Resonance effects in bound states interaction kernels

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Outline

Motivation

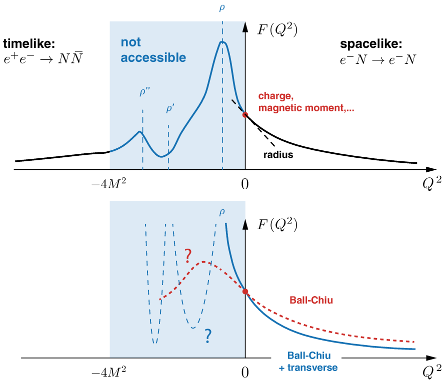
Most hadrons are resonances and they decay

- $\rho \to \pi\pi$
- $\Delta \to N\pi$

A complete description of hadrons should incorporate these properties.

The truncations that are currently employed in functional methods are not yet capable of doing so.

- In order to describe form factors in the timelike region we need to implement $\rho \to \pi\pi$.[1]
- The quark photon vertex carries the resonance dynamics.



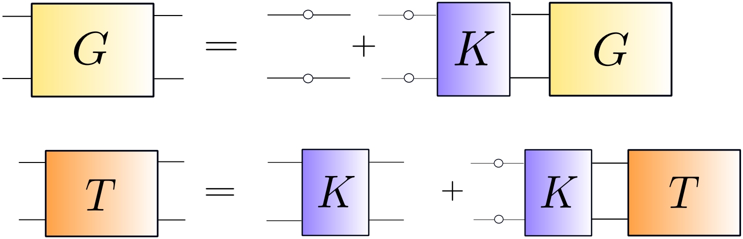
- In rainbow ladder hadrons are stable bound states that do not decay.

[1] Eichmann, Gernot. Probing nucleons with photons at the quark level arXiv:1404.4149 [nucl-th]

Bethe Salpeter equations

Properties of bound states are encoded by a set of n-point Green's functions.

- Mesons as a bound states of $q\bar{q} \longrightarrow$ four point functions.
- Baryons as a bound states of $qqq \longrightarrow$ six point functions.



Bethe-Salpeter Equations

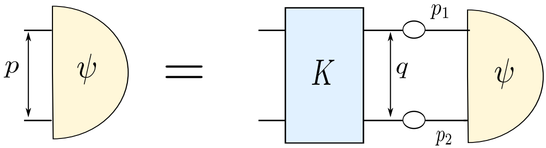
If a system of n-particles form a bound state a pole appears in the Green function for $P^2 = -M^2 + iM\Gamma$,

$$G \to \frac{\Psi\bar{\Psi}}{P^2 + M^2 - iM\Gamma},$$

with Ψ the Bethe-Salpeter amplitude.

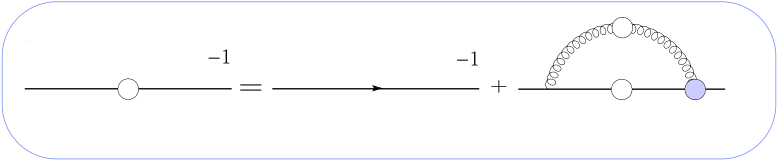
$$\Psi = KG_0\Psi$$

Diagrammatic BSE



Dyson-Schwinger equations (DSEs)

Quark propagator DSE



- In most phenomenological applications, the quark DSE has been truncated.

We use Rainbow ladder (RL) truncation.

