

Production spectra of ${}^3\text{He}(\pi, K)$ reactions with continuum discretized coupled channels

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Outline

1. Introduction - ${}^3_{\Lambda}\text{H}$ puzzle, $nn\Lambda$ bound state?
2. Calculations
 - Continuum $pp\Lambda$ states by the CDCC method
 - Microscopic $(pp)\text{-}\Lambda$ folding-model potentials
 - Fermi-averaged amplitudes for $\pi\text{N-}\Lambda\text{K}$ reactions
3. Results and Discussion
 - ${}^3\text{He}(\pi^+, \text{K}^+)pp\Lambda$ at 1.20GeV/c
 - ${}^3\text{He}(\pi^-, \text{K}^0){}^3_{\Lambda}\text{H}, pn\Lambda$ at 1.20GeV/c
4. Summary

1. Introduction

■ The ${}^3_{\Lambda}\text{H}$ lifetime puzzle

- Hypertriton (${}^3_{\Lambda}\text{H}$) with $B_{\Lambda}=130\pm 50$ keV is expected to have lifetime within a few % of the free Λ lifetime ($\tau_{\Lambda}=263.2\pm 2.0$ ps), which is supported by precise theoretical calculations.
- Recently, three heavy ion experiments found surprisingly short lifetime for ${}^3_{\Lambda}\text{H}$ by STAR, HypHI and ALICE.

$$\tau({}^3_{\Lambda}\text{H}) \text{ w.av} = 185^{+28}_{-23} \text{ ps} \quad (-30\% \text{ with reference to } \tau_{\Lambda})$$

→ To solve the puzzle, experimental measurements of ${}^3_{\Lambda}\text{H}$ lifetime are planned at J-PARC.



■ Is there a $nn\Lambda$ bound/resonant state ?

- The HypHI collaboration reported a bound $nn\Lambda$ system.
- Theoretical calculations suggest that no ${}^3_{\Lambda}\text{n}$ bound state exists.



In this talk,

We investigate theoretically the inclusive spectrum of the **${}^3\text{He}(\pi, \text{K})$ reaction at 1.20 GeV/c** in the distorted-wave impulse approximation (DWIA), using the Continuum-Discretized Coupled-Channel (CDCC) method.

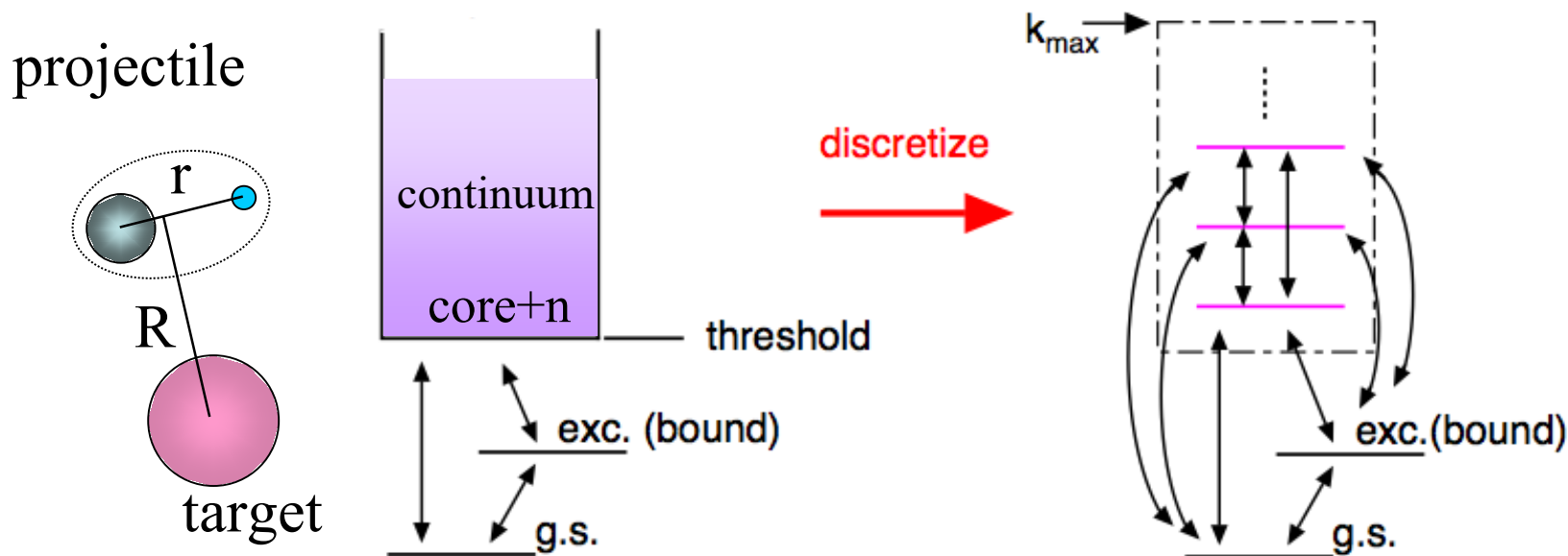
I will focus on

- i. the Λ production spectra with CDCC in order to well describe the pp continuum states above the $p+p+\Lambda$ breakup threshold. **${}^3\text{He}(\pi^+, \text{K}^+)pp\Lambda$**
- ii. the production cross section of the ${}^3_{\Lambda}\text{H}_{\text{g.s.}} (1/2^+)$ bound state and continuum states, considering the framework of the CDCC. **${}^3\text{He}(\pi^-, \text{K}^0)pn\Lambda$**

2. Calculations

Continuum-Discretized Coupled-Channel Method (CDCC)

M. Kamimura et al., Prog. Theor. Phys. Suppl. 89, 1 (1986)

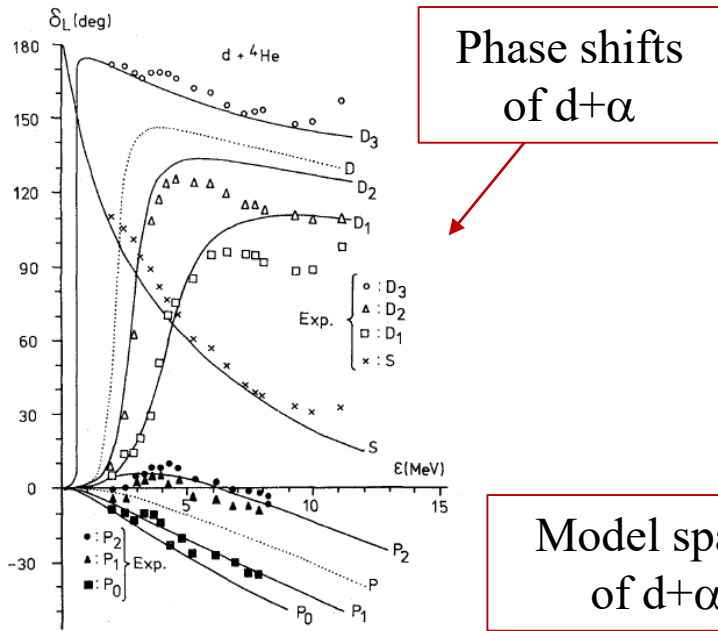


by Takashina

- We can describe the spectra and cross sections taking into account the continuum couplings together with the nuclear 3-body breakup processes.

CDCC calculations for ${}^6\text{Li} + {}^{58}\text{Ni}$ elastic scattering

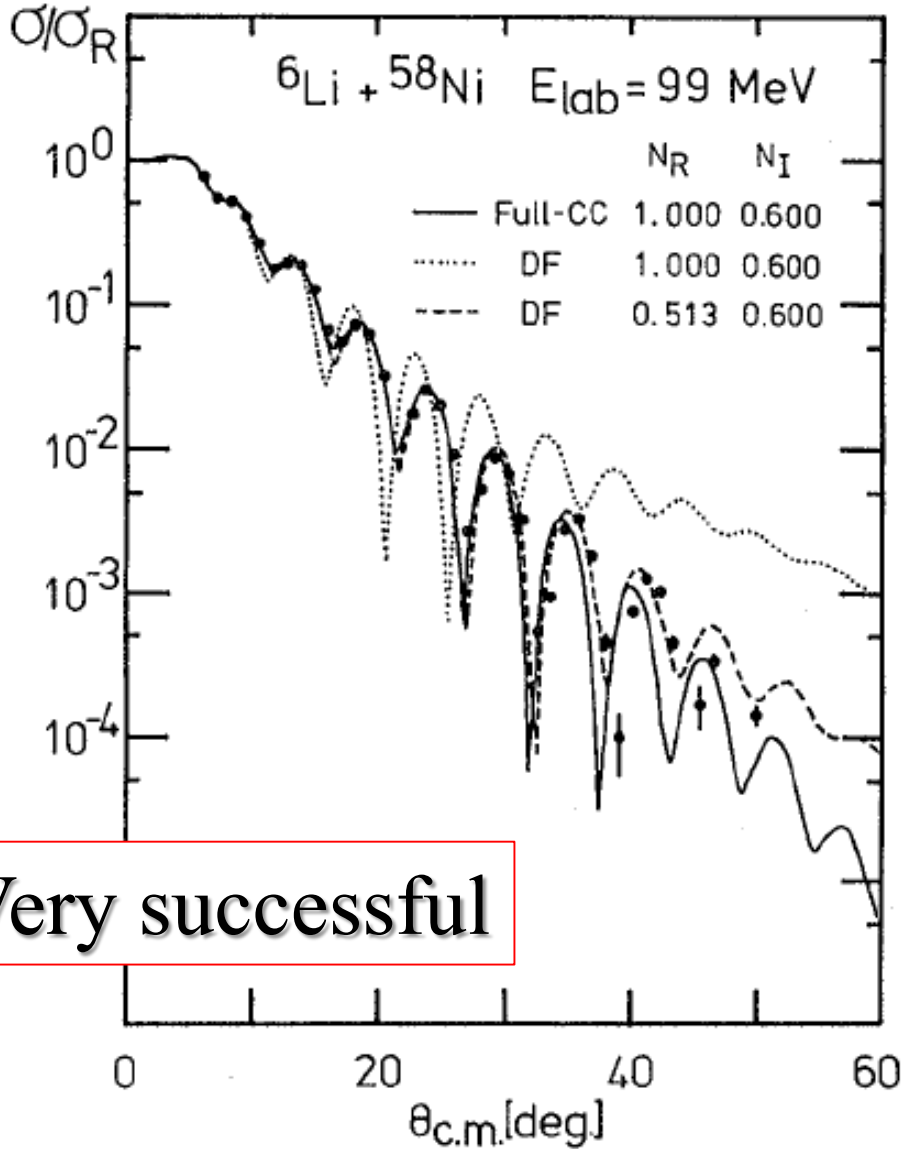
Y. Sakuragi et al., PTP. Suppl. 89, 136 (1986)



