Exploring the finite density QCD based on the complex Langevin method

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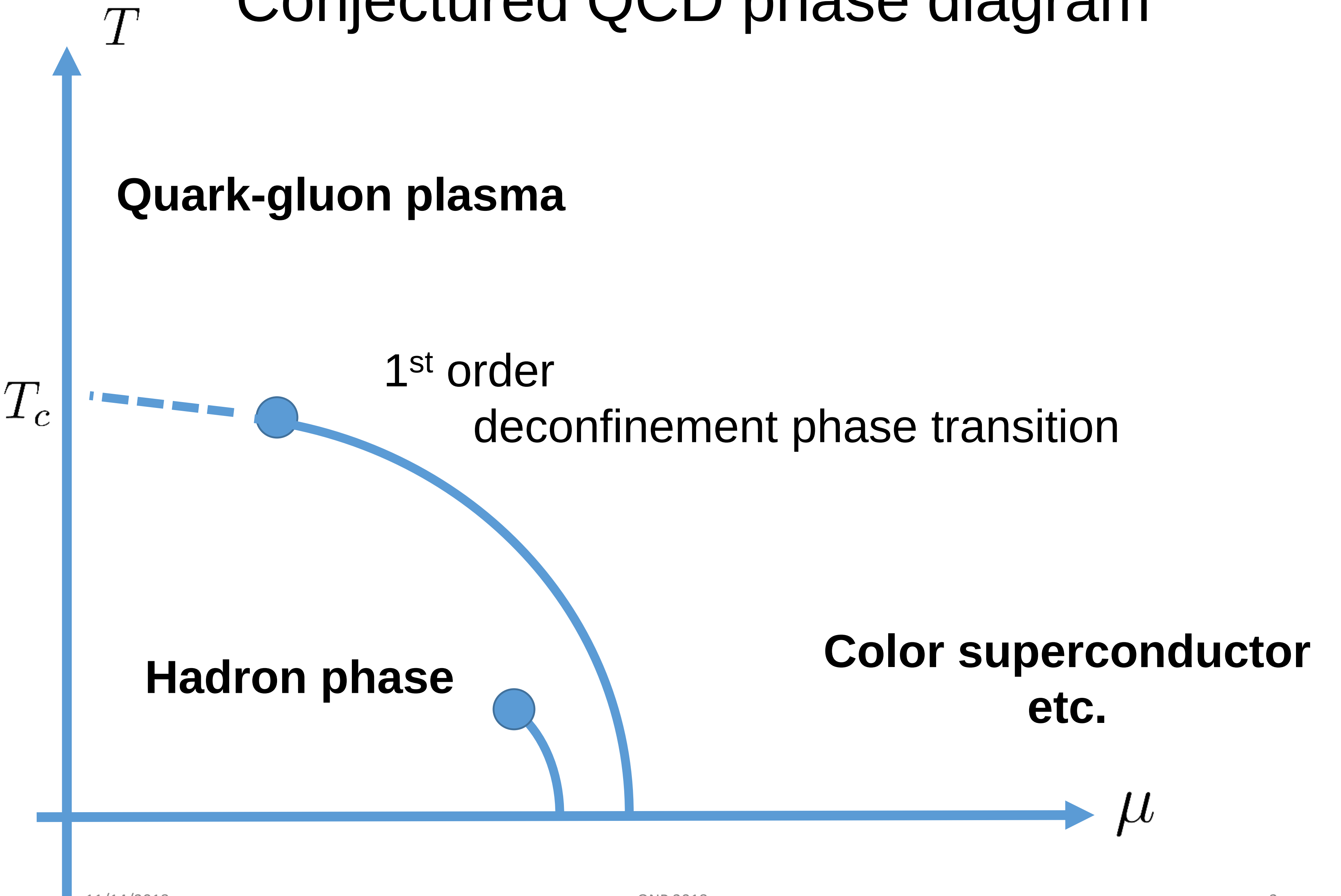
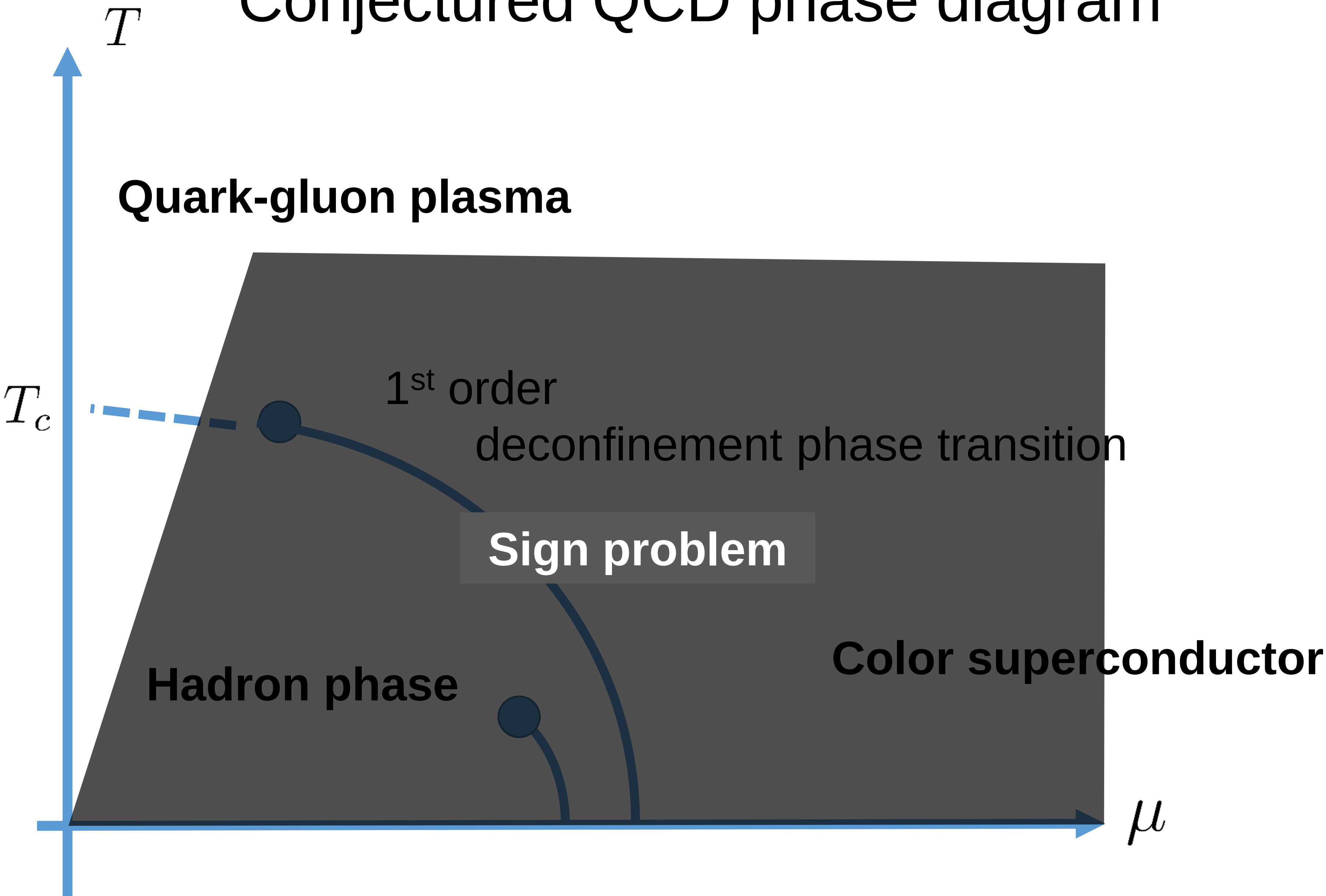
Conjectured QCD phase diagram



Conjectured QCD phase diagram



Finite density QCD

QCD partition function

$$Z = \int dU \det M[U,\mu] \mathrm{e}^{-S_g[U]}$$

$$M = D + m$$

The origin of the sign problem

$\det M$ is complex when $\mu \neq 0$

A promising way to solve the sign problem:

Complex Langevin method (CLM)

Complex Langevin method for QCD

$$Z = \int dU \det M[U, \mu] e^{-S_g[U]}$$

[Parisi '83], [Klauder '84]
[Aarts, Seiler, Stamatescu '09]
[Aarts, James, Seiler, Stamatescu '11]
[Seiler, Sexty, Stamatescu '13]
[Sexty '14] [Fodor, Katz, Sexty, Torok '15]
[Sinclair, Kogut '16]
[Nishimura, Shimasaki '15]
[Nagata, Nishimura, Shimasaki '15]

Complexification

$$U_{x\mu} \in SU(3) \rightarrow \mathcal{U}_{x\mu} \in SL(3, \mathbb{C}) \qquad S(U) \rightarrow S(\mathcal{U})$$

The complex Langevin eq. of QCD

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i\left(-\epsilon \boxed{\mathcal{D}_{x\mu} S[\mathcal{U}]} + \sqrt{\epsilon}\,\eta_{x\mu}(t)\right)\right]\mathcal{U}_{x\mu}(t)$$

Drift term

Setup

- $N_f = 4$, staggered fermion
- Lattice size: $8^3 \times 16$
- $\beta = 5.7$
- $\mu a = 0.0 – 0.5$
- Quark mass: $m_q a = 0.01$
- Number of Langevin steps = $10^4 – 10^5$
- Computer resources: K computer

Lattice spacing: a ~ 0.045 fm

Criterion of correctness

The CLM sometimes gives incorrect results.

Exponential falloff of the drift distribution

Complex Langevin is reliable

Power-law falloff of the drift distribution

Complex Langevin gives incorrect answer

[Nagata, Nishimura, Shimasaki ‘15]

The main causes of the power-law falloff:

Excursion problem : large deviation of the link variables from SU(3)

Singular drift problem: nearly zero eigenvalues of the fermion matrix generate an unreasonably large drift term

Criterion of correctness

Excursion problem:

We have checked

1. Gauge field contributions to the drift term
2. Time dependence of the unitarity norm (distance on SL(3,C))

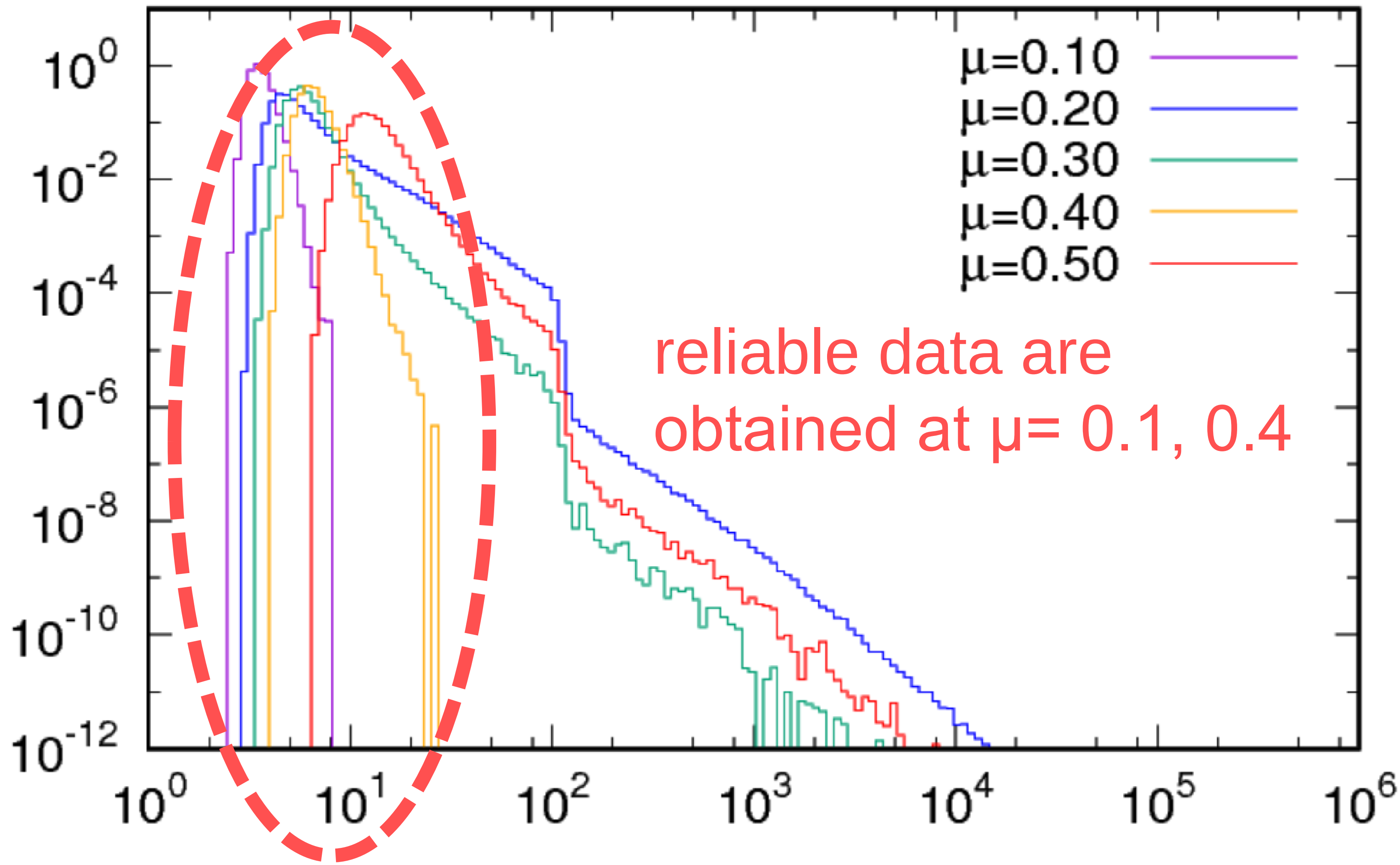
Singular drift problem:

We have checked

1. Fermion contributions to the drift term
2. A snap shot of the eigenvalue distribution of (D+m)

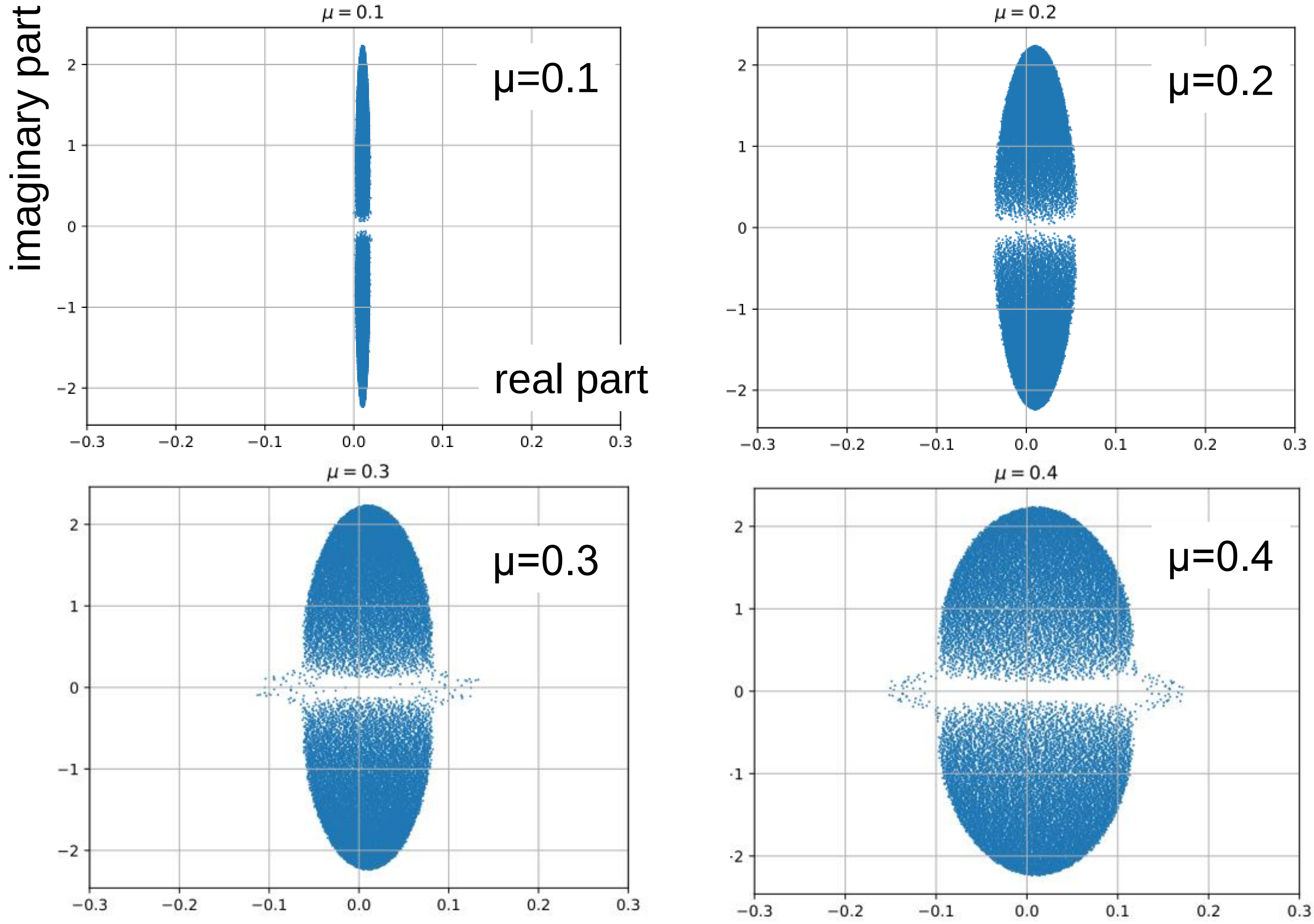
This is the first time to show for the full QCD configurations generated by complex Langevin method.

Histogram of the drift term

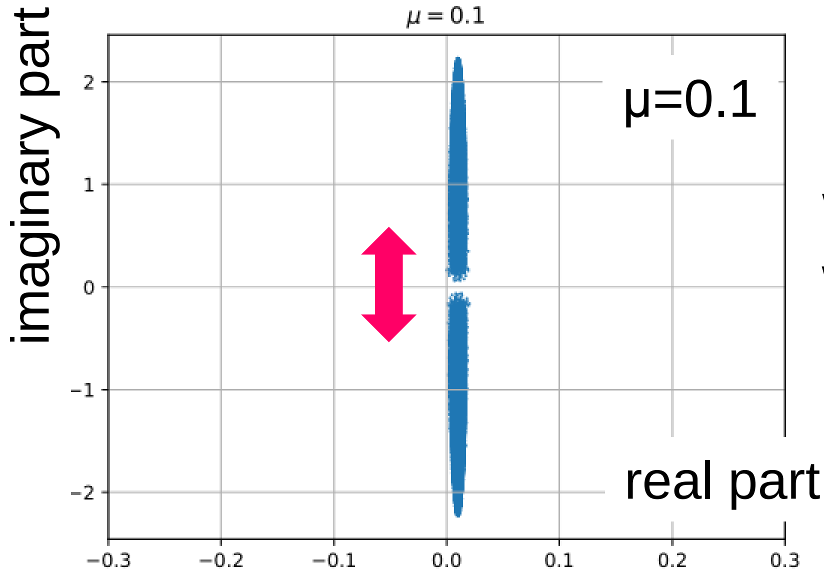


(*) Fermionic contribution is shown.

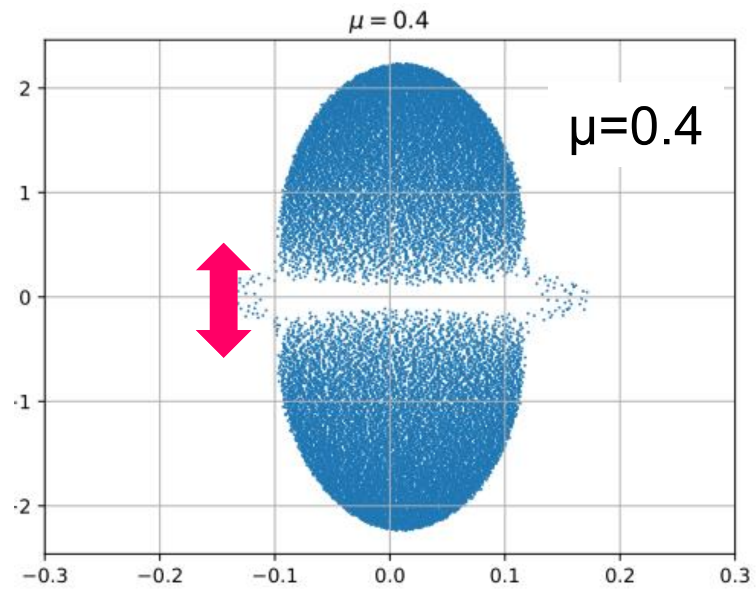
Eigenvalue distribution of (D+m)



Eigenvalue distribution for reliable data



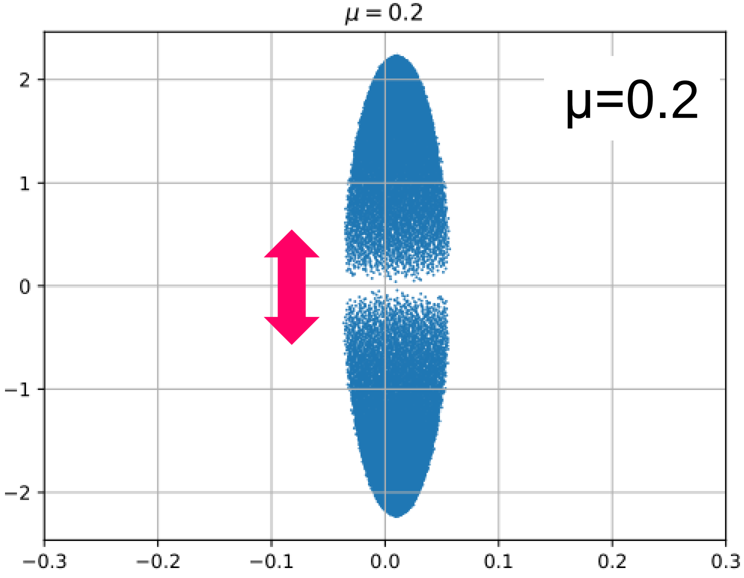
We find **gapped** distribution at μ=0.1, 0.4, where the singular drift problem does not occurs.

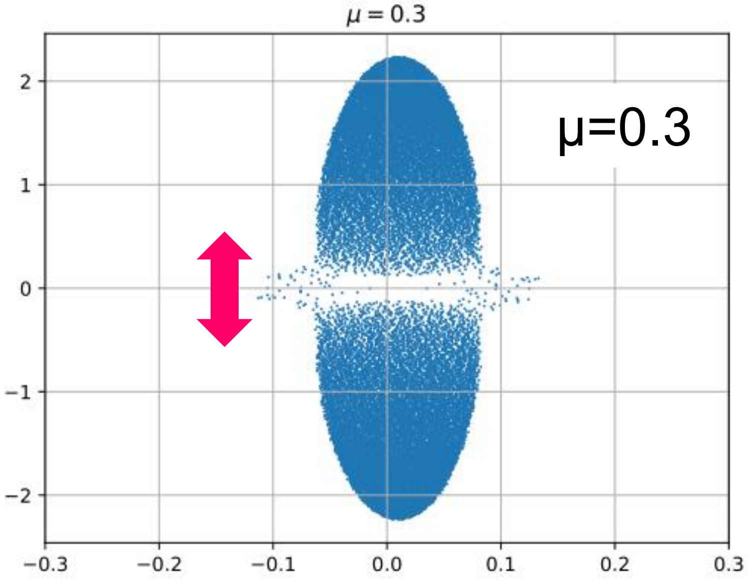


Eigenvalue distribution for unreliable data

Distributions at μ=0.2, 0.3 are also gapped.

