Gluon Wigner distributions at small x with sub-nucleonic fluctuations

Yoshikazu Hagiwara (Kyoto U.)

Nucleon tomography

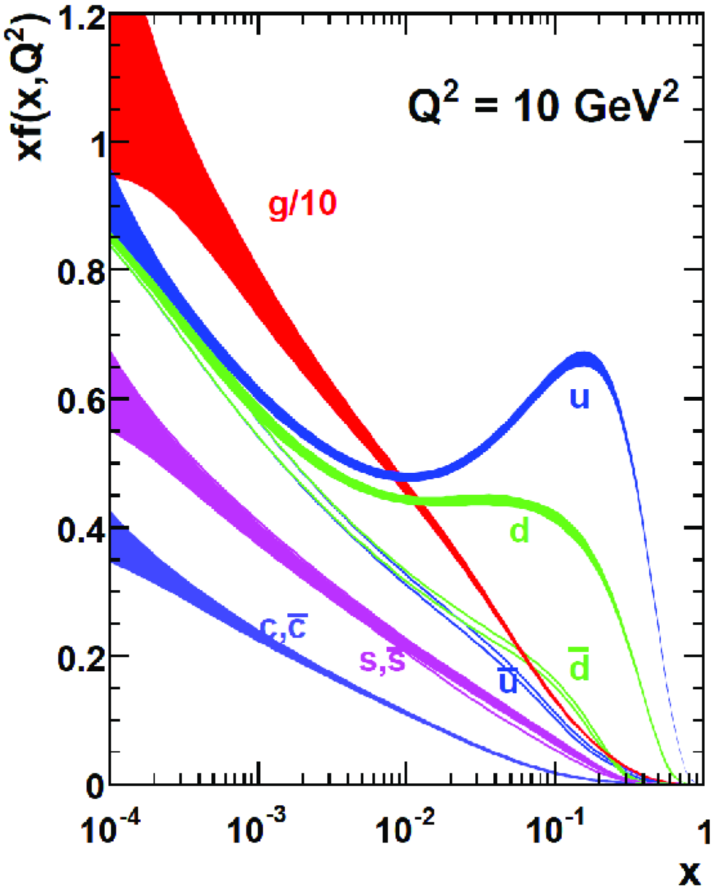
- Current interests in nucleon tomography
  - multi-dimensional phase space structure of the nucleon
    3-dim : GPD, TMD
    5-dim : GTMD, Wigner

- Nucleon is composed of quarks and gluons
  - Does the constituent quarks affect the structure of the parton distribution? ← sub-nucleonic effects

↓

We include the sub-nucleonic effects to the Wigner distribution.

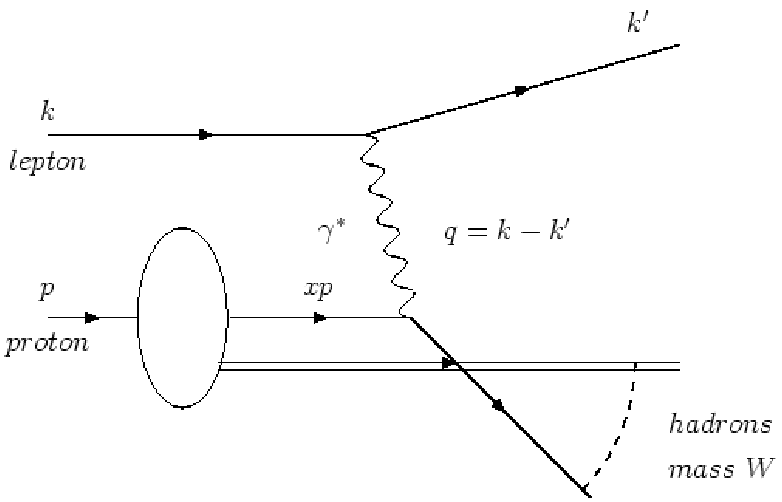
Parton distribution function (PDF)



Martin, A.D. et al. Eur.Phys.J. C63 (2009) 189-285

$f(x,Q^2)$: parton distribution function

x : momentum fraction wrt p
Q : momentum transfer($-q^2 = Q^2$)



Phase space distributions

Generalized TMD $\longleftrightarrow$ (F.T.) Wigner dist.

