Chiral symmetry breaking by monopole condensation

Aiichi Iwazaki
Nishogakusha University

Introduction

Lattice simulations show that the transition temperature T_{chiral}
of chiral symmetry breaking
is nearly equal to the de-confinement temperature T_{con}

Question $\boxed{T_{chiral} \approx T_{con} \text{ Accidental or not ?}}$

Answer $\boxed{\text{Not accidental}}$

Monopole condensation leads to the chiral symmetry breaking

Our assumptions

strong assumption

Quark confinement is caused by QCD monopole condensation

QCD monopoles are not present in weakly coupled QCD. But they have been discussed to play important roles in strong coupled QCD such as in quark gluon plasma near or below the transition temperature. J. Liao (2007) J. Xu, J. Liao and M. Gyulassy (2015)

weak assumption

Abelian dominance holds in low energy QCD

Relevant degrees of freedom; abelian gluon fields A_μ^3, A_μ^8 QCD monopoles, massless quarks.

Irrelevant degrees of freedom;
massive off diagonal gluons $A_\mu^a, \ a \neq 3,8$

Main result

Chiral asymmetric quark pair production takes place
in monopole condensed vacuum
when a color charge is put in the vacuum
(Schwinger mechanism with chiral symmetry breaking)

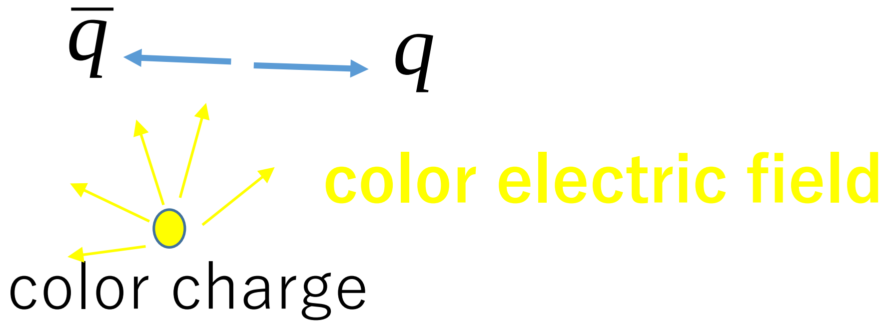
A. I. (2017)

$$\boxed{\left\langle \frac{dQ_5}{dt} \right\rangle = \pm\sqrt{\frac{4\pi c'^2 g^2 L}{\sqrt{2}} \langle Q_m \rho_m(0) \rangle}\,,}$$

Q_m ; magnetic charge

$Q_5 \equiv N_R - N_L$; chiral imbalance

pair production of massless quark



It vanishes in the vacuum such as

$$Q_m \,|\, no\ \ monopole \rangle = 0$$

Hereafter we consider quark monopole scattering in SU(3) gauge theory.
So, we explain QCD monopoles

QCD (SU(3)) monopoles

A. I. (2018)

three types of the monopoles

characterized by SU(3) root vectors

$$\vec{\varepsilon}_1 = (1,0), \quad \vec{\varepsilon}_2 = (-1/2, -\sqrt{3}/2), \quad \vec{\varepsilon}_3 = (-1/2, \sqrt{3}/2)$$

generated by the gauge fields

$$\vec{\varepsilon}_i \cdot \vec{A}_\mu \equiv \varepsilon_i^1 A_\mu^3 + \varepsilon_i^2 A_\mu^8 = A_\mu (Dirac\ monopole)$$

$$\vec{\varepsilon}_1 \cdot \vec{A}_\mu, \qquad \vec{\varepsilon}_2 \cdot \vec{A}_\mu, \qquad \vec{\varepsilon}_3 \cdot \vec{A}_\mu \qquad \vec{A}_\mu = (A_\mu^3, A_\mu^8)$$

coupled with **<u>three types of quark doublet</u>** $q_i ;\, i = 1, 2, 3$

$$q_1 = \begin{pmatrix} q_+ \\ q_- \\ 0 \end{pmatrix}, \qquad q_2 = \begin{pmatrix} q_+ \\ 0 \\ q_- \end{pmatrix}, \qquad q_3 = \begin{pmatrix} 0 \\ q_+ \\ q_- \end{pmatrix}$$

