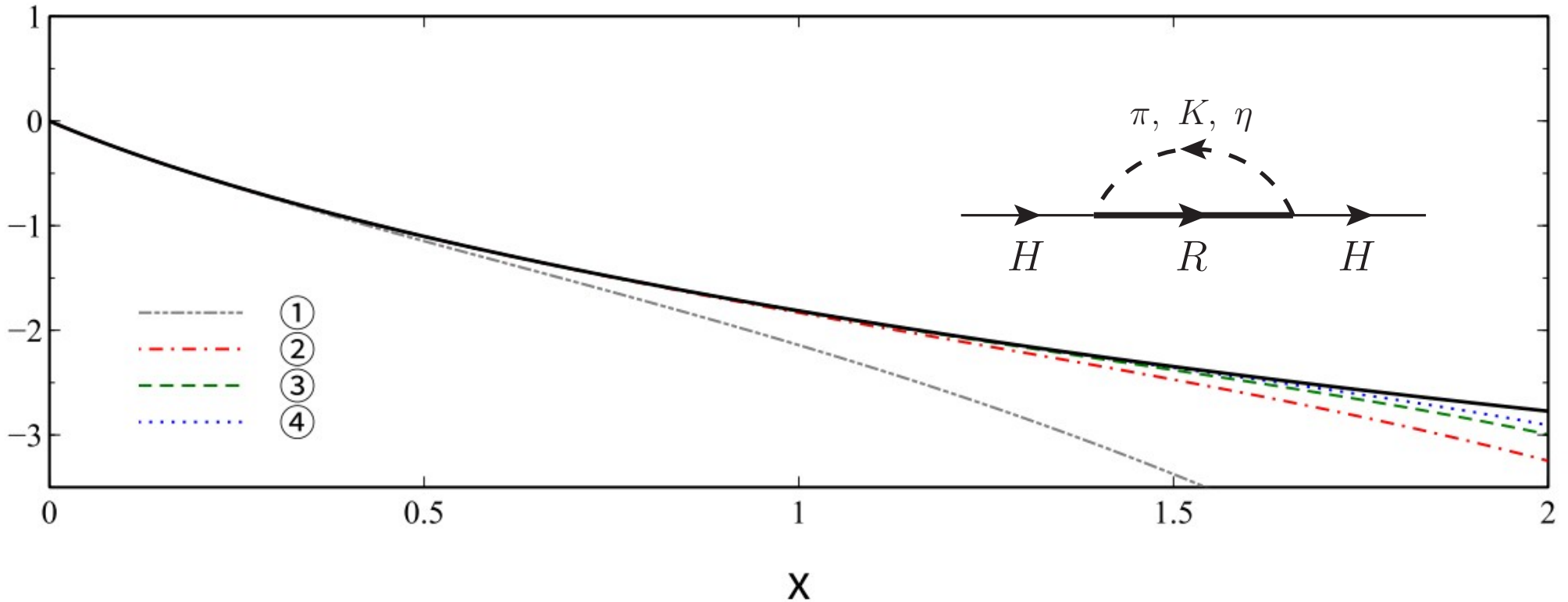
Phys.Rev. D51 (1995) 3697

Phys.Rev. D85 (2012) 034001

Phys.Rev. D86 (2012) 091502

Phys.Rev. D90 (2014) 054505

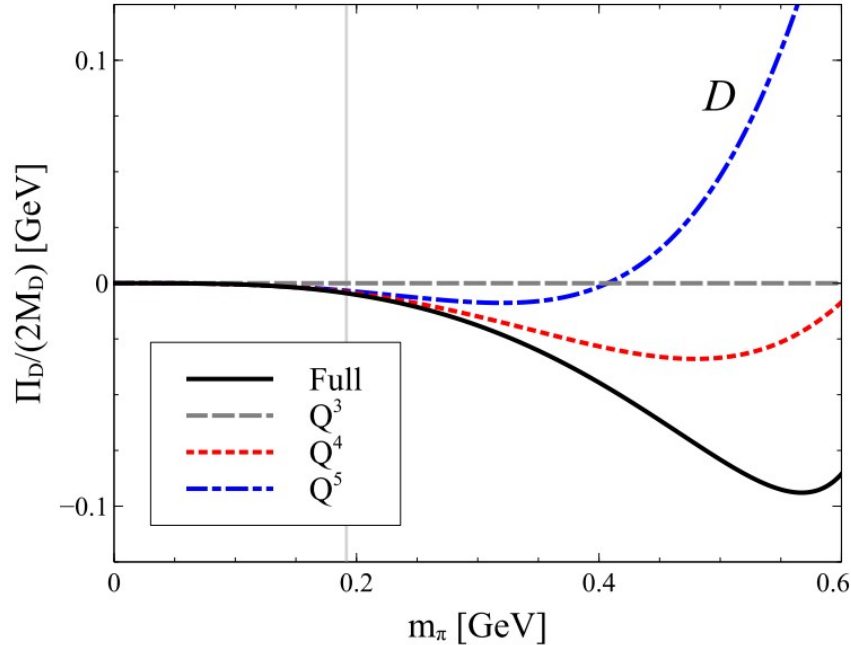
Chiral expansion of the scalar loop function $(4\pi)^2 \bar{I}_{QR}$