$F(x, \mathbf{k}_\perp, \Delta_\perp)$

S. Meissner, A. Metz and M. Schlegel, (2009)

$W(x, \mathbf{k}_\perp, \mathbf{b}_\perp)$

Belistky, Ji, Yuan (2004) , Ji (2003)

$\int d^2\mathbf{k}_\perp$ $\rightarrow$ GPD

$\Delta_\perp \to 0$ $\rightarrow$ TMD

Gaussian smearing

GPD

$G(x, \Delta_\perp)$

TMD

$f(x, \mathbf{k}_\perp)$

Husimi dist.

$H(x, \mathbf{k}_\perp, \mathbf{b}_\perp)$

$\Delta_\perp \to 0$

$\int d^2\mathbf{k}_\perp$

Probability distribution

Y. H, Y. Hatta, (2015)

PDF：

$f(x)$

Wigner distribution in QM

E. Wigner. *Phys. Rev.* 40:749 (1932)

Wigner distribution

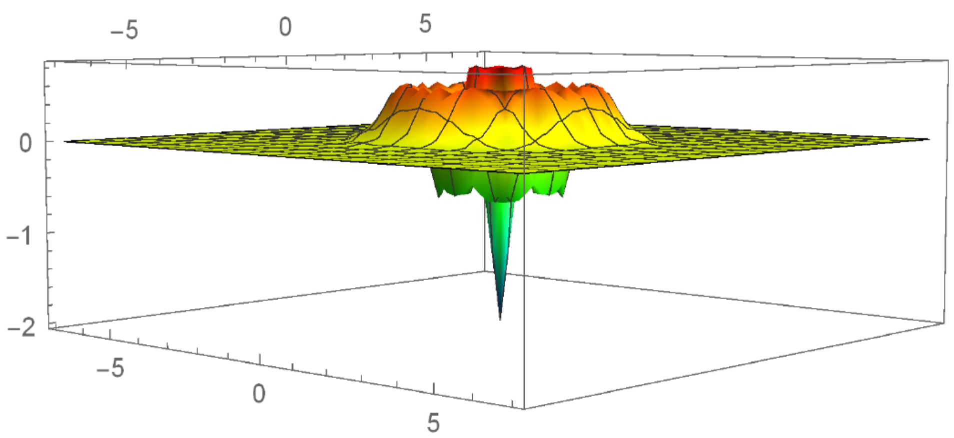
$$W(x,p) = \int d\xi e^{ip\xi} \psi^*(x-\xi/2)\psi(x+\xi/2)$$

$\psi(x)$: wave function

Ex. Harmonic Oscillator in 1D

$$W(q,p)^{(n)} = 2(-1)^n e^{-\frac{2H}{\hbar\omega}} L_n\left(\frac{4H_O}{\hbar\omega}\right)$$

$n = 3$



$$H_O = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

Wigner distribution in QCD

- Quark Wigner distribution

Belistky, Ji, Yuan (2004) , Ji (2003)

$$W_\Gamma(\vec{r},k) = \frac{1}{2}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \langle \vec{q}/2|\hat{\mathcal{W}}_\Gamma(0,k)|-\vec{q}/2\rangle$$

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int d^4\xi e^{ik\cdot\xi}\bar{\Psi}(\vec{r}-\xi/2)\Gamma\Psi(\vec{r}+\xi/2)\delta(\xi^+)2\pi$$

Gluon : $\bar{\Psi}(\vec{r}-\xi/2)\Gamma\Psi(\vec{r}+\xi/2) \to F^{+\nu}(\vec{r}-\xi/2)F_\nu^+(\vec{r}+\xi/2)$

