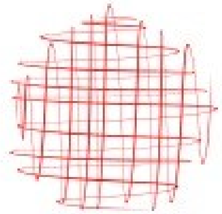
The spectra and decay widths of hybrid quarkonia in a hyperspherical-coordinate framework

T. Miyamoto & S. Yasui



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Any net number of quarks (# of q - # of anti-q) that is divisible by 3, plus any number of valence gluons (except for a single gluon with no quarks) can form a colour singlet.

Meson

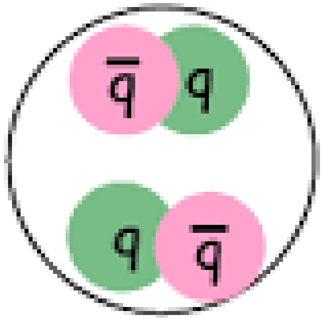
$$N_q = 1$$

$$N_{\bar{q}} = 1$$

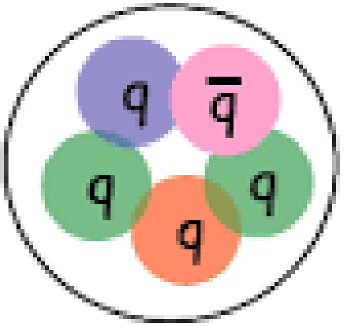
Baryon eg proton (uud)

$$N_q = 3$$

$$N_{\bar{q}} = 0$$



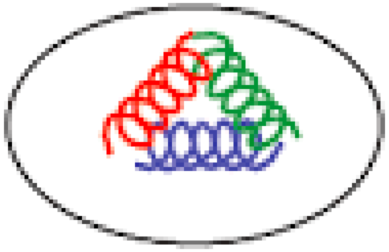
molecules



pentaquark

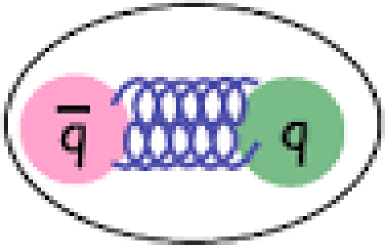
$$N_q = 4$$

$$N_{\bar{q}} = 1$$



glueball meson

$$N_g \geq 2$$



hybrid meson

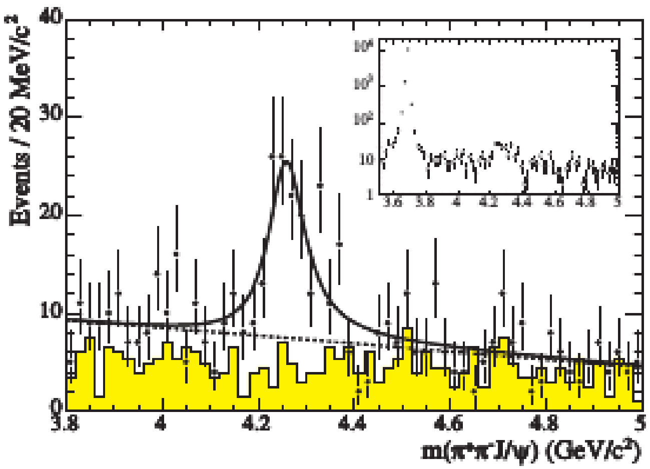
$$N_q = N_{\bar{q}} = 1$$

$$N_g \geq 1$$

Various exotic meson candidates X(3872), Y(4260), Y(4360), etc

Y(4260)

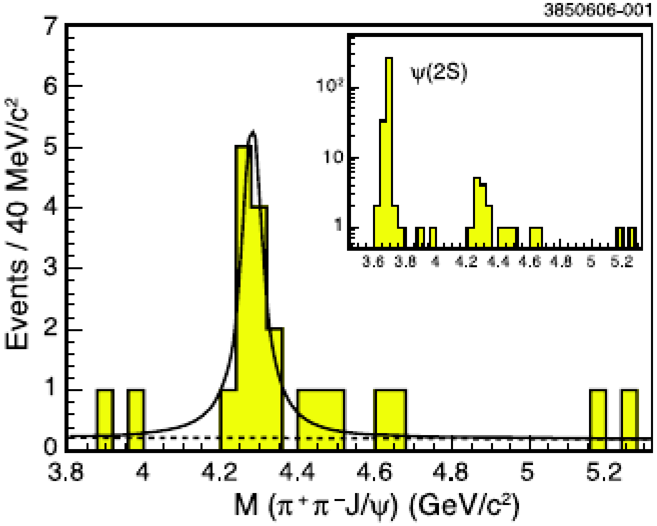
$$e^+e^- \to \gamma\pi^+\pi^- J/\psi$$

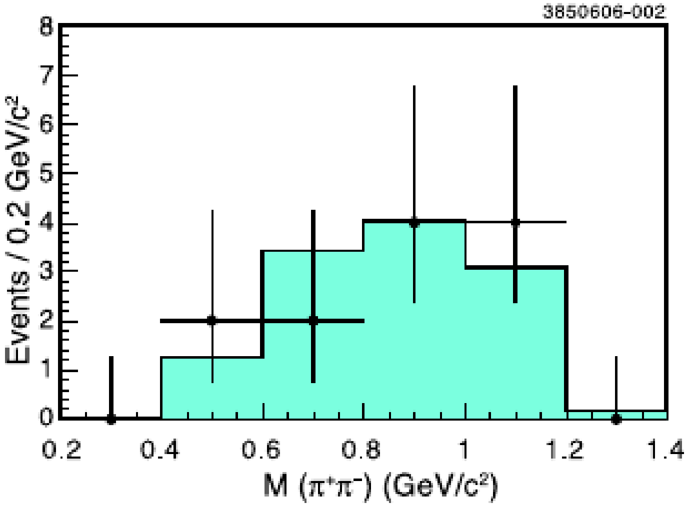


Discovered by the BaBar group through initial state radiation (ISR) events

Another evidence provided by the Cleo collaboration

Width ~ 90 MeV





Relativistic single resonance Breit-Wigner fit

The measured dipion mass distribution agreed with the theoretical Monte Carlo *S*-wave phase space model

$$\boxed{\mathrm{J}^{\mathrm{PC}} = 1^{--}}$$

B. Aubert et al, Phys. Rev. Lett 95 142001 (2005)
Q. He et al, Phys. Rev. D 74 091104 (2006)
J. Beringer et al, Phys. Rev. D 86 010001 (2012)