Phase shifts
of $d+\alpha$

Model space
of $d+\alpha$

k (fm^{-1})	($l=0$) S_{bu}	($l=1$) P_{bu}	($l=2$) $D_{bu}^{(off)}$	k (fm^{-1})
1.500	(MeV) 26.99	(MeV) 26.99	(MeV) 27.80	1.500
1.125	13.86	13.86	15.69	1.166
0.750	5.11	5.11	7.06	0.833
0.375	0.75	0.75	2.23	0.500
0.000	-1.47 g.s.		$D_{bu}^{(res)}$	$\alpha + d$

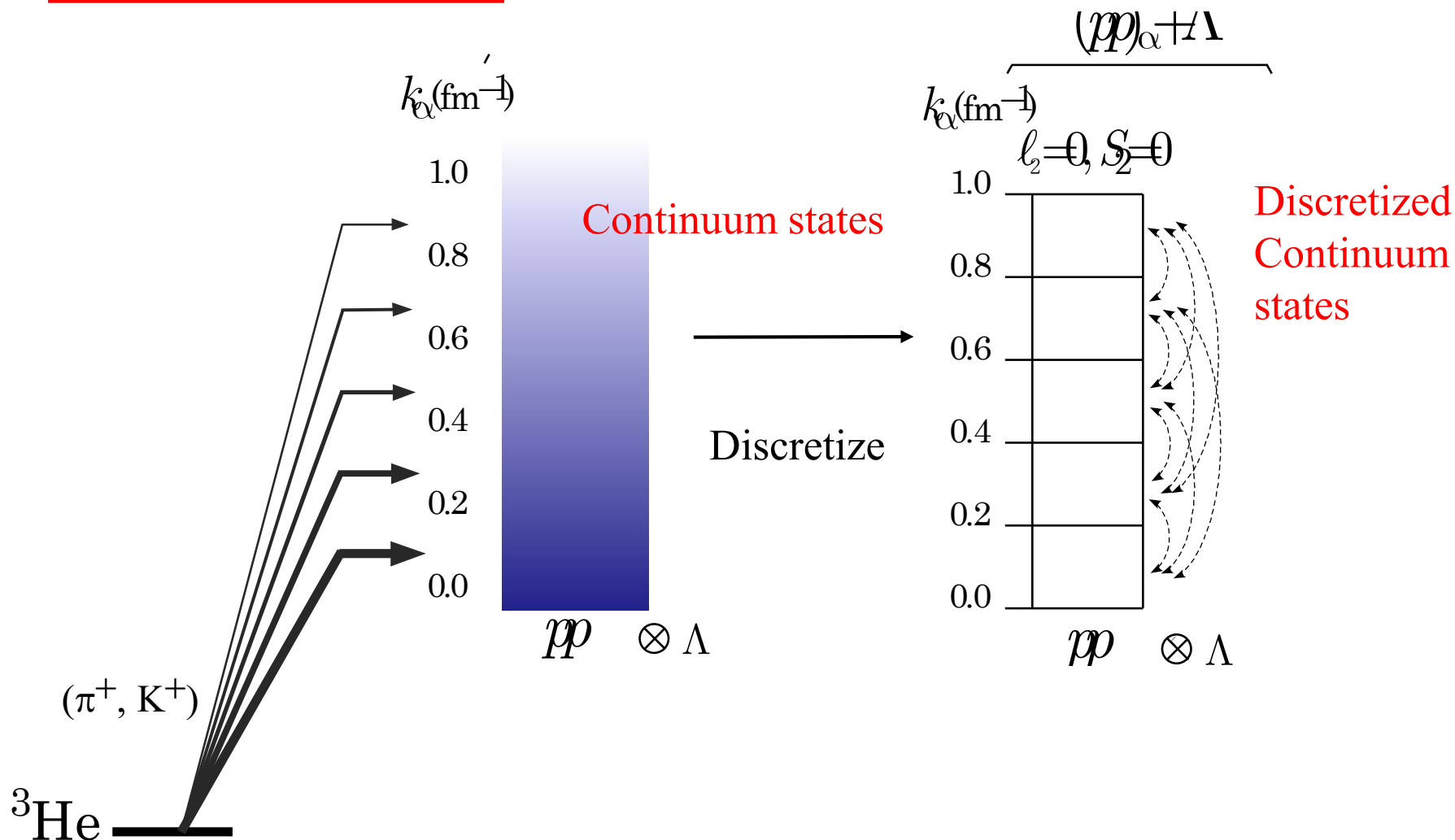


Very successful

${}^6\text{Li}$

Continuum-Discretized Coupled-Channel Method (CDCC)

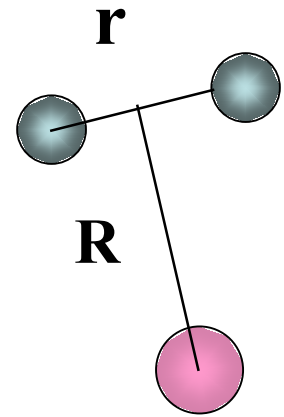
$${}^3\text{He} (\pi^+, \text{K}^+) pp\Lambda$$



■ Wavefunction of the initial state for a 3He target nucleus

$$|\Psi_A\rangle = \hat{\mathcal{A}} \left[\left[\phi_0^{(2N)} \otimes \varphi_0^{(N)} \right]_{L_A} \otimes X_{T_A, S_A}^A \right]_{J_A}^{M_A},$$

$$X_{T_A, S_A}^A = \left[\chi_{I_2, S_2}^{(2N)} \otimes \chi_{1/2, 1/2}^{(N)} \right]_{1/2, 1/2},$$



■ Wavefunctions of final states for pp Λ

$$\Psi_B \simeq \Psi_B^{\text{CDCC}}(\mathbf{r}, \mathbf{R})$$

$$= \sum_{\alpha=1}^{N_{\max}} \left[\left[\tilde{\phi}_{\alpha, l_2}^{(2N)}(\mathbf{r}) \otimes \varphi_{\alpha, l_\Lambda}^{(\Lambda)}(\mathbf{R}) \right]_{L_B} \otimes X_{I_\alpha, S_\alpha}^B \right]_{J_B}^{M_B}$$