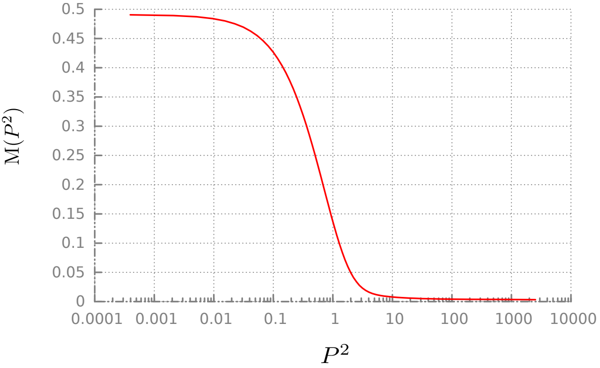
- Preserve chiral symmetry,
- Quark-gluon vertex $\Gamma^{\mu} \sim \gamma^{\mu}$
- Collect the dressings in an effective coupling $\alpha(k^2)$.
- One frequently used effective interaction is the Maris-Tandy model[2]

[2] P. Maris and C. Tandy, Phys. Rev. C60 (1999) 055214

Dressed propagator is given by

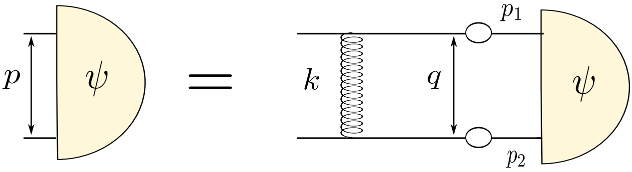
$$S(p) = \frac{1}{A(p^2)} \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)} = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2);$$

Renormalization conditions: $A(\mu^2) = 1, \quad M(\mu^2) = m_q, \quad \mu = 19 GeV$



- The quark mass function encodes dynamical chiral symmetry breaking and displays the transition from constituent quark mass to current quark mass.

We need to specify the interaction kernel. We use the BSE with rainbow ladder truncation



- The resulting BSE kernel is a gluon exchange.
- Can be solved numerically.

$$\Psi(p,P) = \int \frac{d^4q}{(2\pi)^4} \gamma^\mu S(p_1) \Psi(q,P) S(p_2) \gamma^\nu D_{\mu\nu}(k)$$

- The BSE is a parametric eigenvalue equation with discrete solutions at $P^2 = -M_n^2$

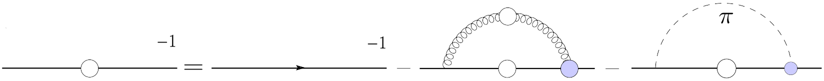
- Rainbow ladder truncation + Maris-Tandy model works very well for ground states. [3] [4]
- In this truncation, hadrons are stable bound states and they do not decay.
- Nevertheless, most hadrons are resonances and they do decay. In order to get a complete description of hadrons we must incorporate these features.

[3] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer. Baryons as relativistic three quark bound states. (arXiv:1606.09602 [hep-ph])

[4] T. Hilger, M. Gmez-Rocha, A. Krassnigg, W. Lucha. Aspects of open-flavour mesons in a comprehensive DSBSE study. arXiv:1702.06262 (2017)

The t-channel pion exchange

We will introduce explicit pionic degrees of freedom in the system, in addition to quarks and gluons. Besides the gluon part of the quark DSE, an emission and absorption of the pion appears[5,6]



- The resulting DSE for the quark propagator is given by

$$S^{-1}(p) = S^{-1}(p)^{RL} - 3\int\frac{d^4q}{(2\pi)^4}\left[Z_2\gamma_5 S(q)\Gamma_\pi\left(\frac{p+q}{2}, q-p\right) + Z_2\gamma_5 S(q)\Gamma_\pi\left(\frac{p+q}{2}, p-q\right)\right]\frac{D_\pi(k)}{2}$$

[5] H. Sanchis-Alepuz, C. S. Fischer, S. Kubrak, Phys. Lett. B733,151 (2014)

[6] C. S. Fischer, R. Williams, Phys. Rev. D78, 074006 (2008)

The Bethe Salpeter vertex of the pion can be represented by

$$\Gamma^i_\pi(p,P) = \tau^i\gamma_5(E_\pi(p,P) - i\not{P}F_\pi(p,P) - i\not{p}p\cdot PG_\pi(p,P) - \left[\not{P},\not{p}\right] H_\pi(p,P))$$

with four independent dressing function $E_\pi, F_\pi, G_\pi, H_\pi$.

