Fully coupled-channel study of K⁻pp resonance

KEK Theory Center
Akinobu Doté

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   - *Fully coupled-channel Complex Scaling Method for "K^-pp"*
   - *Self-consistency for energy-dependent potential in coupled-channel case*
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Collaborators:
- *Takashi Inoue (Nihon university)*
- *Takayuki Myo (Osaka Institute of Technology)*

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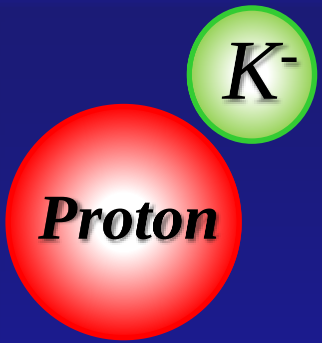
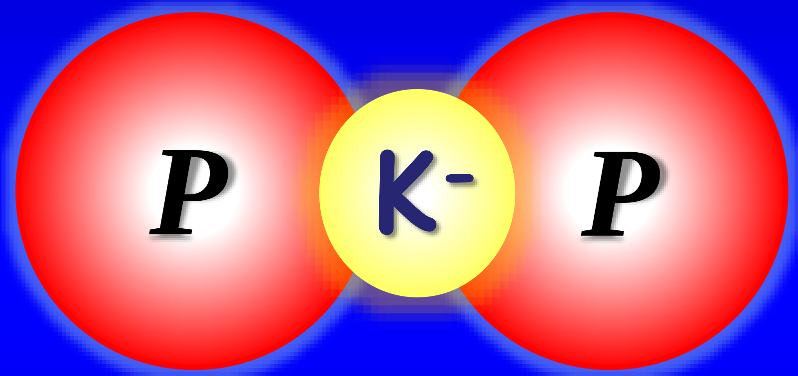
1. Introduction



Excited hyperon $\Lambda(1405)$ ... $K^{bar}N$ quasi-bound state
= $K^{bar}N$ two-body system

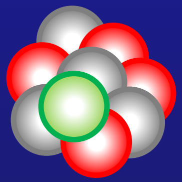
→ Strong $K^{bar}N$ attraction

T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)



K^- pp = the prototype of kaonic nuclei

... bridge from $\Lambda(1405)$ to general kaonic nuclei



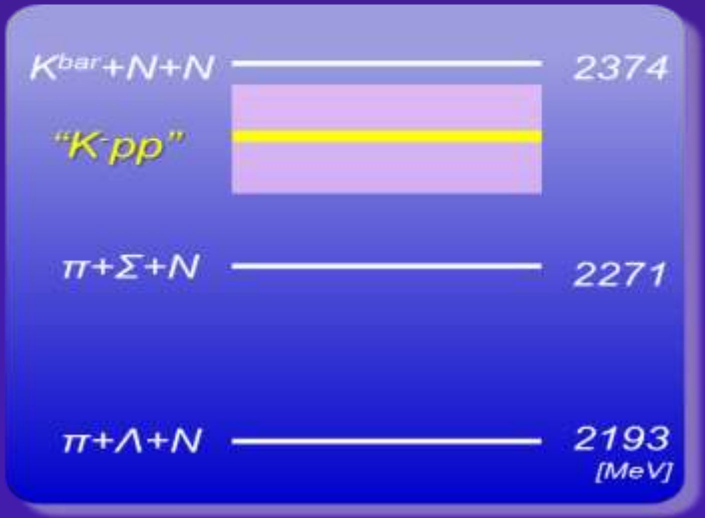
Kaonic nuclei = Nuclear many-body system
with antikaons

Doorway to dense matter?

→ Chiral symmetry restoration in dense matter??

According to many *theoretical* studies of "K^-pp" ...

"K^-pp" ($J^\pi=0^-$, $T=1/2$)



- Doté, Hyodo, Weise, PRC79, 014003(2009).
- Akaishi, Yamazaki, PRC76, 045201(2007)
- Ikeda, Sato, PRC76, 035203(2007).
- Shevchenko, Gal, Mares, PRC76, 044004(2007)
- Barnea, Gal, Liverts, PLB712, 132(2012)

→ Summarized in
A. Gal, E. V. Hungerford, D. J. Millener, Rev. Mod. Phys. 88, 035004 (2016).

Resonant state of
$K^{bar}NN$-$\pi\Sigma N$-$\pi\Lambda N$ coupled channel
three-body system

Resonance & Channel coupling

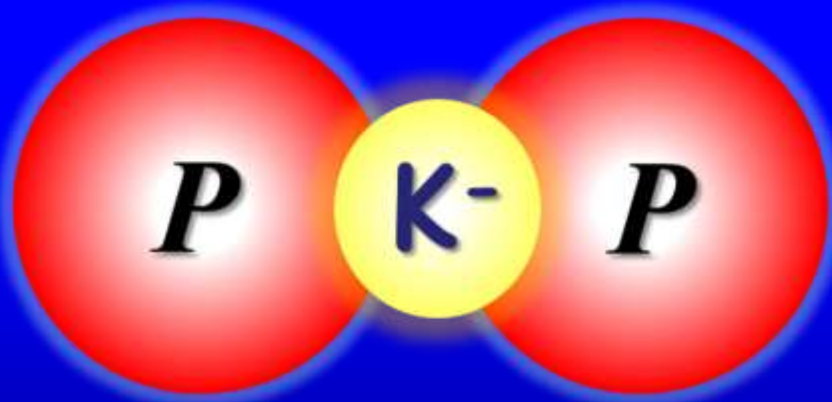
⇒ **"Fully coupled-channel Complex Scaling Method"**

A. Dote, T. Inoue, T. Myo, PRC 95, 062201(R) (2017)

2. Formalism

- *Fully coupled-channel Complex Scaling Method for "K^-pp"*

- *Self-consistency for energy-dependent potential in coupled-channel case*



"K^-pp" =

$K^{bar}NN - \pi\Sigma N - \pi\Lambda N$ ($J^\pi = 0^-$, $T=1/2$)

Wave function ...Treat all channels explicitly!