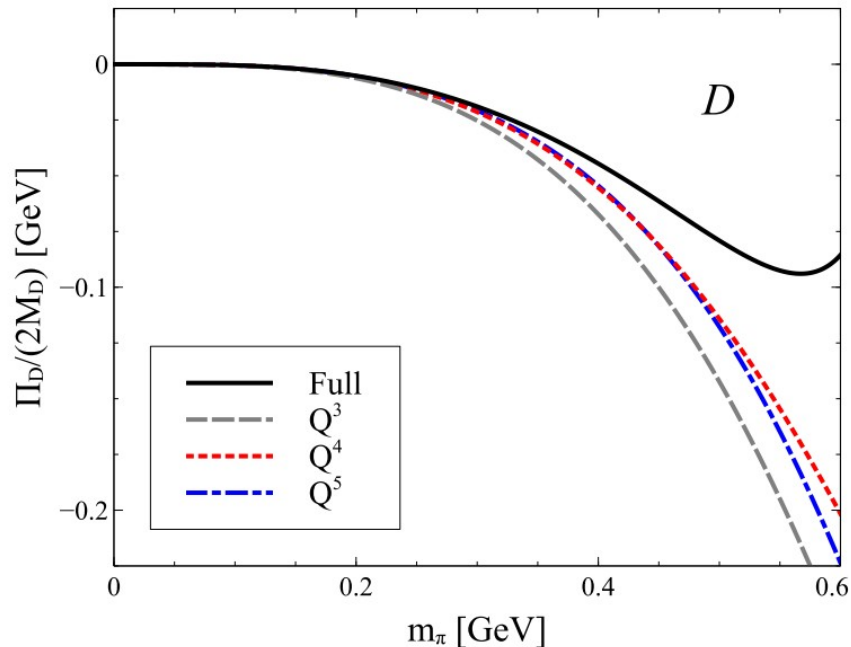
✓ Convergence study for $M_R = M_H$ and $x = m_Q/M_R$

- $(4\pi)^2 \bar{I}_{QR} = -\pi \sqrt{x^2} f_1(x^2) + x^2 f_2(x^2) - \frac{1}{2} x^2 f_3(x^2) \log x^2$
- the functions $f_n(x^2)$ are analytic in x^2 for $|x| < 2$
 $f_n(x^2) = 1 + \#x^2 + \#x^4 + \dots$
- good convergence even for $m_K = M_N$ with $x \simeq 1$!

Chiral decomposition of the loop: how to powercount



- flavour limit with $m_u = m_d = m_s$
- use bare masses $M \simeq 1.91$ GeV and $\Delta \simeq 0.19$ GeV
- expansion with $m_\pi/\Delta \sim Q$
- not useful for physical quark masses

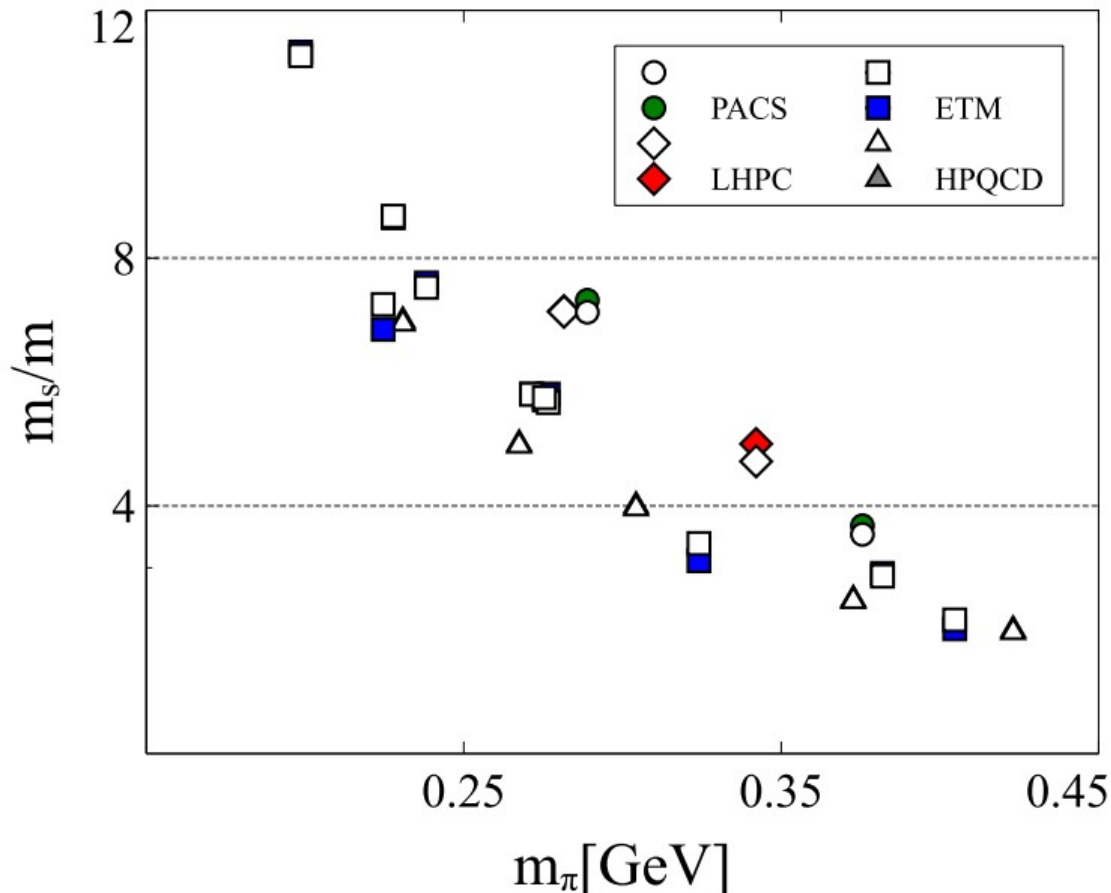


- flavour limit with $m_u = m_d = m_s$
- use bare masses $M \simeq 1.91$ GeV and $\Delta \simeq 0.19$ GeV
- expansion with $m_\pi/\Delta \sim Q^0$
- not useful for physical quark masses
- expansion with on-shell masses well converging!

Predictions for quark-mass ratios in lattice ensembles

✓ How to fit the lattice data?

- take pion and kaon mass of the ensemble \rightarrow compute quark masses
- this requires the low-energy constants $L_4 - 2L_6, L_5 - 2L_8, L_8 + 3L_7$
- we do not fit to the quark-mass ratios given by the lattice groups!