- Wigner distribution at high energy

Lorce, Pasquini (2011)

Using infinite Momentum Frame

$$W_\Gamma(\boldsymbol{b}_\perp,k) = \frac{1}{2}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot\boldsymbol{b}_\perp} \langle \Delta_\perp/2|\hat{\mathcal{W}}_\Gamma(0,k)|-\Delta_\perp/2\rangle$$

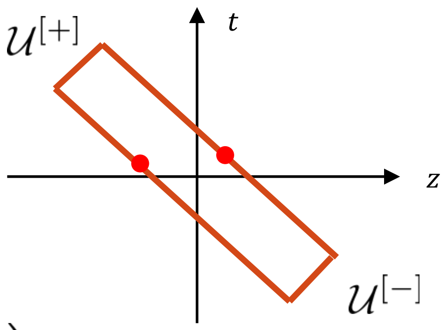
The gluon Wigner distribution

Operator definition of the gluon Wigner distribution

$$xW^g(x,\mathbf{k}_\perp,\mathbf{b}_\perp)=\int\frac{d^2\Delta_\perp}{(2\pi)^2}e^{-i\Delta_\perp\cdot\mathbf{b}_\perp}\int\frac{d\xi^-d^2\xi_\perp}{(2\pi)^3P^+}e^{-ixP^+\xi^--i\mathbf{k}_\perp\cdot\xi_\perp}$$
$$\times\left\langle P+\frac{\Delta_\perp}{2}\right|\mathrm{Tr}\left[F^{+j}\left(\mathbf{b}_\perp+\frac{\xi}{2}\right)\mathcal{U}^{[\pm]\dagger}F^{+j}\left(\mathbf{b}_\perp-\frac{\xi}{2}\right)\mathcal{U}^{[\pm]}\right]\left|P-\frac{\Delta_\perp}{2}\right\rangle$$

$$\mathcal{U}^{[-]}:=U[0,-\infty;0]U[-\infty,\xi^-;\xi_\perp]$$

$$\mathcal{U}^{[+]}:=U[0,\infty;0]U[\infty,\xi^-;\xi_\perp]$$

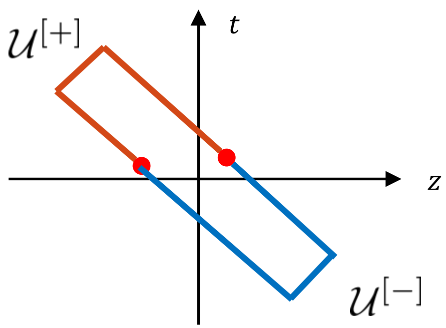


$$U[x_1^-,x_2^-;\boldsymbol{x}_\perp]\equiv\mathcal{P}\exp\left(ig\int_{x_1^-}^{x_2^-}dx^-T^cA_c^+(x^-,\boldsymbol{x}_\perp)\right)$$:Wilson line

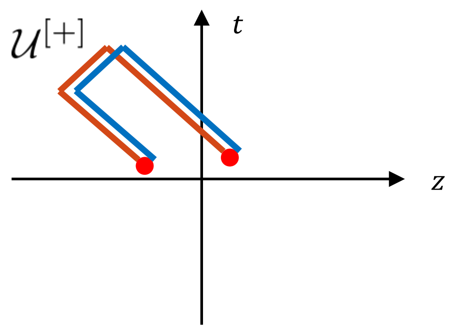
The gluon Wigner distribution

Operator definition of the gluon Wigner distribution

$$xW^g(x,\mathbf{k}_\perp,\mathbf{b}_\perp)=\int\frac{d^2\Delta_\perp}{(2\pi)^2}e^{-i\Delta_\perp\cdot\mathbf{b}_\perp}\int\frac{d\xi^-d^2\xi_\perp}{(2\pi)^3P^+}e^{-ixP^+\xi^- -i\mathbf{k}_\perp\cdot\xi_\perp}$$
$$\times\left\langle P+\frac{\Delta_\perp}{2}\right|\mathrm{Tr}\left[F^{+j}\left(\mathbf{b}_\perp+\frac{\xi}{2}\right)\mathcal{U}^{[\pm]\dagger}F^{+j}\left(\mathbf{b}_\perp-\frac{\xi}{2}\right)\mathcal{U}^{[\pm]}\right]\left|P-\frac{\Delta_\perp}{2}\right\rangle$$