Y(4260) $\psi(4260)$

Tetraquark $c\bar{c}s\bar{s}$ interpretation

needs the channel of $D_s + \bar{D}_s$

DD_1 molecule interpretation

Y4260 close to DD_1

$$e^- e^+ \to \begin{cases} \pi^+\pi^- J/\psi \\ \pi^+\pi^- h_c \\ \omega\chi_{c0} \end{cases}$$

The single-resonance assumption is naïve to determine the mass & the width of Y(4260). → Average the mass and width determinations in the 3 channels
- Y4260 label "retired"
(Olsen 2017)

$D\bar{D}_1$ B.E. soars to 66 MeV ← $M(Y4220) = 4222 \pm 3$

$$\Gamma_{\text{tot}} = 48 \pm 7$$

Hybrid meson

$H_B : c\bar{c} +$ P-wave gluon

Selection rule to restrict the decay of the hybrid
(The decay into two S-wave open charm mesons is prohibited)

Gui-Jun Ding, Phys. Rev. D79, 014001 (2009)
E. Kou & O Pene, Phys. Lett. B 631 (2005) 164

The main motivation for searching for hybrid quarkonia

$$\Gamma = 55 \pm 19 \qquad \psi(4260) \to J/\psi + \pi + \pi, \qquad J/\psi + K^+ + K^-$$

$$\Gamma = 96 \pm 7 \qquad \psi(4360) \to \psi(2S) + \pi^+ + \pi^-$$

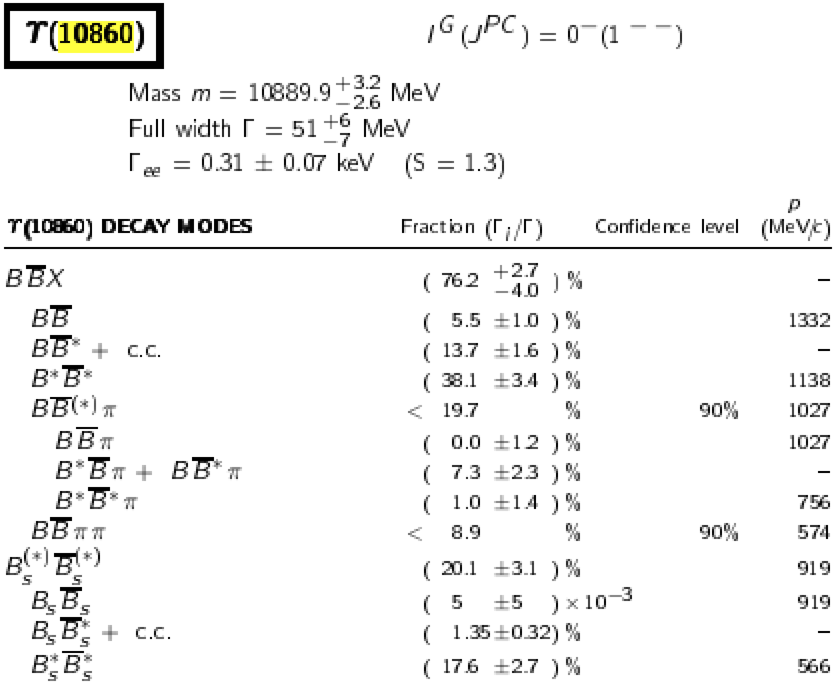
$$\Gamma = 62 \pm 20 \qquad \psi(4415) \to D\bar{D}, \qquad D^*\bar{D} + cc, \qquad D^*\bar{D}^*$$

etc

$$\Gamma = 72 \pm 11 \qquad \psi(4660) \to \psi(2S) + \pi^+ + \pi^-$$

MeV

At the end of the day, are they hybrid quarkonia?



Υ(10860) $\quad I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 10889.9^{+3.2}_{-2.6}$ MeV
Full width $\Gamma = 51^{+6}_{-7}$ MeV
$\Gamma_{ee} = 0.31 \pm 0.07$ keV (S = 1.3)

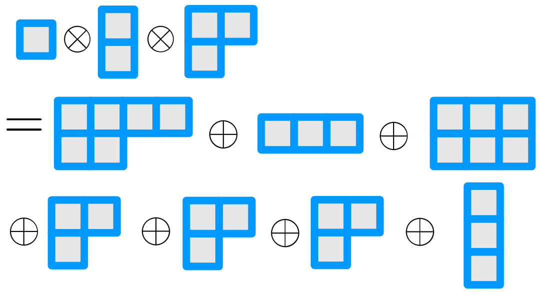
Υ(10860) DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$B\overline{B}X$	$(76.2^{+2.7}_{-4.0})$ %		–
$B\overline{B}$	(5.5 ± 1.0) %		1332
$B\overline{B}^* +$ c.c.	(13.7 ± 1.6) %		–
$B^*\overline{B}^*$	(38.1 ± 3.4) %		1138
$B\overline{B}^{(*)}\pi$	< 19.7 %	90%	1027
$B\overline{B}\pi$	(0.0 ± 1.2) %		1027
$B^*\overline{B}\pi + B\overline{B}^*\pi$	(7.3 ± 2.3) %		–
$B^*\overline{B}^*\pi$	(1.0 ± 1.4) %		756
$B\overline{B}\pi\pi$	< 8.9 %	90%	574
$B_s^{(*)}\overline{B}_s^{(*)}$	(20.1 ± 3.1) %		919
$B_s\overline{B}_s$	$(5 \pm 5) \times 10^{-3}$		919
$B_s\overline{B}_s^* +$ c.c.	(1.35 ± 0.32) %		–
$B_s^*\overline{B}_s^*$	(17.6 ± 2.7) %		566

