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Light meson masses using AdS/QCD modified soft wall model

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Based on Phys. Rev. D **96**, no. 10, 106002 (2017) and work in progress

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Outline

AdS/CFT Correspondence

MAGOO, 1998.
Witten, 1998.

A possible definition...

A strongly coupled QFT living in $d+1$ dimensions (boundary) is equivalent to a weakly coupled gravity theory living in $d+2$ dimensions (bulk).

Implications

- Space–time data encoded into QFT (V. Hubbeny).
- Saddle point approx.: Classical Gravity can be used to explore non-pertubative QFT. (MAGOO, 1999).
- Every field ϕ in the bulk is a *Schwinger source* of an operator $\mathcal{O}$ at the boundary.
- Bulk physics is equivalent to boundary physics.

Summarizing:

$$e^{W[\phi]}\Big|_{\text{Boundary}} = \langle e^{\int \phi\, \mathcal{O}} \rangle\Big|_{\text{QFT}} \tag{1}$$

With $W[\phi]$ the functional generator for the n-point functions of $\mathcal{O}$:

$$\langle \mathcal{O} \ldots \mathcal{O} \rangle = \frac{\delta^n W}{\delta \phi^n}\Bigg|_{\phi=0,\text{ evaluated at the boundary}} \tag{2}$$

Holographic Algorithm

- Define a gravitational action for the bulk physics.
- Solve the equations of motion and obtain the on–shell boundary action.
- Use (2) to obtain the n-point function.
- Find the map between the observables in the QFT and the bulk quantities (i.e. the holographic dictionary).

Holographic Dictionary

Boundary Operator	Bulk Field
Stress Tensor $T_{\mu\nu}$	Metric g_{MN}
Global Current J_μ	Maxwell Field A_M
Bosonic Operator	Klein–Gordon field
Fermionic Operator	Dirac field
Scaling dimension operator	Mass of the field
Global symmetry	Local Symmetry

AdS/QCD soft Wall Model *Karch et. al. 2005.*

It is a phenomenological model introduced as a form to include confinement in holography by means of a static dilaton field $\Phi(z) = c^2 z^2$. This dilaton profile breaks softly the conformal symmetry by introducing the energy scale c.
The model is defined as follows

$$I_{\mathrm{SW}} = \frac{1}{k^2} \int d^5 x \sqrt{-g}\, e^{-c^2 z^2} \mathcal{L}_{\mathrm{Hadron}} \tag{3}$$

As a consequence of the dilaton, we obtain linear Regge trajectories with the excitation number given by

$$M_n^2 = A\, c^2 \,(n + B)\,, \tag{4}$$

where n is the excitation number, A and B are specific numbers given by $\mathcal{L}_{\mathrm{Hadron}}$for each kind of particle defined in the action.

Modified Soft Wall Model with UV cutoff

N. R. F. Braga, M. A. Martin, S. Diles.
EPJ C 76(11):598, 2016

Consider the AdS_5 geometry cut at some UV scale z_0:

$$dS^2 = g_{MN}\, dx^M\, dx^N = \frac{R^2}{z^2}\left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right]\Theta\left(z - z_0\right), \qquad (5)$$

where $\Theta(x)$ is the Heaviside step function and z_0 is the locus of the boundary.
As in the SWM, hadrons are modeled by an action principle that includes a static quadratic dilaton field

$$I_{\text{Modified}} = \frac{1}{k^2}\int d^5x\,\sqrt{-g}\,e^{-\kappa^2 z^2}\,\mathcal{L}_{\text{Hadron}} \qquad (6)$$

This model has two energy scales: κ and z_0. These two parameters will define the Regge trajectories.

How do the mesons emerge in this model?

According to the Field/Operator duality, operators that create mesons should be dual to bulk field living on AdS_5. Thus

- Scalar states will be generated by scalar bulk field.
- Vector states will be generated by vector bulk fields.

Action for the bulk fields

The associated action reads

$$I = I_{\text{Scalar}} + I_{\text{Vector}}, \tag{7}$$

with

$$\begin{aligned} I_{\text{Scalar}} &= -\frac{1}{2\,g_S^2}\int d^5x\,\sqrt{-g}\,e^{-\kappa^2 z^2}\left[g^{MN}\,\partial_M S\,\partial_N S + M_5^2\,S^2\right], \\ I_{\text{Vector}} &= -\frac{1}{2\,g_V^2}\int d^5x\,\sqrt{-g}\,e^{-\kappa^2 z^2}\left[\frac{1}{2}F_{MN}\,F^{MN} + \tilde{M}_5^2\,g^{MN}\,A_M\,A_N\right], \end{aligned}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength related to the U(1) field $A_M(z, x^\mu)$, the coupling $g_{S\,(V)}$ is a constant that fixes units on the scalar (vector) sector, and M_5 ($\tilde{M}_5$) is the bulk mass that fixes the hadronic identity for scalar (vector) states.

How do we obtain meson masses?

Algorithm

1. Define an action principle for the objects dual to mesons (or any other hadronic state).
2. Solve the equation of motion for these objects.
3. Obtain the On-Shell Boundary action.
4. Construct the holographic 2-point function from these solutions and boundary action.

$$\Pi\left(q^2\right) = \sum \frac{f_n^2}{q^2 - m_n^2 + i\,\epsilon}. \qquad (8)$$

5. Calculate the poles of the 2-point function, that define the mass spectrum.
6. Compare to experimental results.

What does defines the meson identity?

