
# Interpretation of transverse momentum and rapidity dependence on angular distributions of Z-boson production at LHC

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collaborating with

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# Outline

- Lepton angular distributions of Drell-Yan processes
- Comparison with the pQCD calculations
- Interpretations from the geometric picture
  - Transverse momentum distributions
  - Rapidity dependence
- Summary

# The Drell-Yan Process

S.D. Drell and T.M. Yan, PRL 25 (1970) 316

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, $Q^2/s$ finite, $Q^2$ and $s$ being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function $\nu W_2$ near threshold.

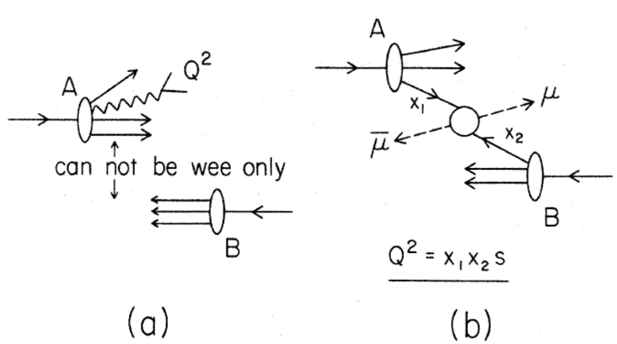



FIG. 1. (a) Production of a massive pair $Q^2$ from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.

$$\tau = \frac{Q^2}{s} = x_1 x_2$$

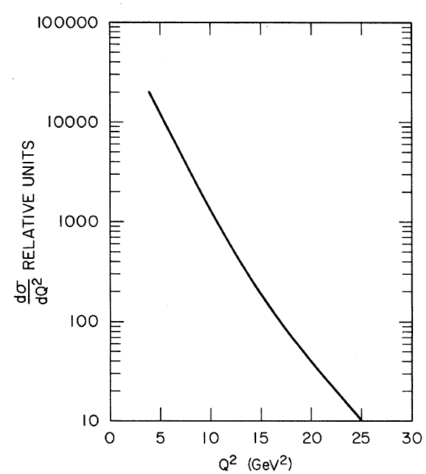
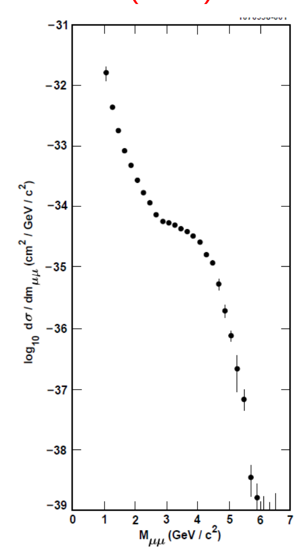



FIG. 2. $d\sigma/dQ^2$ computed from Eq. (10) assuming identical parton and antiparton momentum distributions and with relative normalization.

PRL 25 (1970) 1523

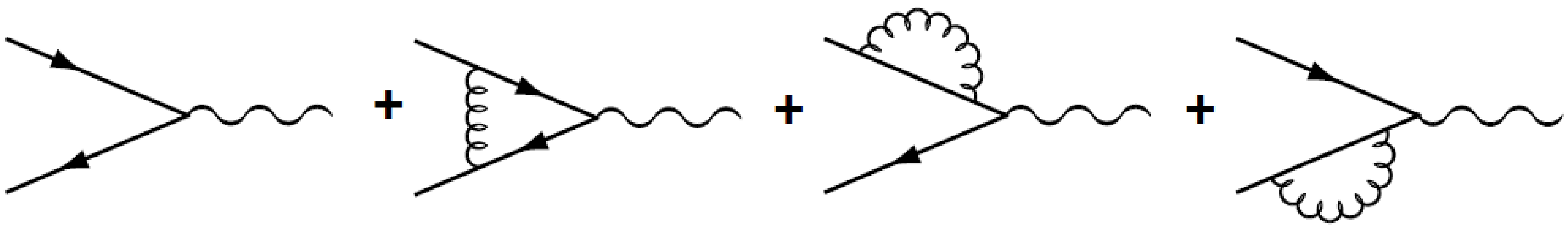



$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\left(\frac{1}{Q^2}\right)\mathfrak{F}(\tau) = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\left(\frac{1}{Q^2}\right)\int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \tau)\sum_a \lambda_a{}^{-2} F_{2a}(x_1) F_{2\bar{a}}{}'(x_2),$$

# Angular Distribution in the "Naïve" Drell-Yan Model

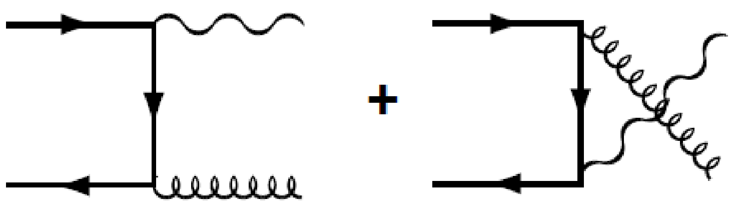
(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where $\theta$ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

# Drell-Yan Process with the $O(\alpha_s^1)$ QCD Effect



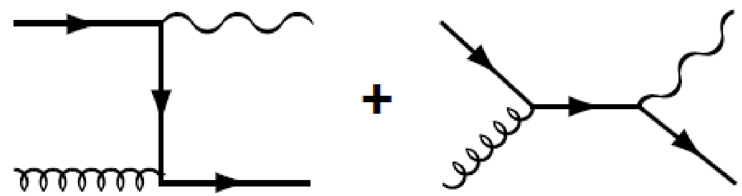
**( a )**

Quark-antiquark ($\mathrm{q}\bar{q}$) annihilation with the virtual gluon correction



**( b )**

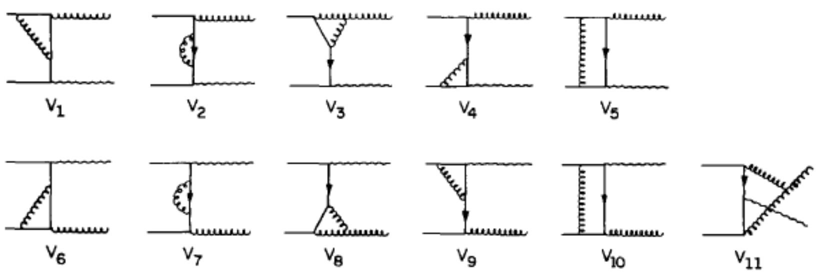
Quark-antiquark ($\mathrm{q}\bar{q}$) annihilation with one real gluon



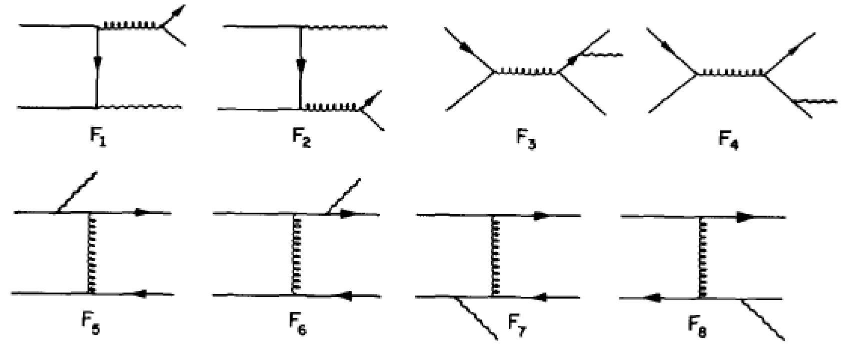
**( c )**

Quark-gluon ($qG$) Compton scattering

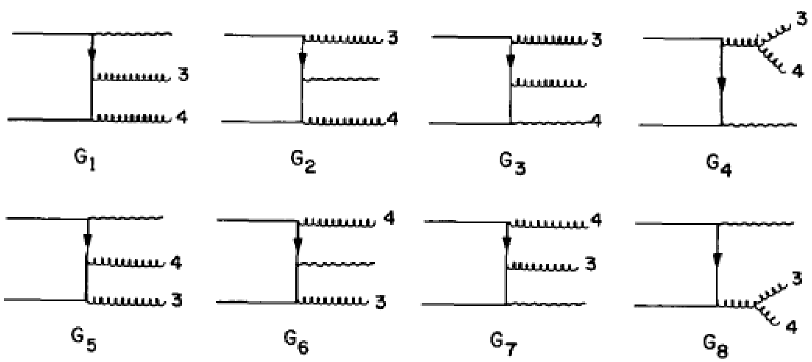
# Drell-Yan Process with the $O(\alpha_s^2)$ QCD Effect