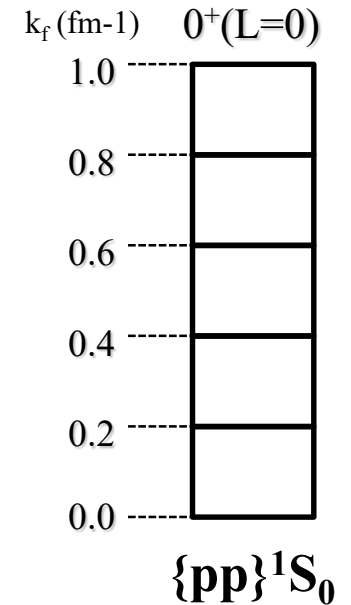
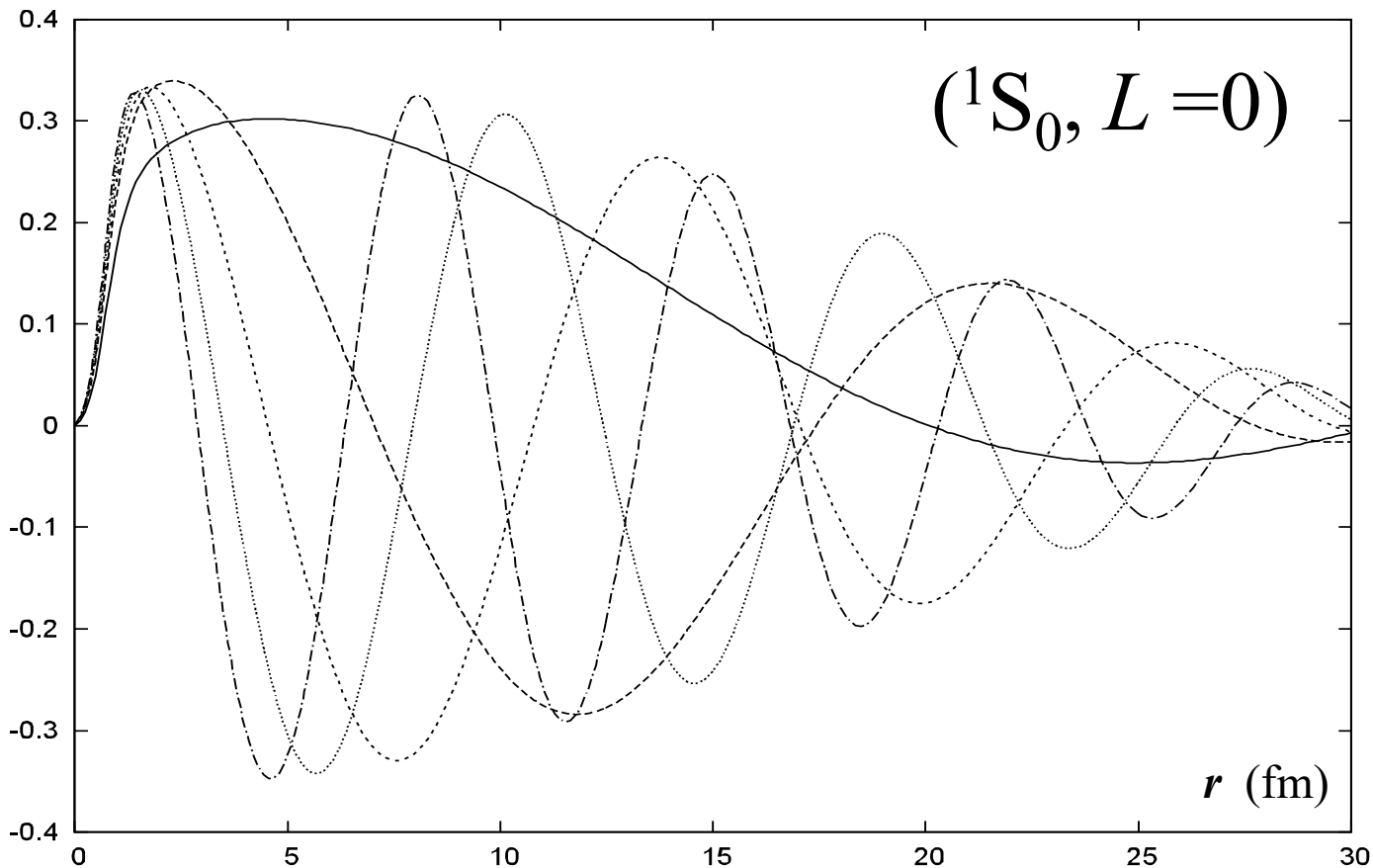
discretized w.f. for continuum pp states

$$X_{I_\alpha, S_\alpha}^B = \left[\chi_{I_2, S_2}^{(2N)} \otimes \chi_{0, 1/2}^{(\Lambda)} \right]_{I_\alpha, S_\alpha},$$

■ Method of momentum bins (discretized for the pp-systems)

$$\tilde{\phi}_{\alpha, l_2}^{(2N)}(\mathbf{r}) = \frac{1}{\sqrt{\Delta k}} \int_{k_\alpha}^{k_{\alpha+1}} \phi_{l_2}^{(2N)}(k, \mathbf{r}) dk$$

$$\int d\mathbf{r} \tilde{\phi}_{\alpha, l_2}^{(2N)}(\mathbf{r}) \tilde{\phi}_{\alpha', l_2}^{(2N)}(\mathbf{r}) = \delta_{\alpha\alpha'}$$



■ Coupled equations for the (pp)- Λ systems

$$\left[-\frac{1}{2\mu_\alpha} \nabla^2 + U_{\alpha\alpha}(\mathbf{R}) - (E - \bar{\varepsilon}_\alpha) \right] \varphi_{\alpha, l_\Lambda}^{(\Lambda)}(\mathbf{R})$$

$$= - \sum_{\alpha' \neq \alpha} U_{\alpha\alpha'}(\mathbf{R}) \varphi_{\alpha', l_\Lambda}^{(\Lambda)}(\mathbf{R}), \quad \bar{\varepsilon}_\alpha = \frac{\bar{k}_\alpha^2}{2\mu_\alpha} - \frac{1}{2}i\Gamma_\alpha$$

■ Microscopic (pp)- Λ folding-model potentials

$$U_{\alpha\alpha'}(\mathbf{R}) = \int \rho_{\alpha\alpha'}(\mathbf{r}) (\bar{v}_{\Lambda N}(\mathbf{r}_1) + \bar{v}_{\Lambda N}(\mathbf{r}_2)) d\mathbf{r}$$

Nucleon or transition density $\rho_{\alpha\alpha'}(\mathbf{r}) = \langle \tilde{\phi}_{\alpha, l_2}^{(2N)} | \sum_i \delta(\mathbf{r} - \mathbf{r}_i) | \tilde{\phi}_{\alpha', l_2}^{(2N)} \rangle$

Spin-averaged ΛN potentials $\bar{v}_{\Lambda N} = \frac{1}{4}v_{\Lambda N}^s + \frac{3}{4}v_{\Lambda N}^t$

Simple Gaussian form

$$v_{\Lambda N}^{s,t}(r) = v_{s,t}^{(0)} \exp(-(r/b_{s,t})^2) \leftarrow$$

NSC97f	$a_{s,t}$	$r_{s,t}^{\text{eff}}$
Λn singlet	-2.51 fm	3.01 fm
Λn triplet	-1.75 fm	3.30 fm

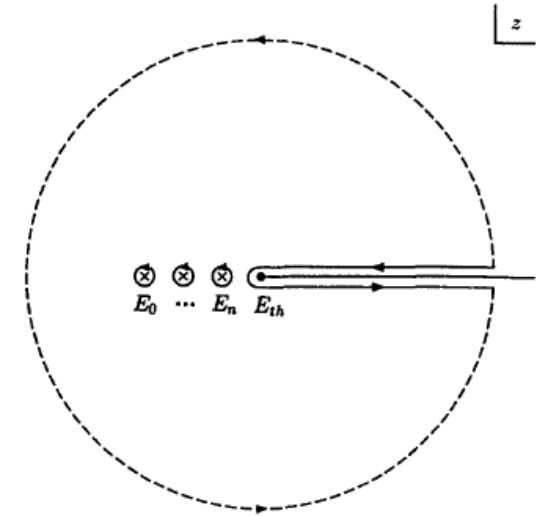
Coupled-channels DWIA calculation for Λ production

Coupled-channel Green's function

T.Harada, NPA672(2000)181

$$\hat{\mathbf{G}}(E_f) = \hat{\mathbf{G}}^{(0)}(E_f) + \hat{\mathbf{G}}^{(0)}(E_f) \hat{\mathbf{U}} \hat{\mathbf{G}}(E_f)$$

$$\hat{\mathbf{G}}^{(0)}(E_f) = \begin{bmatrix} G_{\Lambda_0}^{(0)} & & & \\ & G_{\Lambda_1}^{(0)} & & \\ & & \ddots & \\ & & & G_{\Lambda_N}^{(0)} \end{bmatrix} \quad \hat{\mathbf{U}} = \begin{bmatrix} U_{0,0} & U_{0,1} & \cdots & U_{0,N} \\ U_{1,0} & U_{1,1} & & \vdots \\ \vdots & & \ddots & \\ U_{N,0} & \cdots & & U_{N,N} \end{bmatrix}$$



Green's function method

$$\sum_B |\Psi_B\rangle \langle \Psi_B| \delta(E - E_B) = -\frac{1}{\pi} \text{Im} \hat{\mathbf{G}}(E)$$

Morimatsu, Yazaki, NPA483(1988)493

Strength function

$$\begin{aligned} S(E_B) &= \sum_B |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(E_\pi + E_B - E_K - E_A) \\ &= (-) \frac{1}{\pi} \text{Im} \sum_{\alpha\alpha'} \int d\mathbf{R} d\mathbf{R}' F_\alpha^\dagger(\mathbf{R}) G_{\alpha\alpha'}(E_B; \mathbf{R}, \mathbf{R}') F_{\alpha'}(\mathbf{R}') \end{aligned}$$

Green's function

■ Double differential cross sections within the DWIA

$$\frac{d^2\sigma}{dE_K d\Omega_K} = \beta \frac{1}{[J_A]} \sum_{M_A} \left[\sum_B |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(E_K + E_B - E_\pi - E_A) \right] = S(E_B)$$

■ Production operators with zero-range interaction

$$\hat{F} = \int d\mathbf{r} \chi_\pi^{(-)*}(\mathbf{p}_K, \mathbf{r}) \chi_K^{(+)}(\mathbf{p}_\pi, \mathbf{r}) \sum_{j=1}^A \bar{f}_{\pi N \rightarrow \Lambda K} \delta(\mathbf{r} - \mathbf{r}_j) \hat{O}_j$$