The important point

Chirality is not conserved in the quark monopole scattering

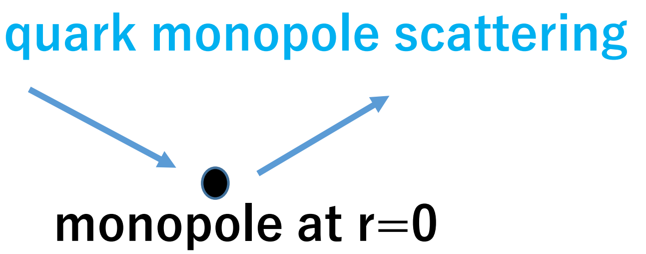
We consider the **quark doublet** with charges $q = \begin{pmatrix} +g \\ -g \end{pmatrix}$ scattering on a monopole with magnetic charge g_m

$$\left(g g_m = \frac{1}{2} \right)$$

The conserved angular momentum is given by

$$\vec{J} = \vec{L} + \vec{S} - g g_m \frac{\vec{r}}{r}$$

$\vec{L} = \vec{r} \times \vec{p}$ spin



from the angular momentum conservation, we find the flip of spin “S” generates the flip of charge “g” in the scattering

$$\Delta(\vec{r}\cdot\vec{S})-\Delta(gg_m)r=0$$

$$(\Delta(Q)\equiv Q_{final}-Q_{initial})$$

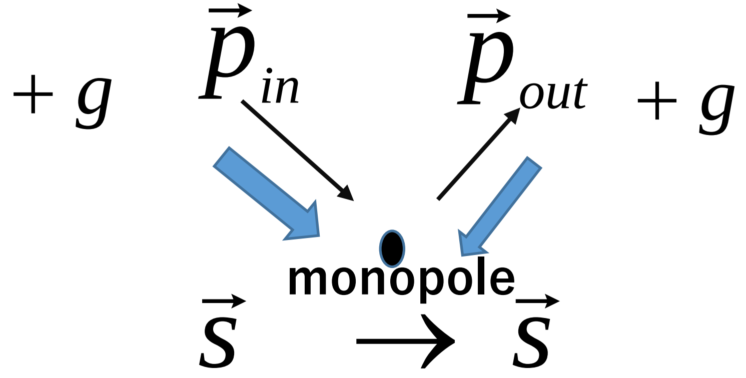
change of “Q” in the scattering

We note

$$\Delta(\vec{r}\cdot\vec{J})=\Delta\left(\vec{r}\cdot(\vec{L}+\vec{S}-gg_m\frac{\vec{r}}{r})\right)=0$$

chirality

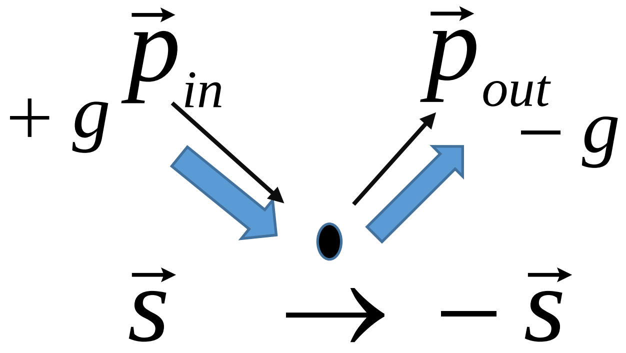
$$\frac{\vec{p}\cdot\vec{s}}{|\vec{p}\cdot\vec{s}|}$$



case 1 (charge conserved)

$$\Delta(\vec{r}\cdot\vec{S})=0 \longrightarrow \Delta(gg_m)r=0$$

chirality non conserved



case 2 (chirality conserved)

$$\Delta(\vec{r}\cdot\vec{S})\neq 0 \longrightarrow \Delta(gg_m)r\neq 0$$

charge non conserved

angular momentum conservation

$$\Delta(\vec{r} \cdot \vec{S}) - \Delta(gg_m)r = 0$$

In the quark monopole scattering
we have either

charge conservation (chirality non conserved)
or
chirality conservation (charge non conserved)

We may choose a boundary condition at the monopole, either chirality or charge conserved boundary condition. In any boundary conditions We see charge are conserved, but, chirality is not.