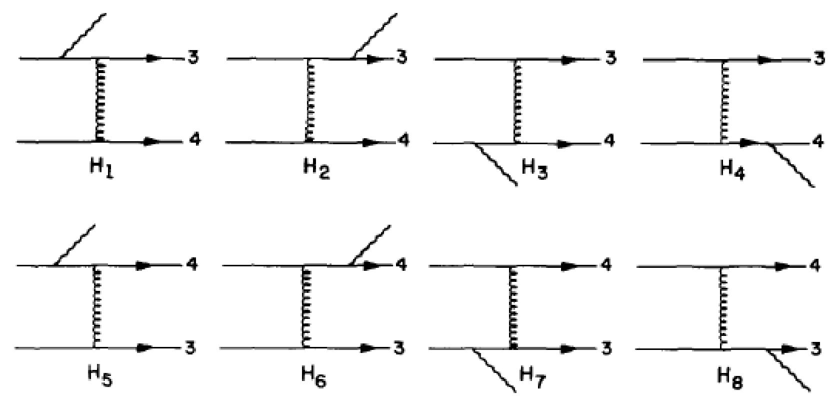

$q\bar{q} \to G\gamma^*$




$q\bar{q} \to q\bar{q}\gamma^*$



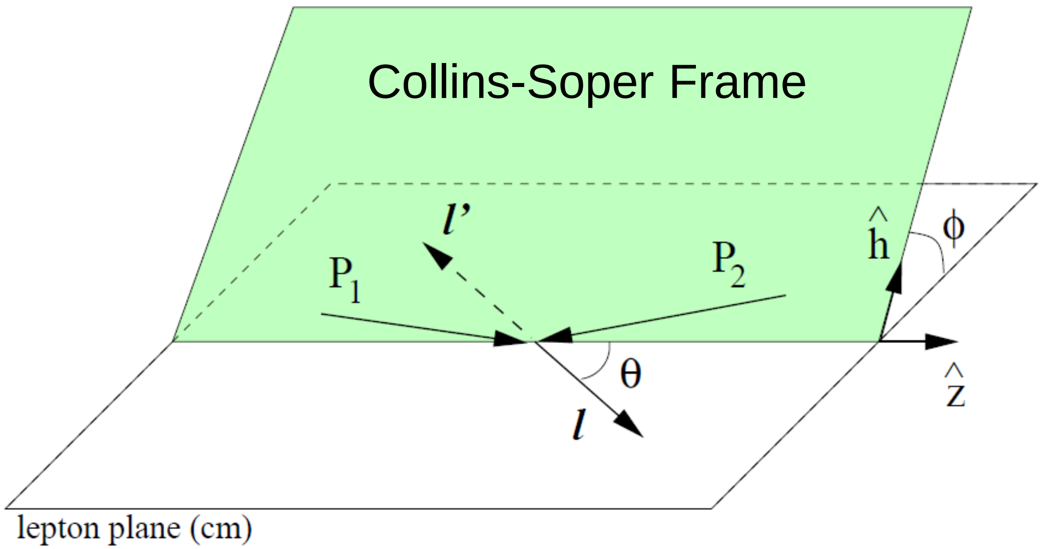

$q\bar{q} \to GG\gamma^*$




$qq \to qq\gamma^*$

NPB319 (1989) 37

# Angular Distributions of Lepton Pairs




$$\lambda = \frac{2-3A_0}{2+A_0}$$

$$\mu = \frac{2A_1}{2+A_0}$$

$$\nu = \frac{2A_2}{2+A_0}$$

$$\frac{d\sigma}{d\Omega} \propto (1+\lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi)$$

$$\propto [(1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + \frac{A_2}{2}\sin^2\theta\cos 2\phi]$$

$q\bar{q}$ annilation parton model: $O(\alpha_s^0)$ $\lambda=1,\ \mu=\nu=0;\ A_0 = A_2 = 0$

pQCD: $O(\alpha_s^1)$, ; $1-\lambda-2\nu = \frac{4(A_0 - A_2)}{2+A_0} = 0$ ; $A_0 = A_2$

Lam-Tung Relation [PRD 18 (1978) 2447]

# Angular Distributions of Lepton Pairs from $Z/\gamma^*$

$$\frac{d\sigma}{d\Omega} \propto [(1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi + \frac{A_2}{2}\sin^2\theta \cos 2\phi$$
$$+ A_3 \sin\theta \cos\phi + A_4 \cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi]$$

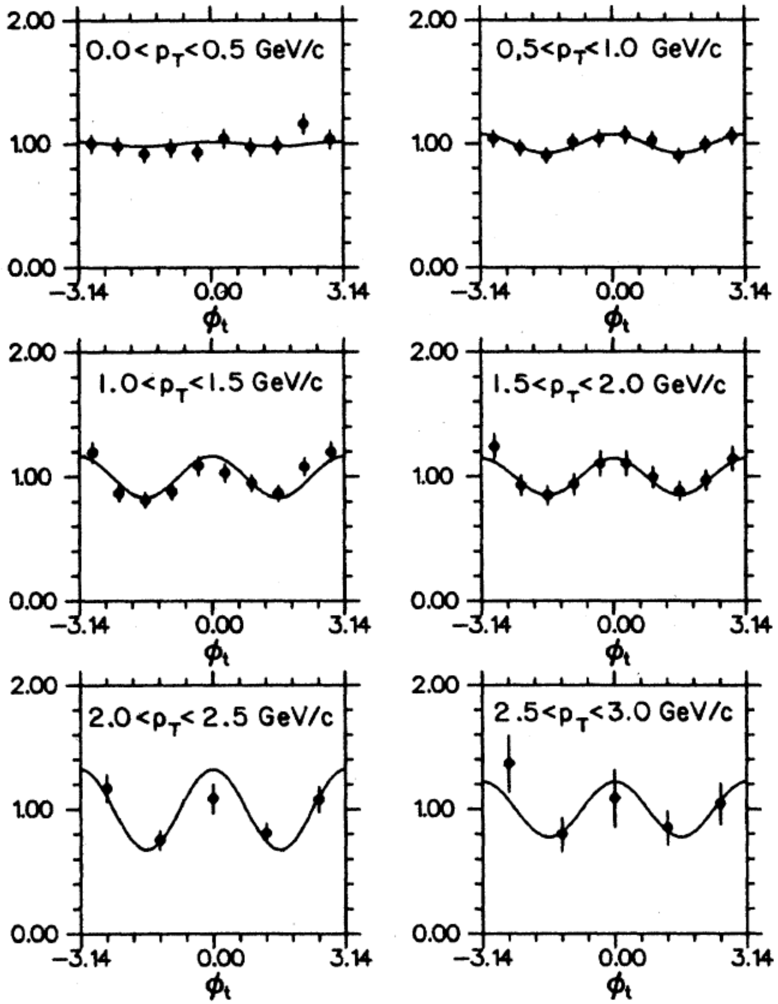
$A_3, A_4 : \gamma^* / Z$ interference, sensitive to $\sin^2\theta_W$

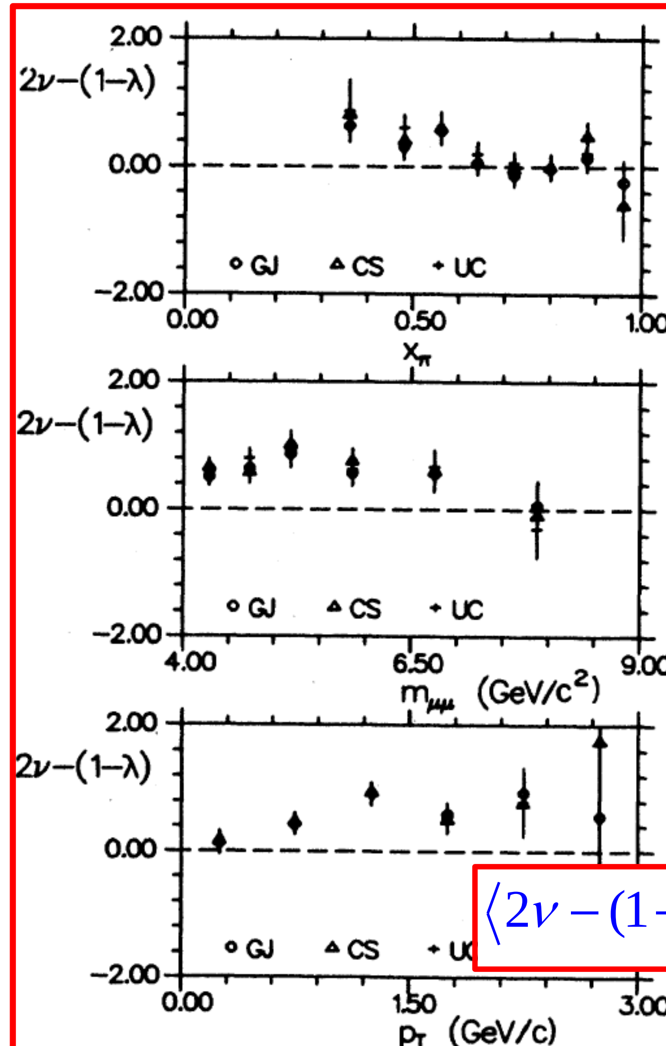
$A_5, A_6, A_7$:=0, up to $O(\alpha_s^1)$