They have similarities in decay patterns, masses & parity and C-parity

$$J^{\rm PC} = 1^{--}$$

Hybrid meson

Hybrid charmonium as a bound state of $c\bar{c}$ plus a gluon



Gluon carries the adjoint representation **8** of SU(3) – it can be linked to a quark & antiquark to form a colour singlet object.

For a magnetic (transverse electric) gluon:

$$P = (-1)^{(L_{q\bar{q}}+J_g)} \, , \quad C = (-1)^{L_{q\bar{q}}+S_{q\bar{q}}+1} \qquad L_g = J_g$$

The lowest state: $L_{q\bar{q}} = 0 \, , \quad J^{PC} = 1^{--}$

For an electric (transverse magnetic) gluon:

$$P = (-1)^{(L_{q\bar{q}}+J_g+1)} \, , \quad C = (-1)^{L_{q\bar{q}}+S_{q\bar{q}}+1} \qquad L_g = J_g \pm 1$$

A. Le Yaouanc et al, Z. Phys. C – Particle & Physics 28, 309 (1985)

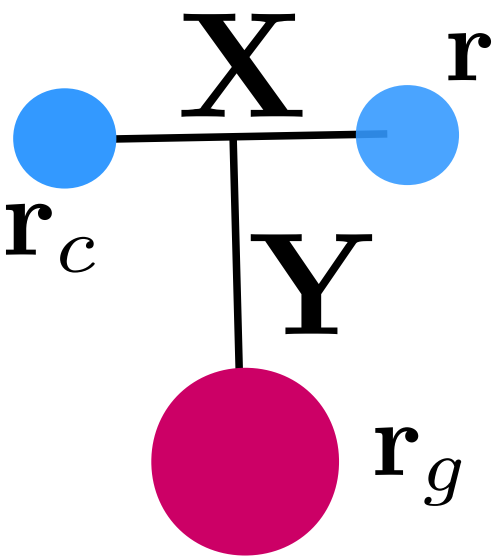
Note:

Cf: electric,magnetic photon (radiation) carries parity of: $(-1)^l, (-1)^{l+1}$

The parity of a meson:

$$P = (-1)^{L+1}$$

Hyperspherical Formalism



$$\mathcal{Y}^{\mathrm{L_xL_y}}_{\mathrm{KLM_L}}(\Omega_5) \overset{\text{def}}{=} \mathcal{N}(K,L_x,L_y)A_n^{L_xL_y}(\alpha)\left[Y_{L_x}(\Omega_x)\otimes Y_{L_y}(\Omega_y)\right]_{LM_L}$$

$$A_n^{L_xL_y}(\alpha) = (\cos\alpha)^{L_x}(\sin\alpha)^{L_y}P_n^{L_y+1/2,L_x+1/2}(\cos 2\alpha)$$

$$\mathcal{N}(K,L_x,L_y) = \sqrt{2(K+2)}\sqrt{\frac{\Gamma(n+1)\Gamma(L_x+L_y+n+2)}{\Gamma(L_x+n+3/2)\Gamma(L_y+n+3/2)}}$$

$$\Psi_{JM_J}(\mathbf{x},\mathbf{y}) = \frac{1}{\rho^{5/2}}\sum_{K,\gamma}\chi_{K,\gamma}(\rho)\Upsilon^{JM_J}_{K\gamma}(\Omega_5)$$

$$\Upsilon^{JM_J}_{K\gamma}(\Omega_5) = \left[\mathcal{Y}^{\mathrm{L_xL_y}}_{\mathrm{KL}}\otimes|S\rangle\right]_{JM_J}$$

$$\left[\frac{\mathbf{p}_x^2}{2m_x}+\frac{\mathbf{p}_y^2}{2m_y}+(\ldots)\right]\Psi = E\Psi$$

$$-\frac{\hbar^2}{2m_b}\left[\frac{\partial}{\partial\rho^2}-\frac{(K+3/2)(K+5/2)}{\rho^2}+(\ldots)\right]\chi = E\chi$$

A. Novoselsky et al. Phys. Rev. A 49, 833 (1994)

The Hamiltonian for the hybrid quarkonia

Auxiliary field methods
-quantisation
-massless particles
*variational approaches

$$H(\mu,\mu_g) = H_0 + V$$

$$H_0 = \mu + \frac{\mu_g}{2} + \frac{m^2}{\mu} + \frac{\mathbf{p}_X^2}{\mu} + \frac{\mathbf{p}_Y^2}{2\phi}$$

$$V = \sigma\,|r_{qg}| + \sigma\,|r_{\bar{q}g}| + V_\mathrm{C}$$

$$V_C = -\frac{3\alpha_s}{2r_{qg}} - \frac{3\alpha_s}{2r_{\bar{q}g}} + \frac{\alpha_s}{6r_{q\bar{q}}}$$