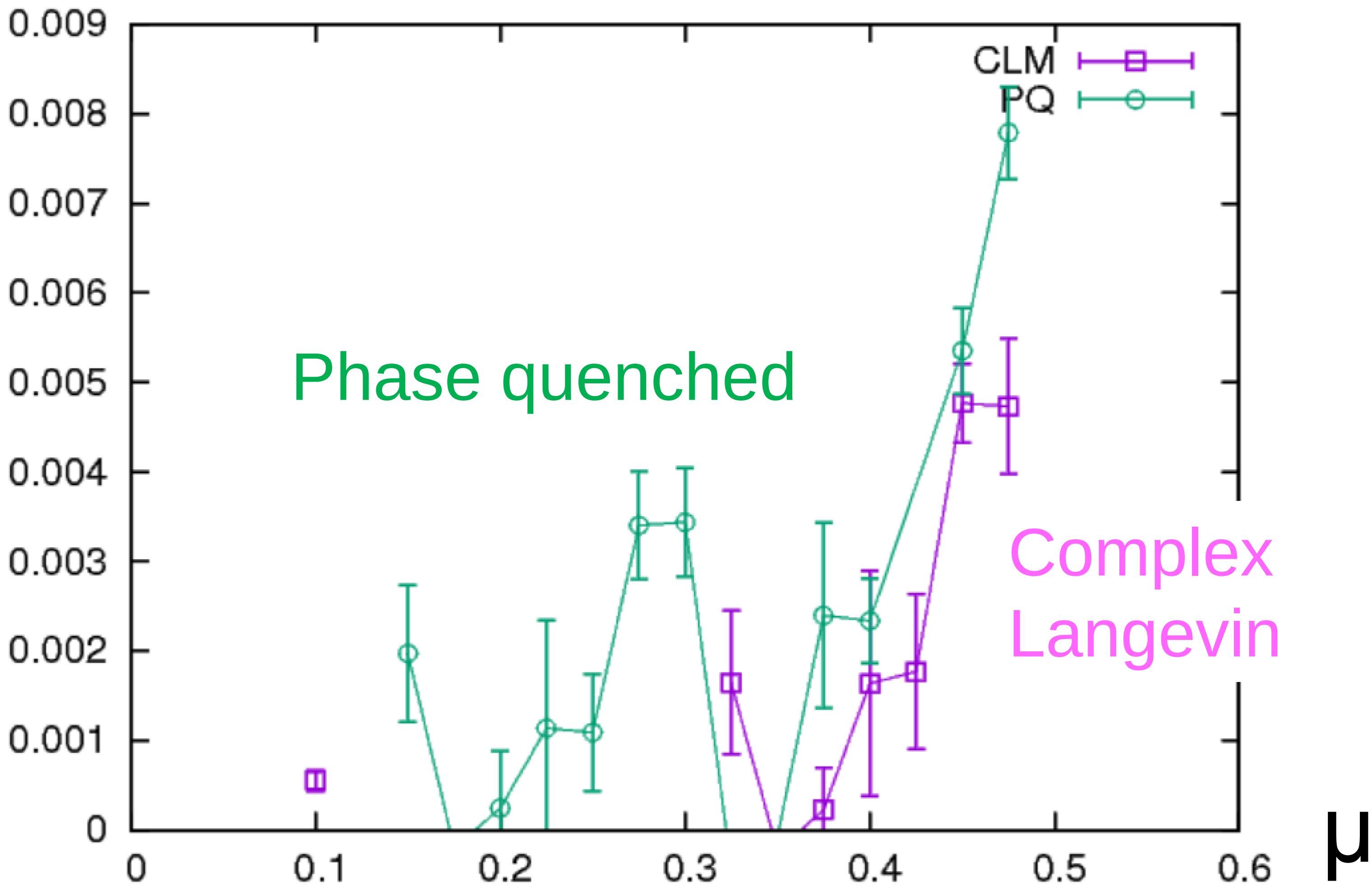
Why?





- These are just snap shots.
- Eigenvalue distribution may have large fluctuations in the vicinity of the phase transition line.

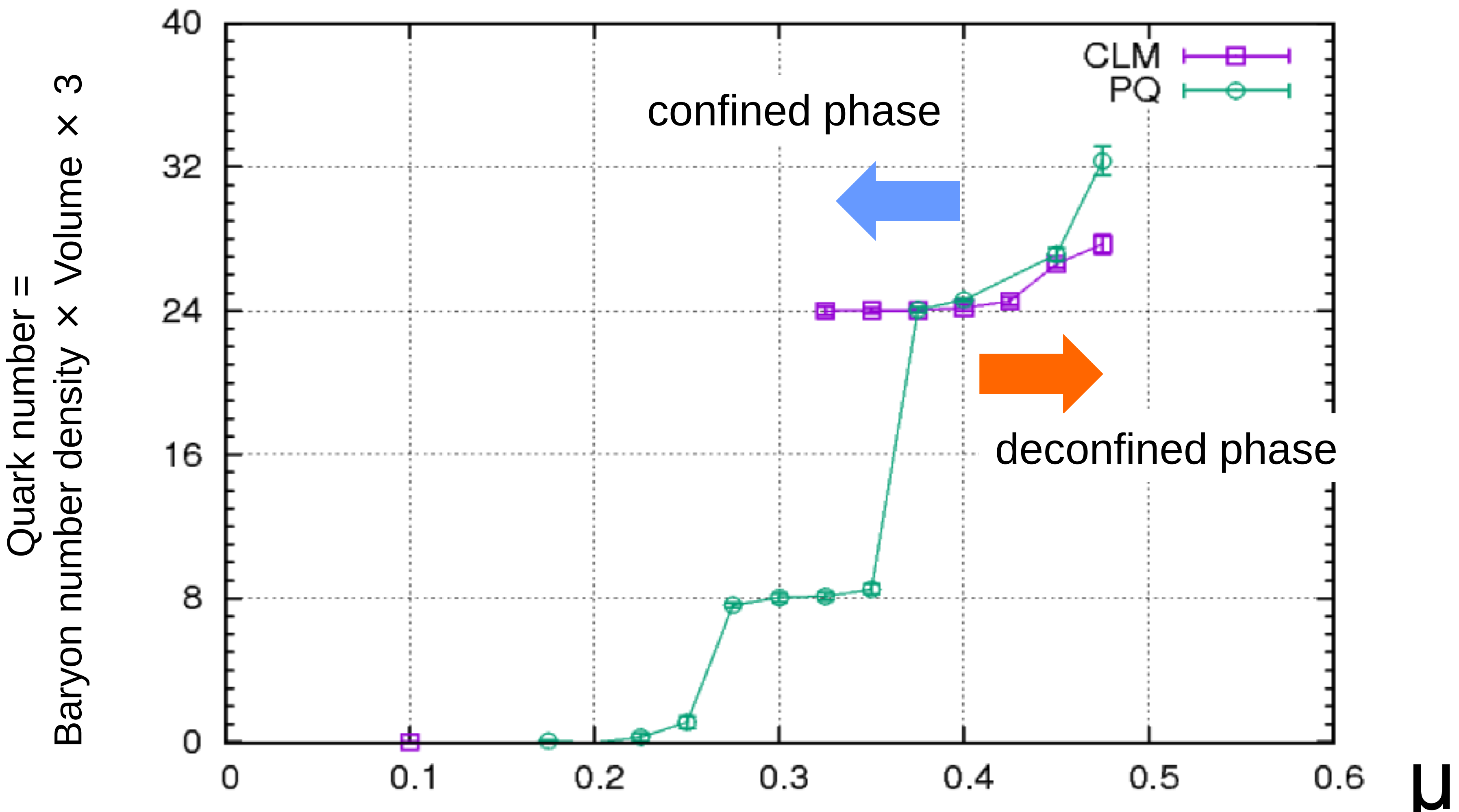
Polyakov loop



confined phase (due to the finite spatial volume effect)
∗ Physical temperature is above Tc.