# E615 @ FNAL: Violation of LT Relation

*PRD 39, 92 (1989)*

252-GeV $\pi^-$+W



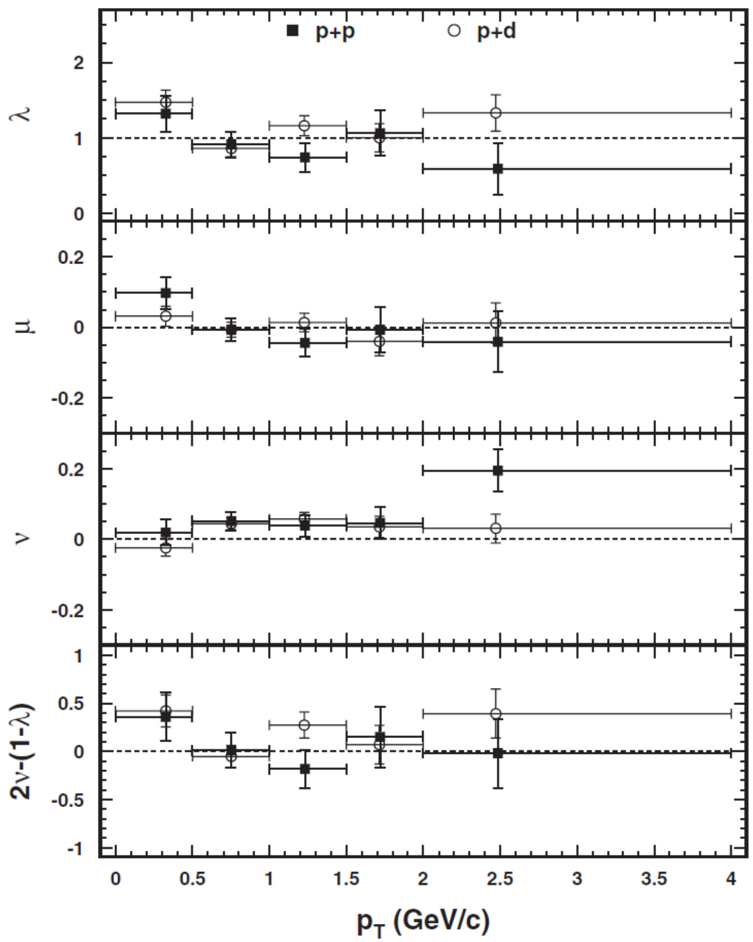

$$\langle 2\nu-(1-\lambda)\rangle_{\pi+p} > 0$$

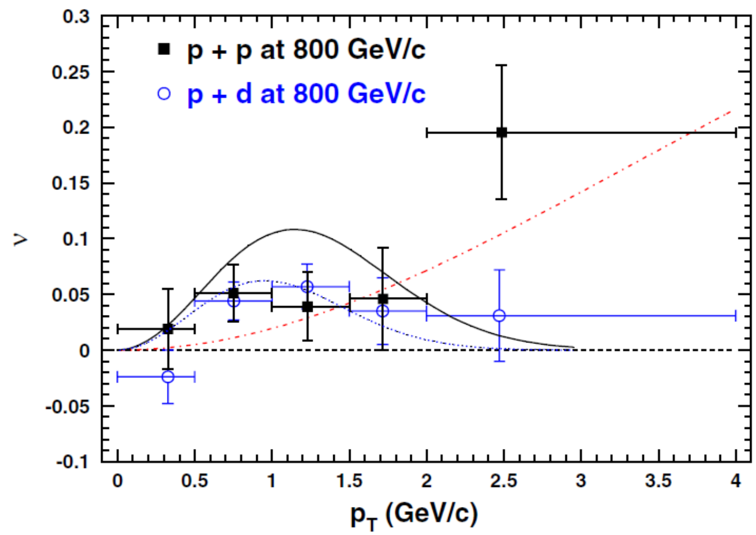
$\cos 2\phi$ modulation at large $p_T$

# E866 @ FNAL: Violation of LT Relation

*PRL 99, 082301 (2007), PRL 102, 182001 (2009)*

## 800-GeV p+p, p+d



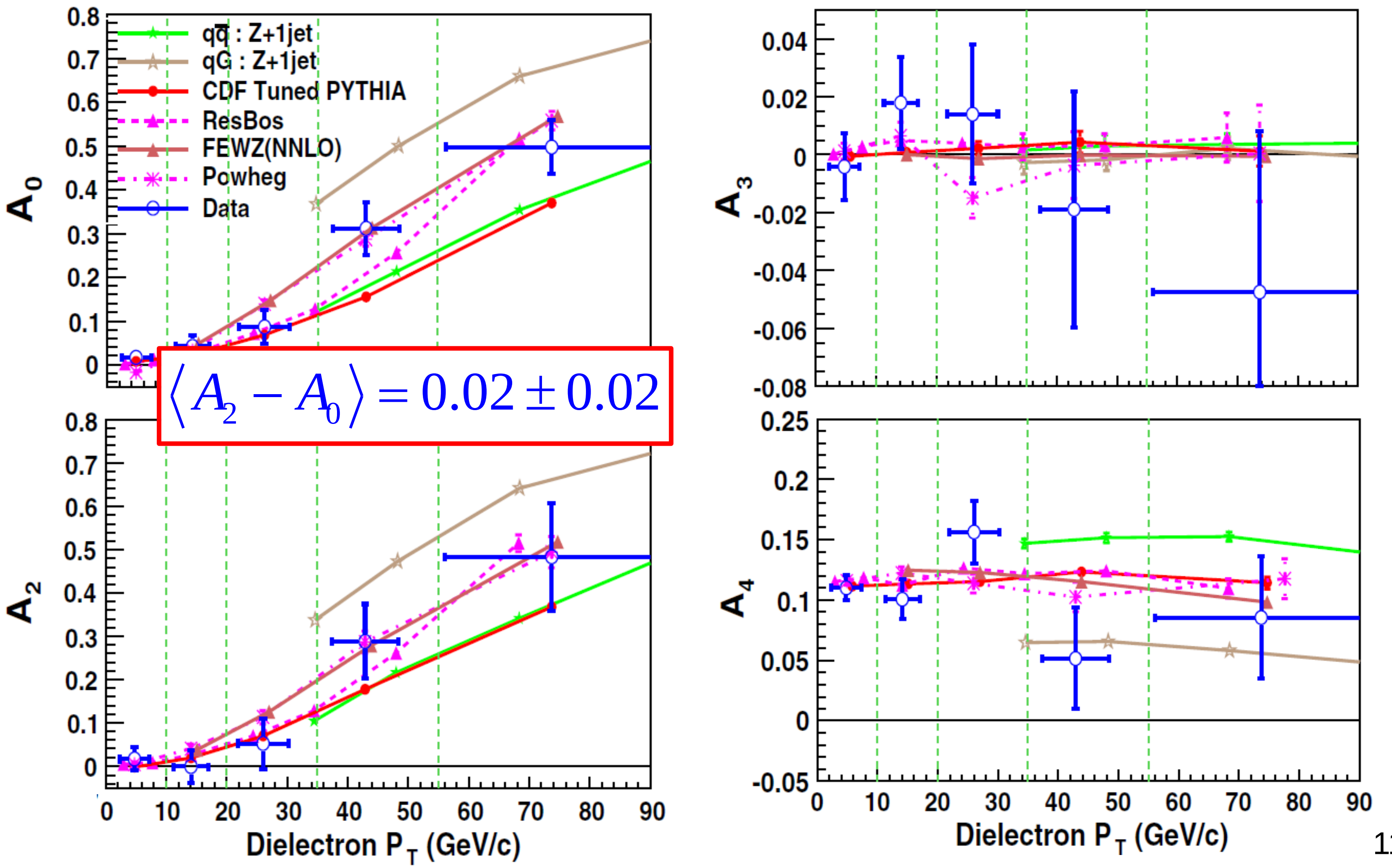




| | $p + p$ 800 GeV/$c$ (E866) | $p + d$ 800 GeV/$c$ (E866) |
|---|---|---|
| $\langle\lambda\rangle$ | $0.85 \pm 0.10$ | $1.07 \pm 0.07$ |
| $\langle\mu\rangle$ | $-0.026 \pm 0.019$ | $0.003 \pm 0.013$ |
| $\langle\nu\rangle$ | $0.040 \pm 0.015$ | $0.027 \pm 0.010$ |
| $\langle 2\nu - (1-\lambda)\rangle$ | $-0.07 \pm 0.10$ | $0.12 \pm 0.07$ |

$$\langle 2\nu-(1-\lambda)\rangle_{p+p} = -0.07 \pm 0.10, \langle 2\nu-(1-\lambda)\rangle_{p+d} = 0.12 \pm 0.07$$

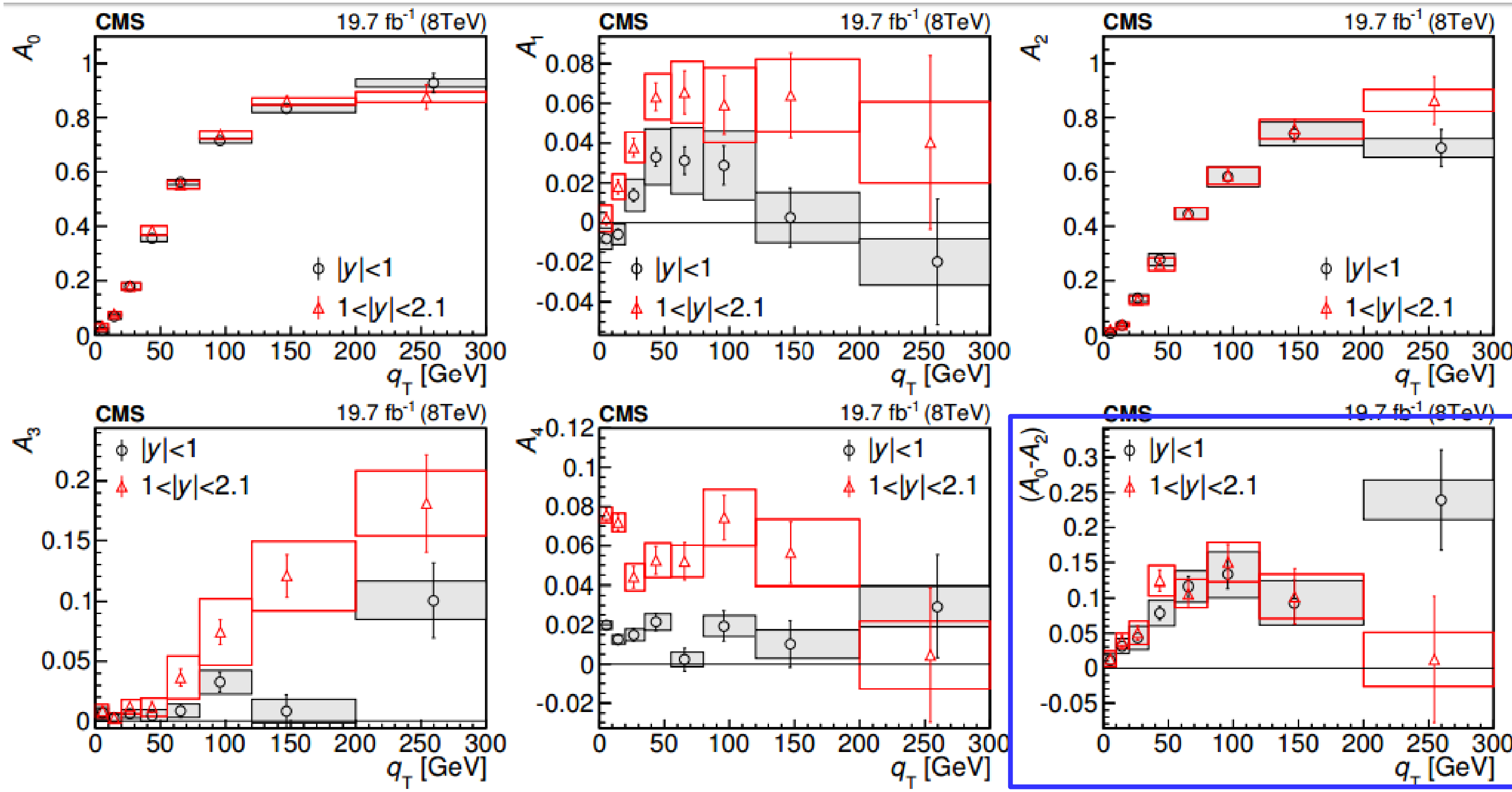
# Angular Distributions of Z Production

CDF, PRL 106, 241801 (2011)