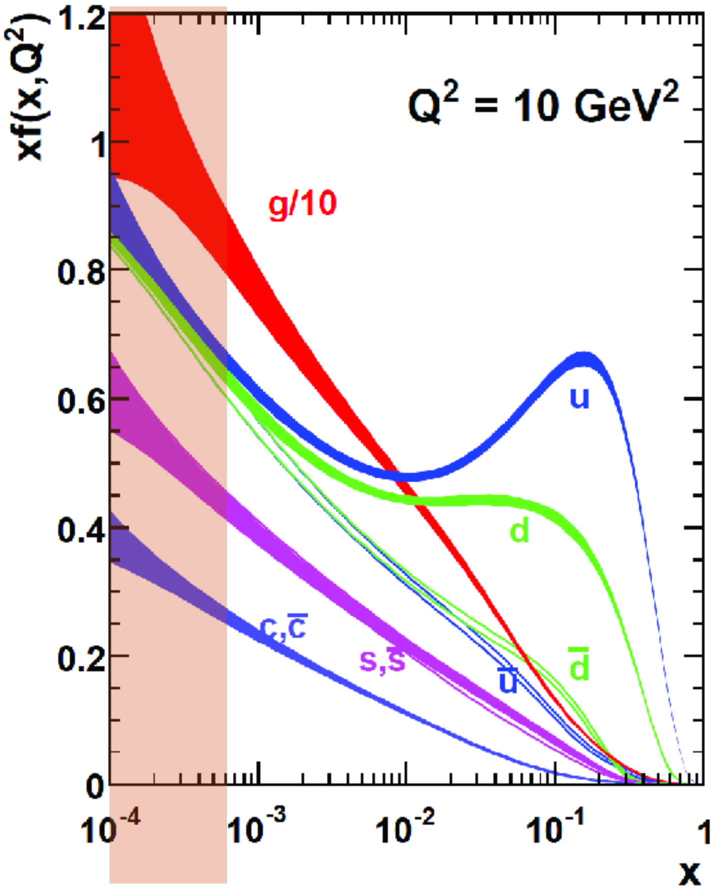
Dipole type



Weizsäcker-Williams type

Small x region

Small x



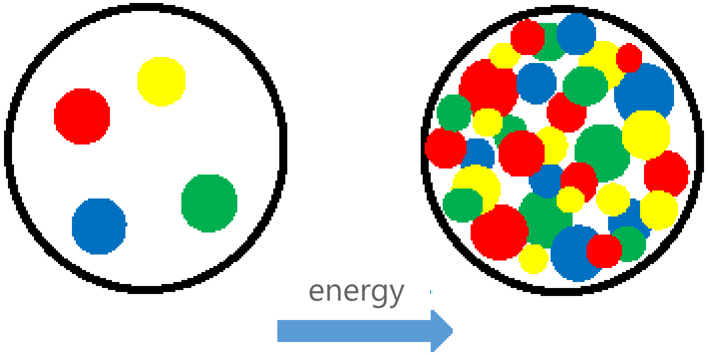
Martin, A.D. et al. Eur.Phys.J. C63 (2009) 189-285

Gluon saturation

Increase the CM energy (x becomes small)