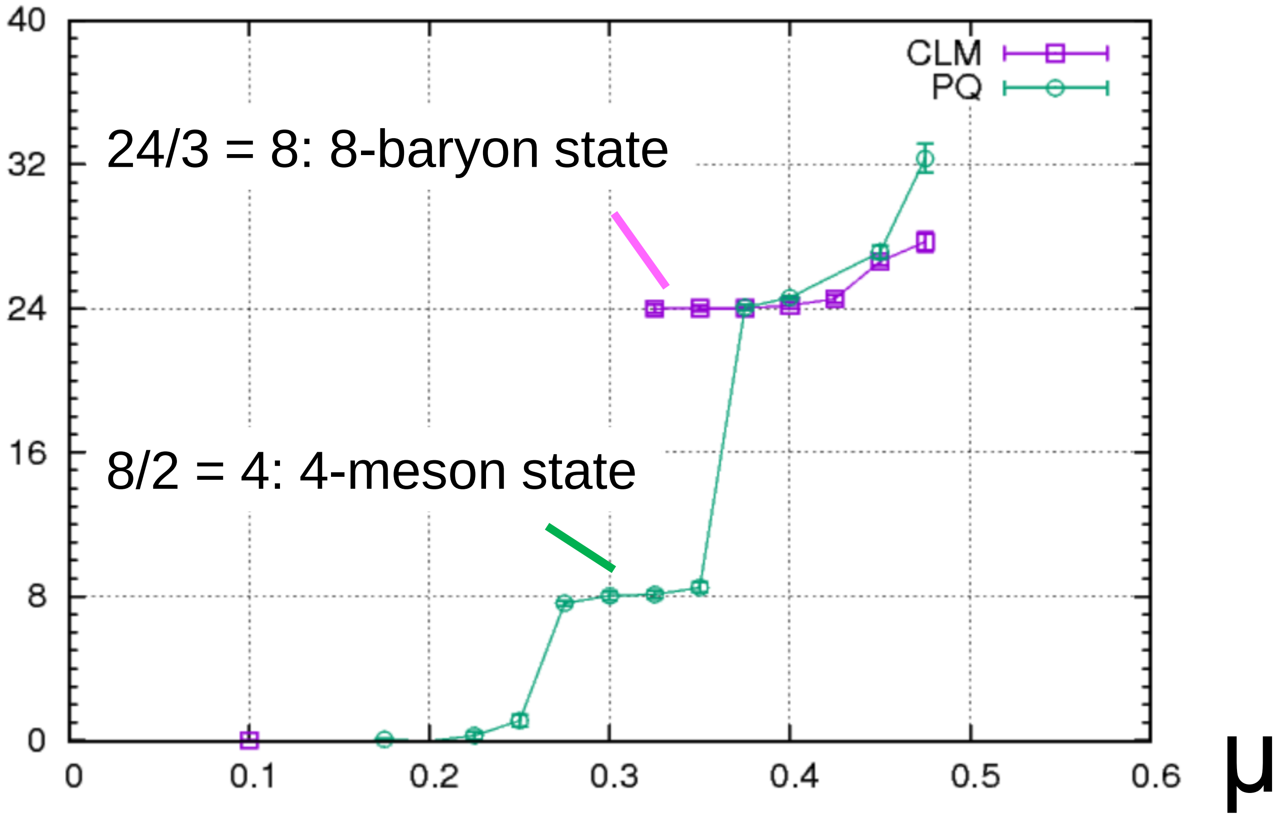
deconfined phase

Quark number



What is the origin of the plateaus?

Quark number



Phase quenched (PQ): μ plays a role of "isospin chemical potential".
→ Meson state is produced.

Complex Langevin: Quark number at the plateau can be divided by 3.
→ Baryon state is produced.

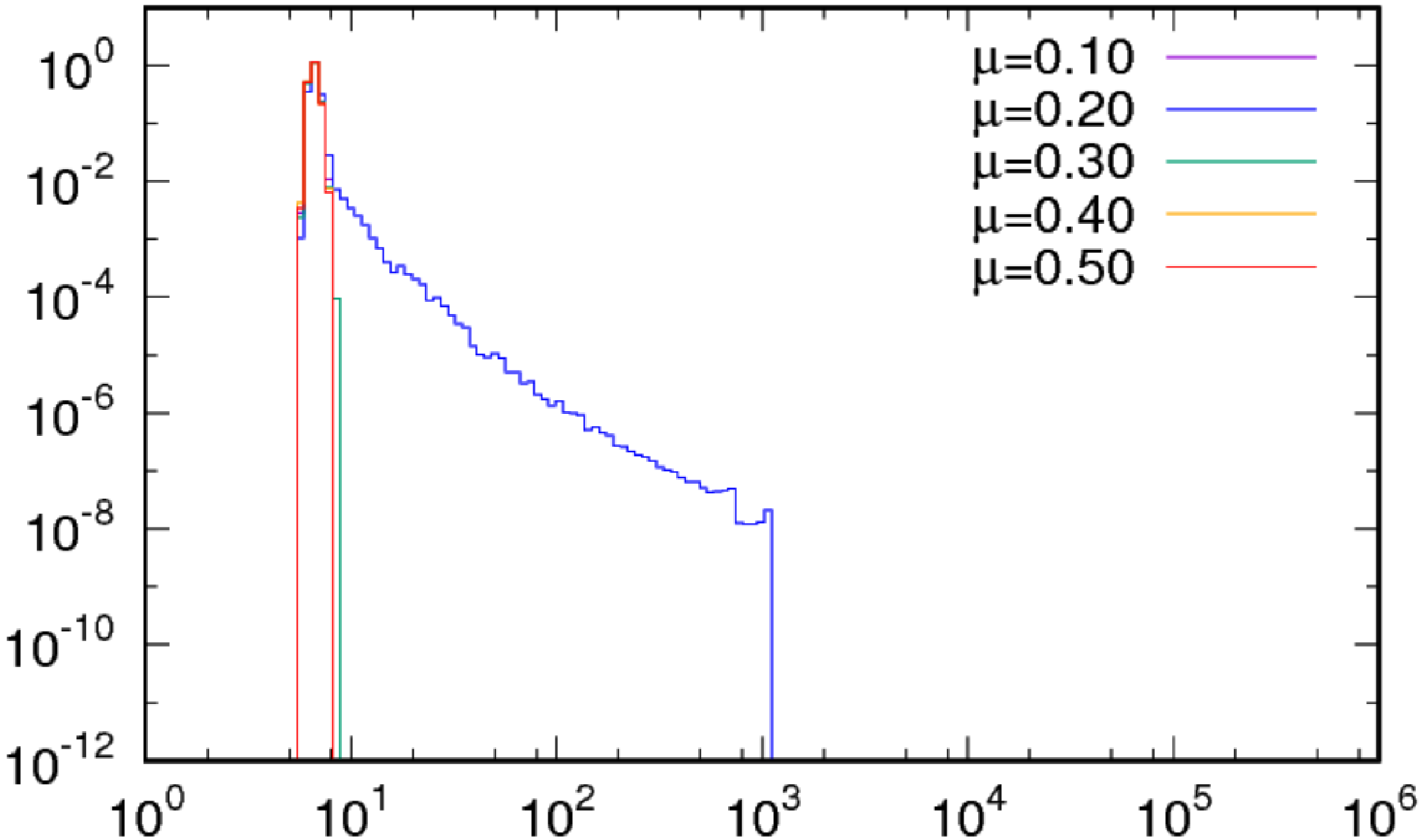
Summary and outlook

- ➢ Complex Langevin method is applied to 4-flavor QCD in finite density region.
- ➢ We have confirmed that the eigenvalue distribution of (D+m) has a gap at the origin when the singular drift problem does not occur.
- ➢ The origin of the plateau of the quark number can be regarded as a baryon state in a (small) box.