✓ A fit to the D meson masses

- renormalization scale $\mu = 0.77$ GeV

$$\begin{array}{l|l} 10^3 (L_4 - 2L_6) & -0.1575 \\ 10^3 (L_5 - 2L_8) & -0.0370 \\ 10^3 (L_8 + 3L_7) & -0.5207 \end{array}$$

$$m_s/m \quad \Bigg| \quad 26.600$$

- at physical quark masses our ratio compares well with lattice result

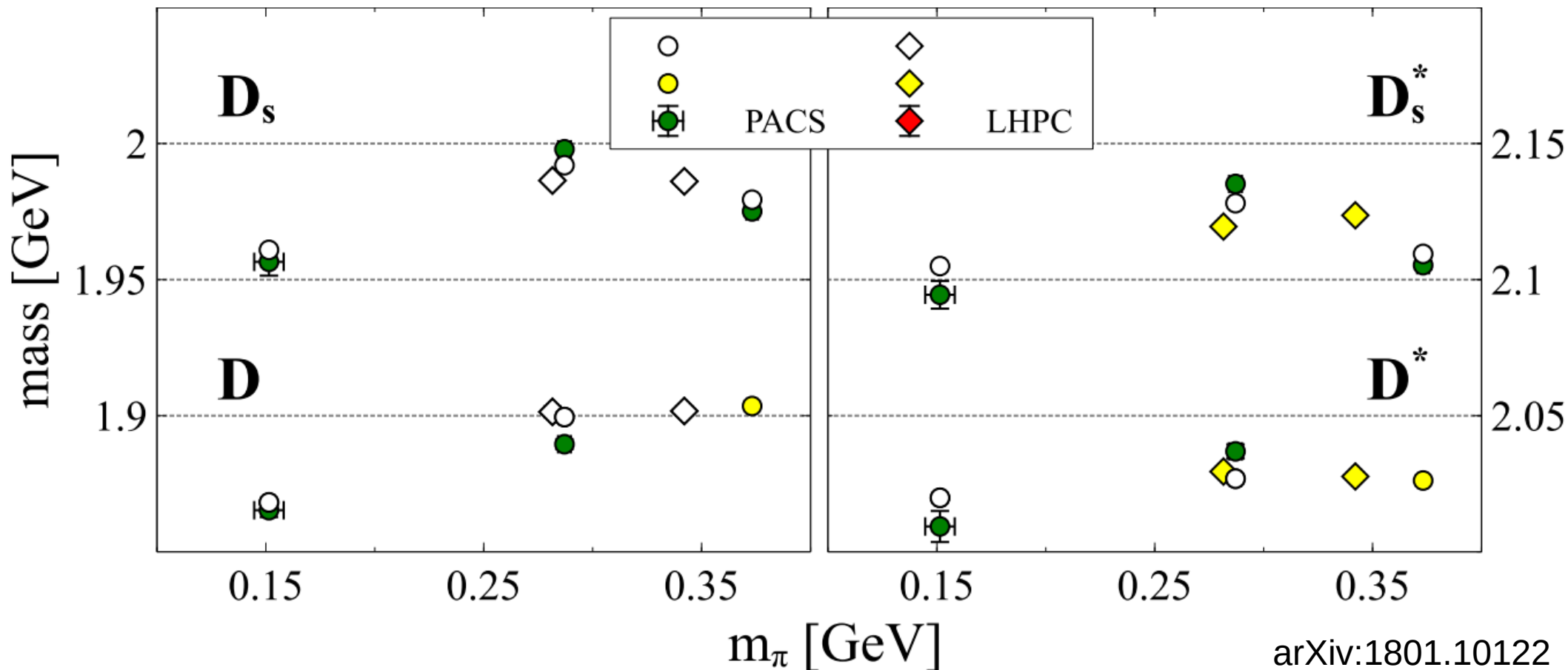
$$m_s/m = 26.66(32) \quad \text{from ETMC}$$

in Nucl. Phys. B887, 19 (2014)

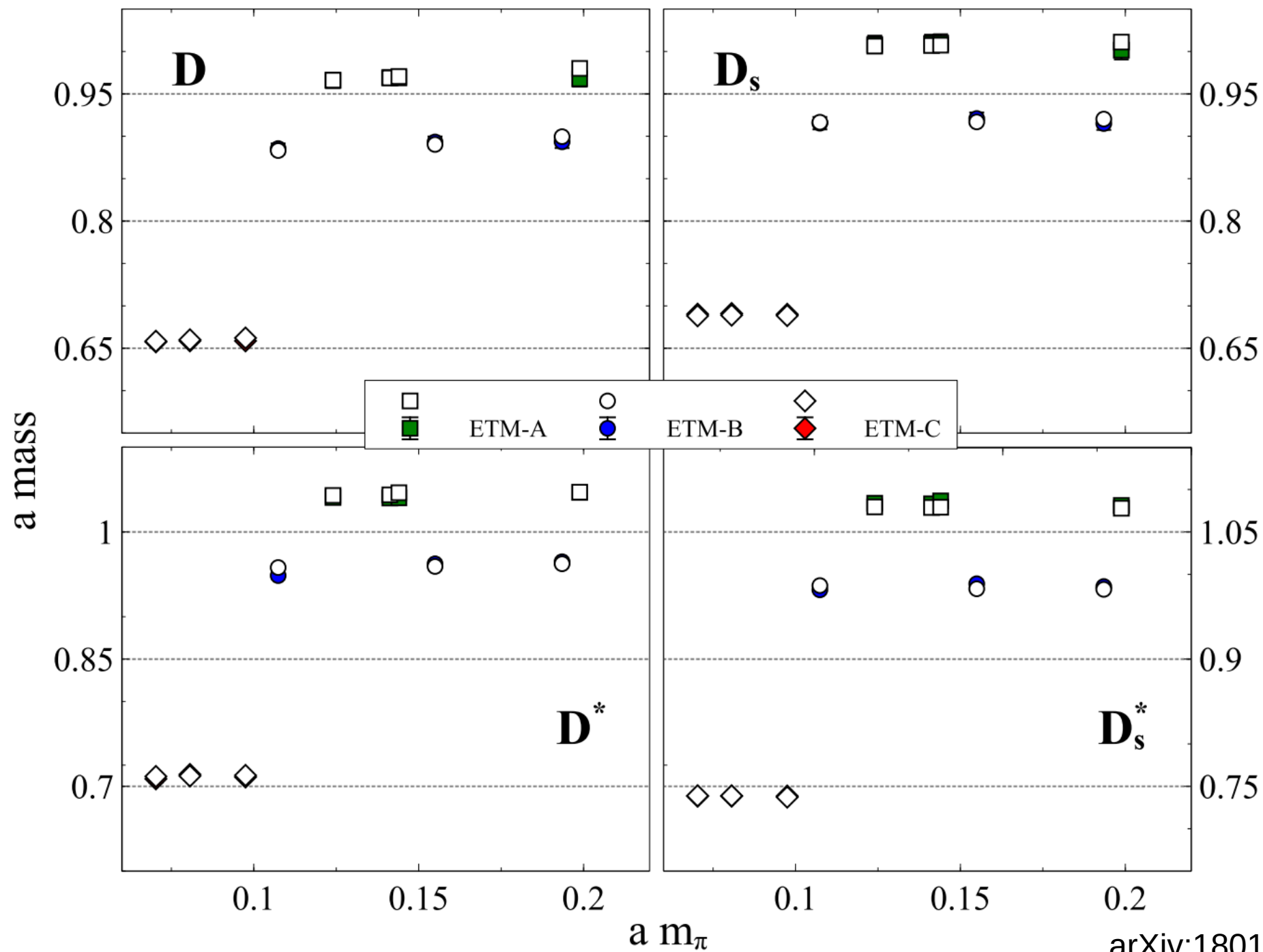
Predictions for D meson masses in lattice ensembles