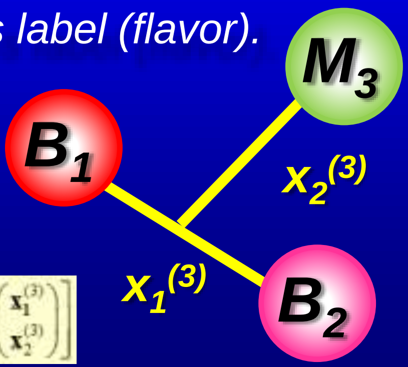
$$
\begin{aligned}
|"K^- pp"\rangle =& \sum_a C_a^{(K\{NN\}+)} G_a^{(K\{NN\}+)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{NN}=0\rangle \left|\left[K[NN]_1\right]_{T=1/2,\,Tz=1/2}\right\rangle \\
&+ \sum_a C_a^{(K\{NN\}-)} G_a^{(K\{NN\}-)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{NN}=0\rangle \left|\left[K[NN]_0\right]_{T=1/2,\,Tz=1/2}\right\rangle \\
&+ \sum_a C_a^{(\pi\{\Sigma N\}+)} G_a^{(\pi\{\Sigma N\}+)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{\Sigma N}=0\rangle \left|\left[[\pi\Sigma]_0 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_S\right\rangle \\
&+ \sum_a C_a^{(\pi\{\Sigma N\}-)} G_a^{(\pi\{\Sigma N\}-)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{\Sigma N}=0\rangle \left|\left[[\pi\Sigma]_0 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_A\right\rangle \\
&+ \sum_a C_a^{(\pi\{\Sigma N\}+)} G_a^{(\pi\{\Sigma N\}+)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{\Sigma N}=0\rangle \left|\left[[\pi\Sigma]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_S\right\rangle \\
&+ \sum_a C_a^{(\pi\{\Sigma N\}-)} G_a^{(\pi\{\Sigma N\}-)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{\Sigma N}=0\rangle \left|\left[[\pi\Sigma]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Sigma N\}_A\right\rangle \\
&+ \sum_a C_a^{(\pi\{\Lambda N\}+)} G_a^{(\pi\{\Lambda N\}+)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{\Lambda N}=0\rangle \left|\left[[\pi\Lambda]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Lambda N\}_S\right\rangle \\
&+ \sum_a C_a^{(\pi\{\Lambda N\}-)} G_a^{(\pi\{\Lambda N\}-)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\ |S_{\Lambda N}=0\rangle \left|\left[[\pi\Lambda]_1 N\right]_{T=1/2,\,Tz=1/2}, \{\Lambda N\}_A\right\rangle
\end{aligned}
$$

- Ch. 1: $K^{bar}NN$, $NN={}^1E$
- Ch. 2: $K^{bar}NN$, $NN={}^1O$
- Ch. 3: $\pi\Sigma N$, $[\pi\Sigma]_{I=0}$, $\{\Sigma N\}_{Sym.}$
- Ch. 4: $\pi\Sigma N$, $[\pi\Sigma]_{I=0}$, $\{\Sigma N\}_{Asym.}$
- Ch. 5: $\pi\Sigma N$, $[\pi\Sigma]_{I=1}$, $\{\Sigma N\}_{Sym.}$
- Ch. 6: $\pi\Sigma N$, $[\pi\Sigma]_{I=1}$, $\{\Sigma N\}_{Asym.}$
- Ch. 7: $\pi\Lambda N$, $[\pi\Lambda]_{I=1}$, $\{\Lambda N\}_{Sym.}$
- Ch. 8: $\pi\Lambda N$, $[\pi\Lambda]_{I=1}$, $\{\Lambda N\}_{Asym.}$

- *Baryon-Baryon are antisymmetrized on space, spin and isospin as well as label (flavor).*

  Glöckle, Miyagawa, Few-body Systems 30, 241 (2001)

- *Spatial part = Correlated Gaussian function*
  - ✓ *including 3 types of Jacobi coordinates*
  - ✓ *projected onto a parity eigenstate of B_1B_2,*



$$G_a^{(X\pm)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) = G_a^{(X)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) \pm G_a^{(X)}(-\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) \quad \Leftarrow \quad G_a^{(X)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) = N_a^{(X)} \exp\left[-(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)})\, A_a^{(X)} \begin{pmatrix}\mathbf{x}_1^{(3)} \\ \mathbf{x}_2^{(3)}\end{pmatrix}\right]$$