Number of partons increase

The number of partons become saturate because of the gluon recombination process



The gluon Wigner distribution

Y. Hatta, B. W. Xiao, F. Yuan Phys. Rev. Lett. 116, 202301 (2016)

- Small x approximation

$$x \ll 1 \to e^{-ixP^+\xi^-} \approx 1$$

The gluon Wigner distribution at small x

$$xW(x,\boldsymbol{k},\boldsymbol{b}) = \frac{2N_c}{\alpha_s}\int \frac{d^2\boldsymbol{r}}{(2\pi)^2} e^{i\boldsymbol{k}\boldsymbol{r}} \left\langle \frac{1}{N_c} \mathrm{tr}(O_i^\dagger(\boldsymbol{b}+\boldsymbol{r}/2)O^i(\boldsymbol{b}-\boldsymbol{r}/2)) \right\rangle$$

Dipole type → $O_i(\boldsymbol{x}) = \partial_i U(\boldsymbol{x})$

Weizsäcker-Williams type → $O_i(\boldsymbol{x}) = U^\dagger(\boldsymbol{x})\partial_i U(\boldsymbol{x})$

$$U(x_\perp) = U[-\infty,\infty;x_\perp]$$

rapidity evolution

JIMWLK equation for the Wilson lines

$$U_{\boldsymbol{x}}(Y+dY) = \exp\left\{-i\frac{\sqrt{\alpha_s dY}}{\pi}\int d^2\boldsymbol{z}\ \boldsymbol{K}_{\boldsymbol{x}-\boldsymbol{z}}\cdot\left[U_{\boldsymbol{z}}(Y)\ \boldsymbol{\xi}_{\boldsymbol{z}}(Y)\ U^\dagger_{\boldsymbol{z}}(Y)\right]\right\}$$

$$\times U_{\boldsymbol{x}}(Y)\ \exp\left\{i\frac{\sqrt{\alpha_s dY}}{\pi}\int d^2\boldsymbol{z}\ \boldsymbol{K}_{\boldsymbol{x}-\boldsymbol{z}}\cdot\boldsymbol{\xi}_{\boldsymbol{z}}(Y)\right\}$$

$Y = \log(1/x)$

Integral kernel $\boldsymbol{K}_{\boldsymbol{x}} = m|\boldsymbol{x}|K_1(m|\boldsymbol{x}|)\frac{\boldsymbol{x}}{\boldsymbol{x}^2}$

m: infrared cutoff ~ 0.2 GeV

Gaussian noise $\langle\xi^a_{\boldsymbol{x},i}(ndY)\xi^b_{\boldsymbol{y},j}(mdY)\rangle = \delta_{ab}\delta_{ij}\delta^2(\boldsymbol{x}-\boldsymbol{y})\delta_{nm}$

H. Mäntysaari and B. Schenke, Phys. Rev. D 98, no. 3, 034013 (2018)

Initial condition

L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994)
H. Mäntysaari and B. Schenke, Phys. Rev. D 98, no. 3, 034013 (2018)

McLerran Venugopalan model

Color source -> Gaussian distribution

$$g^2\langle\rho_a^i(\boldsymbol{x})\rho_b^j(\boldsymbol{y})\rangle = \frac{(g^2\mu_0)^2}{N_0} S\left(\frac{\boldsymbol{x}+\boldsymbol{y}}{2}\right)\delta_{ab}\delta_{ij}\delta^2(\boldsymbol{x}-\boldsymbol{y})$$

with **sub-nucleonic effects**

$$S(\boldsymbol{x}) = \frac{1}{2\pi R_{CQ}^2}\sum_{n=1}^{N_{CQ}}\exp\left(-\frac{1}{2R_{CQ}^2}(\boldsymbol{x}-\boldsymbol{x}_{CQ}^{(n)})^2\right) \qquad \langle\boldsymbol{x}_{CQ}^2\rangle = R_p^2$$

Round nucleon

$$S(\boldsymbol{x}) = \frac{1}{2\pi R_p^2}\exp\left[-\frac{\boldsymbol{x}^2}{2R_p^2}\right]$$