σ : energy density

α_s : the strong coupling constant

$$\mathbf{X} = \mathbf{r}_q - \mathbf{r}_{\bar{q}}$$

$$\mathbf{Y} = \mathbf{r}_g - \frac{\mathbf{r}_q + \mathbf{r}_{\bar{q}}}{2}$$

$$m_q = m_{\bar{q}} = m$$

$$\mu_q = \mu_{\bar{q}} = \mu$$

V. Mathieu, Phys. Rev. D **80**, 014016 (2009)

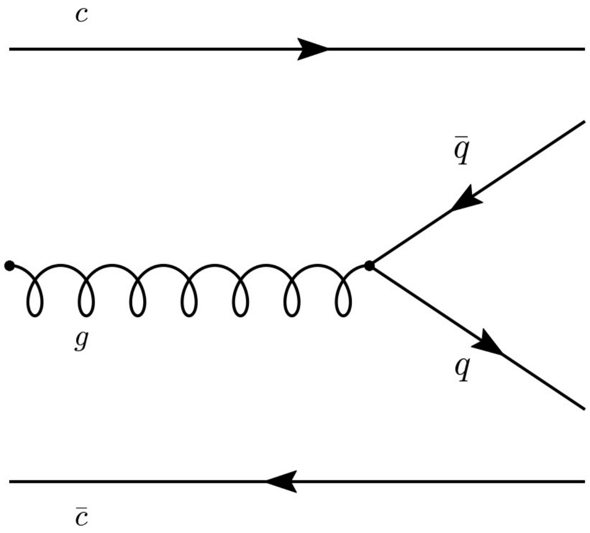
DD or BB decay channels of a hybrid quarkonium

- A hybrid quarkonium contains an electric gluon (and here the S-wave gluon).

- We consider the lowest order decay.

- The initial states are derived directly from the Fourier transformation of the calculated (configuration space) wave functions of hybrid quarkonia.

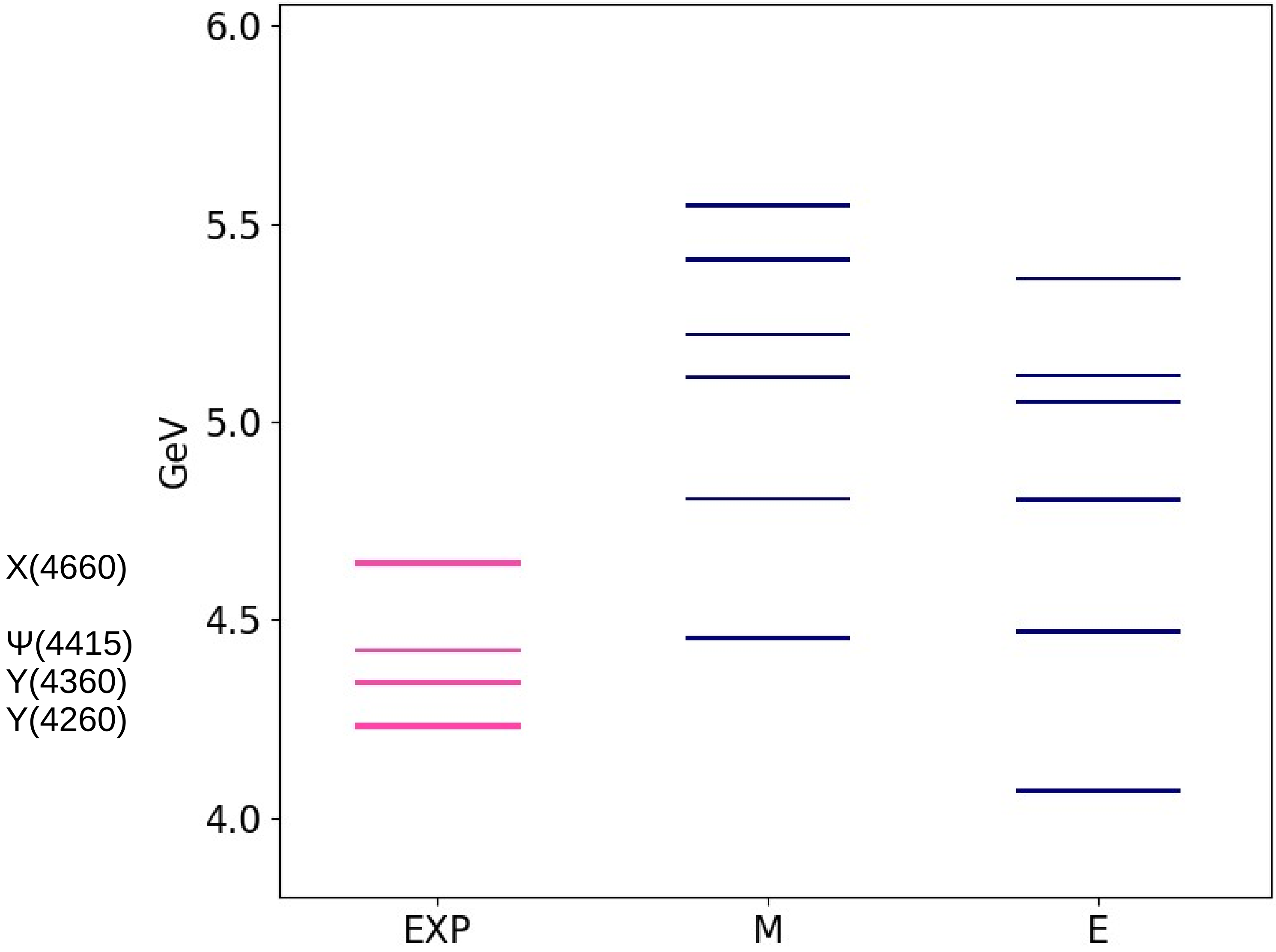
- The final states are treated in a non-relativistic way, where Gaussian-type functions are used for their wave functions.
More suitable for a bottomonium hybrid