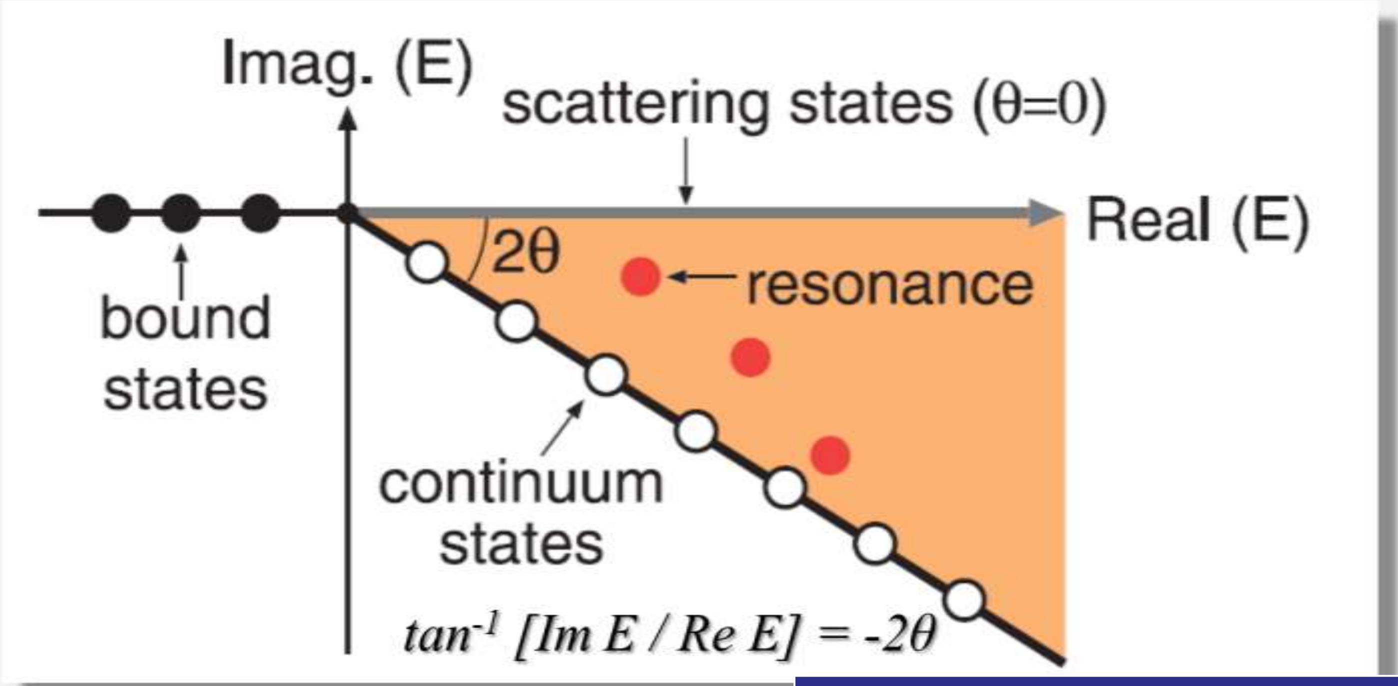
Complex Scaling Method

… Find resonance poles on complex energy plane!

Complex rotation (Complex scaling) of coordinate
Resonance wave function → L^2 integrable