Initial condition

Wilson lines for the MV model

$$U_0(\boldsymbol{x}) = \prod_{i=1}^{N_0} \exp\left(-ig\frac{\rho_a^i(\boldsymbol{x})t^a}{\nabla^2 + m^2}\right)$$

m: infrared cutoff ~ 0.2 GeV

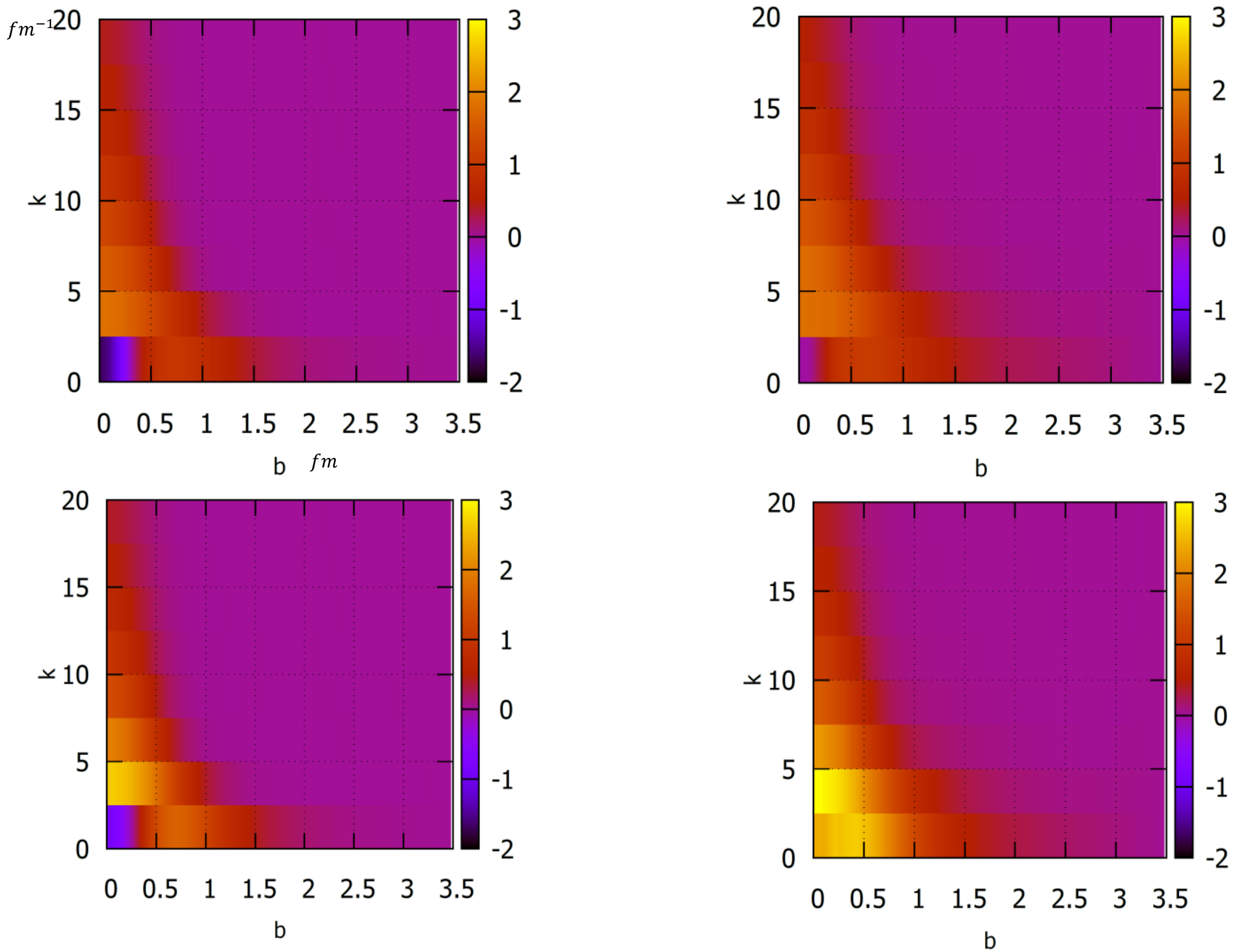
Solving the Poisson equation

Numerical simulation

Including sub-nucleonic effects

DP type

WW type



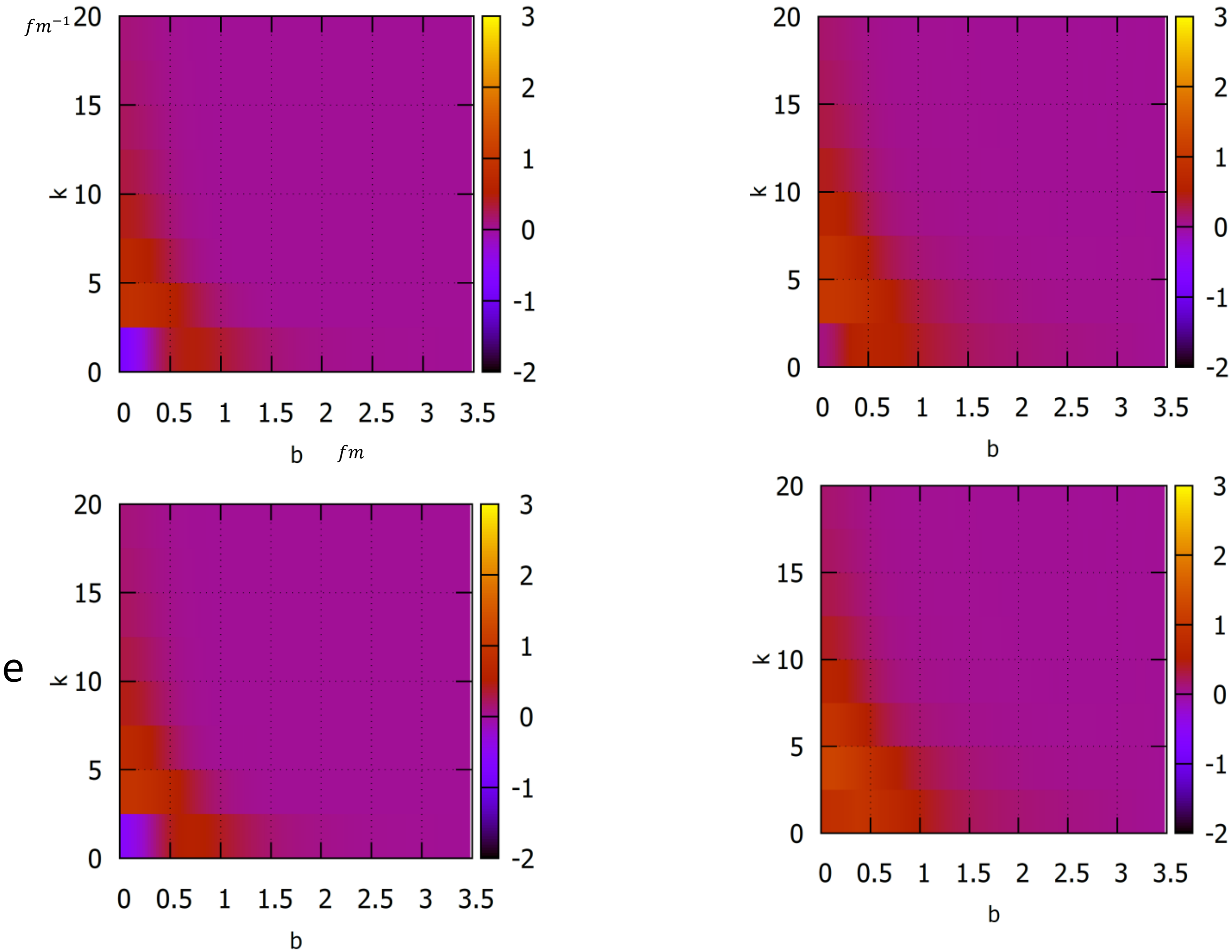
Y=0

Y=10

Round proton results

DP type