✓ How to fit the lattice data?

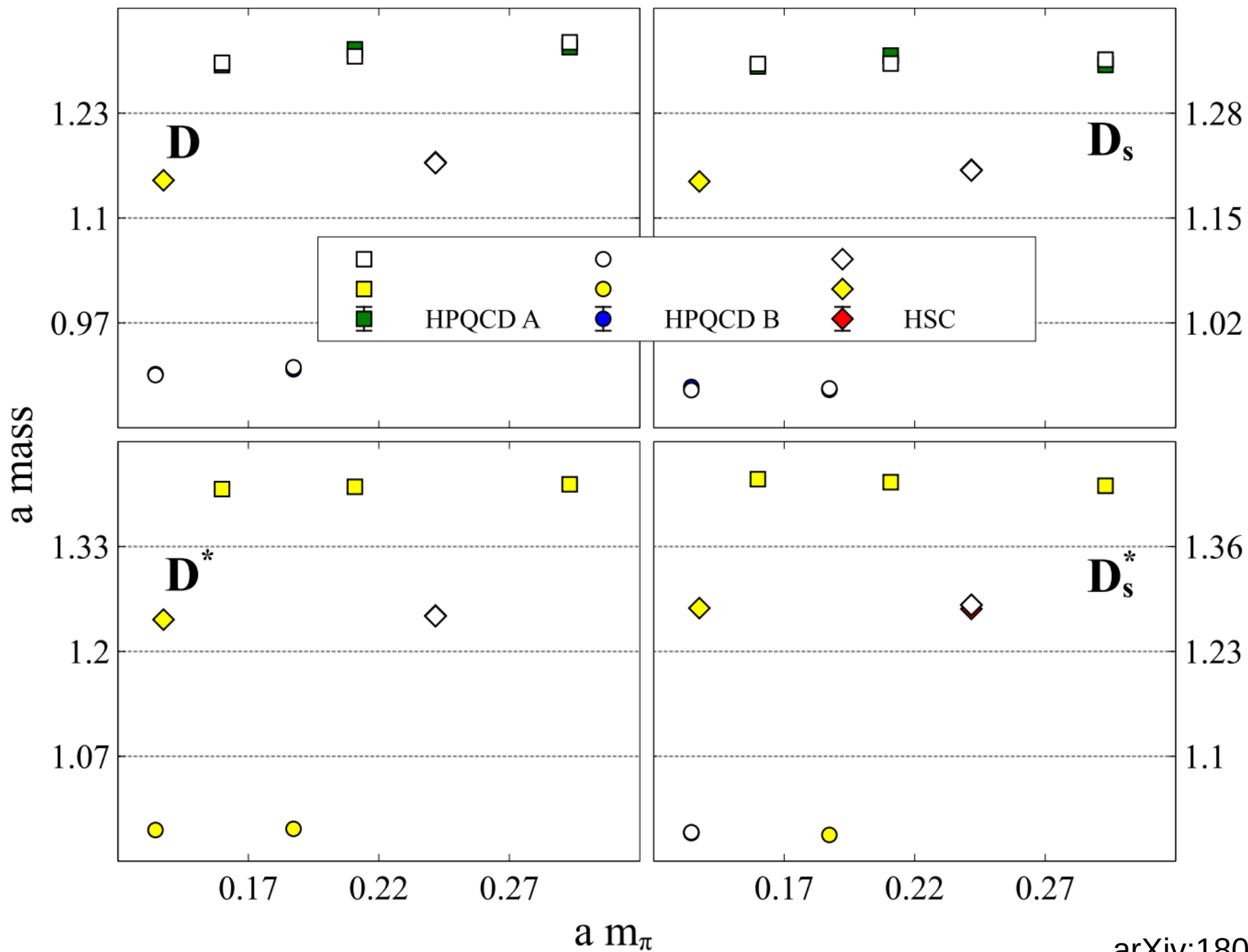
- always reproduce empirical D meson masses from PDG
- lattice scale determination for each β value
- tune the charm quark mass to its physical value
- yellow symbols: theory prediction (no lattice data yet)