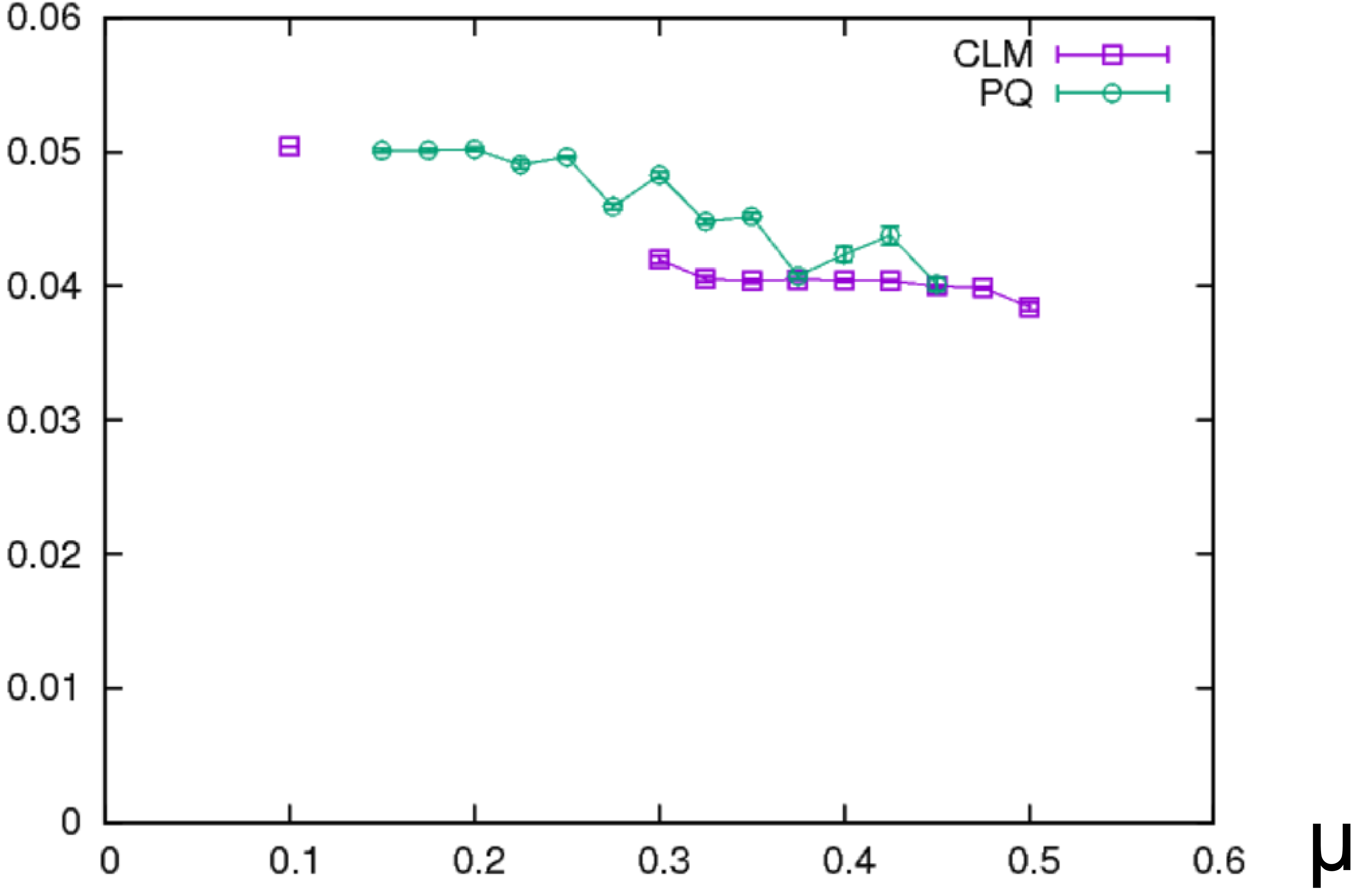
- ◆ We have performed further simulations on 16^4 lattice.
- ◆ We have found that the system is in the deconfined phase in the setup. We have also checked that there in no singular drift problem.
- ◆ There is a window ($0.1<\mu<0.5$) where the complex Langevin works

Appendix

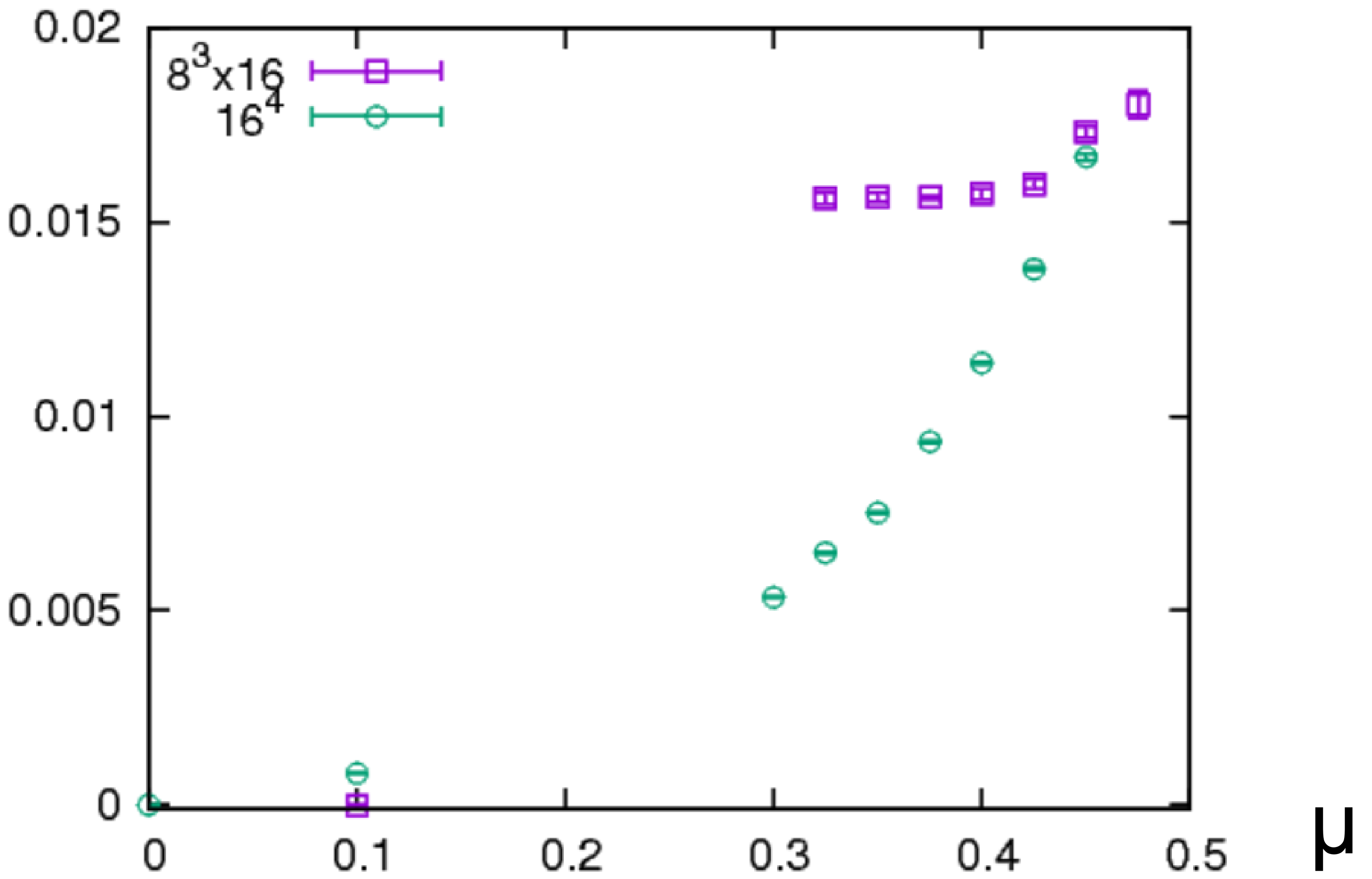
Histogram of the drift term (bosonic part)