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

Diagonalize $H_\theta = U(\theta)\, H\, U^{-1}(\theta)$ with Gaussian base,

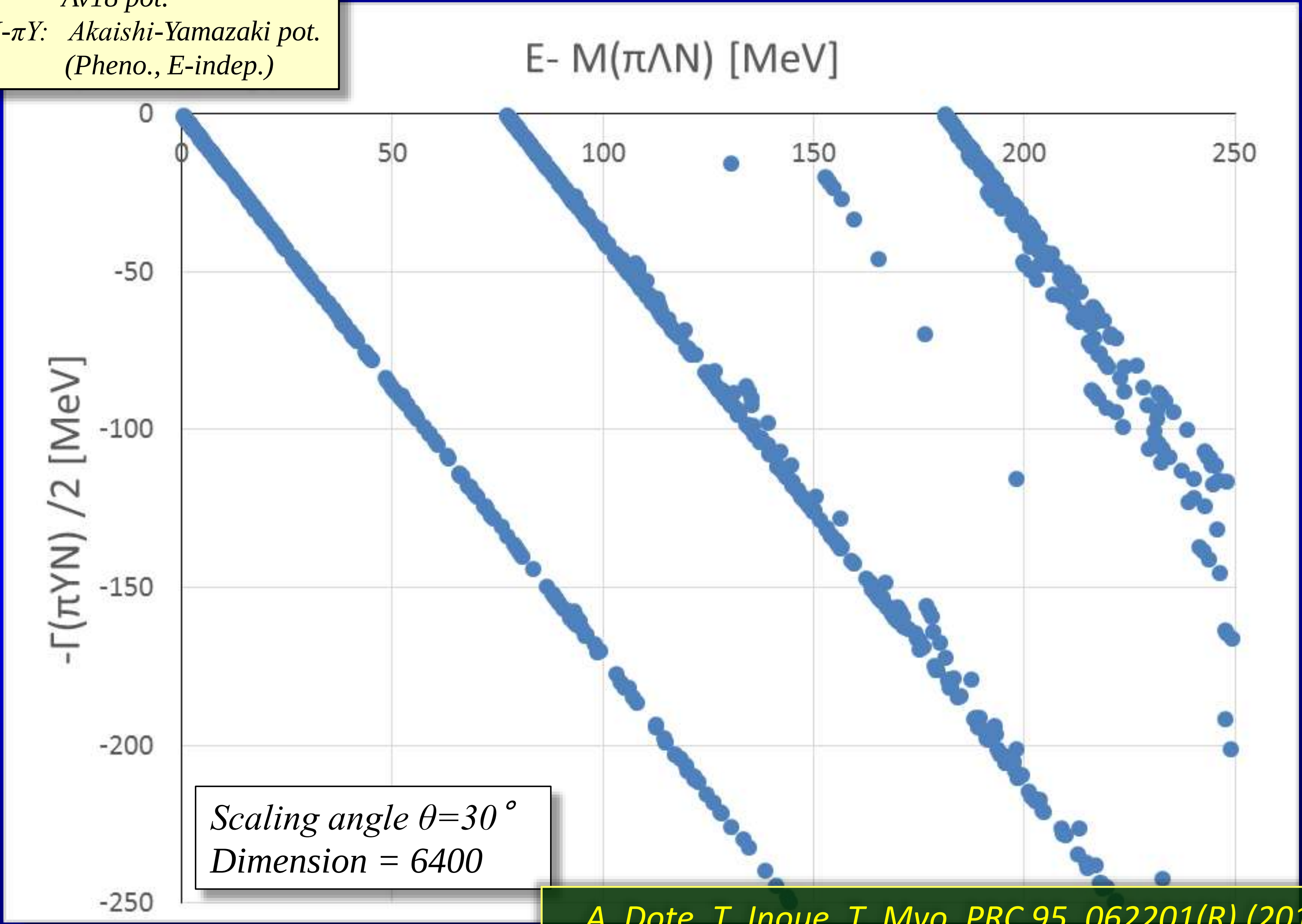


S. Aoyama, T. Myo, K. Kato, K. Ikeda, PTP116, 1 (2006)
T. Myo, Y. Kikuchi, H. Masui, K. Kato, PPNP79, 1 (2014)

- *Continuum state appears on 2ϑ line.*
- *Resonance pole is off from 2ϑ line, and independent of ϑ. (ABC theorem)*

Full ccCSM with a pheno. potential

NN: *Av18 pot.*
$K^{bar}N$-πY: *Akaishi-Yamazaki pot.*
(Pheno., E-indep.)



Scaling angle θ=30°
Dimension = 6400

A. Dote, T. Inoue, T. Myo, PRC 95, 062201(R) (2017)

Full ccCSM with a pheno. potential

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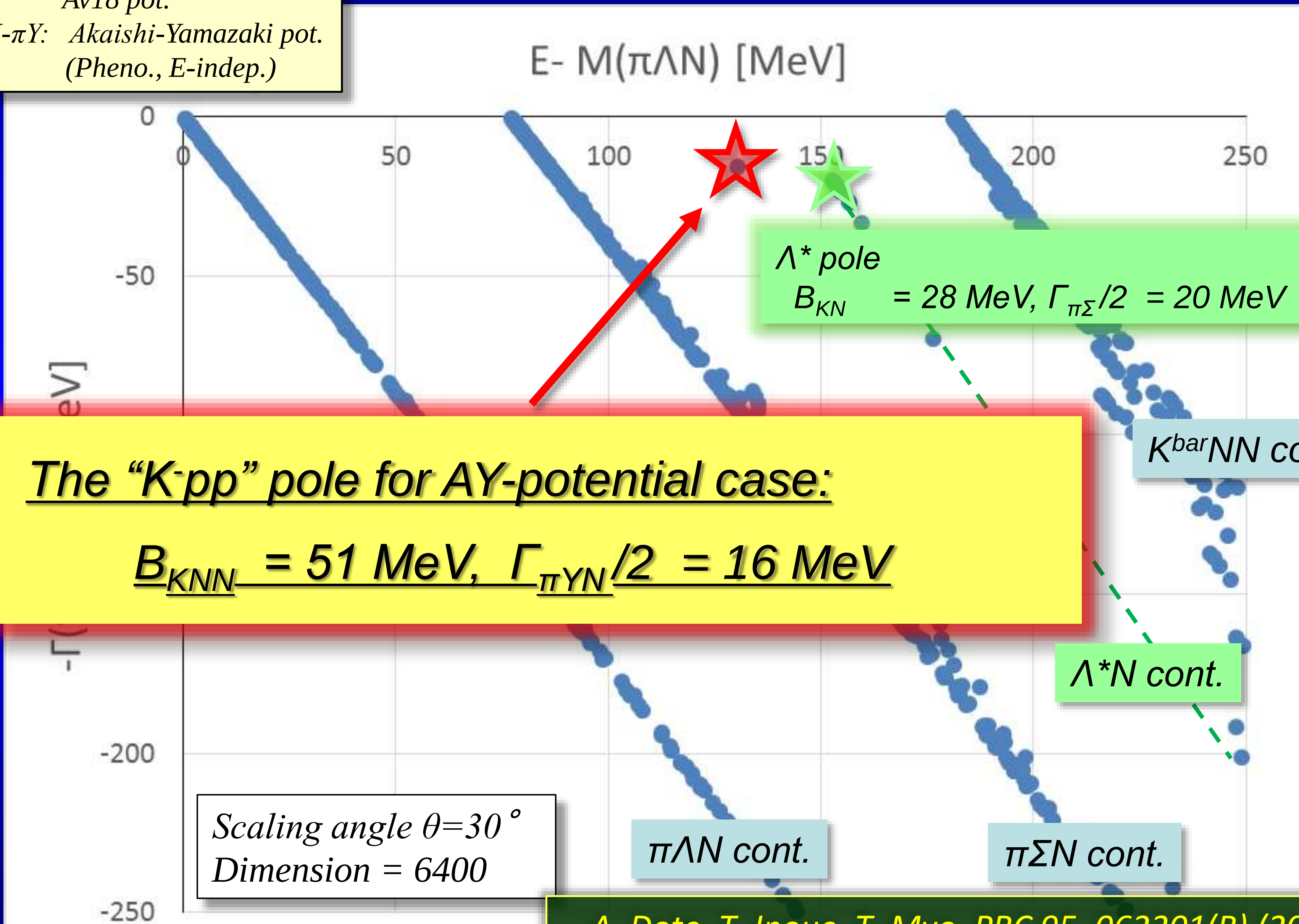