$$\Gamma_{A\to BC} = 4\alpha_s |f_{A\to BC}|^2 \frac{P_B E_B E_C}{m_A} ,$$

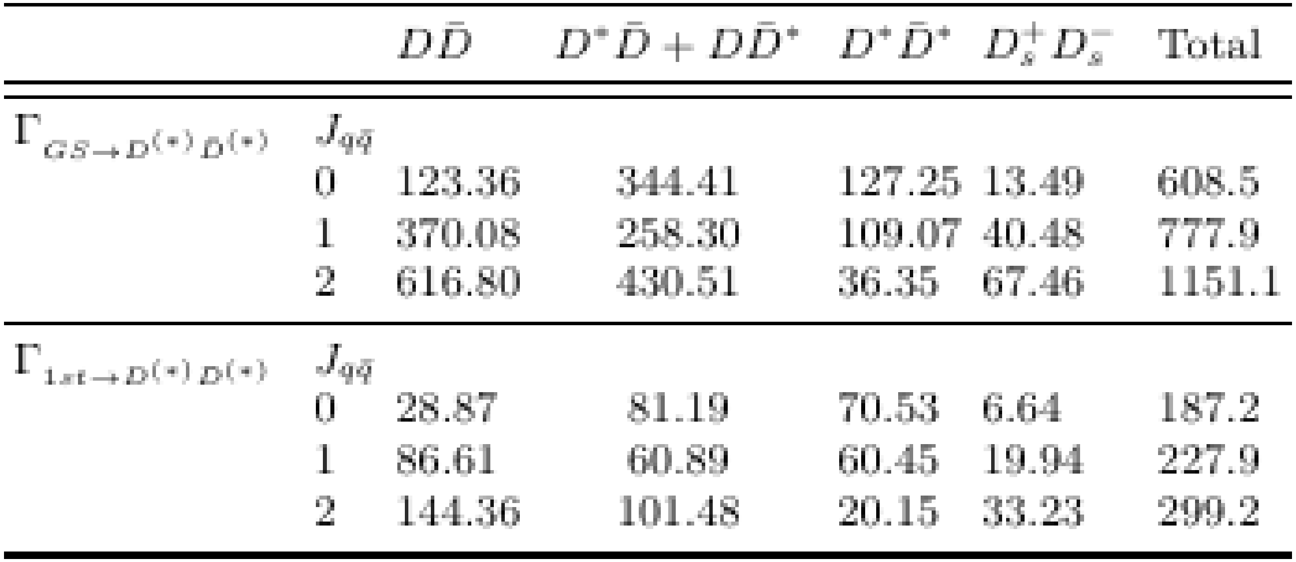
F. Iddir & A. S. Safir, Phys. Lett. B 507 (2001) 183-192

Charmonium Hybrid



The calculated ground state (for the magnetic gluon) consistent with earlier studies (Kalashnikova 2008, Mathieu 2009)

DD decay channels of a hybrid charmonium



		$D\bar{D}$	$D^*\bar{D}+D\bar{D}^*$	$D^*\bar{D}^*$	$D_s^+D_s^-$	Total
$\Gamma_{GS\to D^{(*)}D^{(*)}}$	$J_{q\bar{q}}$					
	0	123.36	344.41	127.25	13.49	608.5
	1	370.08	258.30	109.07	40.48	777.9
	2	616.80	430.51	36.35	67.46	1151.1
$\Gamma_{1st\to D^{(*)}D^{(*)}}$	$J_{q\bar{q}}$					
	0	28.87	81.19	70.53	6.64	187.2
	1	86.61	60.89	60.45	19.94	227.9
	2	144.36	101.48	20.15	33.23	299.2

(MeV)

- That would be much larger than experimental data (~50 MeV)
- It seems unlikely that they are electric-gluon hybrid

$\psi(4260)$ $\quad$ $\psi(4360)$ $\quad$ $\psi(4415)$ $\quad$ $\psi(4660)$

$\Gamma = 55 \pm 19$ $\quad$ $\Gamma = 96 \pm 7$ $\quad$ $\Gamma = 62 \pm 20$ $\quad$ $\Gamma = 72 \pm 11$

Short Summary

- If one of these hybrid meson candidates (Y(4260), Y(4360), Ψ(4415), X(4660)) is a hybrid meson, it should be a magnetic gluon hybrid.