Mesons have dimension $\Delta = 3$. This dimension, according to AdS/CFT dictionary, is dual to the bulk mass of each (vector or scalar) field:

- Scalar: $M_5^2\, R^2 = \Delta\,(\Delta - 4)$.
- Vector: $M_5^2\, R^2 = \Delta\,(\Delta - 4) + 3$.

Thus, fixing the value of Δ will give us the meson identity.

Pseudoscalar and axial mesons

Holographically, the difference between mesons and pseudoscalar (or axial) mesons is the parity behavior. Mesons are invariant under parity transformations. This fact suggests the idea of redefine the dimension Δ as

$$\Delta = \Delta_{\text{Phys}} + \Delta_P \tag{9}$$

where:

- $\Delta_{\text{Phys}} = 3$ for mesons.
- $\Delta_P = 0$ for parity even states, as the f_0 scalar trajectory or the ρ trajectory in the vector mesons.
- $\Delta_P = -1$ defines parity odd states: the η trajectory in the pseudoscalar sector and the a_1 trajectory in the vector axial sector.

Summary of meson identity

Meson Identity	Δ_P	$M_5^2\, R^2$
Scalar meson	0	-3
Vector meson	0	0
Pseudoscalar meson	-1	-4
Axial vector meson	-1	-1

where

- Scalar: $M_5^2\, R^2 = (\Delta_{\text{Phys}} + \Delta_P)\left[(\Delta_{\text{Phys}} + \Delta_P) - 4\right]$.
- Vector: $M_5^2\, R^2 = (\Delta_{\text{Phys}} + \Delta_P)\left[(\Delta_{\text{Phys}} + \Delta_P) - 4\right] + 3$.

Parameters

- z_0: related to the natureness of the strong interaction. Flavor independent.
- κ: related to the mass of the constituents. Flavor dependent.
- Δ_P: Parity of the meson states.

Results for f_0 trajectory

S. Cortes, M. A.M. Contreras, J. R. Roldan.
Phys. Rev. D **96**, no. 10, 106002 (2017).

f_0	M_{th} (GeV)	M_{exp} (GeV)	$\%M$
$f_0(980)$	1.070	0.99	7.46
$f_0(1370)$	1.284	1.370	5.11
$f_0(1500)$	1.487	1.504	1.13
$f_0(1710)$	1.674	1.723	2.93
$f_0(2020)$	1.846	1.992	7.94
$f_0(2100)$	2.153	2.101	2.39
$f_0(2200)$	2.292	2.189	4.49
$f_0(2330)$	2.424	2.314	4.52

Table 1: Mass spectrum for f_0 scalar resonances with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}. Experimental values for the masses are read from PDG 2016.

Results for ρ trajectory

S. Cortes, M. A. M. Contreras, J. R. Roldan. Phys. Rev. D **96**, no. 10, 106002 (2017).

ρ	M_{th} (GeV)	M_{exp} (GeV)	$\%M$
$\rho(775)$	0.975	0.775	20.53
$\rho(1450)$	1.455	1.465	0.66
$\rho(1570)$	1.652	1.570	4.96
$\rho(1700)$	1.829	1.720	5.97
$\rho(1900)$	1.992	1.909	4.15
$\rho(2150)$	2.142	2.153	0.50

Table 2: Mass spectrum for ρ vector mesons with $\kappa = 0.45$ GeV and $z_0 = 5$ GeV^{-1}. Experimental values are obtained from PDG 2016.

Results for η trajectory

η mesons	M_{Exp} (MeV)	M_{Th} (MeV)	$\%M$
$\eta(550)$	547.86 ± 0.017	975.25	43.8
$\eta(1295)$	1294 ± 4	1233.6	4.90
$\eta(1405)$	1408.8 ± 1.8	1455.3	3.18
$\eta(1475)$	1476 ± 4	1652.9	10.65
$\eta(1760)$	1760 ± 11	1829.2	3.78
$\eta(2225)$	2216 ± 21	1992.7	11.3

Table 3: Mass spectrum for η pseudoscalar mesons with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}. Experimental values are obtained from PDG 2018. For the $\eta(1760)$ and $\eta(2225)$ states, their masses are taken from Wang et. al (2017).

Results for a_1 trajectory

a_1 mesons	M_{Exp} (MeV)	M_{Th} (MeV)	$\%M$
$a_1(1260)$	1230 ± 40	808.96	52.2
$a_1(1420)$	$1414^{\pm 15}_{\pm 13}$	1114.7	26.9
$a_1(1640)$	1654 ± 19	1351.3	22.4

Table 4: Mass spectrum for a_1 axial mesons with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}. Experimental values are obtained from PDG 2018. For the $a_1(1420)$ state, its mass is read from Adolph (2015).

Conclusions and outlook

Conclusions

- It was possible to fit 23 states, (6 pseudoscalars, 3 axials, 8 scalars and 6 vectors mesons) with 3 parameters: κ, z_0 and Δ_P.
- The RMS error for this fitting was close to 21.5%.
- Chiral symmetry was considered broken in an implicit form. We do not consider a specific mechanism for SB.
- Axial vector mesons were not well fitted.
- In the case of pseudoscalar mesons, it is possible that κ should be modified since the chiral symmetry is broken.

Outlook

- To introduce explicitly the chiral symmetry effects.
- To extend these ideas to other hadronic states.
- To explore the finite temperature and finite density realms.

Thank you!