Lattice ensembles from ETMC at three different β values



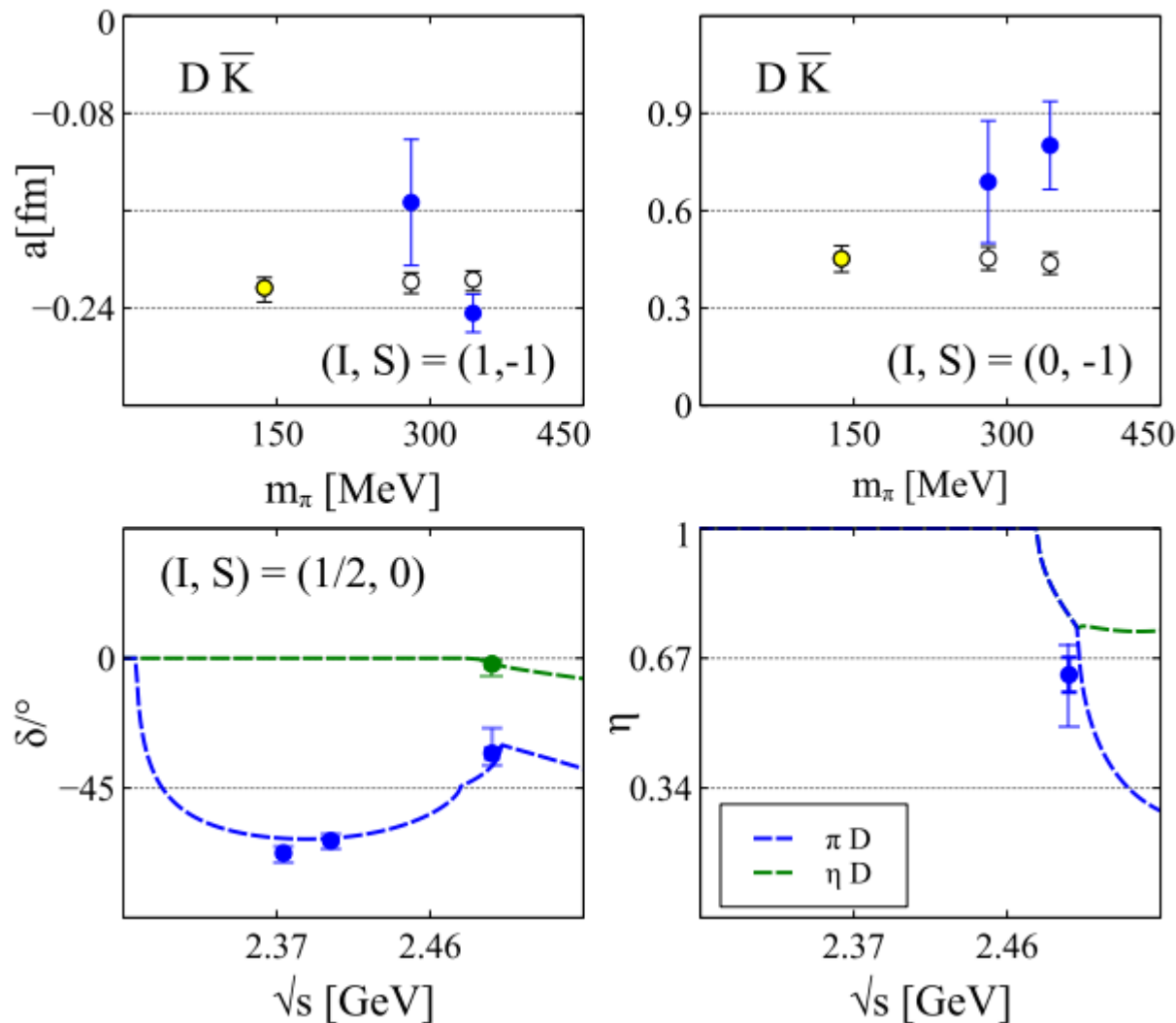
Lattice ensembles from HPQCD and HSC



S-wave scattering on MILC and HSC ensembles

✓ How to fit the lattice data?

- S-wave scattering lengths included if $m_K < 600$ MeV (from LHPC)
- S-wave πD and ηD phase shifts (from HSC)