$$\begin{aligned} K^{pion}_{tu,sr}(q,p;P) &= \frac{1}{4}[\Gamma^j_\pi]_{ru}\left(\frac{p+q-P}{2};p-q\right)[Z_2\tau^j\gamma^5]_{ts}D_\pi(p-q) \\ &+ \frac{1}{4}[\Gamma^j_\pi]_{ru}\left(\frac{p+q-P}{2};q-p\right)[Z_2\tau^j\gamma^5]_{ts}D_\pi(p-q) \\ &+ \frac{1}{4}[\Gamma^j_\pi]_{ts}\left(\frac{p+q-P}{2};p-q\right)[Z_2\tau^j\gamma^5]_{ru}D_\pi(p-q) \\ &+ \frac{1}{4}[\Gamma^j_\pi]_{ts}\left(\frac{p+q-P}{2};q-p\right)[Z_2\tau^j\gamma^5]_{ru}D_\pi(p-q) \end{aligned}$$

- This is the only kernel that respects the axWTI for the general structure of the Bethe-Salpeter vertex [7]

[7] Beyond the rainbow: Effects from pion back-coupling. Fischer, Christian S. et al. Phys.Rev. D78 (2008) 074006 arXiv:0808.3372 [hep-ph]

- The corresponding Bethe-Salpeter equation that we need to solve is,

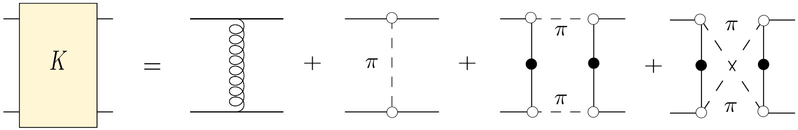
$$\Psi(p;P) = \int \frac{d^4k}{(2\pi)^4} \left[K^{RL}(p,q;P) + K^t(p,q;P)\right] \left[S(k_1)\Psi(k;P)S(k_2)\right]$$

We approximate the pion Bethe-Salpeter amplitude in the quark DSE and the kernel of the BSE by the leading amplitude in the chiral limit,

$$\Gamma_\pi^j(q;P) = \tau^j \gamma_5 \frac{B(p^2)}{f_\pi}$$

	RL	RL + pion	PDG
m_π(MeV)	140	137	138

s- and u-channel pion exchange



In the case of the rho meson the general structure of the Bethe-Salpeter vertex is more complicated [8],

$$\Gamma_\rho(p,P) = \gamma_T^\mu\left(F_1(p,P) - i\not{P}F_2(p,P) - i\not{p}(p\cdot P)F_3(p,P) - \left[\not{P},\not{p}\right]F_4(p,P)\right) + p_T^\mu\left(F_5(p,P) - i\not{P}F_6(p,P) - i\not{p}(p\cdot P)F_7(p,P) - \left[\not{P},\not{p}\right]F_8(p,P)\right)$$

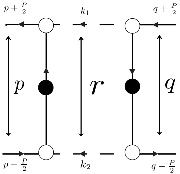
where,

$$\gamma_T^\mu = \gamma^\mu - P^\mu\frac{\not{P}}{P^2} - p_T^\mu\frac{\not{p}_T}{p_T^2},$$

$$p_T^\mu = p^\mu - P^\mu\frac{P\cdot q}{P^2}$$

[8] During the preparation of this work an analogous calculation has appeared, R. Williams, arXiv:1804.11161