** Decay widths too large, for the decays from both the ground & the 1st excited states

** The calculated spectra of Ψ(4415) comparable to the data

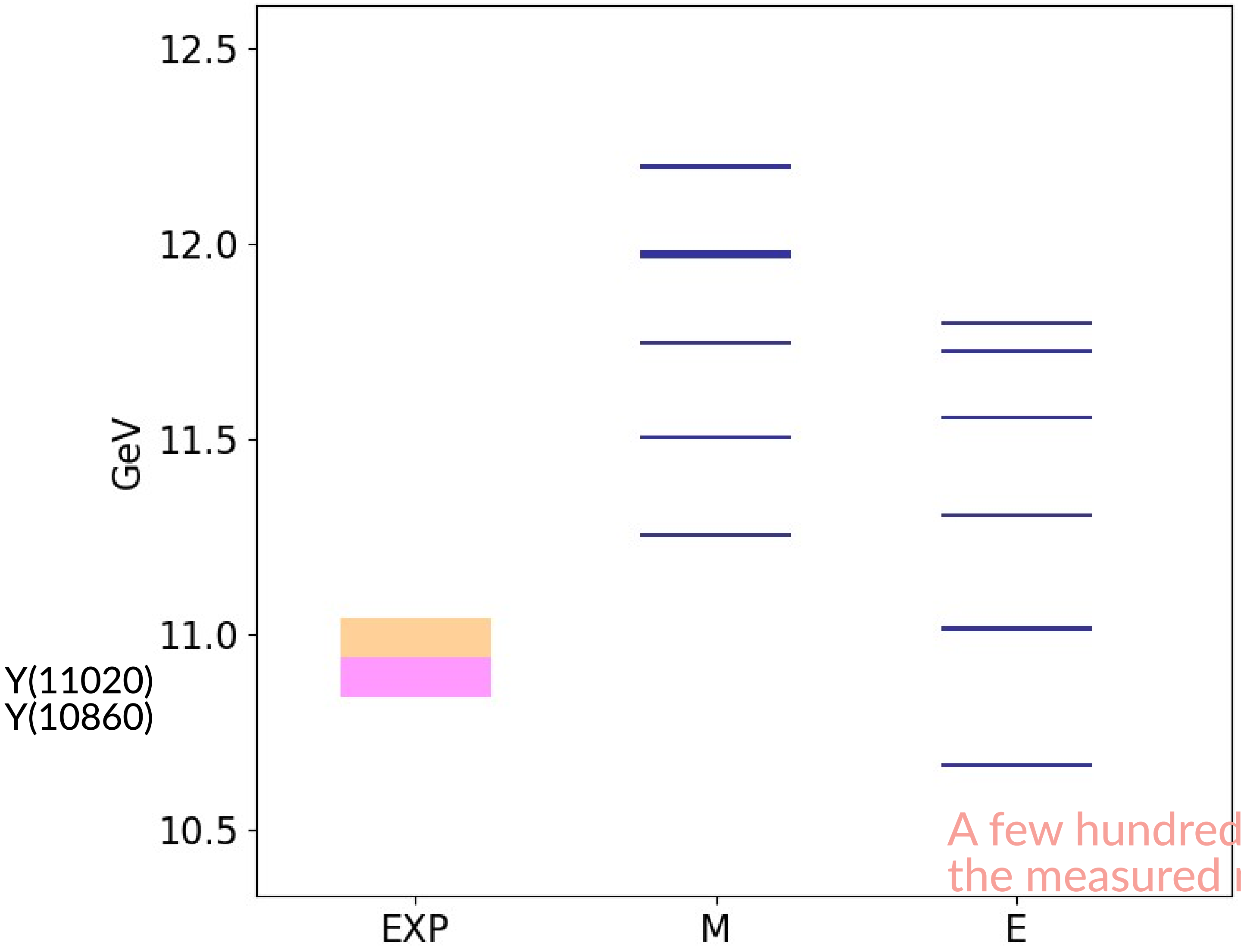
- It seems unlikely that Upsilon(10860) is the ground state of an electric gluon hybrid bottomonium.

- Still, there could be a possibility that Upsilon(10860) is the first excited state of an electric gluon hybrid bottomonium

- Also, we do not rule out the possibility that the first excited state of a bottomonium hybrid can be discovered as a state which is different from Upsilon(10860)

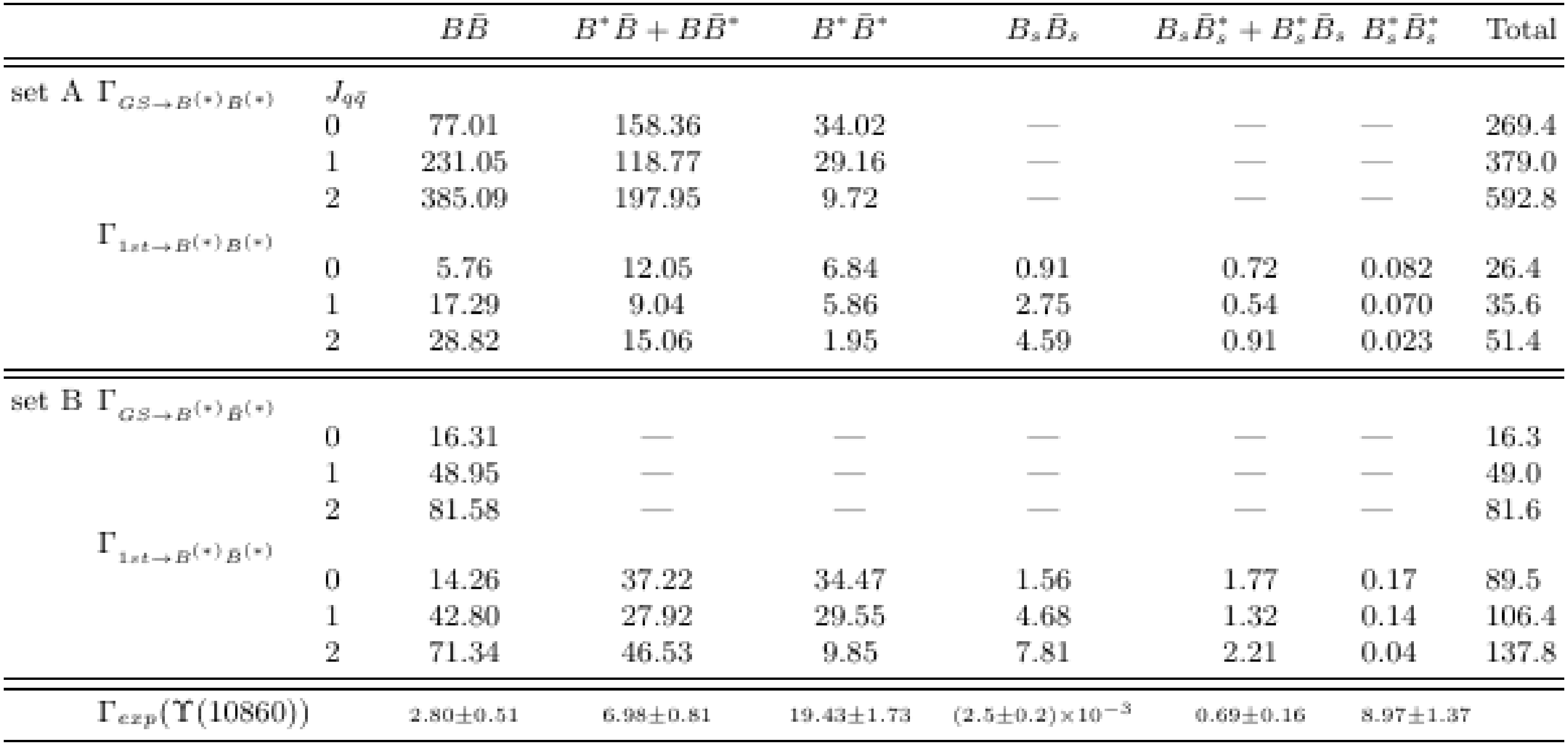
- Something is missing?
** higher components of orbital angular momenta
** higher order decay effects
** higher order potential effects