$\approx j_L^{(+)}(q \frac{M_C}{M_A} R)$ distorted waves
Recoil factor

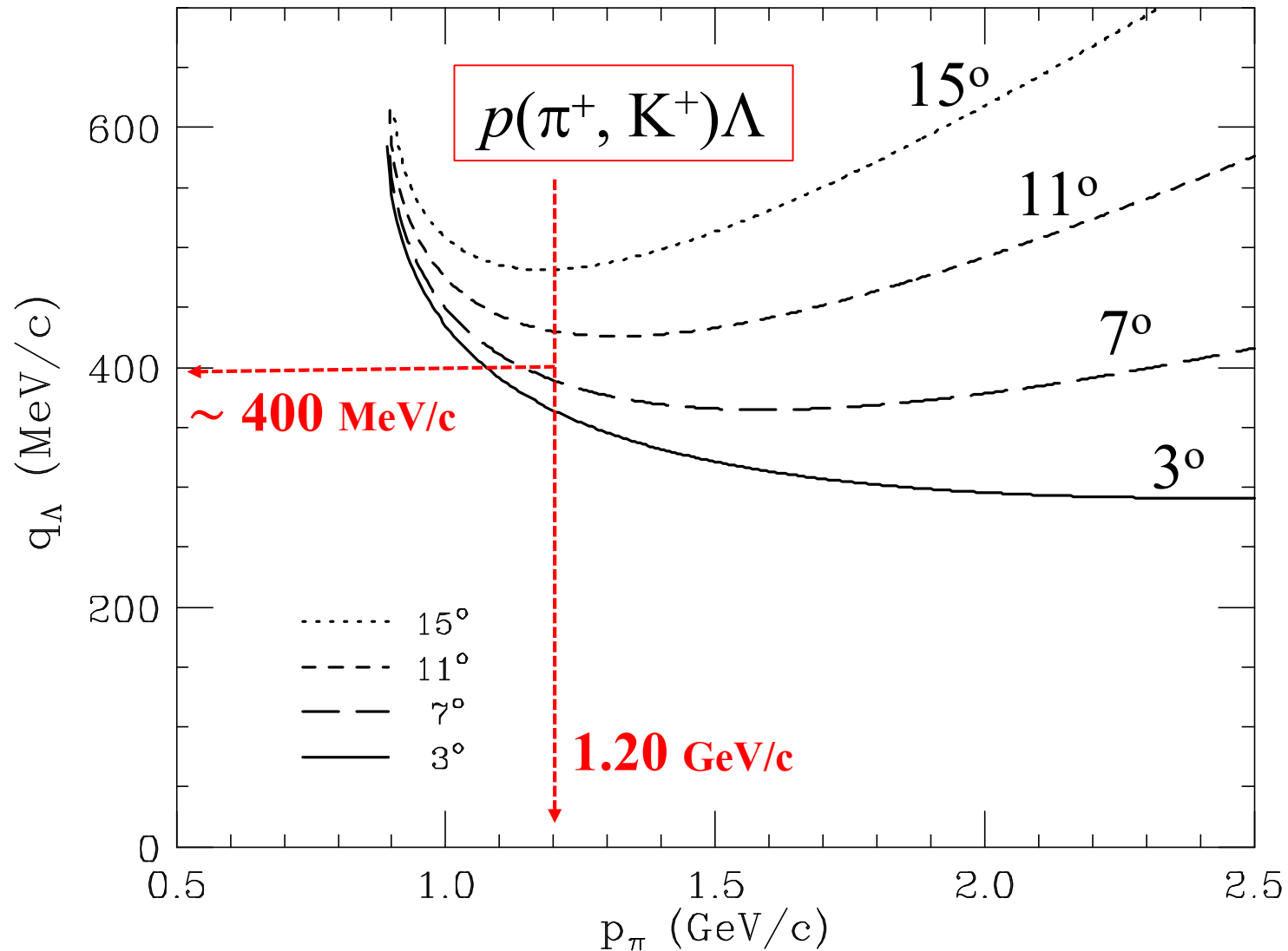
■ Momentum and energy transfer

$$\mathbf{q} = \mathbf{p}_\pi - \mathbf{p}_K, \quad \omega = E_\pi - E_K$$

■ *Optimal* Fermi-averaged amplitude for πN - ΛK reactions

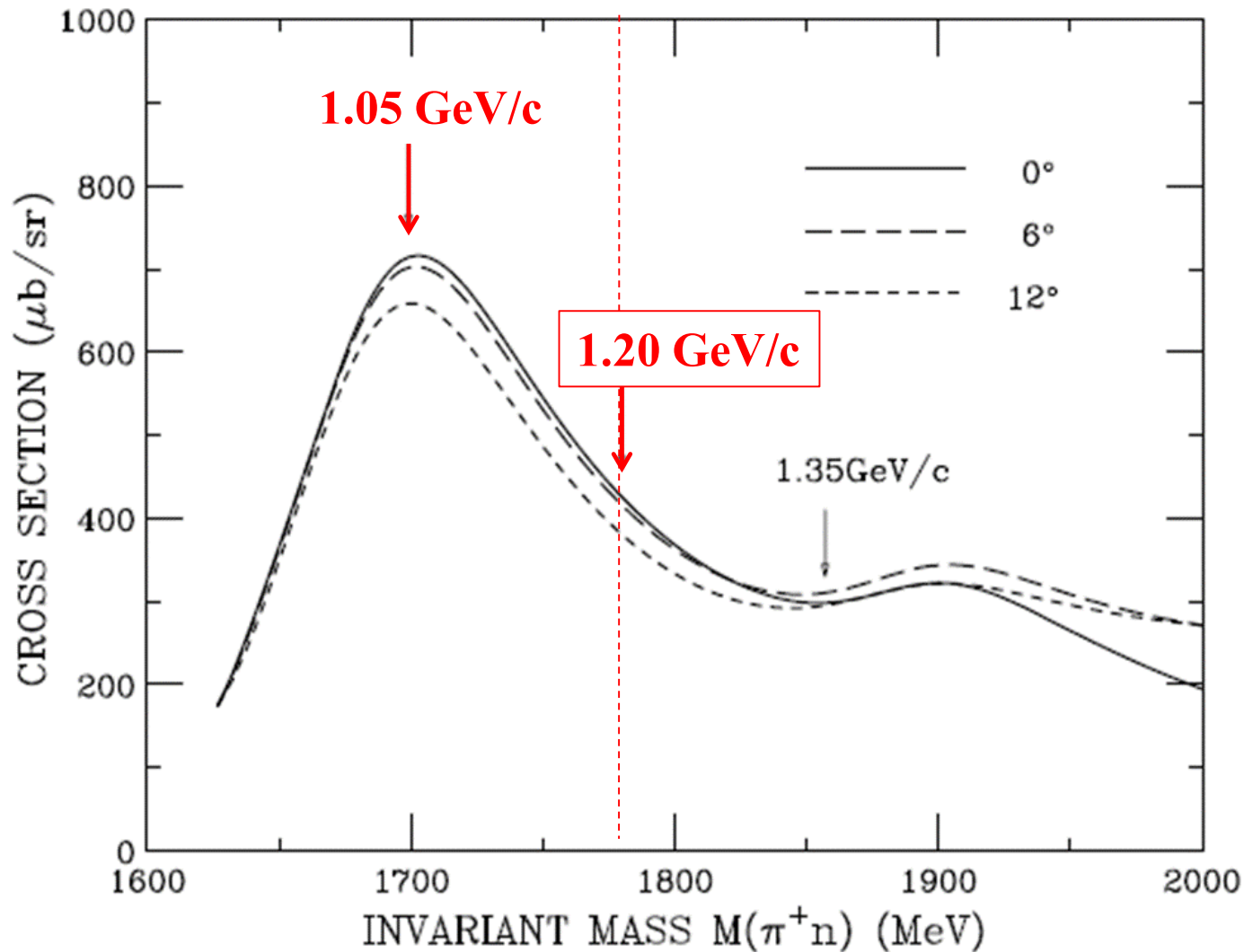
$$\bar{f}_{\pi N \rightarrow \Lambda K} = \frac{1}{2\pi} \sqrt{\frac{p_K E_K}{v_\pi \beta}} \langle t_{\pi N, K\Sigma}^{\text{opt}}(p_\pi; \omega, \mathbf{q}) \rangle$$

Momentum transfer q_Λ for Λ production in (π^+, K^+) reactions



→ The momentum transfer q_Λ becomes very large.