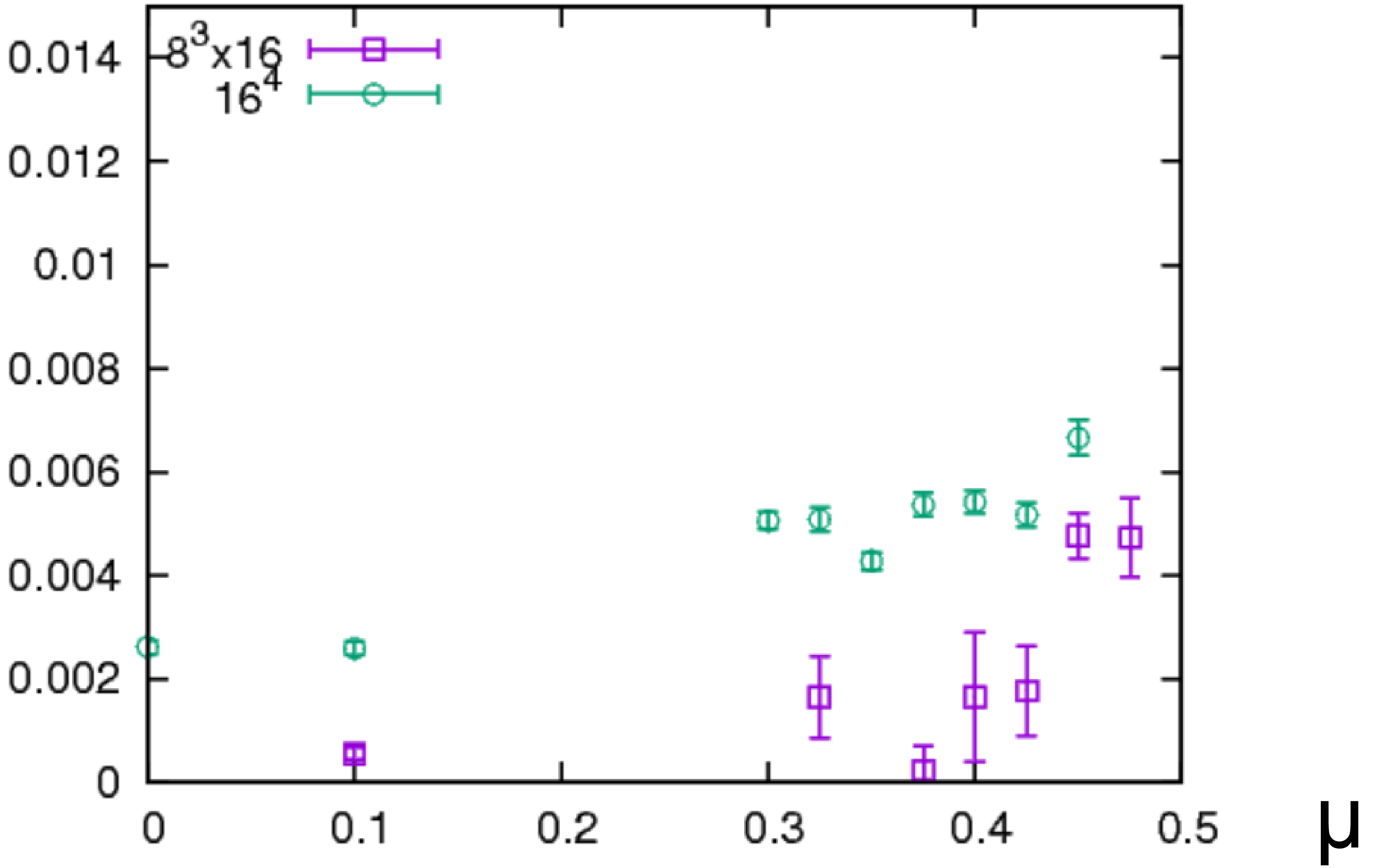
Chiral condensate

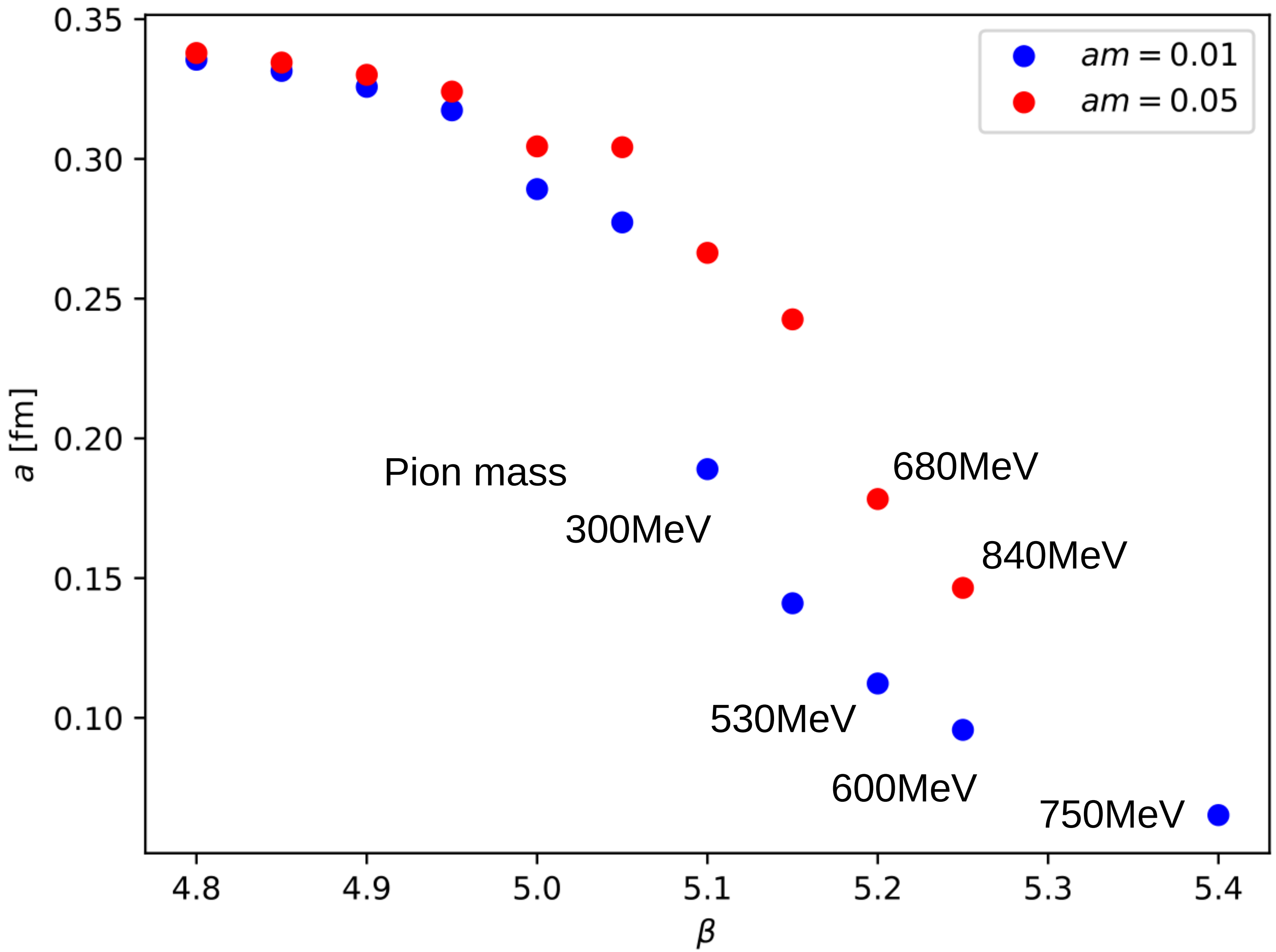


Baryon number density



Polyakov loop




0.35
0.30
0.25
0.20
0.15
0.10
a [fm]
4.8
4.9
5.0
5.1
5.2
5.3
5.4
β
am = 0.01
am = 0.05
Pion mass
300MeV
680MeV
840MeV
530MeV
600MeV
750MeV