# Angular Distributions of Z Production

CMS, PLB750, 154 (2015)




$$\frac{d^2\sigma}{d\cos\theta^* d\phi^*} \propto \Big[(1+\cos^2\theta^*) + A_0\frac{1}{2}(1-3\cos^2\theta^*) + A_1\sin(2\theta^*)\cos\phi^* + A_2\frac{1}{2}\sin^2\theta^*\cos(2\phi^*)$$

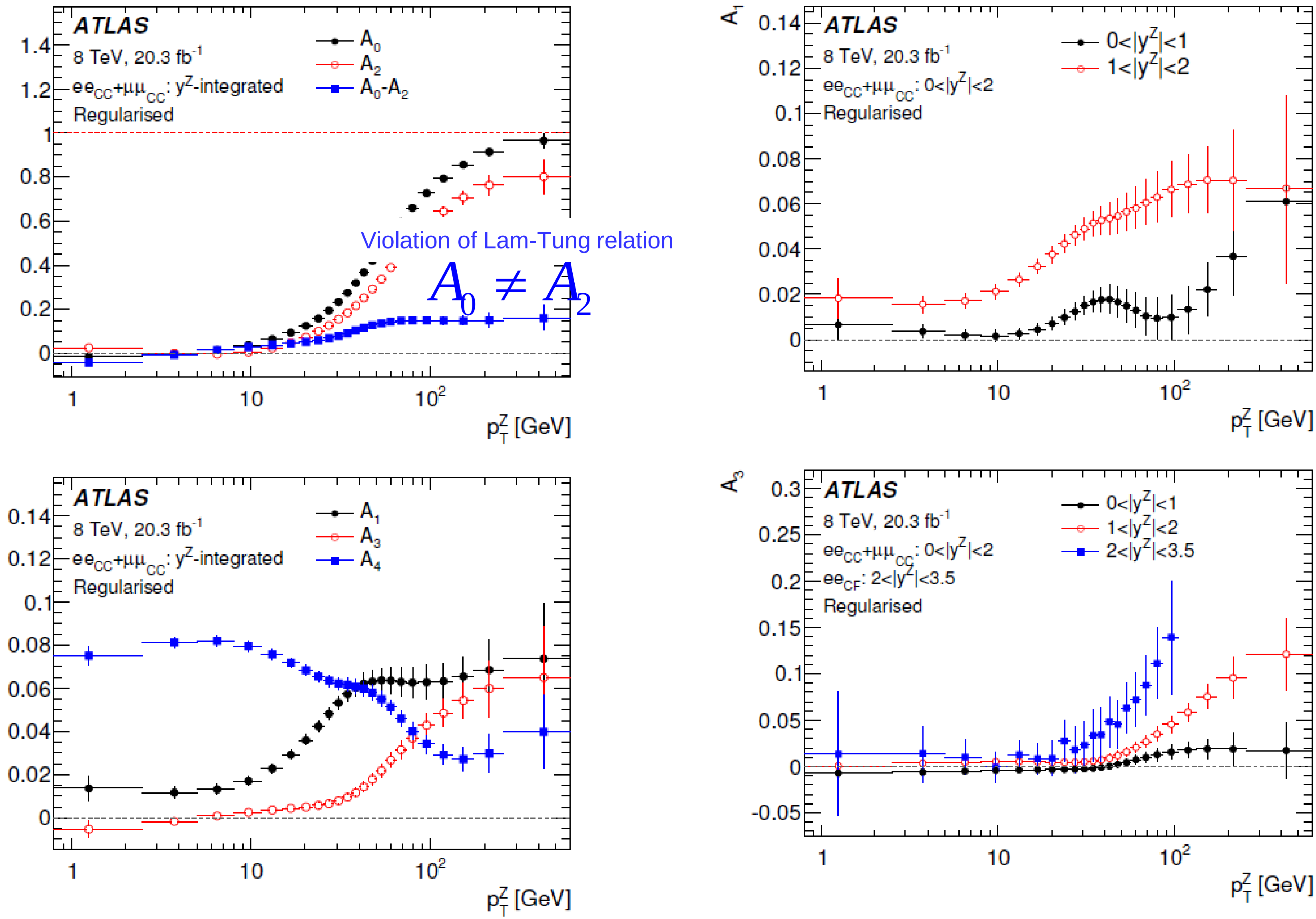
$$+A_3\sin\theta^*\cos\phi^* + A_4\cos\theta^* + A_5\sin^2\theta^*\sin(2\phi^*) + A_6\sin(2\theta^*)\sin\phi^* + A_7\sin\theta^*\sin\phi^*\Big].$$

Violation of Lam-Tung relation

$A_0 \neq A_2$

# Angular Distributions of Z Production

ATLAS, JHEP08, 159 (2016)

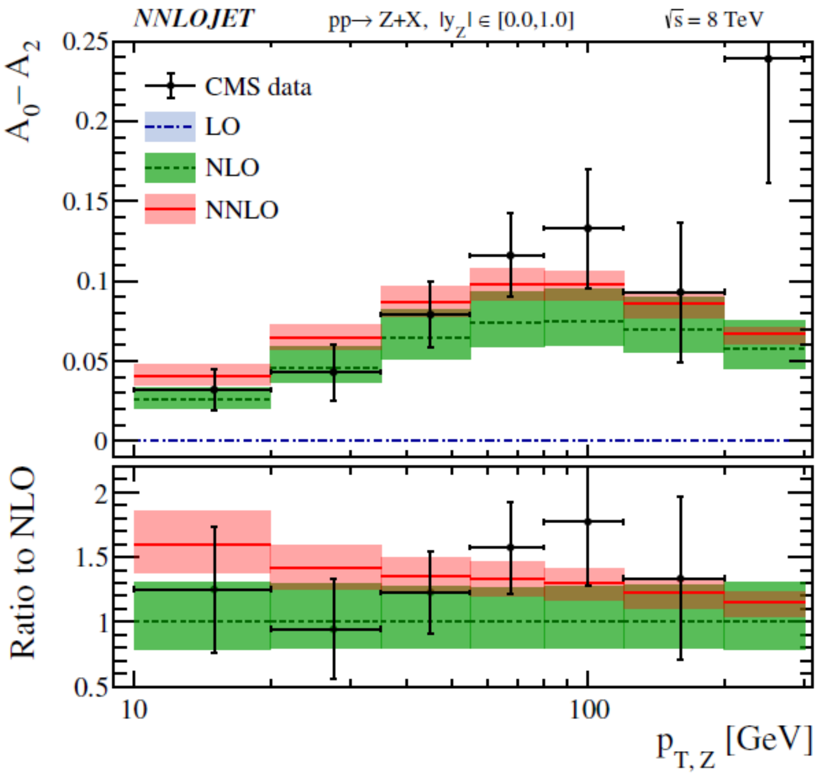


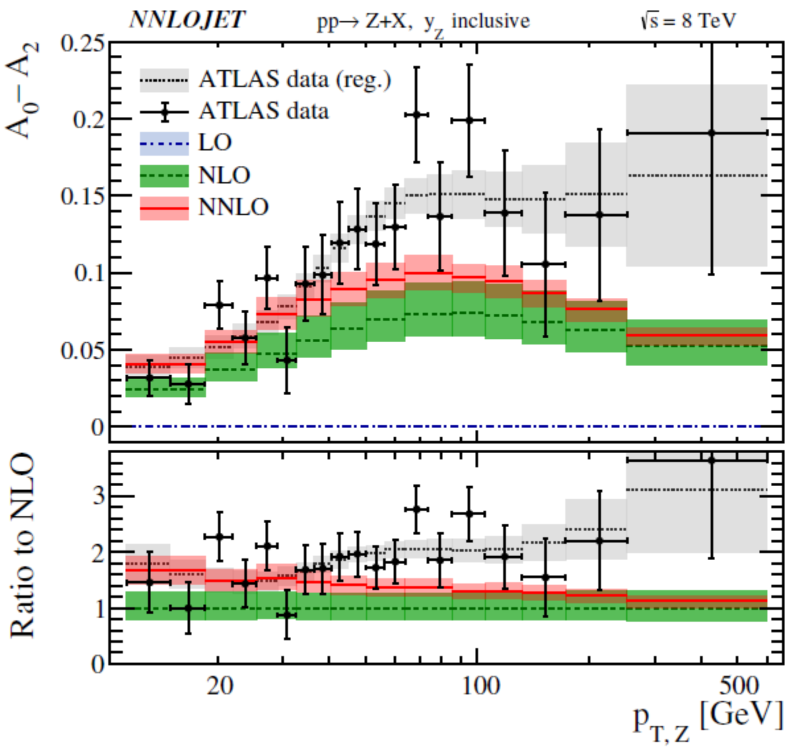
# Drell-Yan Angular Distributions

- Features:
  - Distinctive $q_T$ dependence.
  - Lam-Tung violation:
    - $1 - \lambda - 2\nu \neq 0$ for fixed-target experiments
    - $A_0 - A_2 \neq 0$ for collider experiments.
  - Rapidity dependence for $A_1$, $A_3$, and $A_4$.
- Can we understand these features by pQCD?
- Can we have some simple pictures for interpretation?