Bottomonium Hybrid



A few hundred MeV below the measured mass

BB decay channels of a hybrid bottomonium



	$J_{q\bar{q}}$	$B\bar{B}$	$B^*\bar{B}+B\bar{B}^*$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^*+B_s^*\bar{B}_s$	$B_s^*\bar{B}_s^*$	Total
set A $\Gamma_{GS\to B^{(*)}B^{(*)}}$								
	0	77.01	158.36	34.02	—	—	—	269.4
	1	231.05	118.77	29.16	—	—	—	379.0
	2	385.09	197.95	9.72	—	—	—	592.8
$\Gamma_{1st\to B^{(*)}B^{(*)}}$								
	0	5.76	12.05	6.84	0.91	0.72	0.082	26.4
	1	17.29	9.04	5.86	2.75	0.54	0.070	35.6
	2	28.82	15.06	1.95	4.59	0.91	0.023	51.4
set B $\Gamma_{GS\to B^{(*)}B^{(*)}}$								
	0	16.31	—	—	—	—	—	16.3
	1	48.95	—	—	—	—	—	49.0
	2	81.58	—	—	—	—	—	81.6
$\Gamma_{1st\to B^{(*)}B^{(*)}}$								
	0	14.26	37.22	34.47	1.56	1.77	0.17	89.5
	1	42.80	27.92	29.55	4.68	1.32	0.14	106.4
	2	71.34	46.53	9.85	7.81	2.21	0.04	137.8
$\Gamma_{exp}(\Upsilon(10860))$		2.80 ± 0.51	6.98 ± 0.81	19.43 ± 1.73	$(2.5\pm0.2)\times10^{-3}$	0.69 ± 0.16	8.97 ± 1.37	

- The calculated widths are large enough to suggest that Upsilon(10860) is not the ground state of an electric gluon bottomonium hybrid

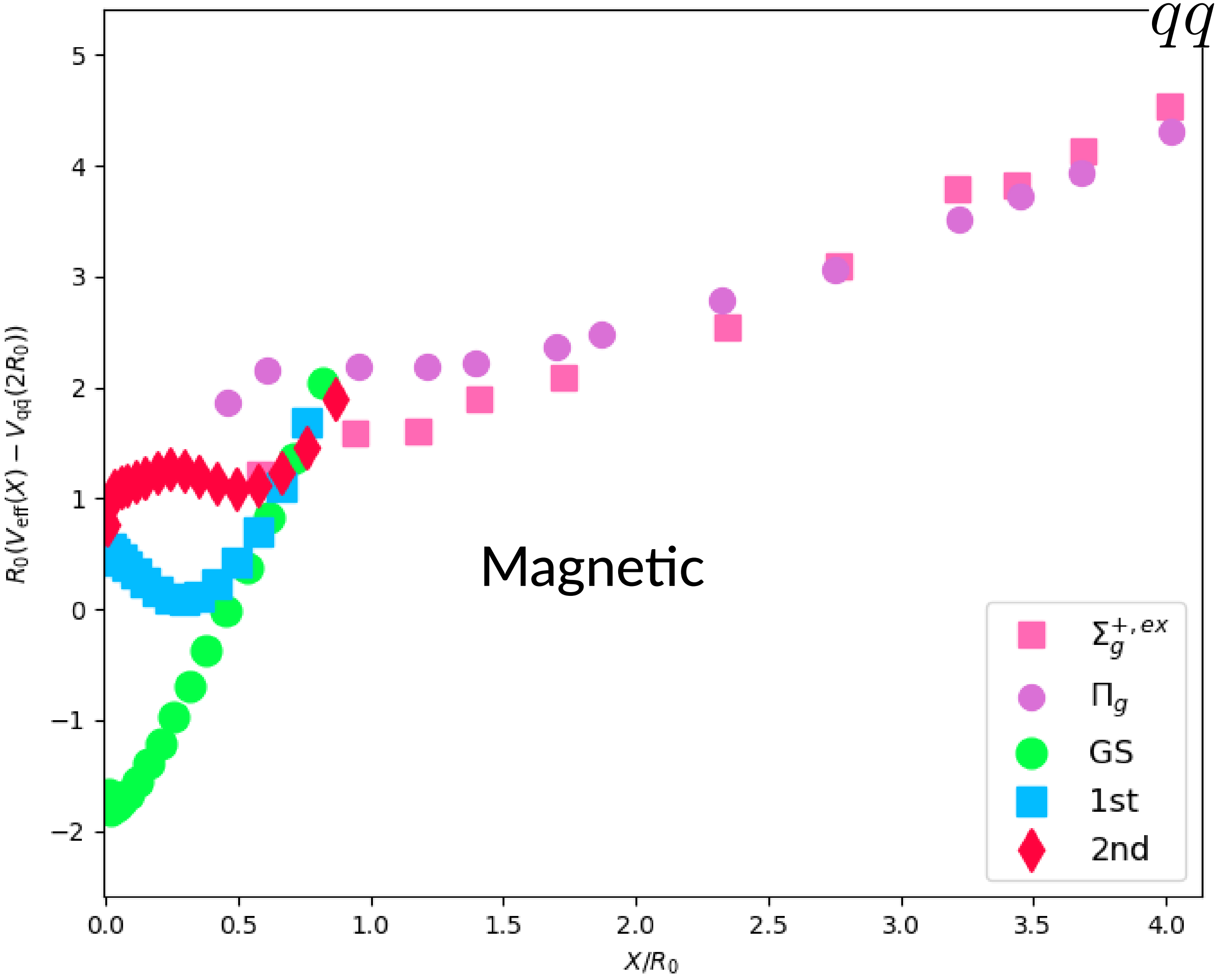
- The widths for the first excited state in set A and set B are comparable to the measured total width.