The charge conservation is strictly preserved in the gauge theory. Thus, the chirality is not conserved. The chiral symmetry is broken around a monopole

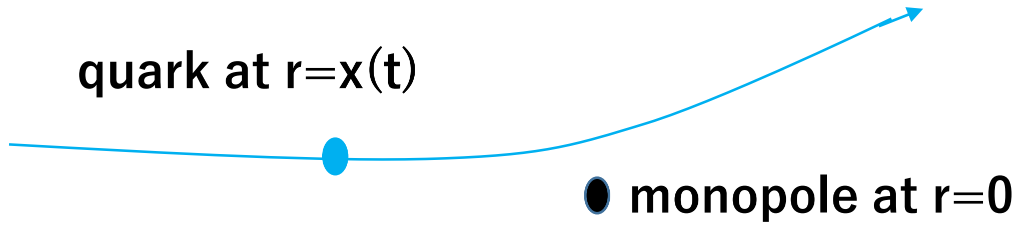
Kazama, Yang, Goldharber (1977)

Another point of view;

The chirality imbalance is produced by <u>chiral anomaly</u>

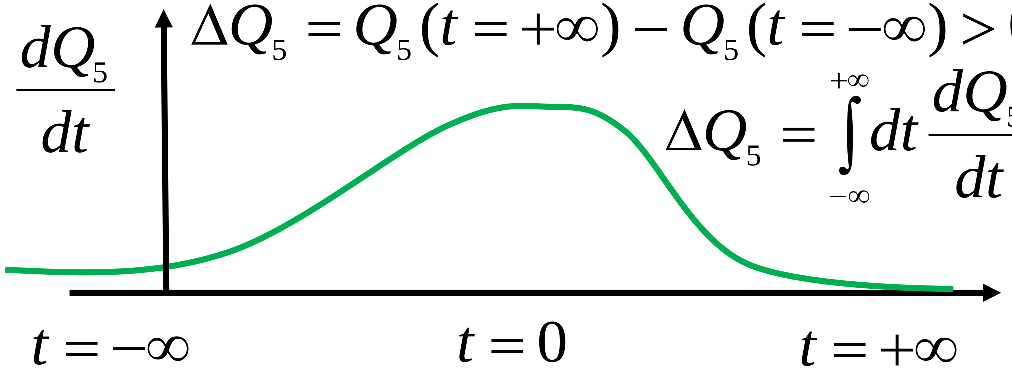
$$Q_5 \equiv N_R - N_L$$

$$\frac{dQ_5}{dt} = c\int d^3r\,\vec{E}\cdot\vec{B} = c\int d^3r\,\vec{E}\cdot\frac{g_m\vec{r}}{r^3} = c\int d^3r\,\overbrace{\frac{g(\vec{r}-\vec{x}(t))}{4\pi|\vec{r}-\vec{x}(t)|^3}}^{\vec{E}}\cdot\overbrace{\frac{g_m\vec{r}}{r^3}}^{\vec{B}} = \frac{cgg_m}{|\vec{x}(t)|}$$



$$\Delta Q_5 = Q_5(t=+\infty) - Q_5(t=-\infty) > 0$$

$$\Delta Q_5 = \int_{-\infty}^{+\infty} dt\,\frac{dQ_5}{dt}$$



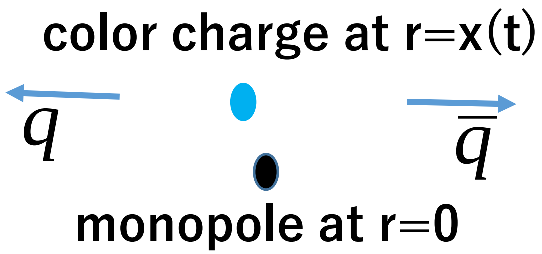
What does the chirality change $\Delta Q_5 > 0$ implies ?