Elementary cross section of the $\pi^+p \rightarrow K^+\Lambda$ reaction

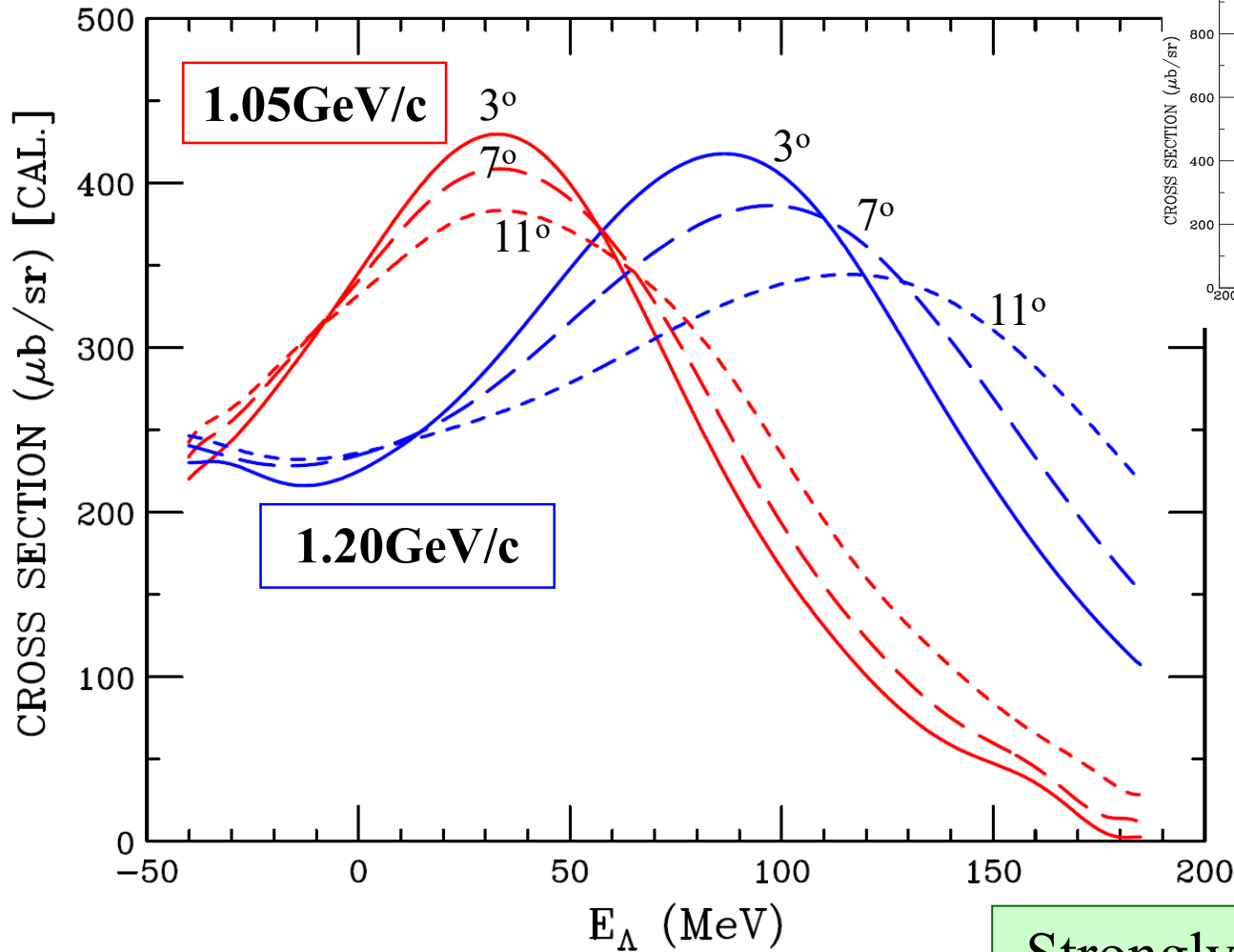


→ The cross section σ_Λ has a strong incident momentum-dep.

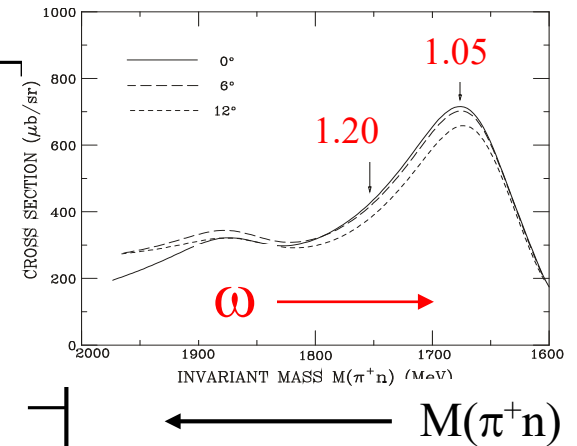
Fermi-averaged X-section of the $\pi^+ + n \rightarrow K^+ + \Lambda$ reaction on ^3He

T. Harada and Y. Hirabayashi, NPA744 (2004) 323.

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{opt}} \equiv \beta |\bar{f}_{\pi N \rightarrow K\Lambda}|^2$$



$\pi^+ + n \rightarrow K^+ + \Lambda$ Cross Section



Strongly E_Λ -dependent

3. Results and Discussion

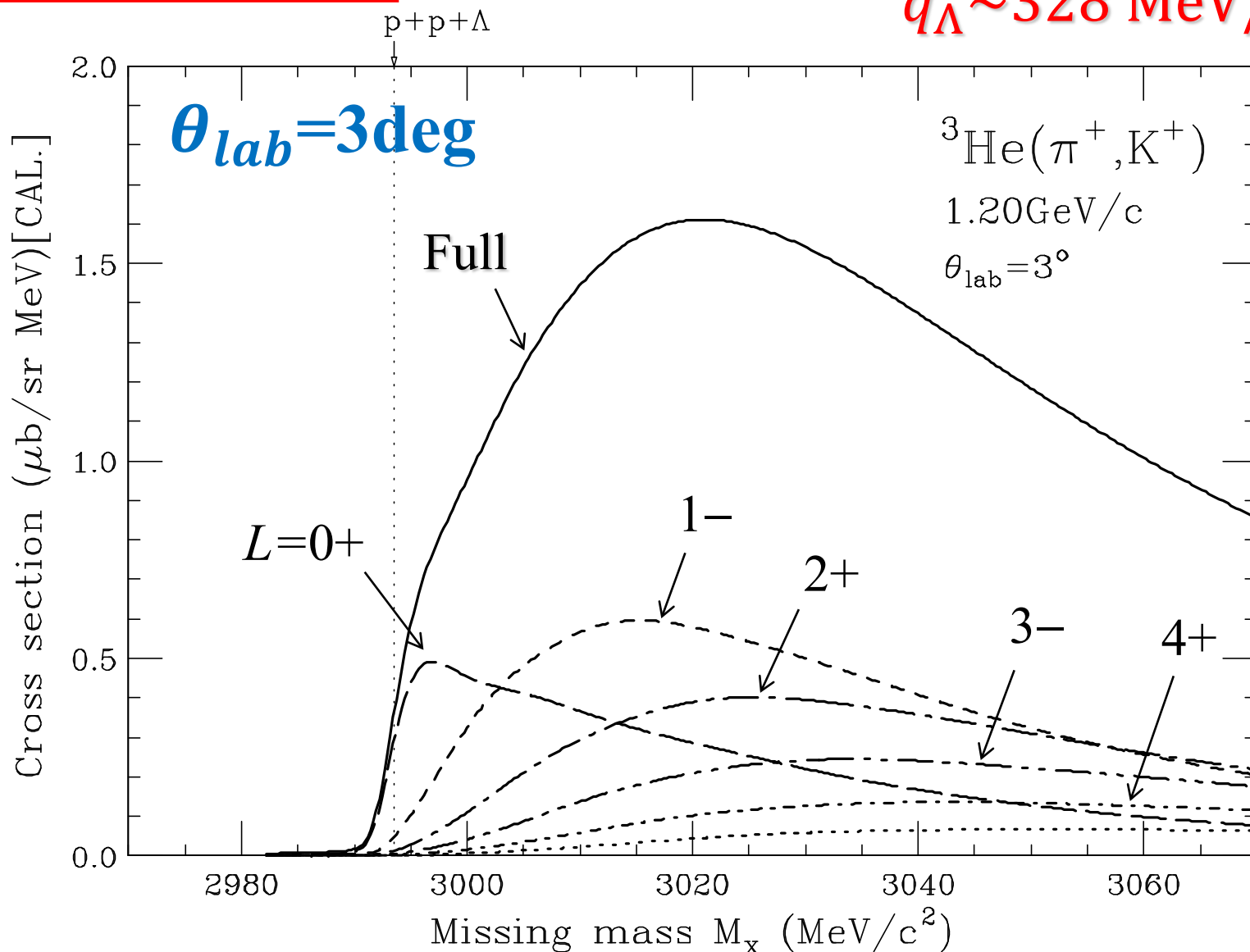
${}^3\text{He} (\pi^+, \text{K}^+) pp\Lambda$

at 1.20 GeV/c

Inclusive spectrum in ${}^3\text{He}(\pi^+, \text{K}^+)pp\Lambda$ at $1.20\text{GeV}/c$