The "K^-pp" pole for AY-potential case:

B_{KNN} = 51 MeV, $\Gamma_{\pi YN}/2$ = 16 MeV

Scaling angle θ=30°
Dimension = 6400

A. Dote, T. Inoue, T. Myo, PRC 95, 062201(R) (2017)

Hamiltonian

$$\hat{H} = \hat{M} + \hat{T} + \hat{V}_{NN} + \sum_{\alpha,\beta=K^{bar}N,\pi\Sigma,\pi\Lambda} \hat{V}_{(MB)\alpha-(MB)\beta}$$

- Kinematics = Non-relativistic
- NN potential = Av18 potential — R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, PRC 51, 38 (1995)
- $K^{bar}N$-πY potential ... Chiral SU(3)-based potential
  - Theoretical and energy-dependent potential
  - Gaussian form potential — A. Dote, T. Inoue, T. Myo, NPA 912, 66 (2013)
  - Constrained by the latest $K^{bar}N$ scattering length

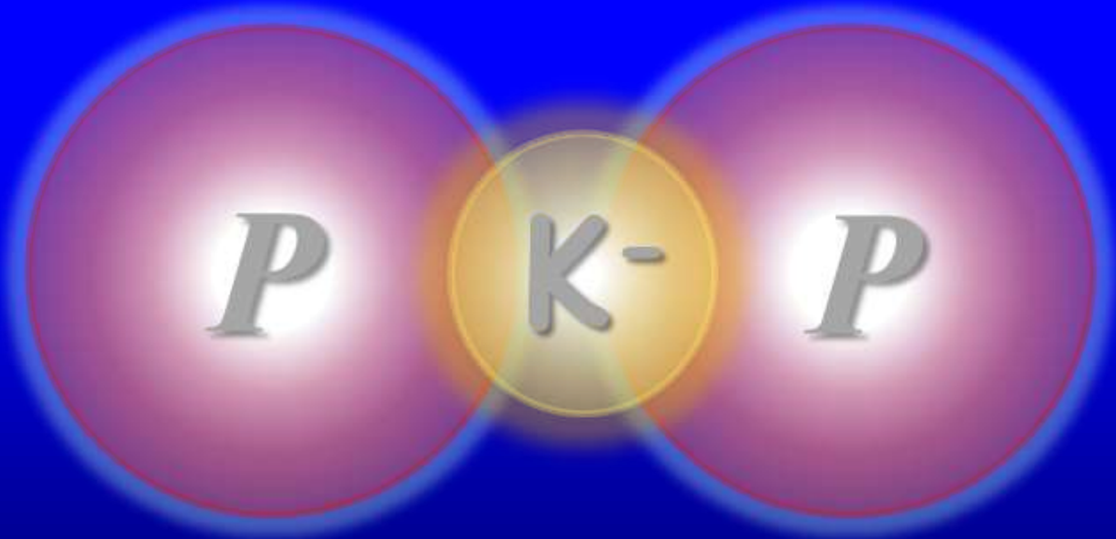
SIDDHARTA K^-p data M. Bazzi et al., NPA 881, 88 (2012)	+	Coupled-channel chiral dynamics Y. Ikeda, T. Hyodo, W. Weise, NPA 881, 98 (2012)

◆ Ignore YN and πN potentials

A. Dote, T. Inoue, T. Myo, PLB 784, 405 (2018)

2. Formalism

- *Fully coupled-channel Complex Scaling Method for "K^-pp"*
- *Self-consistency for energy-dependent potential in coupled-channel case*



How to deal with E-dep. potential?

Chiral SU(3)-based potential = Energy-dependent potential

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8 f_\pi^2}\left(\omega_i + \omega_j\right)\sqrt{\frac{1}{m_i\, m_j}}\; g_{ij}(r)$$

- *How to treat energy-dependent potentials in many-body system?*
- *In addition, in coupled-channels cases???*

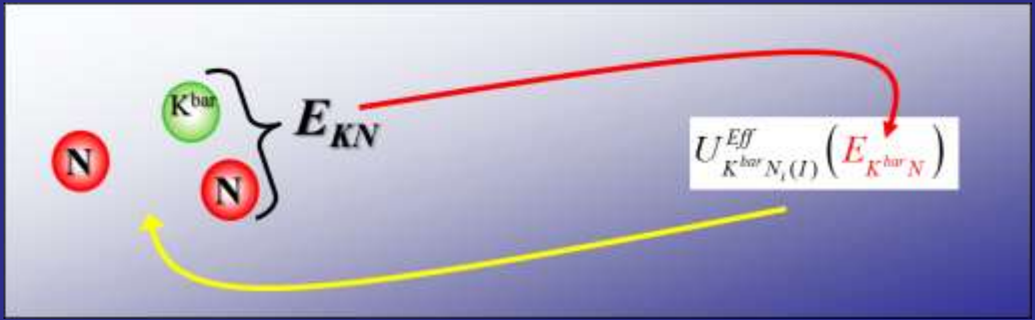
“Self-consistency for Meson-Baryon energy”

... generalize a prescription for single-channel cases

(Dote, Hyodo, Weise, PRC79, 014003 (2009))