WW type



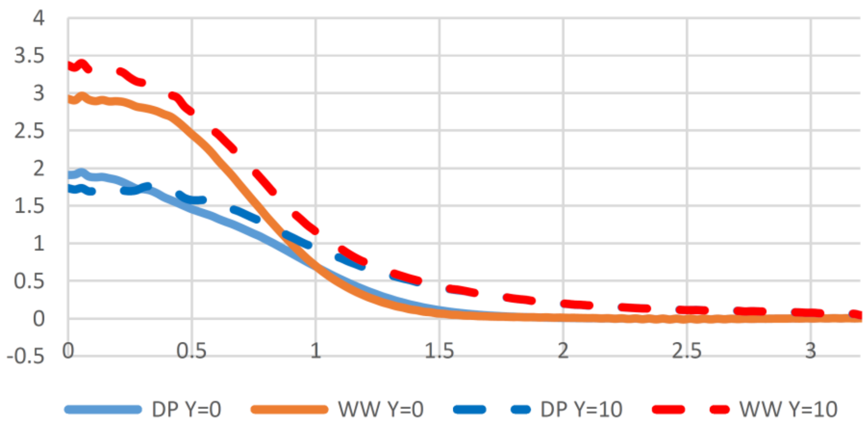
Y=0

Y=10

Fix the momentum

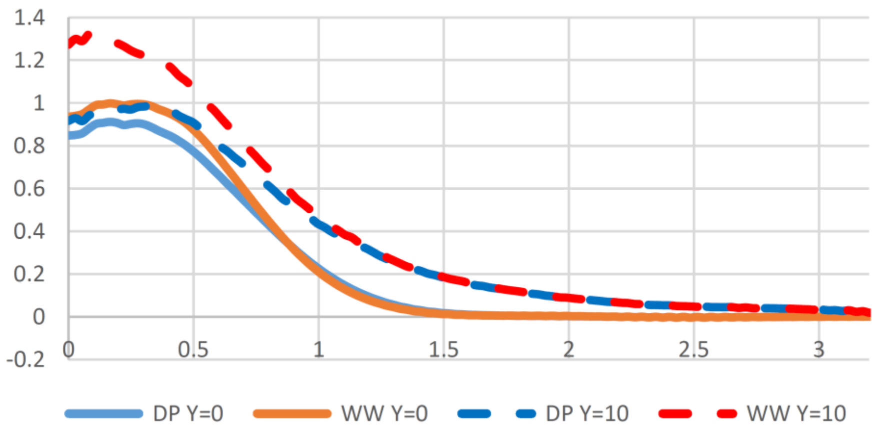
With sub-nucleonic fluctuation

$k = 2.5fm^{-1}$



Round proton

$k = 2.5fm^{-1}$



Summary

We investigate the two gluon Wigner distribution functions.

The DP type Wigner distribution is smaller than the WW type at small b.

The angular independent part of the Wigner distributions have no big different properties between the Wigner functions with sub-nucleonic effects and round ones.

Parameter set

H. Mäntysaari and B. Schenke, Phys. Rev. D 98, no. 3, 034013 (2018)

With sub-nucleonic effects

$g^2\mu$	R_p	R_{CQ}	α_s
$5\sqrt{2.8}\text{fm}^{-1}$	$\frac{\sqrt{3.2}}{5}\text{fm}$	$\frac{\sqrt{0.5}}{5}\text{fm}$	0.21

Round nucleon

$g^2\mu$	R_p	α_s
$5\sqrt{0.8}\text{fm}^{-1}$	$\frac{\sqrt{2.1}}{5}\text{fm}$	0.21