# NNLO: $O(\alpha_s^2)$

$LO: O(\alpha_s^1)$; $NLO: O(\alpha_s^2)$; $NNLO: O(\alpha_s^3)$



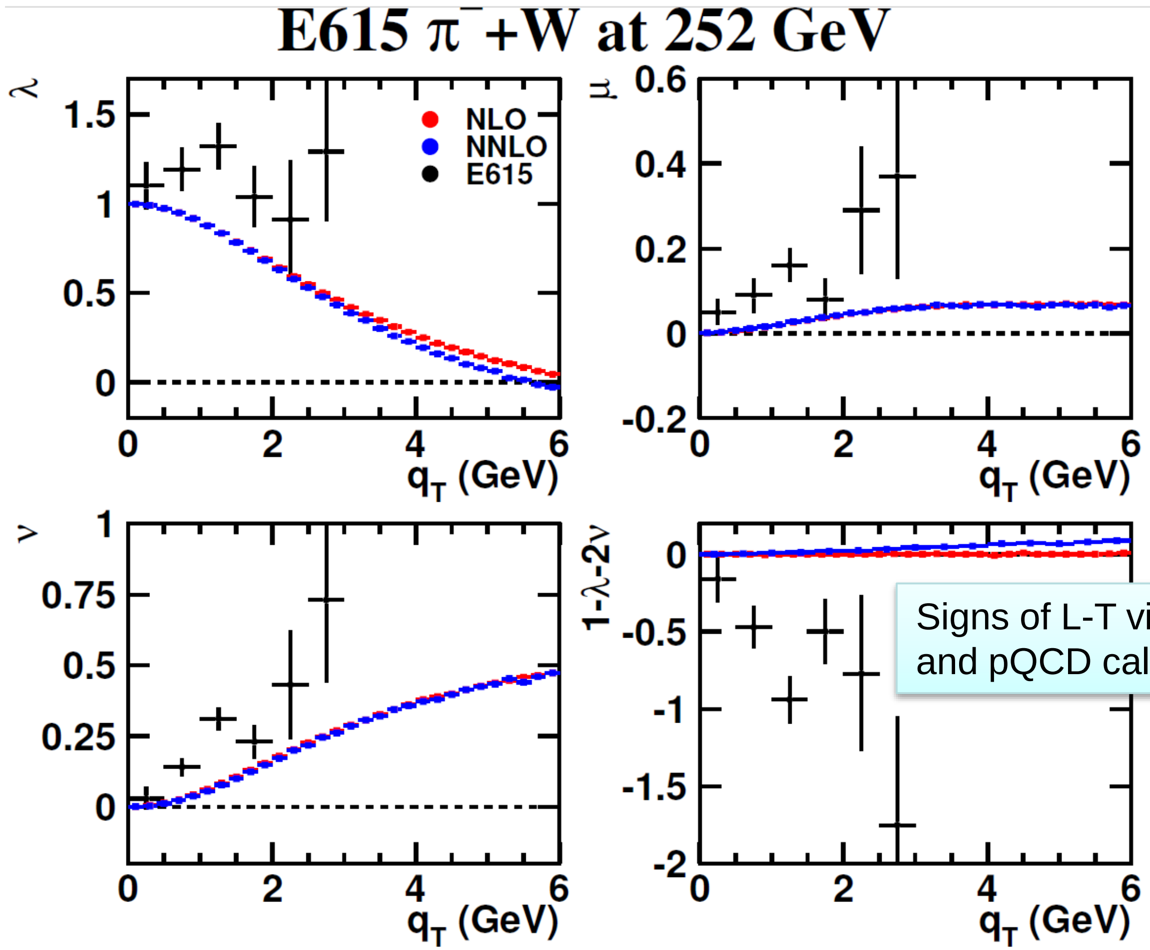




| | | | |
|---|---|---|---|
| NLO (CMS): | $\chi^2/N_{data} = 24.5/14 = 1.75$ | NLO (ATLAS): | $\chi^2/N_{data} = 185.8/38 = 4.89$ |
| NNLO (CMS): | $\chi^2/N_{data} = 14.2/14 = 1.01$ | NNLO (ATLAS): | $\chi^2/N_{data} = 68.3/38 = 1.80$ . |



JHEP11(2017)003

# pQCD NLO and NNLO Calculations: E615




Signs of L-T violation for the data and pQCD calculations are opposite!

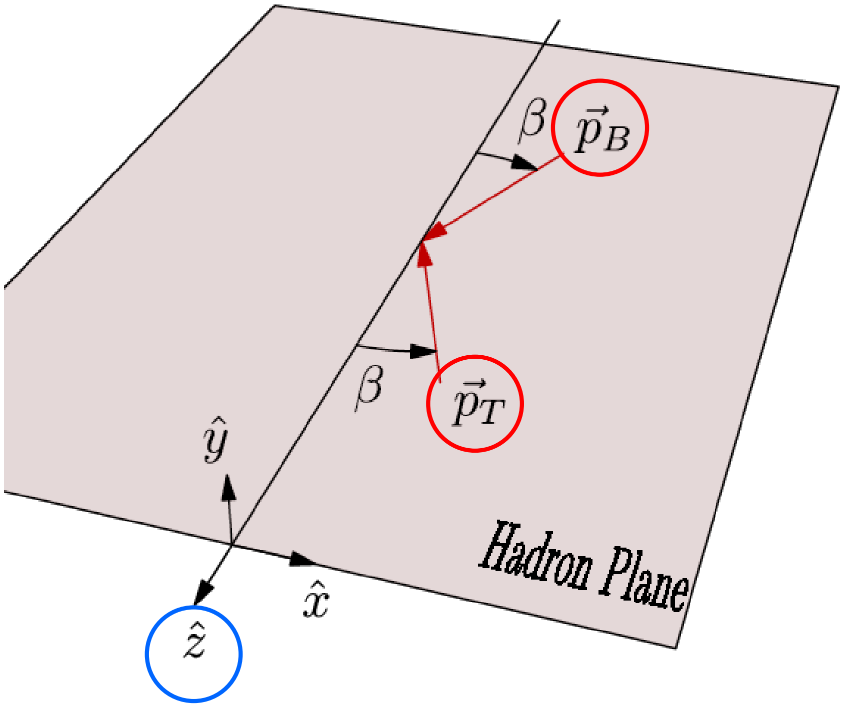
$NLO : O(\alpha_s^1)$; $NNLO : O(\alpha_s^2)$

# Drell-Yan Angular Distributions

- Features:
  - Distinctive $q_T$ dependence.
  - Lam-Tung violation:
    - $1 - \lambda - 2\nu \neq 0$ for fixed-target experiments
    - $A_0 - A_2 \neq 0$ for collider experiments.
  - Rapidity dependence for $A_1$, $A_3$, and $A_4$.
- Can we understand these features by pQCD? **Partially YES!**
- Can we have some simple pictures for interpretation?

A geometric picture:
J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev, PLB 758, 394 (2016)
W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev, PRD 96, 054020 (2017)

# Hadron Plane

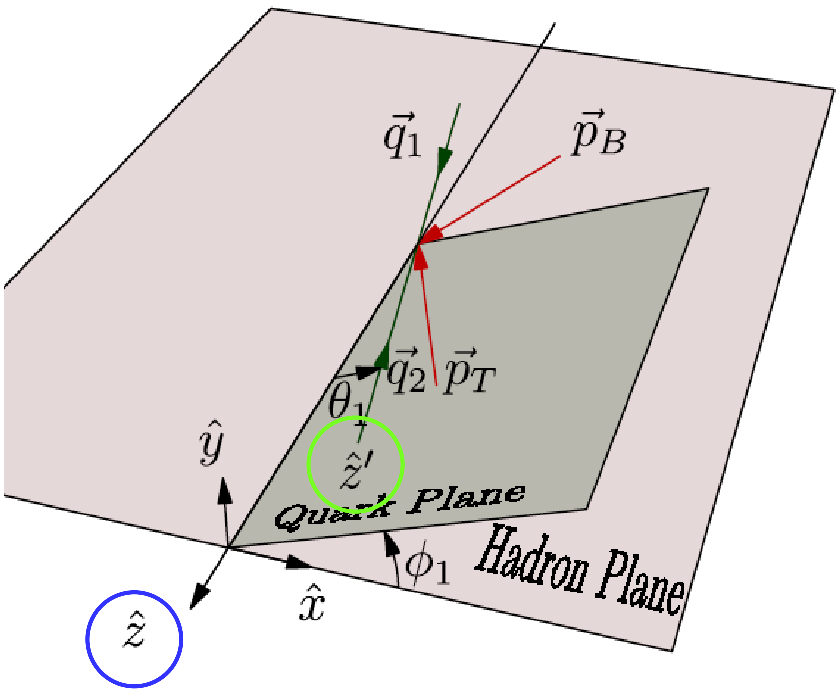



1) Hadron Plane ($\vec{P}_B \times \vec{P}_T$)

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- The ẑ axis of Collins-Soper frame bisects the directions of $\vec{P}_B$ and $\vec{P}_T$ momenta
- Angle $\beta$ satisfies the relation $\tan\beta = q_T / Q$