Chiral asymmetric quark production

The anomaly equation describes chiral asymmetric production around a quark at $\vec{r} = \vec{x}(t)$ when a monopole is located at $\vec{r} = 0$ in a vacuum.

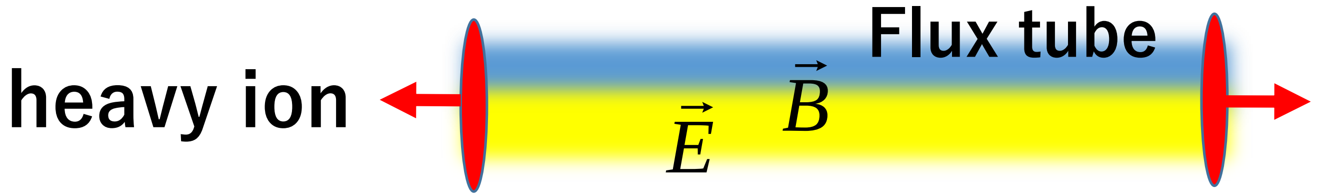
a chiral asymmetric quark pair production when both color charge and magnetic charge are present

$$\frac{dQ_5}{dt} = \frac{c g g_m}{|\vec{x}(t)|}$$



The pair production by the chiral anomaly is known to be coincident with the production by Schwinger mechanism N. Tanji (2010)

The chiral asymmetric pair production has been discussed in glasma decay. (The glasma is a flux tube of color electric and magnetic fields produced by high energy heavy ion collisions.) A. I. (2009), N. Tanji (2018)



$$\frac{dQ_5}{dt} = c\int d^3r\, \vec{E}\cdot\vec{B}$$

Chiral asymmetric quark pair production in monopole condensed vacuum

$$\boxed{\frac{dQ_5(x)}{dt} = c\sum_i \frac{g g_{m,i}}{4\pi|\vec{x}-\vec{r}_i|} = c\int d^3r\,\frac{g\rho_m(\vec{r})}{4\pi|\vec{x}-\vec{r}|},}$$

magnetic charge density

$$\rho_m(\vec{r}) = \sum_i g_{m,i}\delta(\vec{r}-\vec{r}_i)$$

Chiral asymmetric production when many monopoles are present with their density $\rho_m(\vec{r})$.

$$\frac{dQ_5(x)}{dt} = c\int d^3r\,\frac{g\rho_m(\vec{r})}{4\pi|\vec{x}-\vec{r}|},$$

We calculate $\left\langle \frac{dQ_5}{dt} \right\rangle$ when the monopoles condense in vacuum.

A. I. (2017)

We find

$$\left\langle \frac{dQ_5}{dt} \right\rangle \neq 0 \quad when \quad Q_m|vac\rangle \neq 0$$

monopole condensation

$$Q_m = \int d^3r\,\rho_m(\vec{r})$$ magnetic charge

$$\left\langle \frac{dQ_5}{dt} \right\rangle = 0 \quad when \quad Q_m|vac\rangle = 0$$

no monopole condensation

$$\frac{dQ_5(x)}{dt} = c\int d^3r \frac{g\rho_m(\vec{r})}{4\pi|\vec{x}-\vec{r}|}, \quad \rho_m(\vec{y})$$; field operator