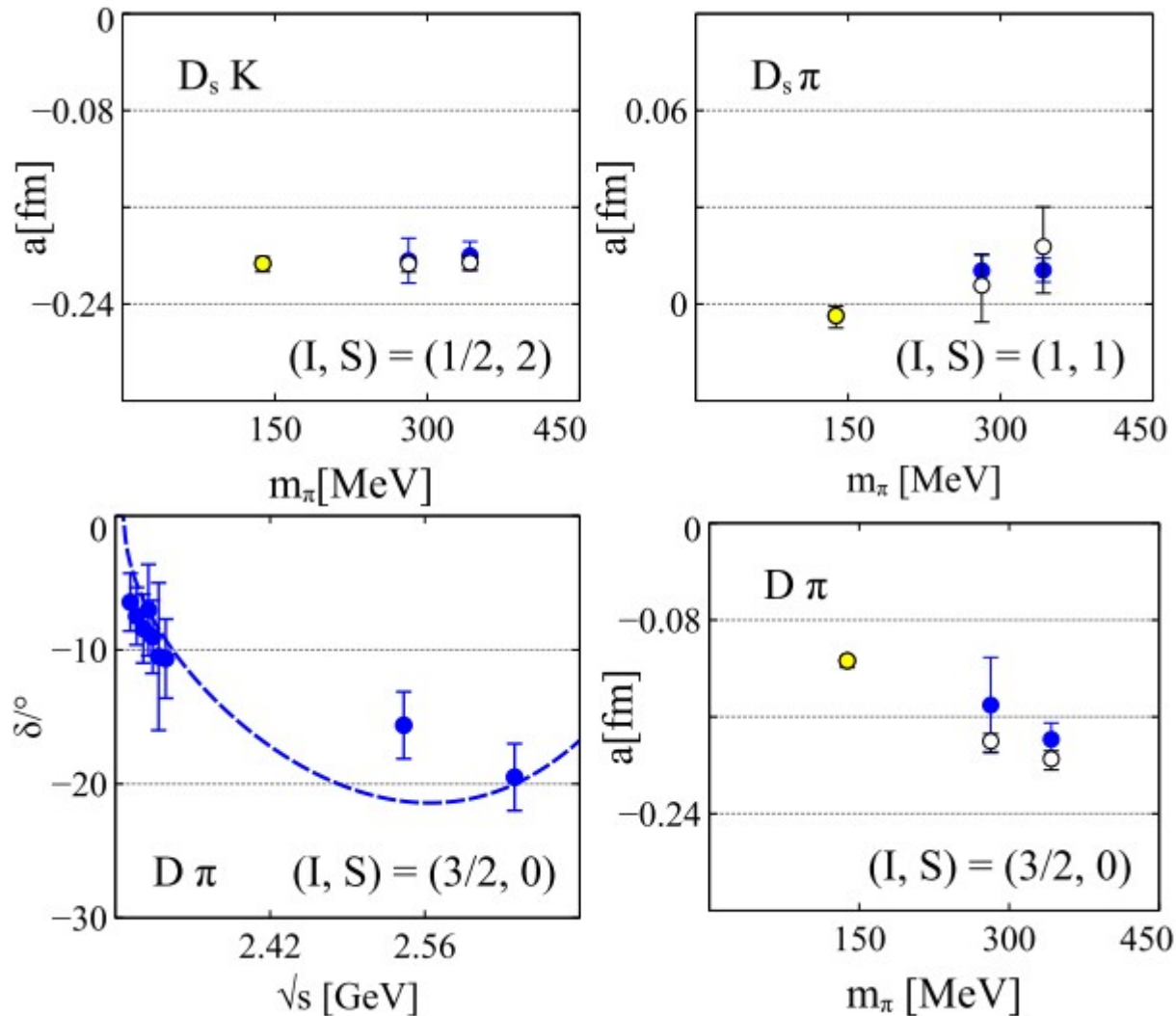


data from
Phys.Rev. D87 (2013) 014508
JHEP 10 (2016) 011

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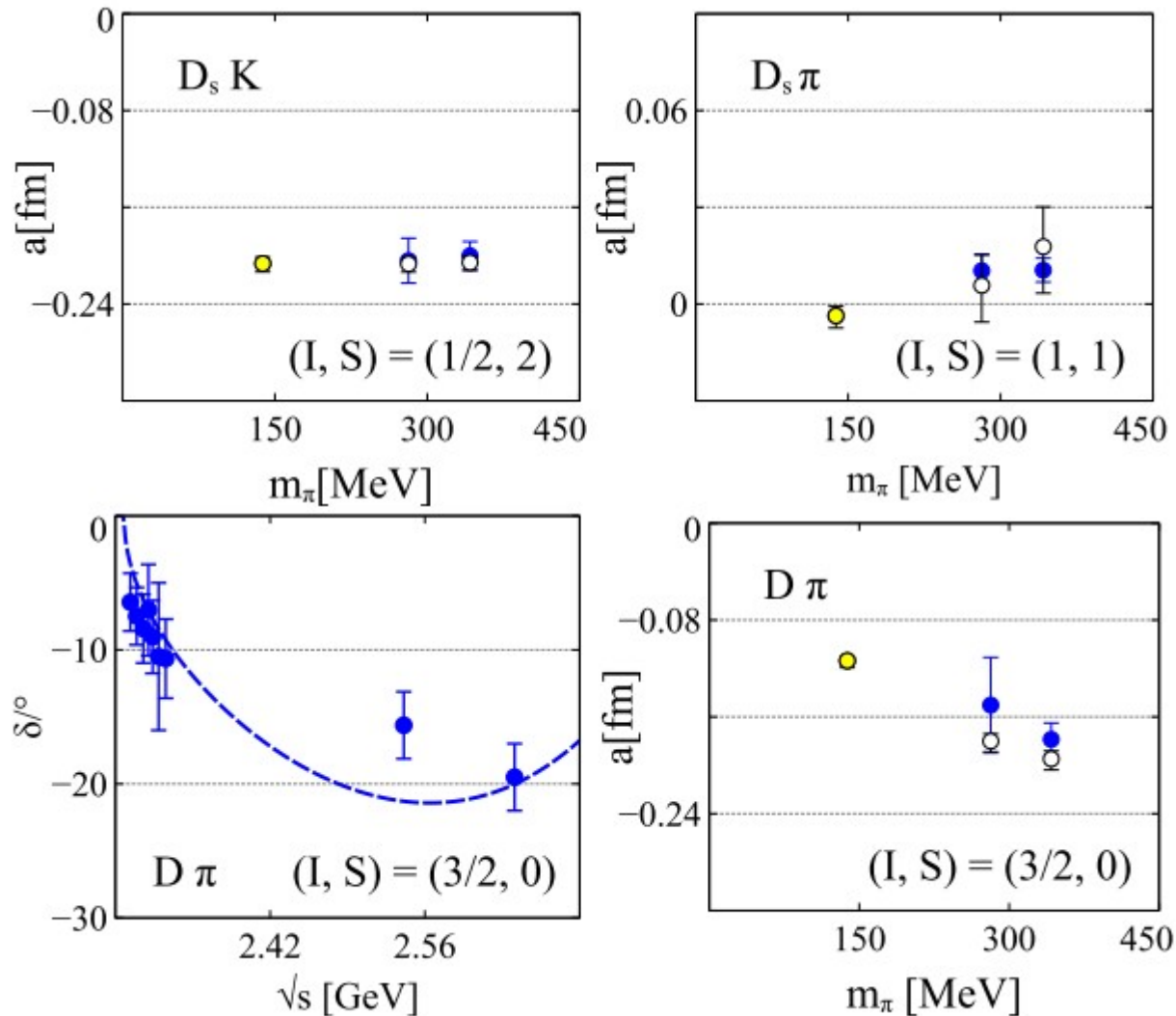