Collins-Soper ($\gamma^*$/Z rest) Frame

# Quark Plane



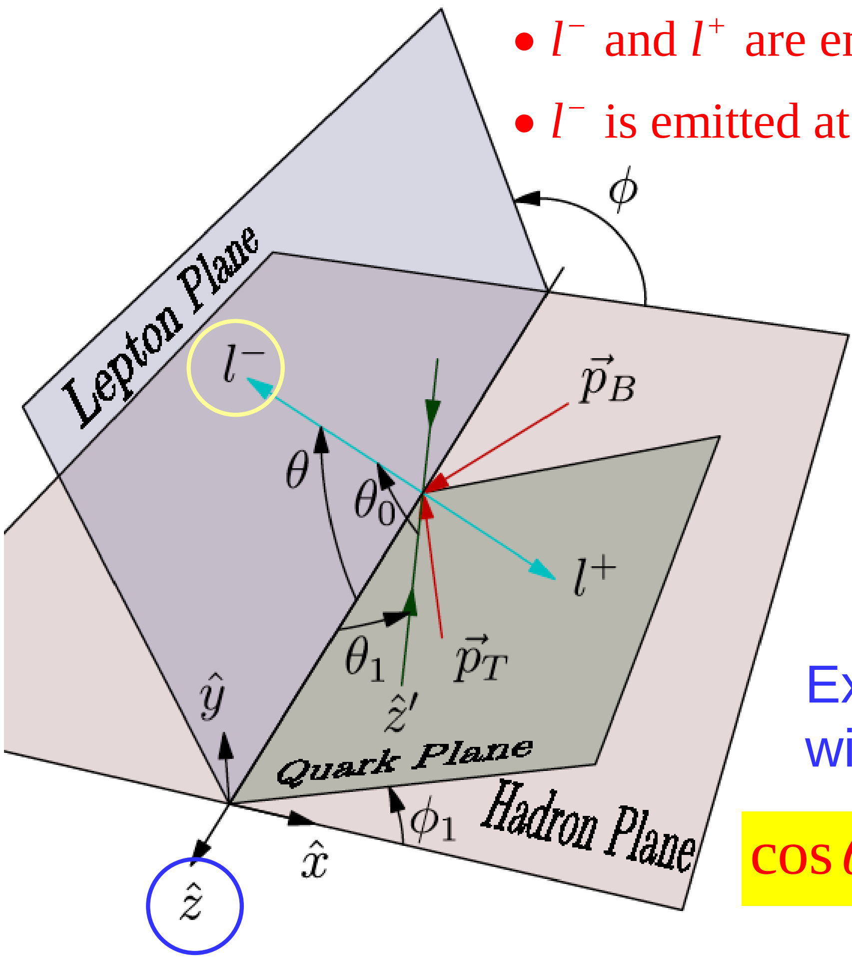

2) Quark Plane ($\hat{z}' \times \hat{z}$)

- $q$ and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles $\theta_1$ and $\varphi_1$ in the C-S frame

# Lepton Plane

3) Lepton Plane $(\overrightarrow{l^-} \times \hat{z})$

- $l^-$ and $l^+$ are emitted back-to-back with equal $|\vec{P}\,|$
- $l^-$ is emitted at angle $\theta$ and $\varphi$ in the C-S frame




Lepton angular distributions with respect to the natural axis $\hat{z}'$:

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Express the lepton angular distributions with respect to the natural axis $\hat{z}$:

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

# Lepton angular distributions w.r.t. the natural axis $\widehat{z'}$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\
& + (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi \\
& + (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi \\
& + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\
& + (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi \\
& + (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi \\
& + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi .
\end{aligned}
$$

$=$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) \\
& + A_1 \sin 2\theta\cos\phi \\
& + \frac{A_2}{2}\sin^2\theta\cos 2\phi \\
& + A_3\sin\theta\cos\phi + A_4\cos\theta \\
& + A_5\sin^2\theta\sin 2\phi \\
& + A_6\sin 2\theta\sin\phi \\
& + A_7\sin\theta\sin\phi
\end{aligned}
$$

**$A_0 - A_7$ are entirely described by $\theta_1, \varphi_1$ and $a$.**

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

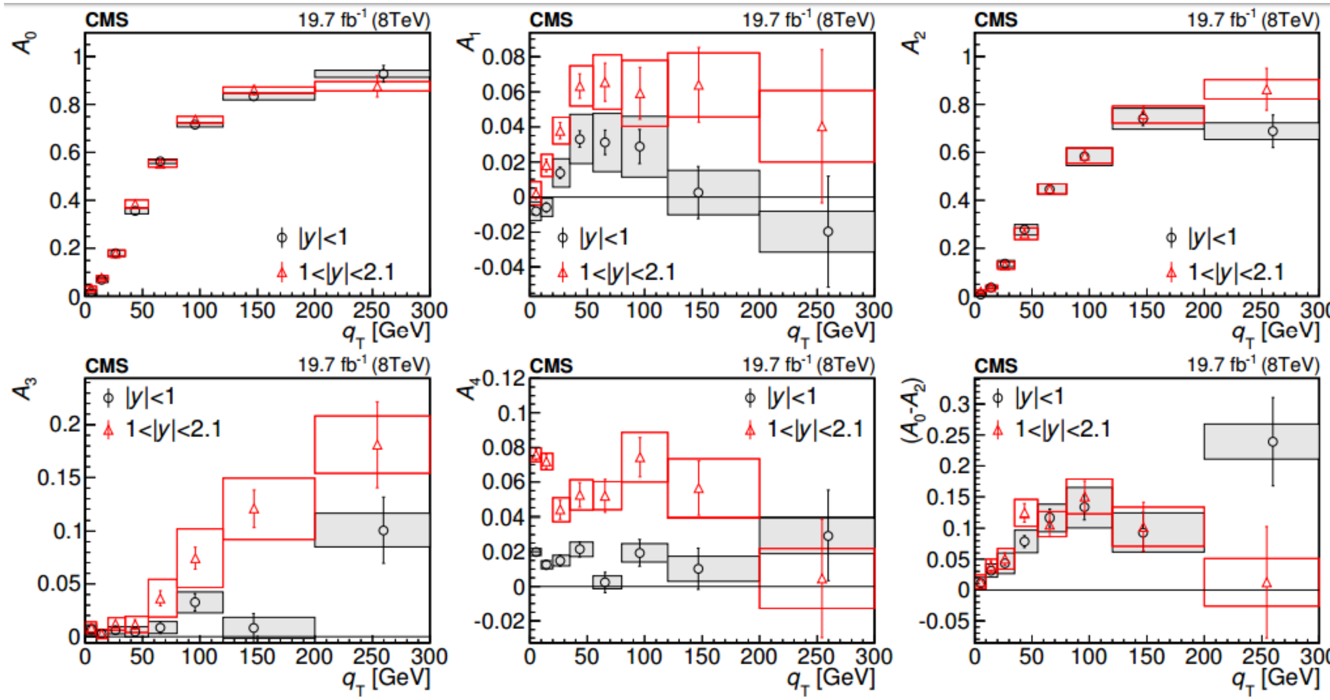
$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

# Some implications of the angular distribution coefficients $A_0 - A_4$




$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

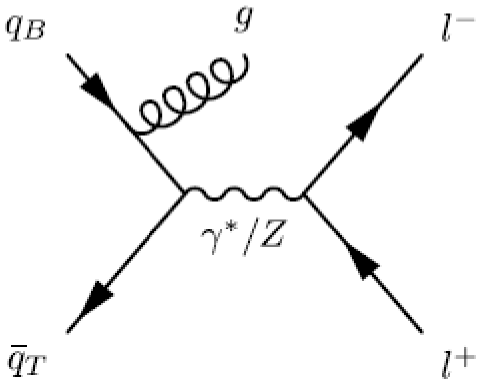
$$A_4 = a\langle \cos\theta_1 \rangle$$

At NLO, $\tan\theta_1 = q_T / Q$

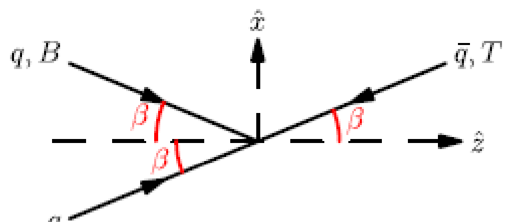
- $A_0 \geq A_2$(or $1 - \lambda - 2\nu \geq 0$). Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$.
- Forward-backward asymmetry, *a*, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for $A_4$.
- $A_0$, $A_2$ and $A_4$ increases with $q_T$ monotonically, while $A_4$ decreases with $q_T$.
- $A_1$ ($\propto \langle \sin 2\theta \rangle$) first increases with $q_T$, reaching a maximum and then decrease.

# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$: $q\bar{q} \to \gamma^*/Zg$

Collins-Soper ($\gamma^*$/Z rest) Frame

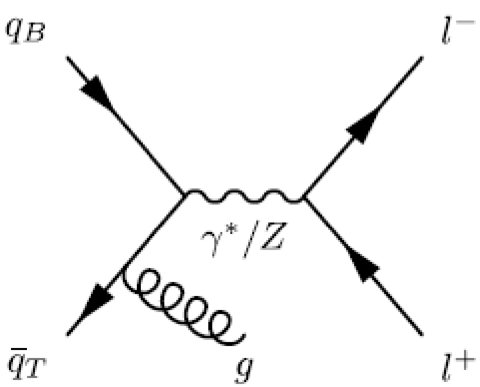



(a)

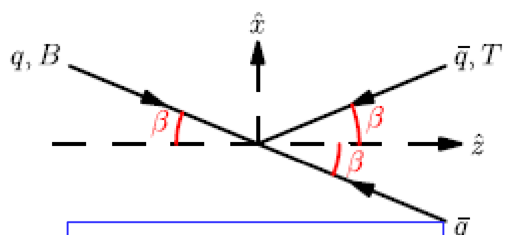



$\theta_1 = \beta;\ \phi_1 = 0$

(b)




(c)




$\theta_1 = \beta;\ \phi_1 = \pi$

(d)

$$A_0^{q\bar{q}} = \frac{q_T^2}{Q^2 + q_T^2} \text{ (Collins 1979)}$$

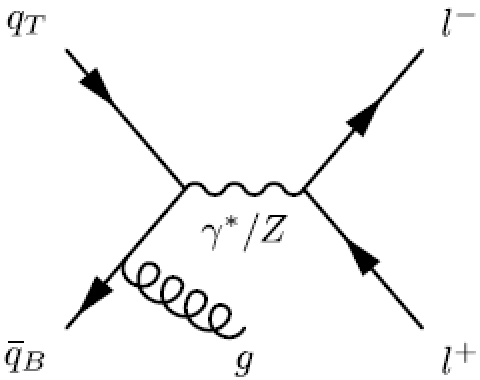
$$A_0 = A_2 = \langle \sin^2\theta_1 \rangle = \frac{q_T^2}{Q^2 + q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$$