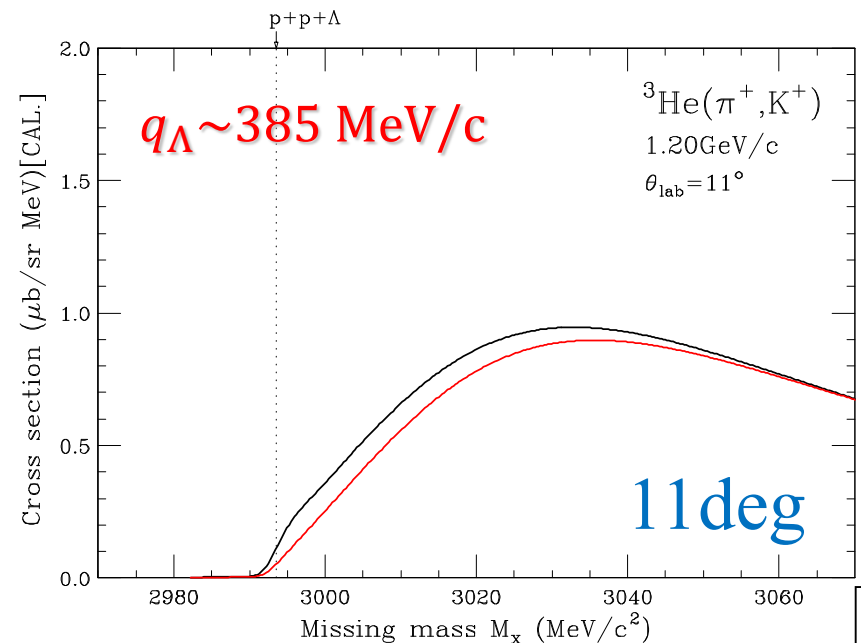
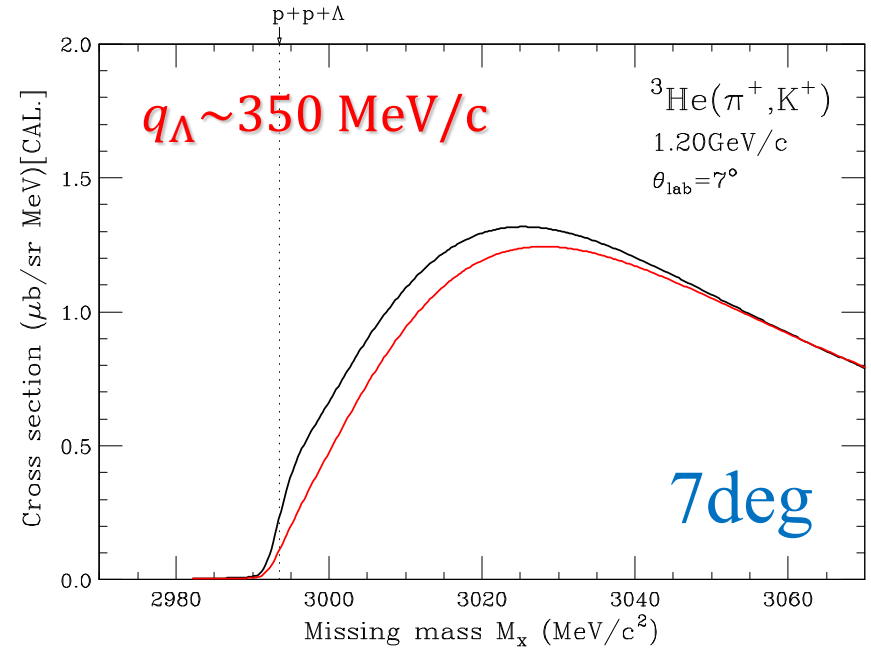
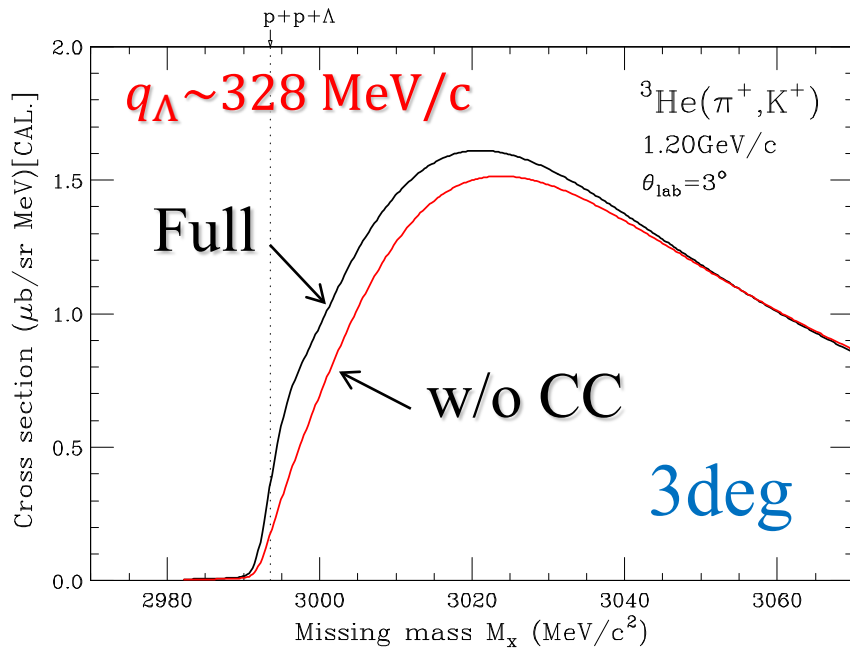
${}^3\text{He}(\pi^+, \text{K}^+)pp\Lambda$

$q_\Lambda \sim 328 \text{ MeV}/c$



Effects of continuum couplings by the CDCC

${}^3\text{He} (\pi^+, K^+) pp\Lambda$



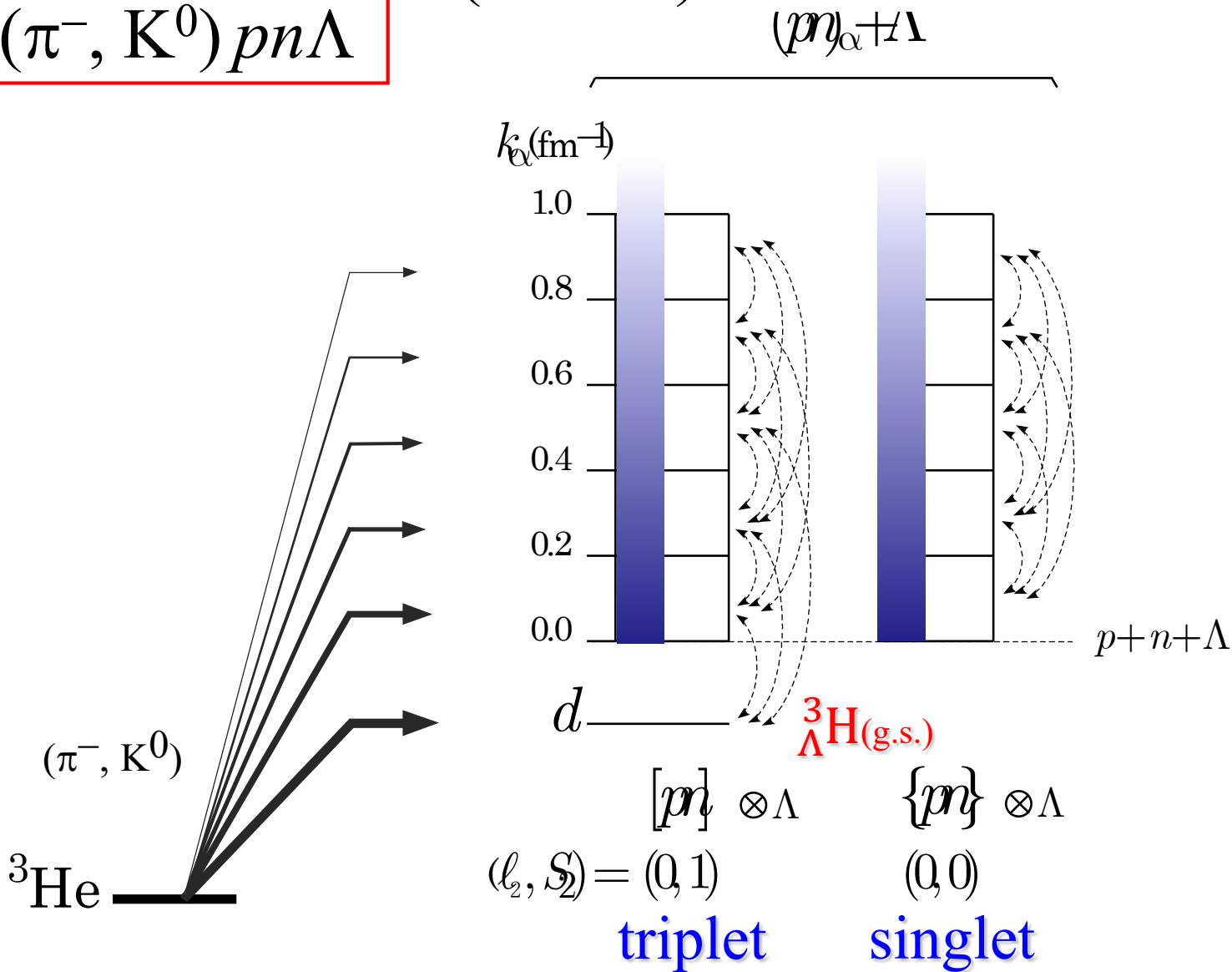
$${}^3\text{He} (\pi^-, \text{K}^0) {}^3_{\Lambda}\text{H}, pn\Lambda$$