Self-consistency for $K^{bar}N$ energy

- **$K^{bar}NN$ single-channel case**



Estimation of the two-body energy in the three-body system

A. D., T. Hyodo, W. Weise, PRC79, 014003 (2009)

1. Kaon's binding energy:

$$B(K) \equiv -\left\{ \left\langle H_{K^{bar}NN} \right\rangle - \left\langle H_{NN} \right\rangle - m_K \right\}$$

H_{NN} : Hamiltonian of 2N

$H_{K^{bar}NN}$: Hamiltonian of 2N+K^{bar}

2. Define a $K^{bar}N$-bond energy in two ways:

$$E(KN) = M_N + \omega_K = \begin{cases} M_N + m_K - B(K) & \text{: Field picture} \\ M_N + m_K - B(K)/2 & \text{: Particle picture} \end{cases}$$

An interacting $K^{bar}N$ pair carries

... ***100%*** *of B(K)*

... ***50%*** *of B(K)*

Self-consistency for **MB** energy

- ***$\bar{K}$NN-πYN coupled-channel case***

1. *Consider the coupled-channel Hamiltonian and mass operators,*
2. *Calculate their **expectation values** with the **obtained wave function**.*
3. *Using these values, we estimate the **averaged** MB energy.*

Estimation of the two-body energy in the three-body system

A. D., T. Inoue, T. Myo, PLB784, 405 (2018)

1. **Meson**'s binding energy:

$$B(M) \equiv -\left\{ \left\langle H_{MB_1B_2} \right\rangle - \left\langle H_{B_1B_2} \right\rangle - \left\langle m_M \right\rangle \right\}$$

$\hat{H}_{B_1B_2}$: Hamiltonian of 2B

$\hat{H}_{MB_1B_2}$: Hamiltonian of 2B+M

2. Define a **MB**-bond energy in two ways:

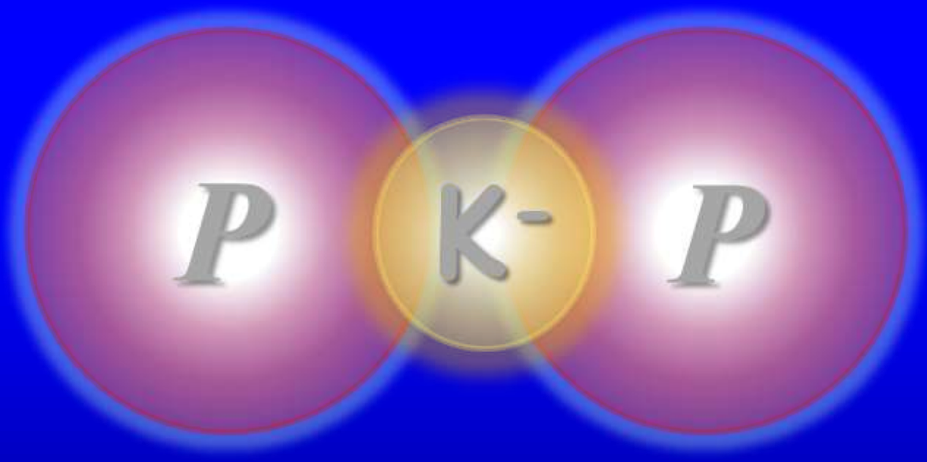
$$E(MB) = \left\langle M_B + \omega_M \right\rangle = \begin{cases} \left\langle M_B + m_M \right\rangle - B(M) & \text{: Field picture} \\ \left\langle M_B + m_M \right\rangle - B(M)/2 & \text{: Particle picture} \end{cases}$$