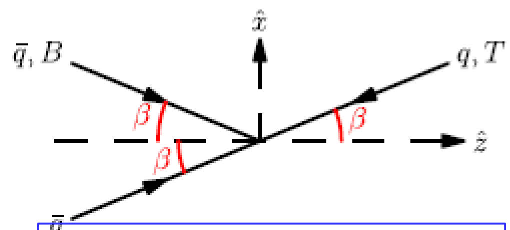
$$\nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$: $q\bar{q} \rightarrow \gamma^*/Zg$

Collins-Soper ($\gamma^*$/Z rest) Frame

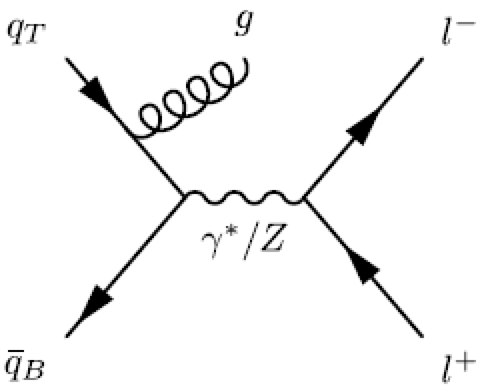



(a)

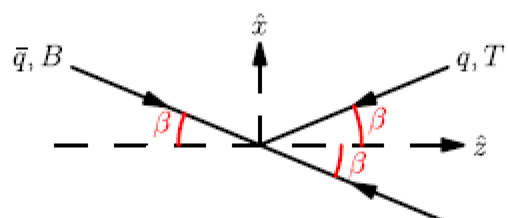



$\theta_1 = \pi - \beta;\ \phi_1 = 0$

(b)




(c)




$\theta_1 = \pi - \beta;\ \phi_1 = \pi$

(d)

$$A_0^{q\bar{q}} = \frac{q_T^2}{Q^2 + q_T^2} \text{ (Collins 1979)}$$

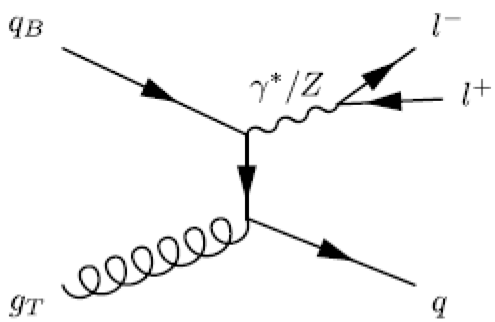
$$A_0 = A_2 = \left\langle \sin^2\theta_1 \right\rangle = \frac{q_T^2}{Q^2 + q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$$

$$\nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

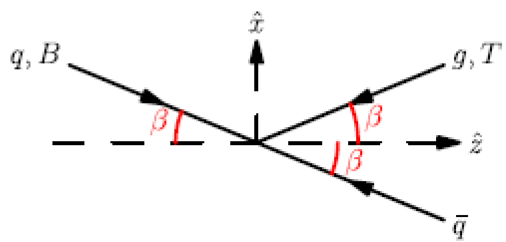
# $\theta_1$ and $\phi_1$ at $O(\alpha_s^1)$: $qg \to \gamma^*/Zq$

Collins-Soper ($\gamma^*$/Z rest) Frame

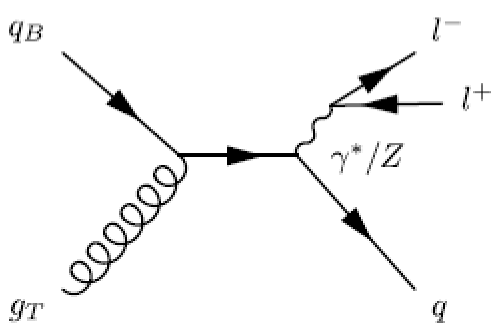



(a)

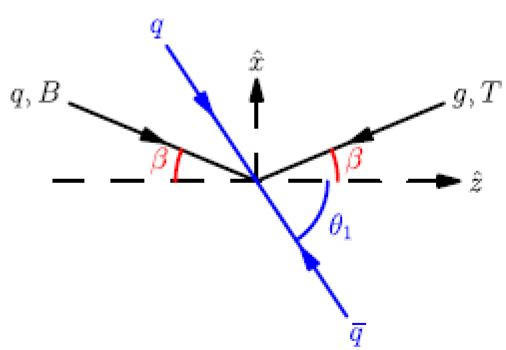
$$\theta_1 = \beta;\ \phi_1 = 0$$




(b)




(c)




(d)

$$\theta_1 > \beta;\ \phi_1 = 0$$

$$A_0^{qg} = \frac{5q_T^2}{Q^2 + 5q_T^2} \text{ (Thews 1979)}$$

$$A_0 = A_2 = \left\langle \sin^2\theta_1 \right\rangle \approx \frac{5q_T^2}{Q^2 + 5q_T^2} > 0$$

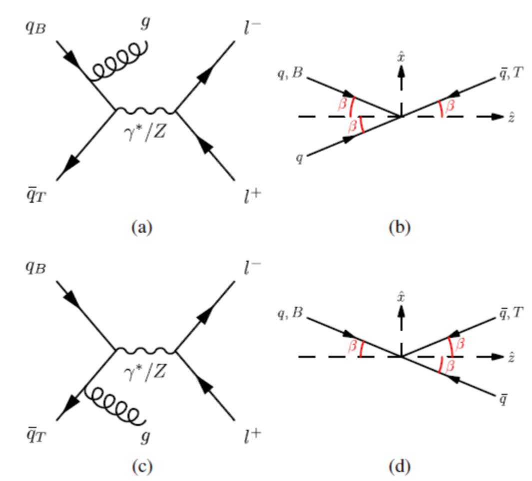
$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}$$

$$\nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

# (Non-)Coplanarity of Quark Plane and Hadron Plane

$A_0 = \langle \sin^2\theta_1 \rangle, A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$

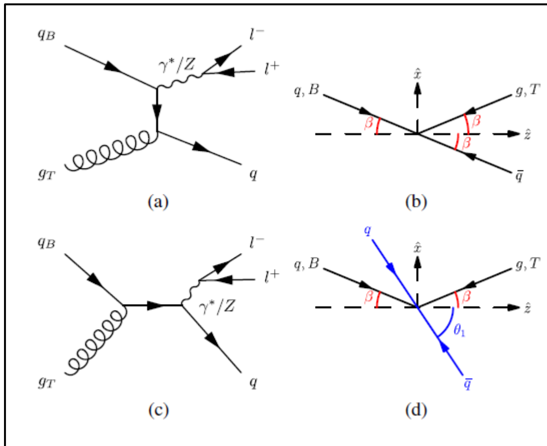
- $O(\alpha_s^1)$: $\phi_1 = 0$ or $\pi$. $A_0 = A_2$




$\theta_1 = \beta;\ \phi_1 = 0$

$q\bar{q} \rightarrow G\gamma^*$

$\theta_1 = \beta;\ \phi_1 = \pi$




$\theta_1 = \beta;\ \phi_1 = 0$

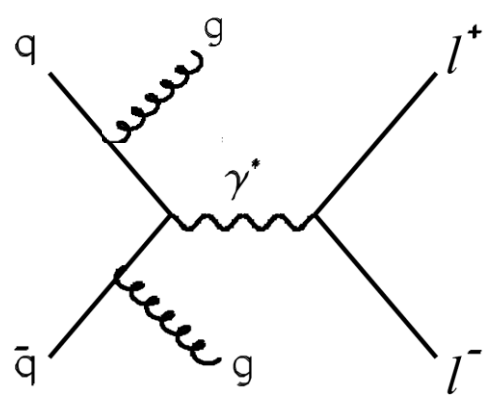
$qG \rightarrow q\gamma^*$

$\theta_1 > \beta;\ \phi_1 = 0$

- $O(\alpha_s^2)$ or higher:

$\phi_1 \neq 0$ or $\pi$

→ $A_0 > A_2$ $(1 - \lambda - 2\nu > 0)$



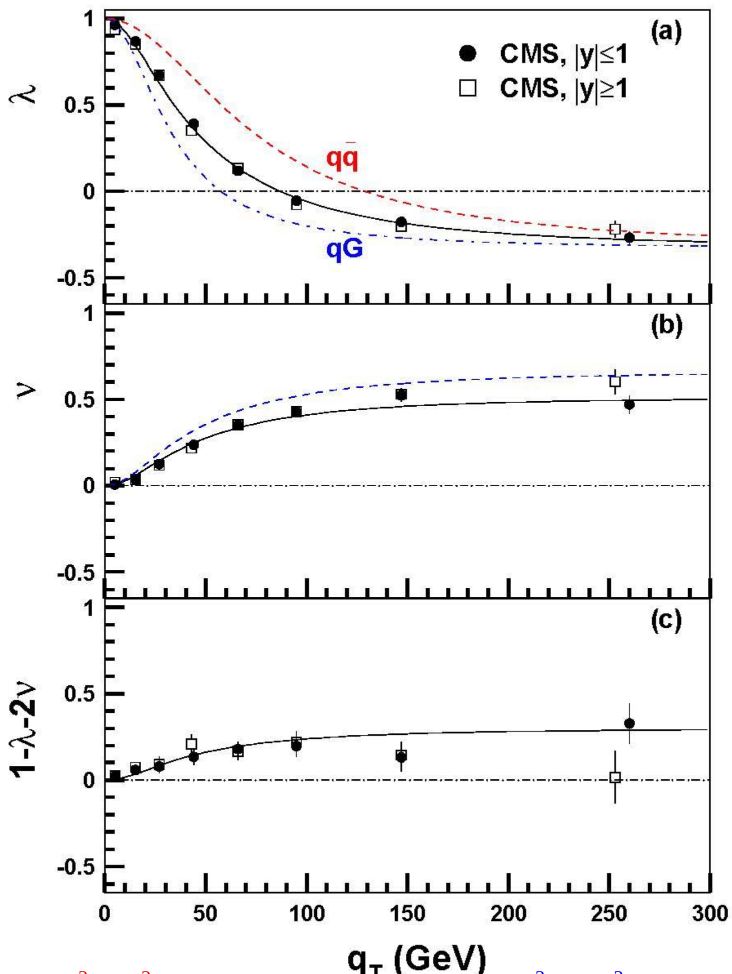

- Intrinsic $k_T$ of interacting partons:

$\phi_1 \neq 0$ or $\pi$.