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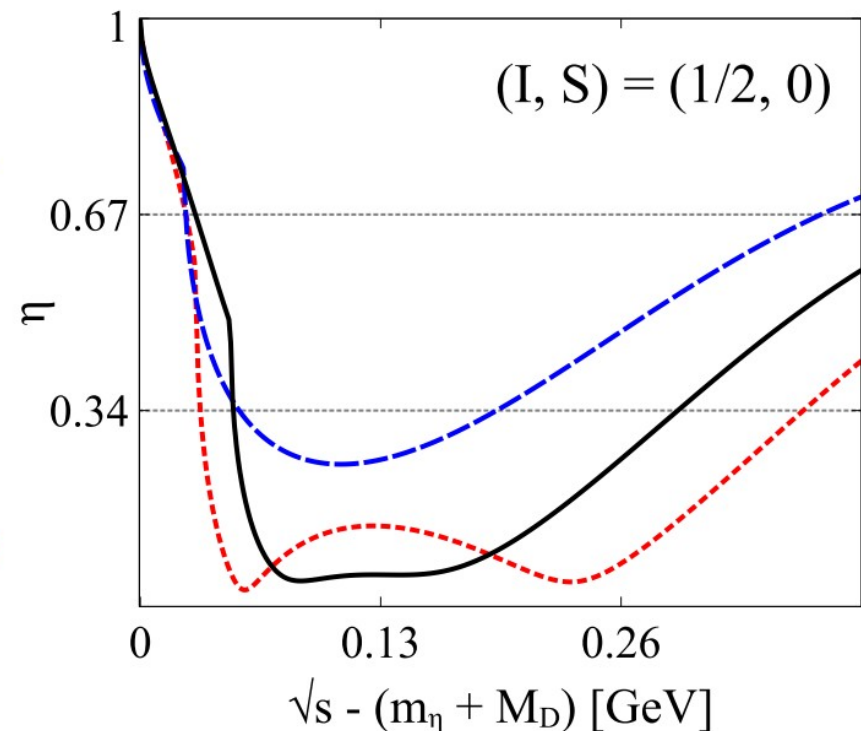
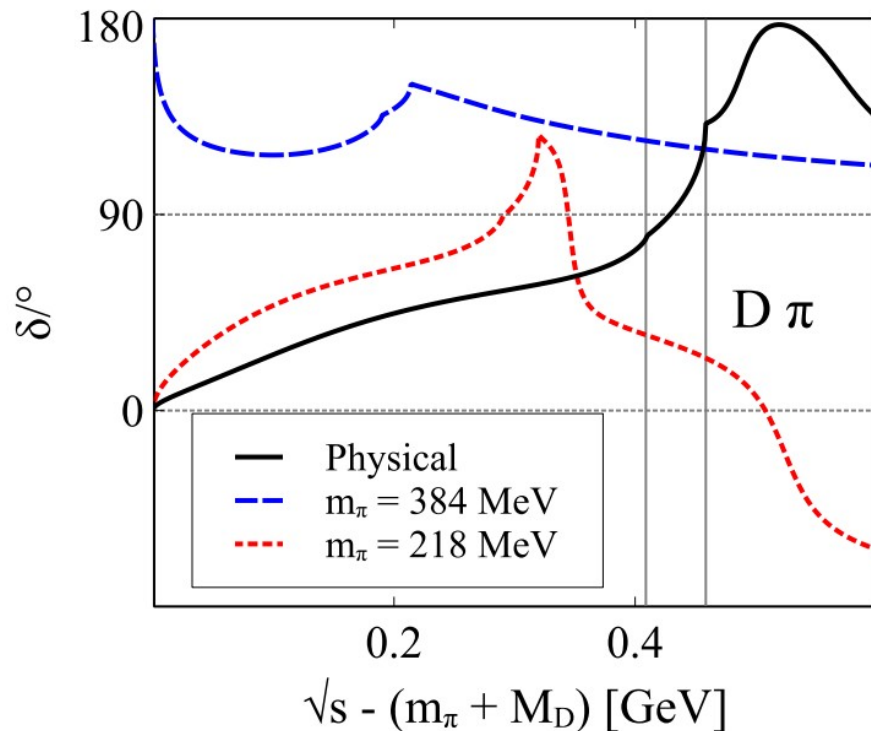
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Phase shifts: from unphysical to physical quark masses

- at unphysical quark masses (published HSC ensemble)
- black line: prediction of phase shifts at physical quark masses
- dashed red lines: prediction of phase shifts (unpublished HSC ensemble)



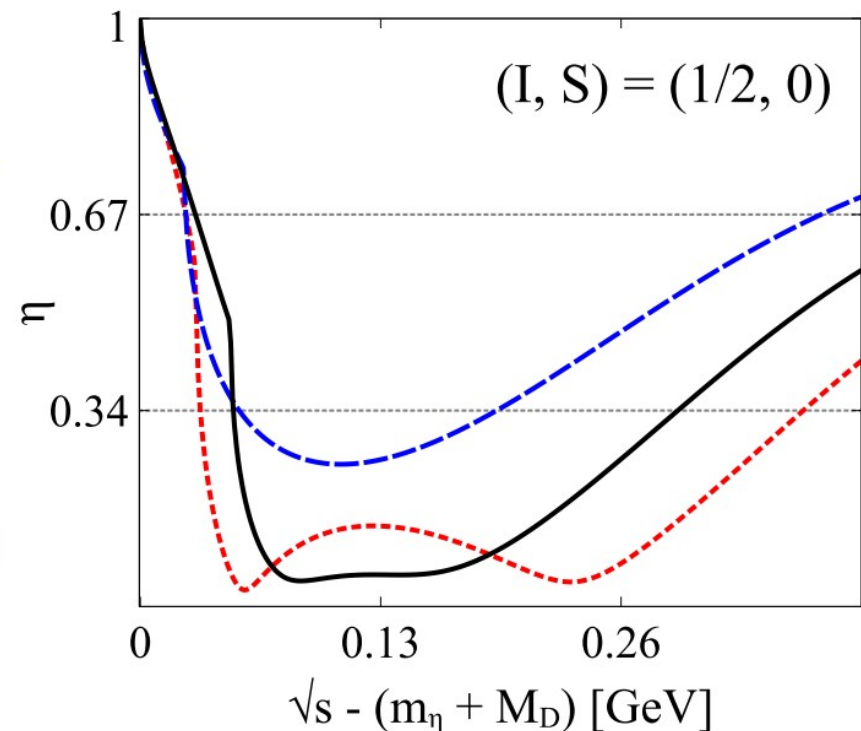
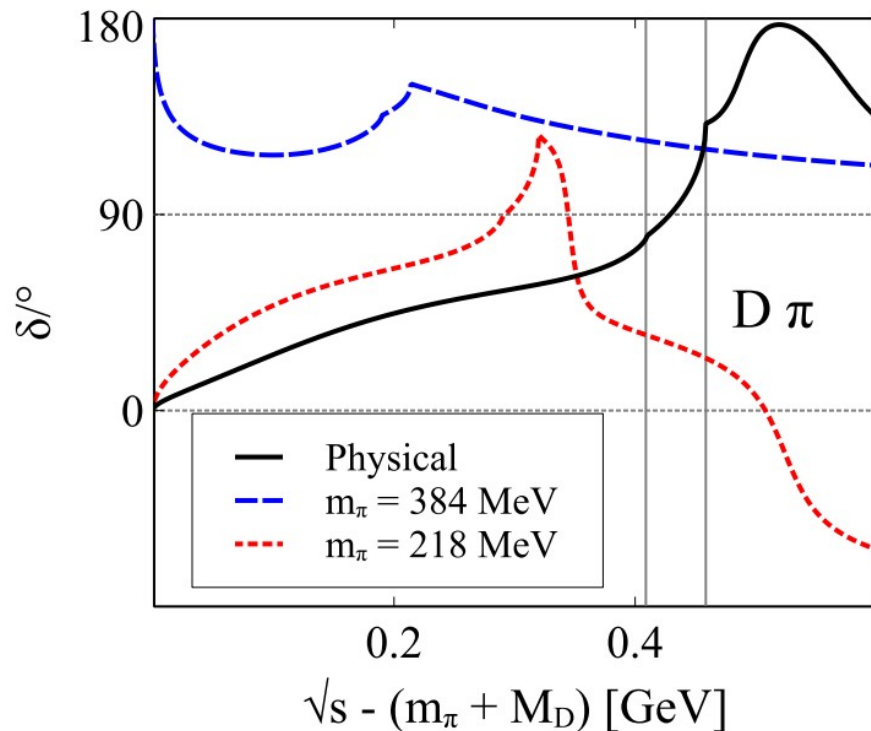
Phase shifts: from unphysical to physical quark masses