$$\lim_{x\to\infty}\left\langle \frac{dQ_5(x)}{dt}\frac{dQ_5(0)}{dt}\right\rangle = \lim_{x\to\infty}\int d^3y d^3y' \frac{c^2g^2\langle\rho_m(\vec{y})\rho_m(\vec{y}')\rangle}{(4\pi)^2|\vec{x}-\vec{y}||\vec{y}'|} = \left\langle\frac{dQ_5}{dt}\right\rangle^2$$

$$= \lim_{x\to\infty}\int d^3y_+ d^3y_- \frac{2c'^2g^2f(\vec{y}_-)}{|\vec{y}_+ + 2\vec{y}_- - \sqrt{2}\vec{x}||\vec{y}_+|}$$

$$f(\vec{y}_-) = f\left(\frac{\vec{y}-\vec{y}'}{\sqrt{2}}\right) \equiv \langle\rho_m(\vec{y})\rho_m(\vec{y}')\rangle$$

$$\vec{y}_\pm \equiv \frac{\vec{y}\pm\vec{y}'}{\sqrt{2}}, \quad c' \equiv 4\pi c$$

$$\cong 8\pi c'^2 g^2 L^2 \lim_{x\to\infty}\int d^3y_- \frac{(1-\exp(-|2\vec{y}_- - \sqrt{2}\vec{x}|/L))f(\vec{y}_-)}{|2\vec{y}_- - \sqrt{2}\vec{x}|}$$

$$c'^2 \equiv c^2/(4\pi)^2$$

Cut off $L;\quad \int_0^L d|y_+|$

$$\cong 8\pi c'^2 g^2 L^2 \lim_{x\to\infty}\int d^3y_- \frac{|2\vec{y}_- - \sqrt{2}\vec{x}|/L\ f(\vec{y}_-)}{|2\vec{y}_- - \sqrt{2}\vec{x}|} = 8\pi c'^2 g^2 L \int d^3y_- f(\vec{y}_-)$$

$$= \frac{4\pi c'^2 g^2 L}{\sqrt{2}}\left\langle\int d^3y \rho_m(\vec{y})\rho_m(0)\right\rangle = \frac{4\pi c'^2 g^2 L}{\sqrt{2}}\langle Q_m\rho_m(0)\rangle$$

Magnetic charge

$$Q_m = \int d^3 r \rho_m(\vec{r})$$

$$\left\langle \frac{dQ_5}{dt} \right\rangle = \pm \sqrt{\frac{4\pi c'^2 g^2 L}{\sqrt{2}} \langle Q_m \rho_m(0) \rangle}$$

when we put a **color charge** in a vacuum

$$\left\langle \frac{dQ_5}{dt} \right\rangle \neq 0 \quad when \quad Q_m |vac\rangle \neq 0$$

in the vacuum with monopole condensation

$$\left\langle \frac{dQ_5}{dt} \right\rangle = 0 \quad when \quad Q_m |vac\rangle = 0$$

in the vacuum with no monopole condensation

$\boxed{\text{We find that the chiral symmetry is broken by the monopole condensation, which suppose to cause the quark confinement.}}$

Transition temperature of chiral symmetry breaking is equal to the de-confinement temperature

A comment

$$\left\langle \frac{dQ_5}{dt} \right\rangle = 0$$

when there is a pair of a **positive charge** and a **negative charge;**
totally neutral

general formula independent of the assumption of abelian dominance

a; index of color charge

external color charge put in a vacuum

chiral anomaly

$$\frac{dQ_5(\vec{x})}{dt} = c\int d^3r\, \vec{E}_a \cdot \vec{B}_a = c\int d^3r \frac{g_a(\vec{r}-\vec{x})}{4\pi|\vec{r}-\vec{x}|^3} \cdot \vec{B}_a(\vec{r}) = c\int d^3r \frac{g_a \rho_m^a(\vec{r})}{4\pi|\vec{x}-\vec{r}|}$$

presence of magnetic charge

$$\vec{B}_a(\vec{r}) = \int d^3y \frac{(\vec{r}-\vec{y})\rho_m^a(\vec{y})}{|\vec{r}-\vec{y}|^3}$$

We derive the formula only by using the two postulates

$$\left\langle \frac{dQ_5}{dt} \right\rangle = \pm\sqrt{\frac{4\pi c'^2 g^2 L}{\sqrt{2}} \left\langle Q_m^a \rho_m^a(0) \right\rangle} \qquad Q_m^a = \int d^3r \rho_m^a(\vec{r})$$

Up to now, we have shown that the chiral asymmetric pair production takes place in the monopole condensed vacuum when we put a classical color charge in the vacuum. But, we have not yet shown the presence of chiral condensate in the vacuum.