at 1.20 GeV/c

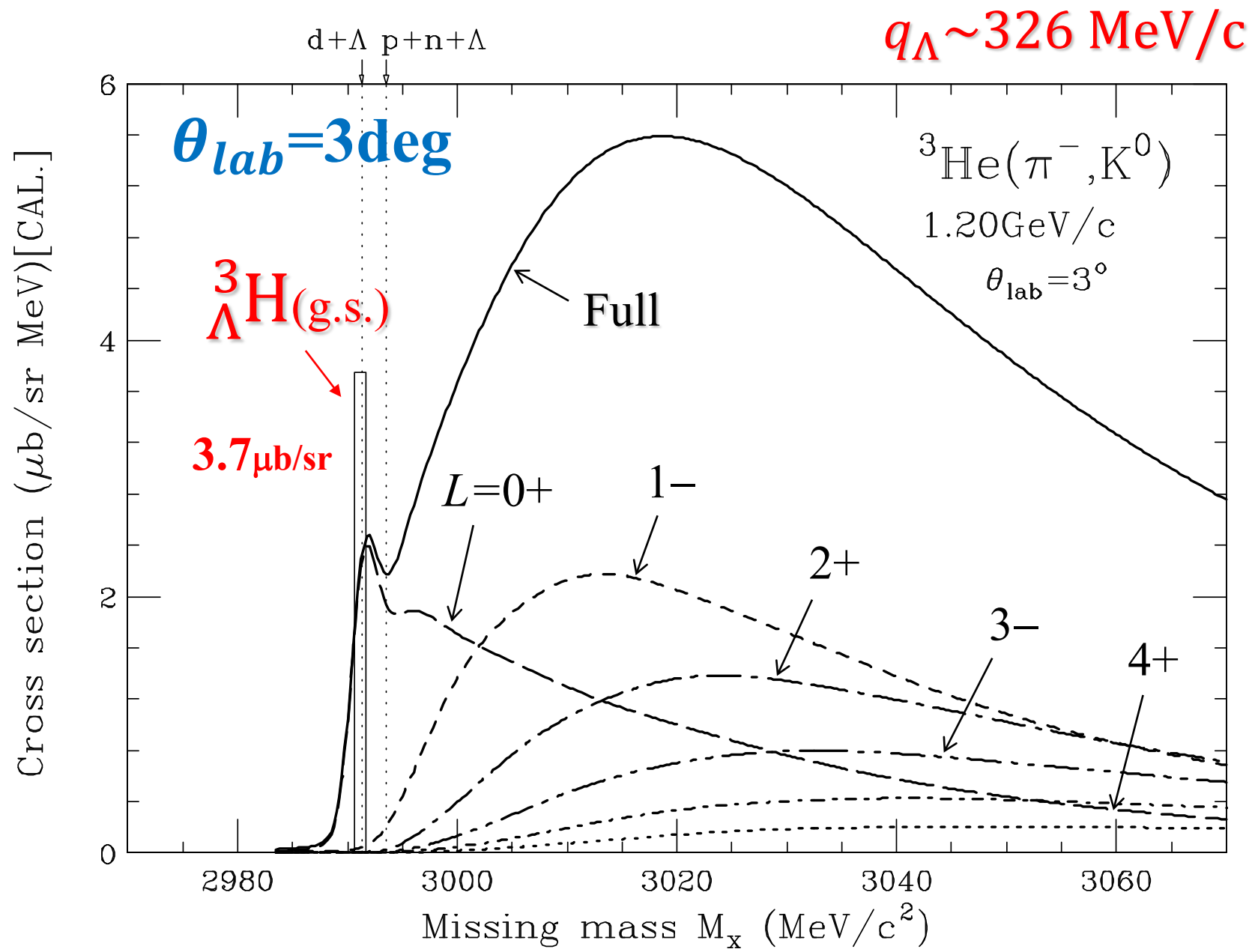
Continuum-Discretized Coupled-Channel Method

(CDCC)

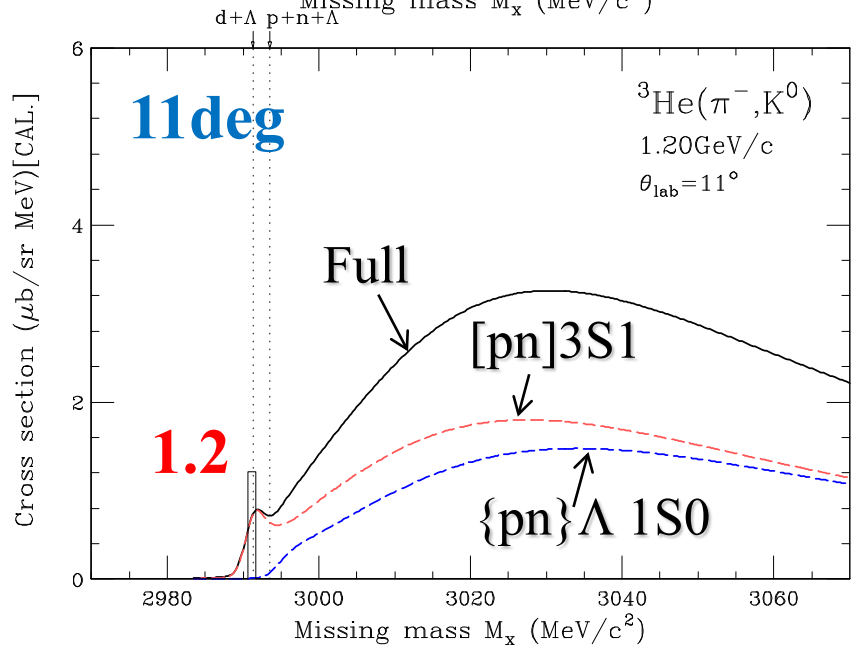
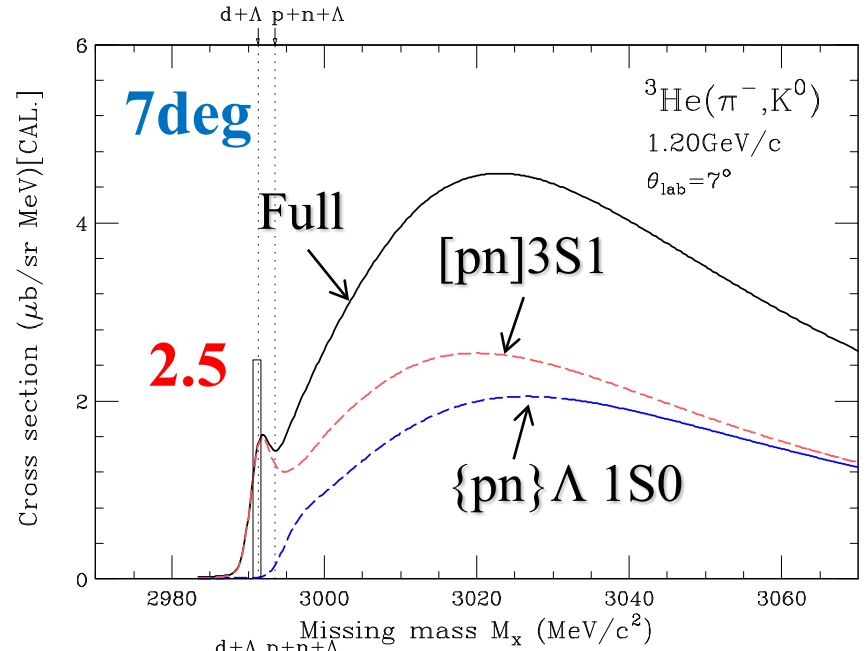
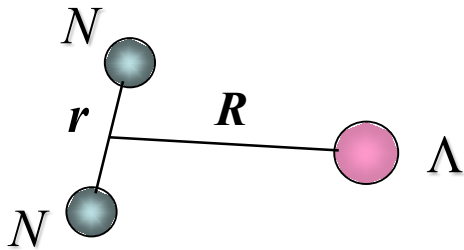
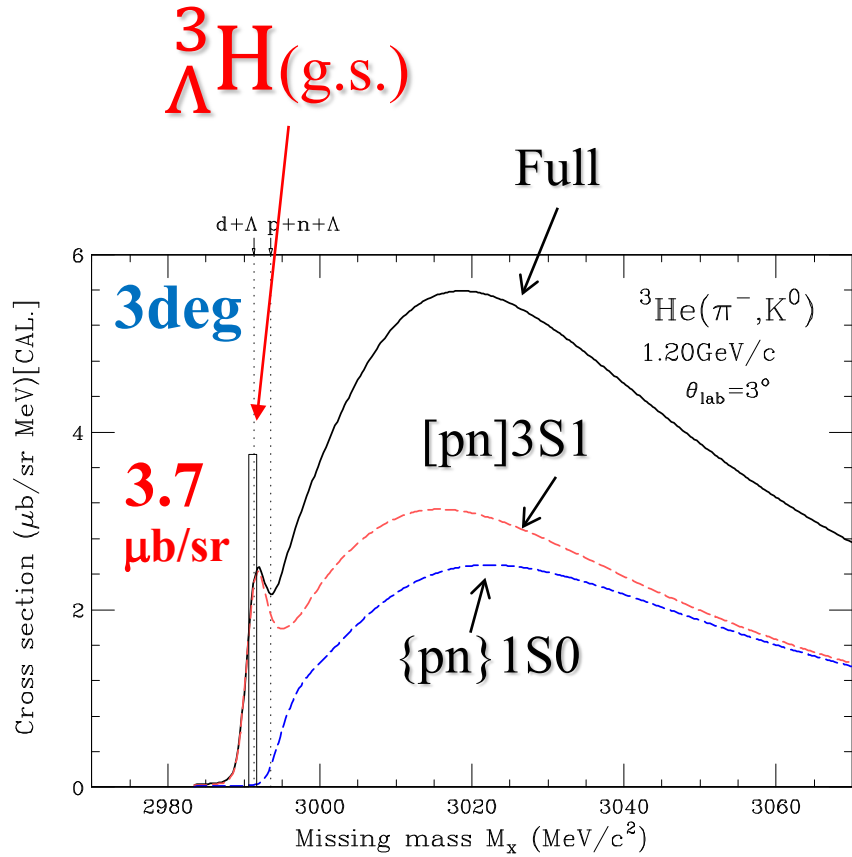
$${}^3\text{He} (\pi^-, K^0) pn\Lambda$$