Basic idea of complex Langevin method

$$Z = \int_{-\infty}^{\infty} dx e^{-S(x)}, \quad S(x) \in \mathbb{C}$$

[Parisi 83], [Klauder 84]
[Aarts, Seiler, Stamatescu 09]
[Aarts, James, Seiler, Stamatescu 11]
[Seiler, Sexty, Stamatescu 13]
[Sexty 14] [Fodor, Katz, Sexty, Torok 15]
[Nishimura, Shimasaki 15]
[Nagata, Nishimura, Shimasaki 15]

Complexification

$$x \in \mathbb{R} \to z \in \mathbb{C} \qquad S(x) \to S(z)$$

Complex Langevin equation

$$\frac{dz}{dt} = -\frac{\partial S(z)}{\partial z} + \eta(t)$$

$$\langle \eta(t) \rangle = 0$$

$$\langle \eta(t)\eta(t') \rangle = 2\delta(t - t')$$

$\langle \ldots \rangle$:noise average

We identify the noise effect as a *quantum fluctuation.*

Justification of complex Langevin method

Associated Fokker-Planck-like equation becomes,

$$\frac{\partial}{\partial t}\Phi(x,y,t) = \left[\frac{\partial}{\partial x_i}\left\{\mathrm{Re}\left(\frac{\partial S}{\partial z_i}\right) + N_R\frac{\partial}{\partial x_i}\right\} + \frac{\partial}{\partial y_i}\left\{\mathrm{Im}\left(\frac{\partial S}{\partial z_i}\right) + N_I\frac{\partial}{\partial y_i}\right\}\right]\Phi(x,y,t)$$

Under *certain conditions*,

$$\int dxdyO(x+iy)\Phi(x,y) = \int dxO(x)P(x)$$

$$\frac{\partial}{\partial t}P(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial S}{\partial x} + \frac{\partial}{\partial x}\right)P(x,t)$$

The stationary solution reads

$$P_{\mathrm{eq}}(x) \propto e^{-S(x)} \qquad \langle O(z(t))\rangle \to \frac{1}{Z}\int dxO(x)e^{-S(x)}, \quad t\to\infty$$

Criterion of correctness

A criterion for the correctness of the complex Langevin method

K. Nagata, J. Nishimura, S. Shimasaki [1508.02377, 1606.07627]

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i\left(-\epsilon\mathcal{D}_{x\mu}S[\mathcal{U}] + \sqrt{\epsilon}\eta_{x\mu}\right)\right]\mathcal{U}_{x\mu}(t)$$

Drift term $v_{x\mu}(\mathcal{U}) = -\mathcal{D}_{x\mu}S[\mathcal{U}]$

Probability distribution of the magnitude of the drift term plays a key role.

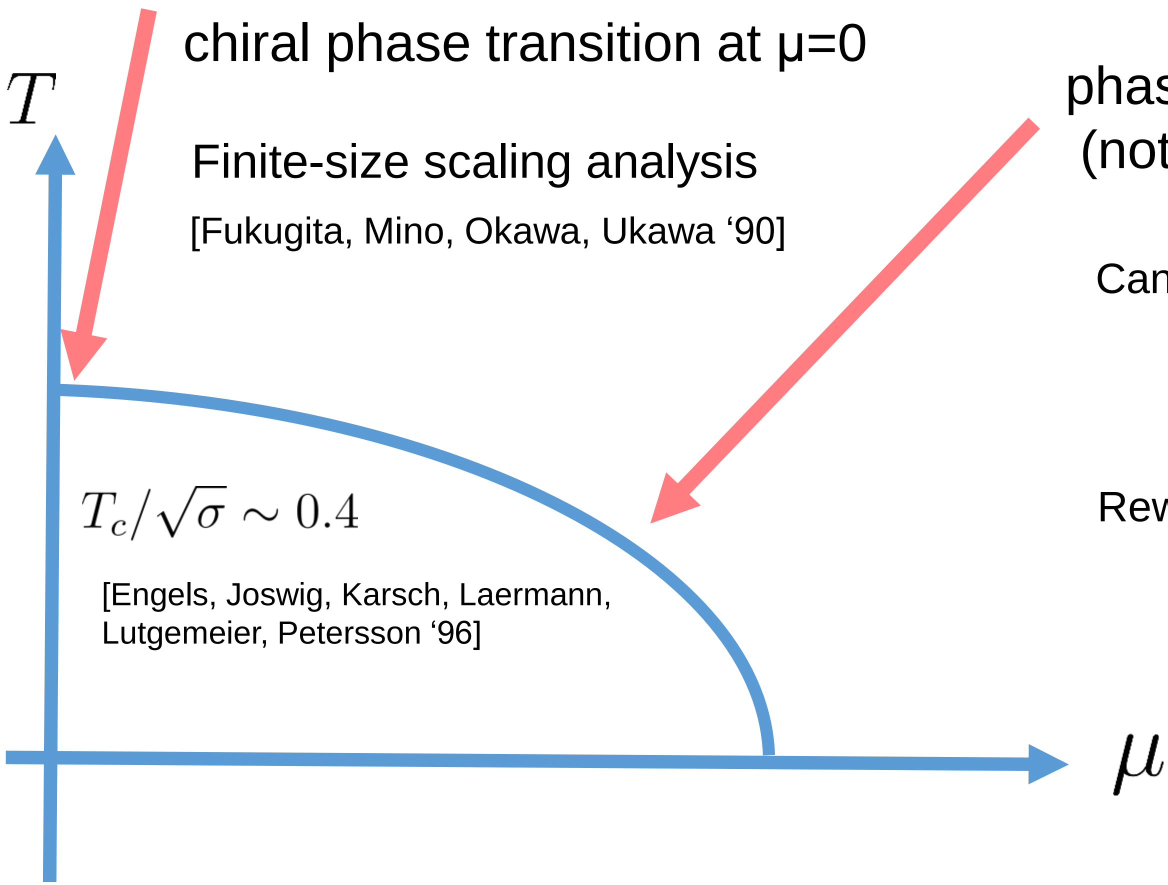
$$p(u) = \frac{1}{4N_V}\left\langle \sum_{x\mu}\delta(u-u_{x\mu})\right\rangle \qquad u_{x\mu} = \sqrt{\frac{1}{N_c^2-1}\,\mathrm{tr}(v_{x\nu}v_{x\nu}^\dagger)}$$

Phase diagram of QCD with 4-flavor staggered fermion

1st order
chiral phase transition at μ=0

Finite-size scaling analysis

[Fukugita, Mino, Okawa, Ukawa ‘90]



$T_c/\sqrt{\sigma} \sim 0.4$

[Engels, Joswig, Karsch, Laermann, Lutgemeier, Petersson ‘96]

phase transition at finite μ
(not completely established)

Canonical method

[de Forcrand, Kratochvila ‘06]
[Li, Alexandru, Liu, Meng ‘10]

Reweighting and complex Langevin

[Fodor, Katz, Sexty, Torok ‘15]