Here we make a comment that a chiral condensate locally arises around a monopole.

chiral condensate $\langle \bar{q}_{\pm} q_{\pm} \rangle = \dfrac{const.}{r^3}$ around a monopole

By taking the quantum effects of gauge fields δA_μ
in the quark monopole scattering, $\left(A_\mu = A_\mu(monopole) + \delta A_\mu \right)$
we find the quark condensate around a monopole at $\vec{r} = 0$

$$\langle \bar{q}_{\pm} q_{\pm} \rangle = \frac{const.}{r^3}$$

It has been shown that the local chiral condensate arises owing to the chiral anomaly. It is a by-product of the analysis of the Rubakov effect.

Rubakov (1982), Callan (1982)
Ezawa and A. I. (1983)

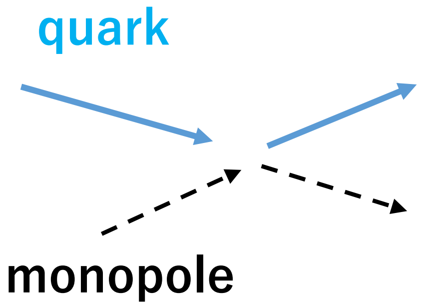
Each monopole carries such a local quark condensate

(baryon decay in nucleon collision with GUT monopole)

The monopole condensation leads to the chiral condensate.

Effective monopole quark interaction

We have shown that
quarks change their chirality in the monopole quark scattering.
Effectively the interaction can be described by



$$g'\phi^{*}\phi\bar{q}q \qquad \phi;\, monopole, \quad g';\, coupling\ con. \quad q=\begin{pmatrix} u \\ d \end{pmatrix}$$

g'=order of Λ_{QCD}^{-1}

Monopole condensation generates
constituent quark mass m_q**; not chiral condensate**

$$g'v^2\bar{q}q = m_q\bar{q}q;\ \ m_q \equiv g'v^2 \qquad \langle\phi\rangle = v$$

When the monopoles are relevant dynamical degrees of freedom to strongly coupled QCD with energy scales $\leq \Lambda_{QCD}$ the chiral symmetry ($SU_A(2)$ and $U_A(1)$) is explicitly broken.

Results from the QCD monopole quark interaction

No chiral magnetic effect

Chiral charges imbalance produced in early stages of high energy heavy ion collisions disappears due to the monopole quark interaction near transition temperature. (The monopoles play important roles in QGP. Liao (2007))

Decrease of hadron masses in dense nuclear matter

Constituent quark masses decrease because the monopole condensate decreases in dense nuclear matter owing to color electric fields
(the monopole condensate is expelled by the color electric fields in dense nuclear matter)

Pion is not Nambu Goldstone boson

Smallness of pion mass comes from smallness of the monopole quark coupling.
Strong coupling constant $\alpha_s > 1$ does not appear in the interaction. $gg_m = 1/2$

conclusion

Chiral asymmetric quark pair productions arise when a classical color charge is put in the monopole condensed vacuum.

$$\left\langle \frac{dQ_5}{dt} \right\rangle = \pm \sqrt{\frac{4\pi c'^2 g^2 L}{\sqrt{2}} \langle Q_m \rho_m(0) \rangle}$$

magnetic charge

$$Q_m = \int d^3 r \rho_m(\vec{r})$$

The equation indicates that
the chiral symmetry is broken by the monopole condensation.