$\hat{M}_B$: Baryon-mass operator

$\hat{m}_M$: Meson-mass operator

A. Dote, T. Inoue, T. Myo, PLB 784, 405 (2018)

3. Result

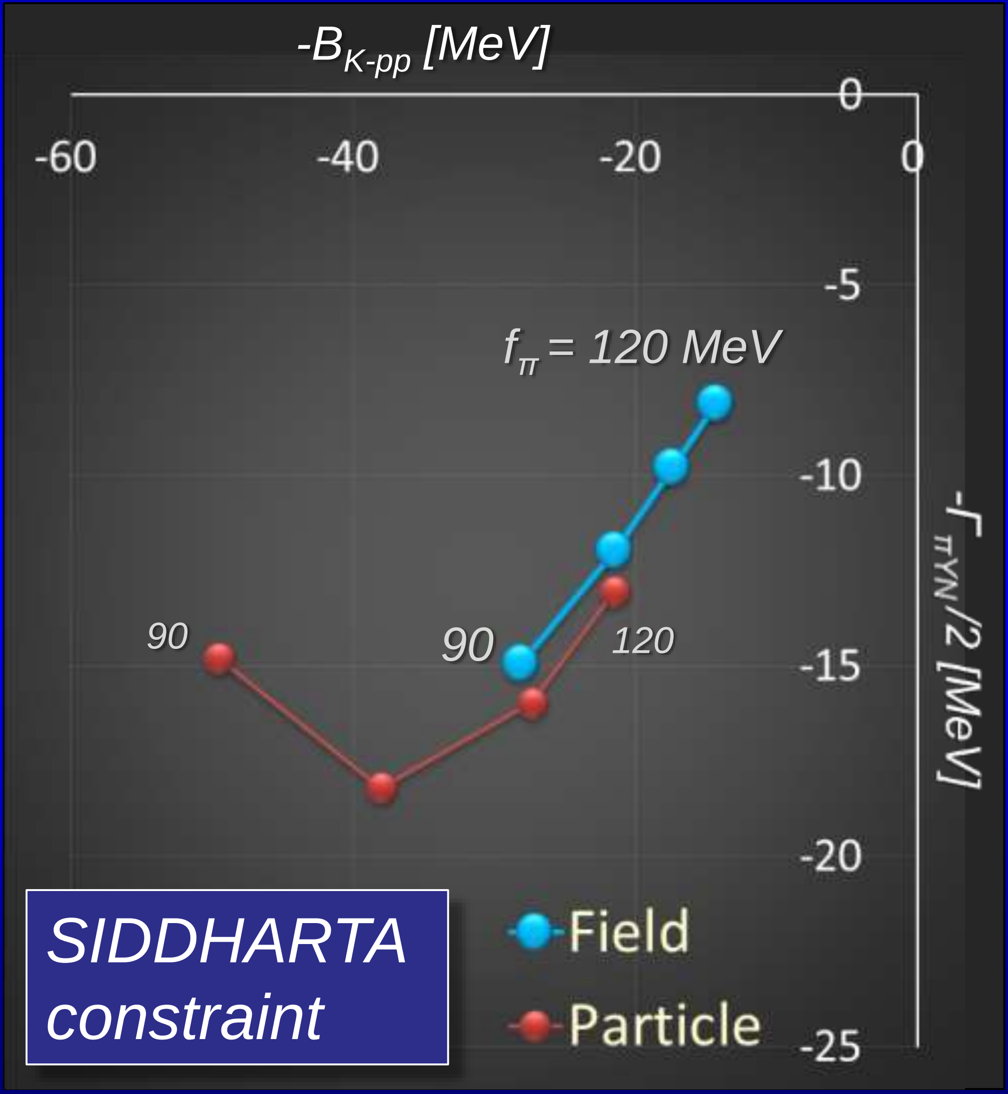


"K^-pp" =

$K^{bar}NN$ – $\pi\Sigma N$ – $\pi\Lambda N$ ($J^\pi = 0^-$, $T=1/2$)

Binding energy and decay width

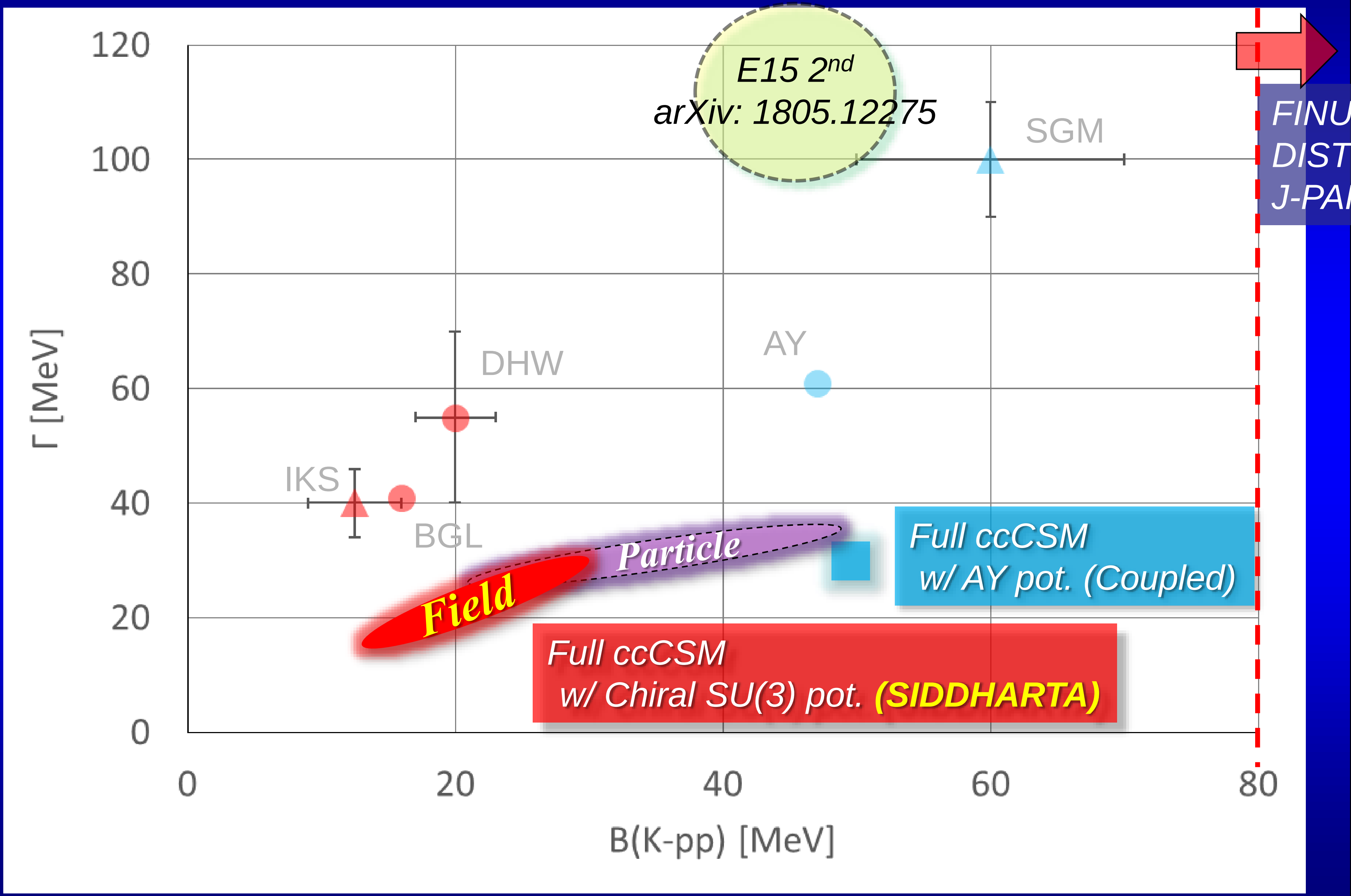
Remark
For each f_π value, range parameters in the potential are tuned to reproduce the $K^{bar}N$ scattering length.