- Still, the calculated partial widths differ from the experimental data.

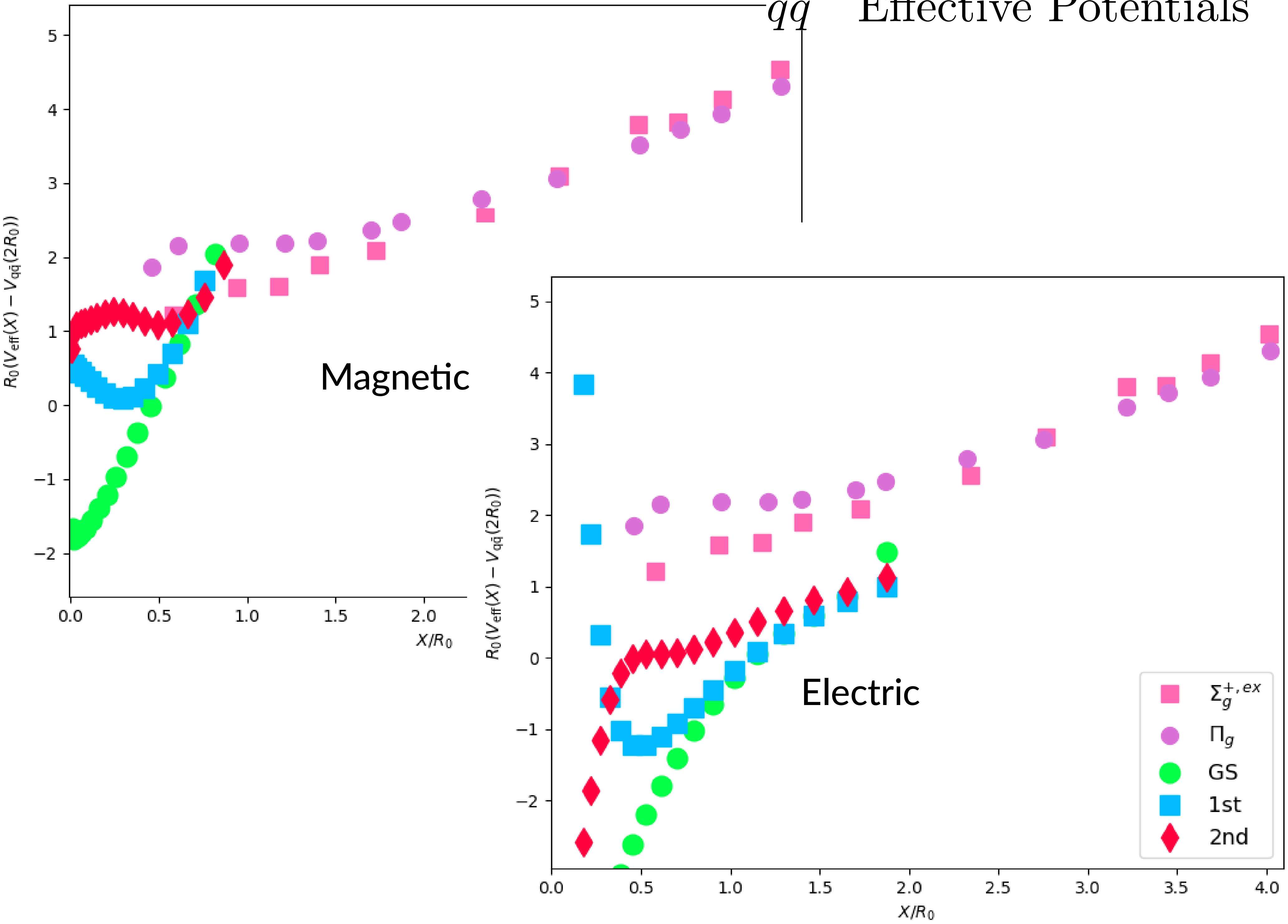
- Other decay channels, or does the running coupling constant matter?

$$B\bar{B}\pi, \quad B\bar{B}\pi\pi$$

$q\bar{q}$ Effective Potentials



$q\bar{q}$ Effective Potentials



Summary

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