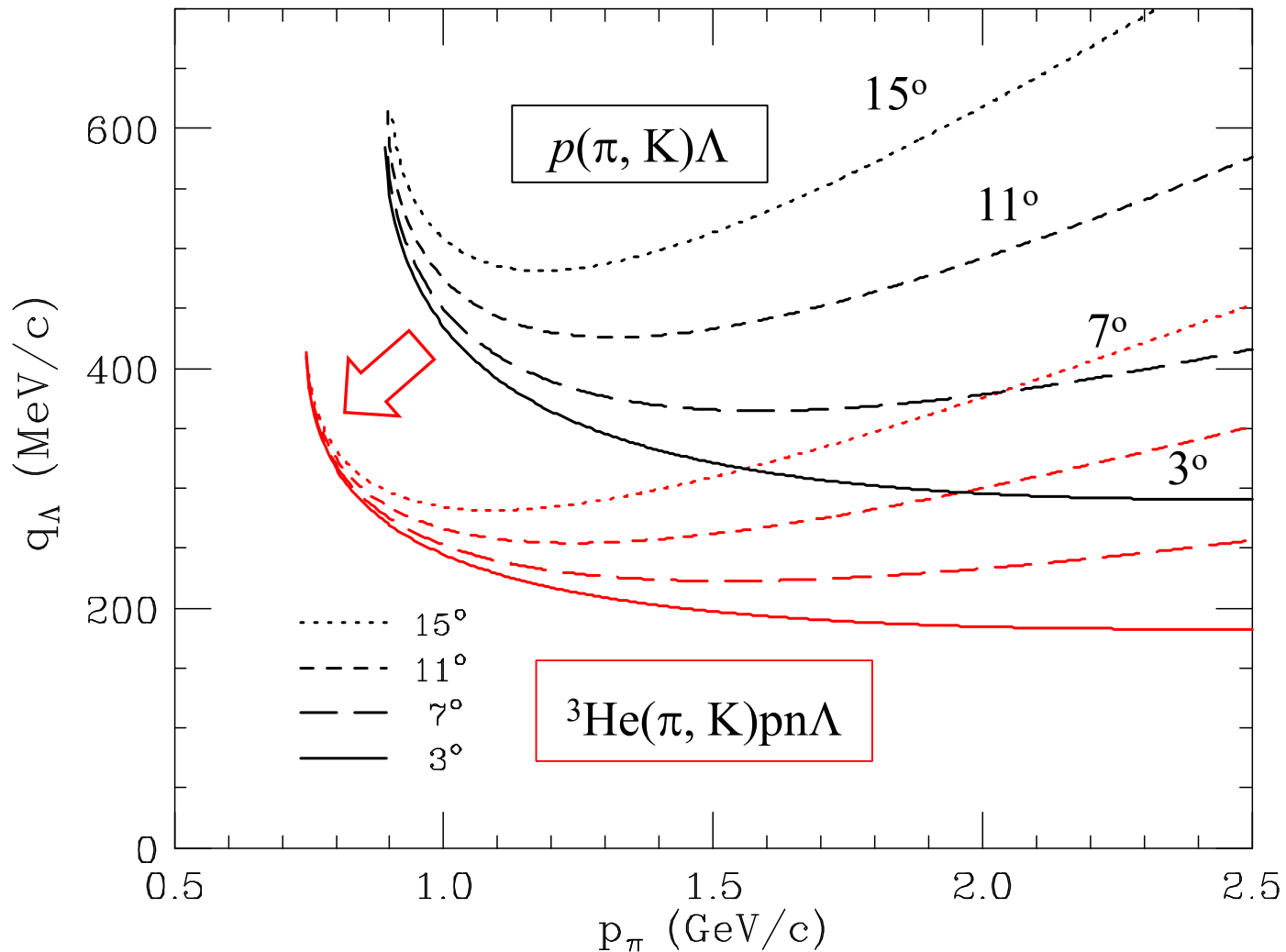
Inclusive spectrum in ${}^3\text{He}(\pi^-, \text{K}^0)p n \Lambda$ at $1.20\text{GeV}/c$



Components of the NN -core in $pn\Lambda$ for the spectra



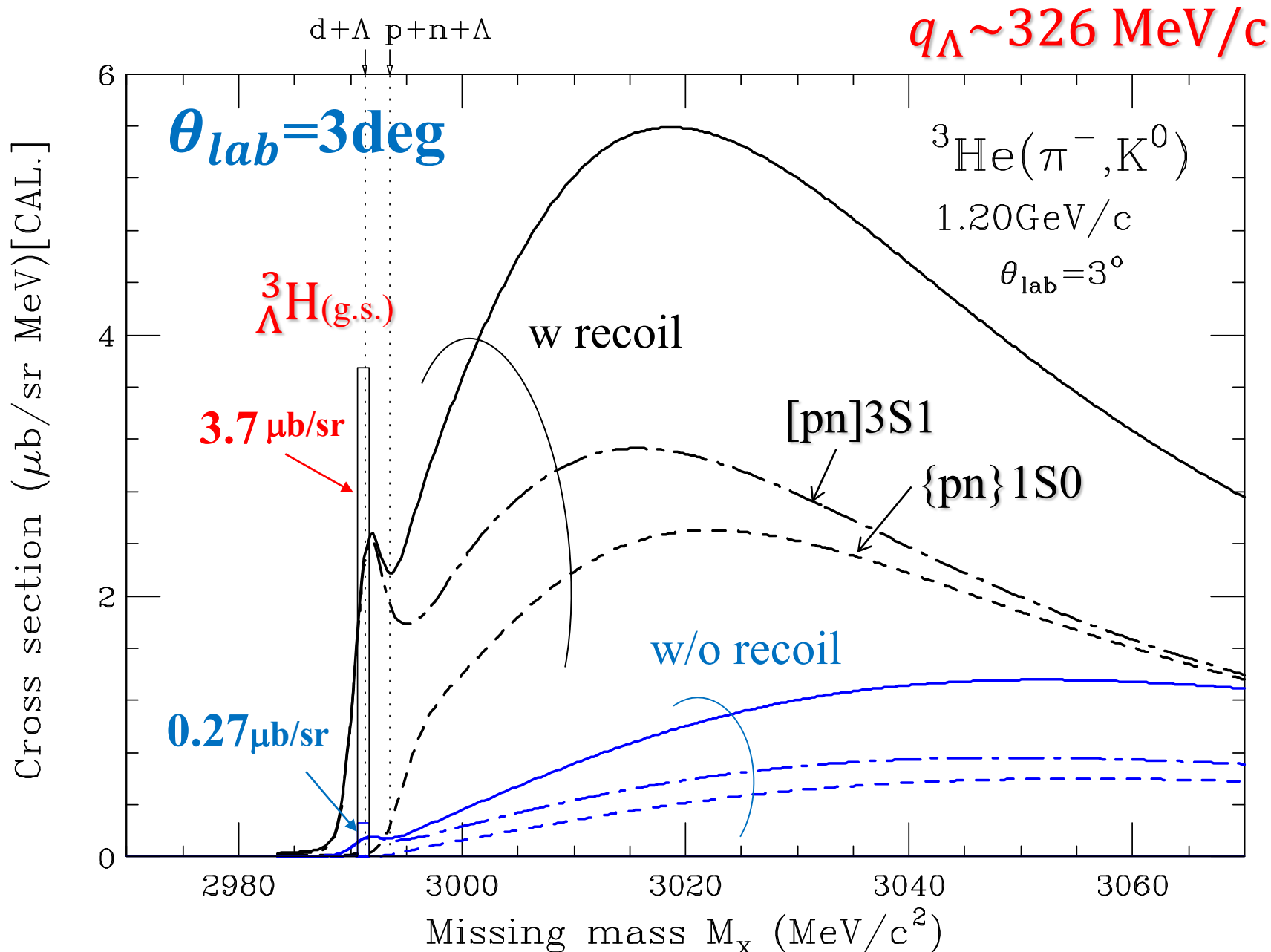
Effective recoil momentum q_{Λ}^{eff} for Λ production in ${}^3\text{He}$



→ The effective recoil momentum becomes

$$q_{\Lambda}^{\text{eff}} \sim \frac{M_C}{M_A} q_{\Lambda} \approx \frac{2}{3} \times 400 \text{ MeV}/c = 267 \text{ MeV}/c$$

Recoil effects of the spectrum in ${}^3\text{He}(\pi^-, \text{K}^0)pn\Lambda$



Summary

- Calculations of the coupled-channel Green's function with the CDCC provide the ability of describing continuum $NN\Lambda$ states including $N+N+\Lambda$ breakup processes.
- The production cross section of ${}^3_{\Lambda}\text{H}(1/2^+)$ accounts for $3.7 \mu\text{b}/\text{sr}$ at $1.2\text{GeV}/c$ (3°) in the ${}^3\text{He}(\pi^-, \text{K}^0)$ reaction.
- The recoil effects are very important to the production with a very light nuclear target as ${}^3\text{He}$.

Future Subjects

- Convergence of the CDCC model space depending on $(k_{\text{max}}, L_{\text{max}})$ parameters should be checked.
- ...

Thank you very much.