→ $A_0 > A_2$ $(1 - \lambda - 2\nu > 0)$

# CMS Data Interpreted by the Geometric Picture

$A_0 = \langle \sin^2\theta_1 \rangle, A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$

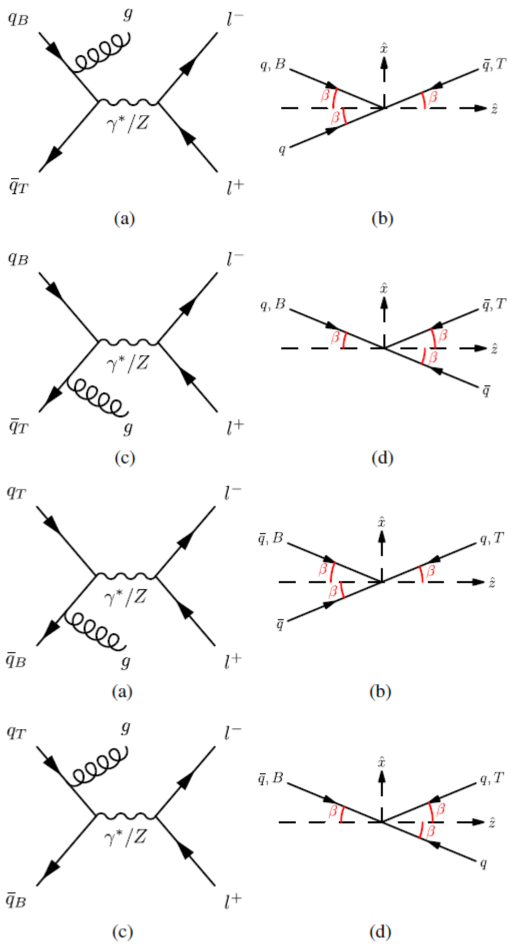



- $\lambda(q_T)$:determine 42% $q\bar{q}$ and 58% $qG$ processes.
- $\nu(q_T)$:determine $\frac{\langle \sin^2\theta_1 \cos 2\phi_1 \rangle}{\langle \sin^2\theta_1 \rangle} = 0.77$ (solid curve).
- Violation of Lam-Tung relation $1-\lambda-2\nu$: is well described

$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$ for $q\bar{q} \to ZG$, $\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}$ for $qG \to Zq$

# Cancelation Effect for $A_1$, $A_3$ and $A_4$

- $O(\alpha_s^1)$: $\theta_1 = \beta, \pi - \beta;\ \phi_1 = 0, \pi$




$\theta_1 = \beta;\ \phi_1 = 0$

$\theta_1 = \beta;\ \phi_1 = \pi$

$\theta_1 = \pi - \beta;\ \phi_1 = 0$

$\theta_1 = \pi - \beta;\ \phi_1 = \pi$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

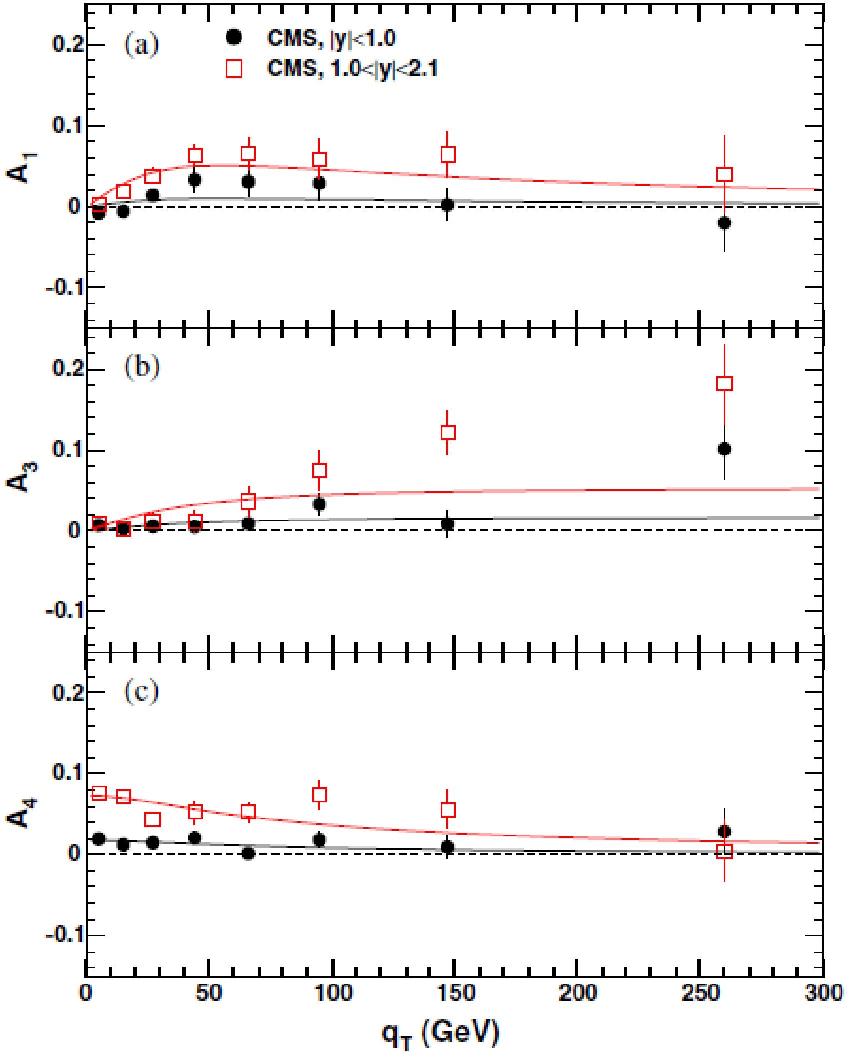
$$A_4 = a\langle \cos\theta_1 \rangle$$

TABLE I. Angles $\theta_1$ and $\phi_1$ for four cases of gluon emission in the $q - \bar{q}$ annihilation process at order-$\alpha_s$. The signs of $A_0$ to $A_4$ for the four cases are also listed.

| Case | Gluon emitted from | $\theta_1$ | $\phi_1$ | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|---|---|---|---|
| 1 | Beam quark | $\beta$ | 0 | + | + | + | + | + |
| 2 | Target antiquark | $\beta$ | $\pi$ | + | − | + | − | + |
| 3 | Beam antiquark | $\pi - \beta$ | 0 | + | − | + | + | − |
| 4 | Target quark | $\pi - \beta$ | $\pi$ | + | + | + | − | − |

A cancelation effect leads to a strong rapidity-dependence of $A_1$, $A_3$ and $A_4$.

# CMS Data Interpreted by the Geometric Picture




- $A_0(q_T)$:determine 42% $q\bar{q}$ and 58% $qG$ processes.
- $$A_1 = r_1\left[ f\frac{q_T Q}{Q^2+q_T^2} + (1-f)\frac{\sqrt{5}q_T Q}{Q^2+5q_T^2}\right]$$

  $$A_3 = r_3\left[ f\frac{q_T}{\sqrt{Q^2+q_T^2}} + (1-f)\frac{\sqrt{5}q_T}{\sqrt{Q^2+5q_T^2}}\right]$$

  $$A_4 = r_4\left[ f\frac{Q}{\sqrt{Q^2+q_T^2}} + (1-f)\frac{Q}{\sqrt{Q^2+5q_T^2}}\right]$$
- Rapidity dependence of Violation of $A_1$, $A_3$ and $A_4$ is well described

# Summary

- The lepton angular distributions in the Drell-Yan process can be used to explore the reaction mechanisms and the parton distributions.
- Fixed-order pQCD calculations could quantitatively describe the data from colliders at large $q_T$. Deviation seen for the data of fixed-target experiments.

# Summary

- Many salient features of the data and the results of fixed-order pQCD calculations could be well understood by a geometric picture.
  - The lepton angular coefficients $A_0$-$A_7$ (or $\lambda$, $\mu$, $\nu$) are described in terms of the polar ($\theta_1$) and azimuthal angles ($\phi_1$) of the natural $\mathrm{q}-\bar{\mathrm{q}}$ axis.
  - The striking $q_T$ dependence of $A_0$, $A_2$ (or $\lambda$, $\nu$) can be well described by the mis-alignment of the $\mathrm{q}-\bar{\mathrm{q}}$ axis and the CS z-axis, i.e. **finite $\boldsymbol{\theta_1}$**.
  - Violation of the Lam-Tung relation ($A_0 \neq A_2$) is described by the non-coplanarity of the $\mathrm{q}-\bar{\mathrm{q}}$ axis and the hadron plane, i.e. **finite $\boldsymbol{\phi_1}$**.
  - The cancelation effect leads to strong rapidity dependence of $A_1$, $A_3$ and $A_4$ (or $\mu$).

# References


- "*Interpretation of Angular Distributions of Z-boson Production at Colliders*", J.C. Peng, W.C. Chang, R.E. McClellan, and O. Teryaev, Phys. Lett. B 758, 394 (2016), arXiv:1511.08932.
- "*Dependencies of lepton angular distribution coefficients on the transverse momentum and rapidity of Z bosons produced in pp collisions at the LHC*", W.C. Chang, R.E. McClellan, J.C. Peng, and O. Teryaev, Phys. Rev. D 96, 054020 (2017), arXiv:1708.05807.
- "*On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production* ", J.C. Peng, D. Boer, W.C. Chang, R.E. McClellan and O. Teryaev, arXiv:1808.04398, Phys. Lett. B to be published.
- "*Lepton Angular Distributions of Fixed-target Drell-Yan Experiments in Perturbative QCD and a Geometric Approach*", W.C. Chang, R.E. McClellan, J.C. Peng, and O. Teryaev, arXiv:1811.03256.