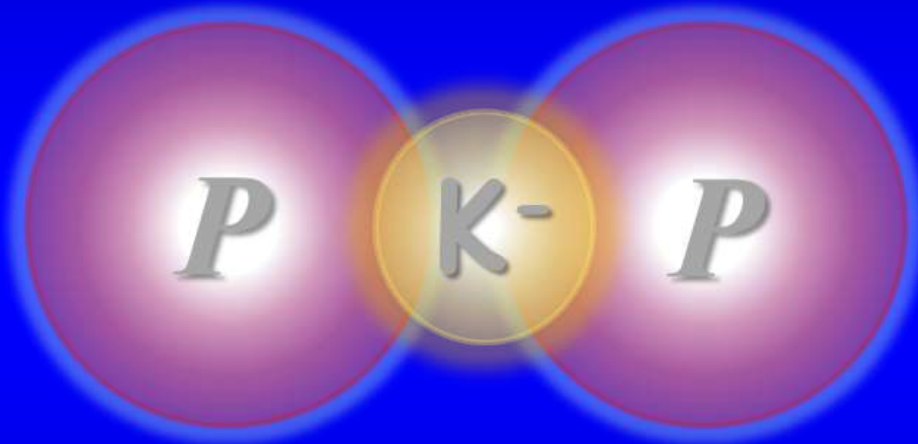
- ***Field picture:***
  $B_{K\text{-}pp} = 14 - 28$ MeV
  $\Gamma_{\pi YN}/2 = 8 - 15$ MeV

- ***Particle picture:***
  $B_{K\text{-}pp} = 21 - 50$ MeV
  $\Gamma_{\pi YN}/2 = 13 - 19$ MeV

Current result of "K-pp"



4. Relativistic effect



Small mass!

→ Is Non-rela. treatment o.k.?

"K^-pp" =

$\bar{K}NN$ – $\pi\Sigma N$ – $\pi\Lambda N$ ($J^\pi = 0^-$, $T=1/2$)

Semi-relativistic treatment of mesons (π and K^{bar})

- *Meson … **Semi-relativistic** kinematics (SR)*
- *Baryon … Non-relativistic kinematics (NR)*

$$H = \sqrt{\mathbf{p}_{Meson}^2 + m_{Meson}^2} + \sum_{i=1}^{2}\left(m_{Baryon(i)} + \frac{\mathbf{p}_{Baryon(i)}^2}{2m_{Baryon(i)}} \right) + V_{NN} + V_{MB(SR)}$$

✓ *$K^{bar}N$-πY potential is reconstructed in SR kinematics.*

Pion appears in decay channels.
→ Change of kinematics may affect the decay width???

Case: SIDDHARTA / Field picture / f_π=110 MeV

- *Non-rela.*
  ($B_{K\text{-}pp}$, $\Gamma_{\pi YN}/2$) = (17.5, 10.0)
  [MeV]

Preliminary

- *Semi-rela.(Meson)*
  ($B_{K\text{-}pp}$, $\Gamma_{\pi YN}/2$) = (30.2, 29.3)

Decay width increases?

5. Summary

and

Future prospects

5. Summary

K^-pp = a prototype of kaonic nuclei

We have developed Fully coupled-channel Complex Scaling Method, by which K^-pp is completely treated as a resonance of $K^{bar}NN$-$\pi\Sigma N$-$\pi\Lambda N$ coupled-channel system.

Binding energy and mesonic decay width of "K^-pp" are obtained as

Chiral SU(3)-based $K^{bar}N$ potential constrained with the latest $K^{bar}N$ data (SIDDAHRTA)

K^-pp ($J^\pi=0^-$, $T=1/2$) … ($B_{K\text{-}pp}$, $\Gamma_{\pi YN}/2$) = (14--28, 8--15) MeV (Field picture)

(21--50, 13--19) MeV (Particle picture)

Self-consistency for meson-baryon energy is considered.

cf) Phenomenological $K^{bar}N$ potential (Akaishi-Yamazaki potential; Energy-independent)

($B_{K\text{-}pp}$, $\Gamma_{\pi YN}/2$) = (51, 16) MeV

Examined Semi-relativistic kinematics for mesons (Preliminary)

Decay width ($\Gamma_{\pi YN}$) seems to become larger, compared with Non-relativistic case.

5. Future prospects

- **Semi-relativistic treatment (Undergoing)**

  ... Pion mass is small. Influence to decay width?

- **Examine other $K^{bar}N$ potential**

  ... A sophisticated version of
  a chiral SU(3)-based $K^{bar}N$-πY local potential
  K. Miyahara, T. Hyodo and W. Weise,
  PRC 98, 025201 (2018)

- **Non-mesonic decay ($K^{bar}NN \rightarrow YN$)**

- **Reaction spectrum**

  ... Direct comparison with experimental data.
  It can be calculated using the Green function obtained with ccCSM.
  ("Morimatsu-Yazaki Green's function method")

- ...

Thank you very much!

References:

- *A. Dote, T. Inoue, T. Myo, PRC 95, 062201(R) (2017)*
- *A. Dote, T. Inoue, T. Myo, PLB 784, 405 (2018)*