- The kernel corresponding to the new contribution is given by,



$$\begin{aligned} K_{da,he}(q,p.r;P) &= \left[\frac{1}{2}[\Gamma^j_\pi]_{dc}\left(p+\frac{P}{4}-\frac{r}{4};\frac{P+r}{2}\right)S_{cb}\left(p-\frac{r}{2}\right)[\Gamma^j_\pi]_{ba}\left(p-\frac{P}{4}-\frac{r}{4};\frac{P-r}{2}\right)\right. \\ &\times \frac{1}{2}[\Gamma^j_\pi]_{hg}\left(q+\frac{P}{4}-\frac{r}{4};\frac{r-P}{2}\right)S_{gf}\left(q-\frac{r}{2}\right)[\Gamma^j_\pi]_{fe}\left(q+\frac{P}{4}-\frac{r}{4};-\frac{P+r}{2}\right) \\ &\times D_\pi\left(\frac{P+r}{2}\right)D_\pi\left(\frac{P-r}{2}\right) \end{aligned}$$

The new BSE to solve is,

$$\begin{aligned}\Psi(p;P) &= \frac{1}{(2\pi)^4}\int r^2 d^2r \int \sqrt{1-z_r}dz_r \int d\phi_r \int dy_r \Big[K^{RL}(p,q;P) + K^t(p,q;P) \\ &+ K^s(p,q,r;P) + K^u(p,q,r;P)\Big]\left[S(q_1)\Psi(q;P)S(q_2)\right]\end{aligned}$$

- The inclusion of the two kernels in BSE calculations is very challenging, as they have a non-trivial analytic structure.
- Knowing the position of the singularities allows to develop effective algorithms for numerical calculations
- For example, the kernel features now branch cuts corresponding to the virtual pions. Those are determined by the zeroes of the denominators and are parametrized by

$$\begin{aligned}y_1(P,zr) &= -m_\pi^2 - P^2 + 2z_r^2P^2 - 2\sqrt{-m_\pi^2 z_r^2 P^2 - z_r^2 P^4 + z_r^4 P^4} \\ y_2(P,zr) &= -m_\pi^2 - P^2 + 2z_r^2P^2 + 2\sqrt{-m_\pi^2 z_r^2 P^2 - z_r^2 P^4 + z_r^4 P^4}\end{aligned}$$

- In order to perform the integration over the relative momentum r to solve the BSE with the new contributions, first we need to deform the contour since the branch cut overlaps the real axis.

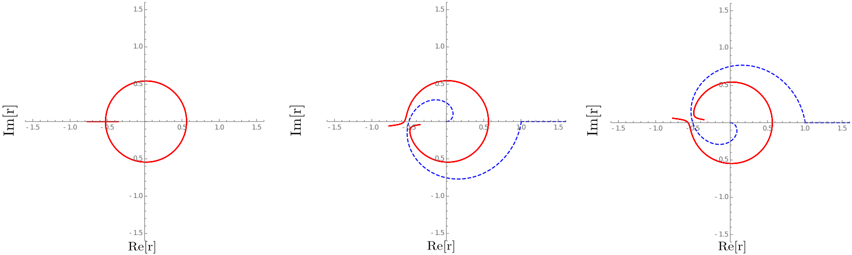
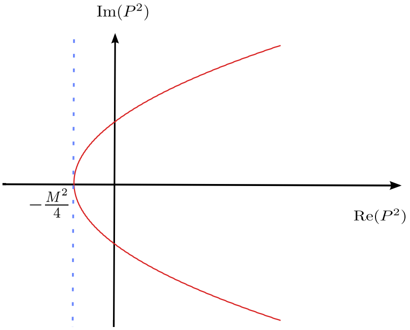


Figure : The solid line corresponds to the branch cuts due to the two pion propagators and the dotted line shows a possible integration path.

We work in euclidean space time.