- predict that QCD forms a flavour sextet of $J^P = 0^+$ states
 expect two states $D_0(2082)$ and $D_0(2439)$
 the extra state just above the ηD threshold
- predict also a flavour sextet of $J^P = 1^+$ states
 expect three states $D_1(2232)$, $D_1(2420)$, $D_1(2575)$
 one extra state just above the ηD^* threshold

arXiv:1801.10122

arXiv:1811.00478

arXiv:1809.01311



Predictions for pole masses at physical quark masses

✓ An exotic flavour sextet of 0^+ states

- LEC derived from global fit to lattice data
- compare pole masses at LO with NLO

	$(I, S) = (1, 1)$	$(I, S) = (1/2, 0)$	$(I, S) = (0, -1)$
WT	$2.488_{-19}^{+22} - 0.083_{-5}^{+14}i$	$2.390_{-17}^{+20} - 0.038_{-1}^{+0}i$	2.335_{+15}^{-43}
NLO	$2.382_{-10}^{+10} - 0.322_{-10}^{+12}i$	$2.439_{-32}^{+42} - 0.092_{+3}^{-7}i$	$2.229_{+3}^{-4} - 0.083_{-11}^{+13}i$

- NLO results in qualitative agreement with LO predictions
- clear signal of an 'unexpected' pole with $(I, S) = (1/2, 0)$
- relevant for B meson decays into πD and ηD channels

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Predictions for pole masses at physical quark masses

✓ An exotic flavour sextet of 1^+ states

- LEC from global fit to lattice data
- compare pole masses at LO with NLO

	$(I, S) = (1, 1)$	$(I, S) = (1/2, 0)$	$(I, S) = (0, -1)$
WT	$2.618_{-19}^{+23} - 0.080_{-5}^{+14}i$	$2.525_{-18}^{+20} - 0.036_{-0}^{+0}i$	2.484_{+12}^{-32}
NLO	$2.564_{-10}^{+10} - 0.290_{-9}^{+12}i$	$2.606_{-30}^{+23} - 0.059_{+13}^{-25}i$	$2.390_{+3}^{-4} - 0.095_{-16}^{+18}i$

- NLO results in qualitative agreement with LO predictions
- clear signal of an 'unexpected' pole with $(I, S) = (1/2, 0)$
- relevant for B meson decays into πD^* and ηD^* channels

Isospin violating decay width of $D_{s0}(2317)$

✓ $D_{s0}(2317)$ from chiral coupled-channel dynamics

- Hadronic decay width from $D_{s0}(2317) \rightarrow D_s(1968) \pi_0$
- LO prediction: $\Gamma = 75 \text{ keV}$ [NPA813(2008)14]
- corrections may significantly change this estimate
 - NLO estimates: $\Gamma = 140 \text{ keV}$ [NPA813(2008)14]
 - $\Gamma = 133 \text{ keV}$ [PRD87(2013)014508]
- global fit to QCD lattice data
 - NLO prediction: $\Gamma = (104 - 116) \text{ keV}$ [arXiv:1801.10122]

✓ Why to measure the width ?

- significant impact on chiral dynamics in QCD
- the low-energy constants are predicted from QCD lattice results
- closely related to exotic flavour sextet states (e.g. ηD - channel)

Summary & Outlook

✓ Chiral extrapolation of hadron masses

- resummed χ PT : use on-shell masses in the loops
 - chiral expansion with up, down and strange quarks is well convergent
- so far we considered baryon and open-charm meson masses at N³LO
 - fits to masses of ground states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ and $J^P = 0^-, 1^-$
 - quantitative reproduction of the world lattice data set (about 400 data points)
- predict a large number of low-energy constants for the chiral Lagrangian of QCD
 - quantitative results for the baryon sigma terms
 - quantitative results for s-wave $\pi D, \eta D$ and $\pi D^*, \eta D^*$ phase shifts \dots
 - predict that QCD forms exotic flavour sextet states with $J^P = 0^+$ and $J^P = 1^+$

✓ QCD spectroscopy with coupled-channel dynamics

- current QCD lattice data provide the counter terms for hadron-hadron scattering
- use as input in systematic coupled-channel computations
- analyze and predict the quark-mass dependence of hadron resonances in QCD

thanks to: Yonggoo Heo and Xiao-Yu Guo