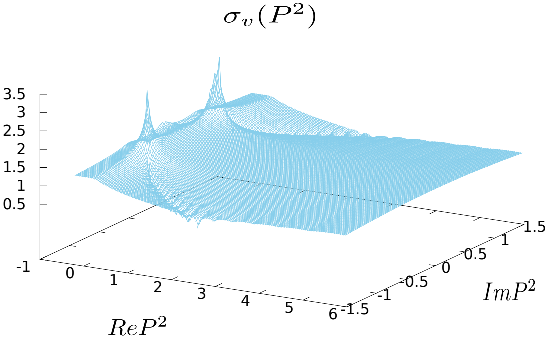
- In euclidean space to get $P^2 = -M^2$ for bound states we need to use complex momentum,
- Therefore we need the propagator for complex momentum.

The propagators are sampled within complex parabolas



The propagator carries a singularity structure

- Complex conjugate poles



- One way to parametrize the quark propagator is as a sum of complex-conjugate poles,

$$S(p) = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2),$$

$$\begin{aligned} \sigma_v &= \sum_i^n \left[\frac{\alpha_i}{p^2+m_i} + \frac{\alpha_i^*}{p^2+m_i^*}\right] \\ \sigma_s &= \sum_i^n \left[\frac{\beta_i}{p^2+m_i} + \frac{\beta_i^*}{p^2+m_i^*}\right], \end{aligned}$$

- The parameters m_i, α_i, β_i can be obtained by fitting the corresponding solution along the real axis of p^2.

- We plot the additional cuts from the quark propagator. Nevertheless, these cuts do not cross the real axis, and do not affect the integration contour.

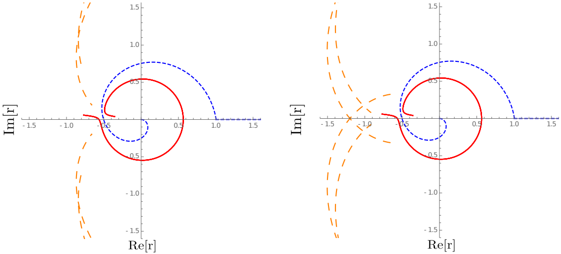
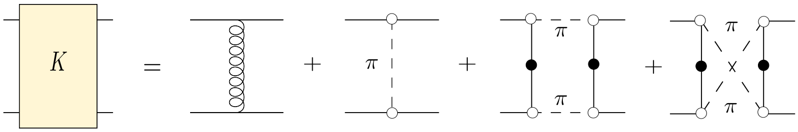


Figure : Branch cuts due to the quark propagators (dashed lines), for two different values in the relative momentum p.

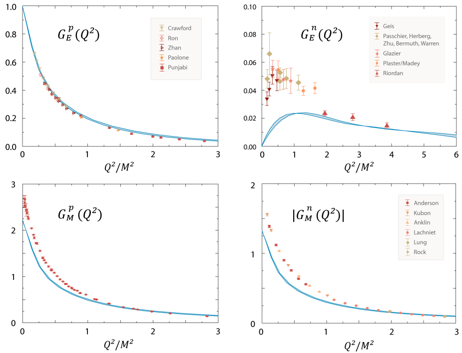
- Solving the homogeneous BSE including the different channels we get the following results,



	RL	RL + pion	RL + pion + decay	PDG
m_ρ (MeV)	740	720	645	776
Γ_ρ (MeV)			103	150

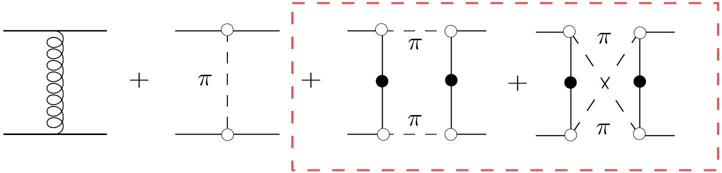
Resonance effects in meson FFs

- We want to explore the effect of pion contributions to meson form factors (FFs).
- The quark photon vertex is needed.
- Rainbow-ladder truncation is useful in describing FFs [9].

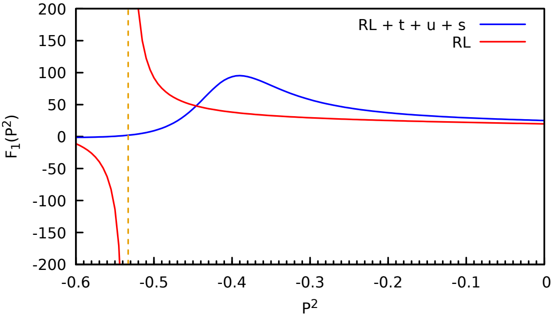


[9] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer. Baryons as relativistic three quark bound states. (arXiv:1606.09602 [hep-ph])

- Pion cloud effects are expected to play an important role in the low momentum behavior of form factors.
- We have to solve an inhomogeneous Bethe-Salpeter equation (BSE) for the vertex; it depends on the kernel where the truncation to rainbow-ladder is made.



- First we solve the inhomogeneous BSE including the s and u channels.
- We solve it for $P^2 > 0$ and then we extrapolate using Padé approximant.



Summary and Outlook

- In order to include the resonant character of bound states in BSE calculations, virtual decay mechanism must be included.
- The appearance of branch cuts entails that the integration contour must be deformed in order to avoid the crossing of the cuts.
- When we add the s and u channels the poles have an imaginary part and we can extract the width of the resonance.

Next step,

- Solve the inhomogeneous BSE for the quark photon vertex.
- Study meson form factor in space-like and time-like region.

Thank you!

Maris-Tandy model

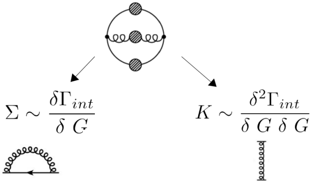
One frequently used effective interaction is the Maris-Tandy model[10]

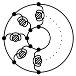
$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 \exp^{-\eta^2 \frac{k^2}{\Lambda^2}} + \alpha_{UV}$$

- Reproduces the one-loop QCD behaviour of the quark propagator at large momenta,
- Enough strength for dynamical chiral symmetry breaking to take place.
- Λ and η fitted to reproduce the decay constant from pion BSE.

[10] P. Maris and C. Tandy, Phys. Rev. C60 (1999) 055214

- A way of defining interaction kernels is using effective action or nPI techniques.





Pion lines appear as the resonant contribution to infinite loop terms

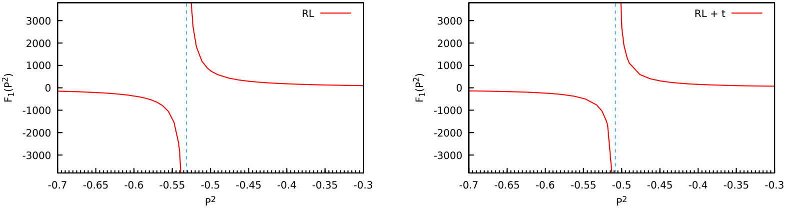
- The Bethe-Salpeter kernel can be constrained via the axial Ward-Takahashi identity (axWTI).

$$[\Sigma(p_+)\gamma_5 + \gamma_5\Sigma(p_-)]_{tu} = \int \frac{d^4k}{(2\pi)^4} K_{tu;sr}(p,k;P)\,[\gamma_5 S(k_-) + S(k_+)\gamma_5]_{rs}$$

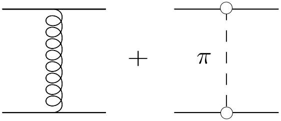
- The axWTI ensures that chiral symmetry is preserved in the chiral limit.

- First we solve an inhomogeneous BSE for the rho meson using RL truncation and including the t channel.

$$\Gamma^\mu = \Gamma_0^\mu + K G_0 \Gamma^\mu$$



- We have additional couplings when we include the pion exchange into DSE/BSE equations.
- In this case the photon also couples with the pion and with the pion vertex.



$J^\mu =$